标准与协变光前方法及其在b味强子半轻衰变中的应用

学科、专业	：	物理学、粒子与原子核物理
研 究 方 向	：	粒子物理理论
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博士学位论文答辩

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Outline

1. 标准和协变光前方法下$P \to P, S, V, A$过程张量形状因子的研究
   - 研究动机
   - 理论框架
   - 数值结果与分析
   - 小结
2. $b \to c\ell^- \bar{\nu}_\ell$引起的矢量介子半轻衰变的研究
   - 研究动机
   - 理论框架与结果
   - 数值结果与分析
   - 小结
3. 工作总结和展望
4. 致谢

- 光前夸克模型(LFQM)是一种基于光前量子化和夸克模型的方法。
  - 标准光前夸克模型(SLF QM)：是一种基于光前形式和QCD光前量子化的相对论性的夸克模型。
    问题：协变性问题
    M. V. Terentev, SJNP 24 (1976) 106; P. L. Chung *et al.*, PLB 205 (1988) 545.
  - 协变光前夸克模型(CLF QM)：Jaus基于显然协变性Bethe-Saltpeter (BS)方法并结合SLF QM 提出的一种模型和计算方法；可以有效地确定zero-mode贡献；
    问题：协变性问题，自洽性问题
    W. Jaus, Phys. Rev. D 60 (1999) 054026; H. Y. Cheng *et al.*, Phys. Rev. D 69 (2004) 074025;
  - 解决方案：通过修改显然协变的BS方法和LFQM的对应关系（type-II）得以解决。
    H. M. Choi *et al.*, Phys. Rev. D 89 (2014) no. 3, 033011;
- 形状因子是必不可少的非微扰输入参数。

主要任务：

$$\mathcal{B} \equiv \langle M''(p'')|\bar{q}_1''(k_1'')\Gamma q_1'(k_1')|M'(p')\rangle\,, \qquad \Gamma = \sigma_{\mu\nu},\ \sigma_{\mu\nu}\gamma_5,\ \ldots \tag{1}$$

The SLF QM :

矩阵元：

$$\begin{aligned}\mathcal{B}_{\rm SLF} = & \sum_{h_1',h_1'',h_2} \int \frac{\mathrm{d}x\,\mathrm{d}^2\mathbf{k}'_\perp}{(2\pi)^3\,2x} \psi''^*(x,\mathbf{k}''_\perp)\psi'(x,\mathbf{k}'_\perp) S''^{\dagger}_{h_1'',h_2}(x,\mathbf{k}''_\perp) \\ & \times C_{h_1'',h_1'}(x,\mathbf{k}'_\perp,\mathbf{k}''_\perp)\, S'_{h_1',h_2}(x,\mathbf{k}'_\perp)\,,\end{aligned} \tag{2}$$

其中，$C_{h_1'',h_1'}(x,\mathbf{k}'_\perp,\mathbf{k}''_\perp) \equiv \bar{u}_{h_1''}(x,\mathbf{k}''_\perp)\Gamma u_{h_1'}(x,\mathbf{k}'_\perp)$；

径向波函数$\psi(x, \mathbf{k}_\perp)$：

$$\psi_s(x, \mathbf{k}_\perp) = 4\frac{\pi^{\frac{3}{4}}}{\beta^{\frac{3}{2}}}\sqrt{\frac{\partial k_z}{\partial x}}\exp\left[-\frac{k_z^2 + \mathbf{k}_\perp^2}{2\beta^2}\right], \tag{3}$$

$$\psi_p(x, \mathbf{k}_\perp) = \frac{\sqrt{2}}{\beta}\psi_s(x, \mathbf{k}_\perp), \tag{4}$$

自旋轨道波函数$S_{h_1,h_2}(x, \mathbf{k}_\perp)$： Phys. Rev. D 41 (1990) 3405;Phys. Rev. D 69 (2004) 074025;

$$S_{h_1,h_2} = \frac{\bar{u}(k_1, h_1)\Gamma_M v(k_2, h_2)}{\sqrt{2}\hat{M}_0}, \tag{5}$$

其中，$\hat{M}_0^2 = M_0^2 - (m_1 - m_2)^2$；$P$，$S$，$V$和$A$介子的顶角算符$\Gamma_M$：

$$\Gamma_P = \gamma_5\,, \quad \Gamma_S = \frac{\hat{M}_0^2}{2\sqrt{3}M_0}\,, \quad \Gamma_V = -\,\not{\hat{\epsilon}} + \frac{\hat{\epsilon}\cdot(k_1 - k_2)}{D_{V,\mathrm{LF}}}\,,$$
$$\Gamma_{1A} = -\frac{1}{D_{1,\mathrm{LF}}}\hat{\epsilon}\cdot(k_1 - k_2)\gamma_5\,, \qquad \Gamma_{3A} = -\frac{\hat{M}_0^2}{2\sqrt{2}M_0}\left[\not{\hat{\epsilon}} + \frac{\hat{\epsilon}\cdot(k_1 - k_2)}{D_{3,\mathrm{LF}}}\right]\gamma_5\,, \tag{6}$$

其中，$D_{V,\mathrm{LF}} = M_0 + m_1 + m_2$，$D_{1,\mathrm{LF}} = 2$，$D_{3,\mathrm{LF}} = \hat{M}_0^2/(m_1 - m_2)$

The CLF QM :

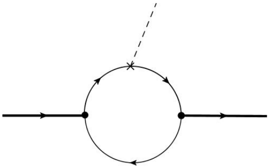


Figure: 矩阵元$\mathcal{B}$费曼图

矩阵元：

$$\mathcal{B}_{\rm CLF} = N_c \int \frac{{\rm d}^4 k_1'}{(2\pi)^4} \frac{H_{M'} H_{M''}}{N_1' N_1'' N_2} iS \cdot (E_{M'} E^*_{M''}) \,, \tag{7}$$

其中，$E_{P,S} = 1$，$E_{V,A} = \epsilon_\mu$，

$$S = {\rm Tr}\left[\Gamma\,(\not k_1' + m_1')\,(i\Gamma_{M'})\,(-\not k_2 + m_2)\,(i\gamma^0\Gamma^\dagger_{M''}\gamma^0)(\not k_1'' + m_1'')\right] \tag{8}$$

$$i\Gamma_P = -i\gamma_5\,, \quad \Gamma_S = -i\,, \quad i\Gamma_V = i\left[\gamma^\mu - \frac{(k_1-k_2)^\mu}{D_{V,\rm con}}\right]\,, \quad i\Gamma_{1A} = i\frac{(k_1-k_2)^\mu}{D_{1,\rm con}}\gamma_5\,,$$

$$i\Gamma_{3A} = i\left[\gamma^\mu + \frac{(k_1-k_2)^\mu}{D_{3,\rm con}}\right]\gamma_5\,, \quad D_{(V,1,3).\rm con} = M + m_1 + m_2, 2, M^2/(m_1 - m_2) \tag{9}$$

Manifestly covariant expression $\xrightarrow{\text{integrating out } k^-}$ LF expression

对$k_1'^-$积分后，矩阵元$\mathcal{B}$的LF形式：

$$\hat{\mathcal{B}}_{\rm CLF} = N_c \int \frac{{\rm d}x{\rm d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{h_{M'} h_{M''}}{\bar{x}\hat{N}_1' \hat{N}_1''} \hat{S} \cdot (E_{M'} E^*_{M''}) \,. \tag{10}$$

应用type-I，-II：

$$N_1 \to \hat{N}_1 = x(M^2 - M_0^2)\,, \qquad S \to \hat{S}\,, \tag{11}$$

$$\begin{aligned} \chi_M = H_M/N_1 \to h_M/\hat{N}_1\,, &\qquad D_{\rm con} \to D_{\rm LF} \quad (\text{type-I}) \\ \chi_M = H_M/N_1 \to h_M/\hat{N}_1\,, &\qquad M \to M_0 \quad (\text{type-II})\,. \end{aligned} \tag{12}$$

P，S，V，A介子顶角的LF形式：

$$
\begin{aligned}
h_P/\hat{N} &= h_V/\hat{N} = \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\psi_s}{\hat{M}_0}\,, && (13)\\
h_S/\hat{N} &= \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\hat{M}_0'^2}{2\sqrt{3}M_0'}\frac{\psi_p}{\hat{M}_0}\,, && (14)\\
h_{^1A}/\hat{N} &= \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\psi_p}{\hat{M}_0}\,, && (15)\\
h_{^3A}/\hat{N} &= \frac{1}{\sqrt{2N_c}}\sqrt{\frac{\bar{x}}{x}}\frac{\hat{M}_0'^2}{2\sqrt{2}M_0'}\frac{\psi_p}{\hat{M}_0}\,. && (16)
\end{aligned}
$$

处理夸克圈Trace需要的代换： Jaus, Phys. Rev. D 60 (1999) 054026;
H. Y. Cheng, Phys. Rev. D 69 (2004) 074025

$$
\begin{aligned}
\hat{k}_1'^{\mu} &\to P^{\mu}A_1^{(1)} + q^{\mu}A_2^{(1)} + \ldots(\omega, \mathbf{C_i^{(j)}})\,,\\
k_1'^{\mu}\hat{N}_2 &\to q^{\mu}\left[A_2^{(1)}Z_2 + \frac{q\cdot P}{q^2}A_1^{(2)}\right]\,, && (17)
\end{aligned}
$$

$$\hat{k}_1'^{\mu}\hat{k}_1'^{\nu} \to g^{\mu\nu}A_1^{(2)} + P^{\mu}P^{\nu}A_2^{(2)} + (P^{\mu}q^{\nu} + q^{\mu}P^{\nu})A_3^{(2)} + q^{\mu}q^{\nu}A_4^{(2)}$$
$$+ \frac{P^{\mu}\omega^{\nu} + \omega^{\mu}P^{\nu}}{\omega \cdot P}B_1^{(2)} + \ldots(\omega, \mathbf{C}_{\mathbf{i}}^{(\mathbf{j})}),$$
$$\hat{k}_1'^{\mu}\hat{k}_1'^{\nu}\hat{N}_2 \to g^{\mu\nu}A_1^{(2)}Z_2 + q^{\mu}q^{\nu}\left(A_4^{(2)}Z_2 + 2\frac{q \cdot P}{q^2}A_2^{(1)}A_1^{(2)}\right)$$
$$+ \frac{P^{\mu}\omega^{\nu} + \omega^{\mu}P^{\nu}}{\omega \cdot P}B_3^{(3)} + \ldots(\omega, \mathbf{C}_{\mathbf{i}}^{(\mathbf{j})}),$$

$$\cdots\cdots$$

其中，$P = p' + p''$，$q = p' - p''$，A和B：

$$A_1^{(1)} = \frac{x}{2}, \qquad A_2^{(1)} = \frac{x}{2} - \frac{\mathbf{k}_{1\perp}' \cdot \mathbf{q}_{\perp}}{q^2},$$
$$A_1^{(2)} = -\mathbf{k}_{1\perp}'^2 - \frac{(\mathbf{k}_{1\perp}' \cdot \mathbf{q}_{\perp})^2}{q^2}, \qquad A_2^{(2)} = (A_1^{(1)})^2$$
$$A_3^{(2)} = A_1^{(1)}A_2^{(1)}, \qquad A_4^{(2)} = (A_2^{(1)})^2 - \frac{1}{q^2}A_1^{(2)}, \qquad B_1^{(2)} = \frac{x}{2}Z_2 + \frac{k_{\perp}^2}{2},$$
$$Z_2 = \hat{N}_1' + m_1'^2 - m_2^2 + (\bar{x} - x)M'^2 + (q^2 + q \cdot P)\frac{\mathbf{k}_{1\perp}' \cdot \mathbf{q}_{\perp}}{q^2}, \tag{18}$$

$\mathbf{P \to P, S}$:

$P \to P, S$过程张量形状因子的定义：

$$\langle P''(p'')|\bar{q}_1''\sigma^{\mu\nu}q_1'|P'(p')\rangle = i(P^\mu q^\nu - P^\nu q^\mu)\frac{F_T(q^2)}{M'+M''}, \tag{19}$$

$$\langle S''(p'')|\bar{q}_1''\sigma^{\mu\nu}\gamma_5 q_1'|P'(p')\rangle = i(P^\mu q^\nu - P^\nu q^\mu)\frac{U_T(q^2)}{M'+M''}, \tag{20}$$

理论结果：

$$\widetilde{F}_T^{\rm SLF} = -\frac{2(M'+M'')(m_1'\mathbf{k}_\perp''\cdot\mathbf{q}_\perp - m_1''\mathbf{k}_\perp'\cdot\mathbf{q}_\perp - x m_2\mathbf{q}_\perp^2)}{\mathbf{q}_\perp^2}, \tag{21}$$

$$\widetilde{F}_T^{\rm CLF} = 2(M'+M'')\left[m_1' - (m_1'+m_1''-2m_2)A_1^{(1)} - (m_1'-m_1'')A_2^{(1)}\right], \tag{22}$$

$$\widetilde{F}_T^{\rm val.} = \widetilde{F}_T^{\rm CLF}, \tag{23}$$

$$\widetilde{U}_T^{\rm SLF} = \frac{\hat{M}_0''^2}{2\sqrt{3}M_0''}\widetilde{F}_T^{\rm SLF}[m_1''\to -m_1''], \tag{24}$$

$$\widetilde{U}_T^{\rm CLF} = \widetilde{F}_T^{\rm CLF}[m_1''\to -m_1''], \tag{25}$$

$$\widetilde{U}_T^{\rm val.} = \widetilde{U}_T^{\rm CLF}. \tag{26}$$

对于$P \to P, S$过程的张量形状因子$\mathcal{O} = F_T(q^2)$，$U_T(q^2)$:

- 没有自洽性问题；
- 没有zero-mode效应；
- $[\mathcal{O}]_{\rm SLF} = [\mathcal{O}]_{\rm val.} = [\mathcal{O}]_{\rm CLF} = [\mathcal{O}]_{\rm full}$。

$\mathbf{P \to V, A}$:

$P \to V$过程张量形状因子的定义：

$$\langle V(p'', \epsilon)|\bar{q}_1''\sigma^{\mu\nu}q_\nu \ q_1'|P(p')\rangle = \varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* P_\alpha q_\beta T_1(q^2), \tag{27}$$

$$\begin{aligned}\langle V(p'', \epsilon)|\bar{q}_1''\sigma^{\mu\nu}\gamma_5 q_\nu \ q_1'|P(p')\rangle =& -i\left[(M'^2 - M''^2)\epsilon^{\mu *} - \epsilon^* \cdot qP^\mu\right] T_2(q^2) \\ & - i\epsilon^* \cdot q\left[q^\mu - \frac{q^2}{M'^2 - M''^2}P^\mu\right] T_3(q^2),\end{aligned} \tag{28}$$

$P \to A$过程张量形状因子的定义：

$$\langle {}^i A(p'', \epsilon)|\bar{q}_1''\sigma^{\mu\nu}\gamma_5 q_\nu \ q_1'|P(p')\rangle = -\varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* P_\alpha q_\beta T_1^{(i)}(q^2), \tag{29}$$

$$\begin{aligned}\langle {}^i A(p'', \epsilon)|\bar{q}_1''\sigma^{\mu\nu}q_\nu \ q_1'|P(p')\rangle =& i\left[(M'^2 - M''^2)\epsilon^{\mu *} - \epsilon^* \cdot qP^\mu\right] T_2^{(i)}(q^2) \\ & + i\epsilon^* \cdot q\left[q^\mu - \frac{q^2}{M'^2 - M''^2}P^\mu\right] T_3^{(i)}(q^2),\end{aligned} \tag{30}$$

$P \to V$过程张量形状因子的理论结果：$\widetilde{T}_{1,2,3}$

$$
\begin{aligned}
\widetilde{T}_1^{\rm SLF} =& \frac{1}{(M'^2 - M''^2 + \mathbf{q}_\perp^2)} \frac{1}{x\bar{x}} \Big\{ 2x(xm_2 + \bar{x}m_1')(xm_2 + \bar{x}m_1'')(M'^2 - M''^2) \\
& + 2x^2(M'^2 - M''^2)\mathbf{k}_\perp' \cdot \mathbf{k}_\perp'' + (xm_2 + \bar{x}m_1')\left[xm_2 + \bar{x}(x - \bar{x})m_1' + 2x\bar{x}m_1''\right]\mathbf{q}_\perp^2 \\
& - [2x^2 m_2^2 + \bar{x}(x - \bar{x})m_1'^2 - \bar{x}m_1''^2]\mathbf{k}_\perp' \cdot \mathbf{q}_\perp \\
& + 2(\bar{x} - x)\mathbf{k}_\perp'' \cdot \mathbf{q}_\perp \mathbf{k}_\perp' \cdot \mathbf{k}_\perp'' + (1 - 2x\bar{x})\mathbf{k}_\perp' \cdot \mathbf{k}_\perp'' \mathbf{q}_\perp^2 \\
& + \frac{2}{D_V''}\Big[x\bar{x}(M'^2 - M''^2)\left[(m_1' + m_1'')\mathbf{k}_\perp' \cdot \mathbf{k}_\perp'' - (xm_2 + \bar{x}m_1')\mathbf{k}_\perp'' \cdot \mathbf{q}_\perp\right] \\
& - (m_1' + m_1'')(\bar{x}m_1' + xm_2)(\bar{x}m_1'' - xm_2)\mathbf{k}_\perp'' \cdot \mathbf{q}_\perp \\
& + \bar{x}(xm_2 + \bar{x}m_1')(\mathbf{k}_\perp'' \cdot \mathbf{q}_\perp)^2 - x\bar{x}\left(m_1' + m_1''\right)\left(\mathbf{k}_\perp' \cdot \mathbf{q}_\perp\right)^2 + x\bar{x}\left(m_1' + m_1''\right)\mathbf{k}_\perp'^2 \mathbf{q}_\perp^2 \\
& + (x - \bar{x})(m_1' + m_1'')\mathbf{k}_\perp'' \cdot \mathbf{q}_\perp \mathbf{k}_\perp' \cdot \mathbf{k}_\perp'' - \bar{x}(\bar{x}m_1'' - xm_2)\mathbf{k}_\perp' \cdot \mathbf{q}_\perp \mathbf{k}_\perp'' \cdot \mathbf{q}_\perp\Big]\Big\}, \\
\widetilde{T}_2^{\rm SLF} =& \widetilde{T}_1^{\rm SLF} + \frac{q^2}{(M'^2 - M''^2)(M'^2 - M''^2 + \mathbf{q}_\perp^2)} \frac{1}{x^2\bar{x}} \{\cdots\}, \\
\widetilde{T}_3^{\rm SLF} =& \frac{M'^2 - M''^2}{q^2}\left[\widetilde{T}_1^{\rm SLF} - \widetilde{T}_2^{\rm SLF}\right] + \frac{2\left(M'^2 - M''^2\right)}{x\bar{x}\left(M'^2 - M''^2 + \mathbf{q}_\perp^2\right)\mathbf{q}_\perp^2}\{\cdots\},
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\widetilde{T}_1^{\rm val.} =& \frac{1}{\bar{x}\left(M'^2 - M''^2 + \mathbf{q}_\perp^2\right)} \Big\{ -2\mathbf{k}'_\perp \cdot \mathbf{q}_\perp \mathbf{k}''^2_\perp + \mathbf{k}'_\perp \cdot \mathbf{k}''_\perp \mathbf{q}^2_\perp - \bar{x}(2x-1)\mathbf{k}''_\perp \cdot \mathbf{q}_\perp M'^2 \\
& + 2\bar{x}\left[m'_1 m''_1 - x(m'_1 - m_2)(m''_1 - m_2)\right]\left(M'^2 - M''^2 + \mathbf{q}^2_\perp\right) + m_2^2 \mathbf{q}^2_\perp \\
& + 2x(M'^2 - M''^2)(\mathbf{k}'_\perp \cdot \mathbf{k}''_\perp + m_2^2) + \mathbf{k}'_\perp \cdot \mathbf{q}_\perp (\bar{x} M''^2 - 2m_2^2) \\
& + \frac{2}{D''_{V,\rm con}}(m'_1 + m''_1)\Big[\mathbf{k}'_\perp \cdot \mathbf{q}_\perp \mathbf{k}''^2_\perp + \mathbf{k}''_\perp \cdot \mathbf{q}_\perp (m_2^2 - \bar{x}^2 M'^2) \\
& + \bar{x}\mathbf{k}'_\perp \cdot \mathbf{k}''_\perp (M'^2 - M''^2)\Big]\Big\}, \\
\widetilde{T}_2^{\rm val.} =& \widetilde{T}_1^{\rm val.} - \frac{q^2}{(M'^2 - M''^2)\left(M'^2 - M''^2 + \mathbf{q}^2_\perp\right)} \frac{1}{\bar{x}} \{\cdots\}, \\
\widetilde{T}_3^{\rm val.} =& \frac{M'^2 - M''^2}{q^2}\left[\widetilde{T}_1^{\rm val.} - \widetilde{T}_2^{\rm val.}\right] + \frac{2\left(M'^2 - M''^2\right)}{\bar{x}\left(M'^2 - M''^2 + \mathbf{q}^2_\perp\right)\mathbf{q}^2_\perp}\{\cdots\},
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
\widetilde{T}_1^{\mathrm{CLF}} = & (2\bar{x}-1)\left(m_1'^2+\hat{N}_1'\right)+m_1''^2+\hat{N}_1''+\mathbf{q}_\perp^2 \\
& +2\left(\bar{x}m_1'm_1''+xm_1'm_2+xm_1''m_2\right) \\
& -8A_1^{(2)}+2\left(M'^2-M''^2\right)\left(A_1^{(1)}+2A_2^{(2)}-2A_3^{(2)}\right) \\
& +2\mathbf{q}_\perp^2\left(A_1^{(1)}-2A_2^{(1)}-2A_3^{(2)}+2A_4^{(2)}\right)-\frac{4}{D''_{V,\mathrm{con}}}(m_1'+m_1'')A_1^{(2)}\,, \\
\widetilde{T}_2^{\mathrm{CLF}} = & \cdots, \\
\widetilde{T}_3^{\mathrm{CLF}} = & \cdots\,.
\end{aligned}
\tag{33}
$$

$P \to^1 A$过程张量形状因子的理论结果：$\widetilde{T}^{(1)}_{1,2,3}$

$$\begin{aligned}
\widetilde{T}^{(1),\mathrm{SLF}}_{1,2,3} &= \widetilde{T}^{\mathrm{SLF}}_{1,2,3}\,[D''\text{-terms only}\,,\,D''_V \to D''_1\,,\,m''_1 \to -m''_1]\,,\\
\widetilde{T}^{(1),\mathrm{val.}}_{1,2,3} &= \widetilde{T}^{\mathrm{val.}}_{1,2,3}[D''\text{-terms only}\,,\,D''_{V,\mathrm{con}} \to D''_{1,\mathrm{con}},\,m''_1 \to -m''_1]\,,\\
\widetilde{T}^{(1),\mathrm{CLF}}_{1,2,3} &= \widetilde{T}^{\mathrm{CLF}}_{1,2,3}[D''\text{-terms only}\,,\,D''_{V,\mathrm{con}} \to D''_{1,\mathrm{con}}\,,\,m''_1 \to -m''_1]\,. \qquad (34)
\end{aligned}$$

$P \to^3 A$过程张量形状因子的理论结果：$\widetilde{T}^{(3)}_{1,2,3}$

$$\begin{aligned}
\widetilde{T}^{(3),\mathrm{SLF}}_{1,2,3} &= \frac{\hat{M}''^2_0}{2\sqrt{2}M''_0}\,\widetilde{T}^{\mathrm{SLF}}_{1,2,3}\,[D''_V \to D''_3\,,\,m''_1 \to -m''_1]\,,\\
\widetilde{T}^{(3),\mathrm{Val.}}_{1,2,3} &= \widetilde{T}^{\mathrm{Val.}}_{1,2,3}\,\left[D''_{V,\mathrm{con}} \to D''_{3,\mathrm{con}}\,,\,m''_1 \to -m''_1\right]\,,\\
\widetilde{T}^{(3),\mathrm{CLF}}_{1,2,3} &= \widetilde{T}^{\mathrm{CLF}}_{1,2,3}\,\left[D''_{V,\mathrm{con}} \to D''_{3,\mathrm{con}}\,,\,m''_1 \to -m''_1\right]\,. \qquad (35)
\end{aligned}$$

CLF QM的自洽性：

$P \to V$张量形状因子T_1的B函数相关贡献：

$$\widetilde{\mathcal{B}}^{\mu}_{B}(\Gamma = \sigma^{\mu\nu} q_\nu) = 4\frac{B_1^{(2)}}{\omega \cdot P}\Big[-\epsilon^{\delta\nu\alpha\beta} P_\delta q_\nu \omega_\alpha \epsilon^*_\beta P^\mu + \epsilon^{\mu\nu\alpha\beta} P_\nu \omega_\alpha \epsilon^*_\beta \left(M'^2 - M''^2\right) + \epsilon^{\mu\nu\alpha\beta} q_\nu \omega_\alpha \epsilon^*_\beta \left(M'^2 - M''^2\right) - \epsilon^{\mu\nu\alpha\beta} P_\nu q_\alpha \omega_\beta (q \cdot \epsilon^*) \frac{m_1' + m_1''}{D''_{V,\mathrm{con}}}\Big], \tag{36}$$

$$\widetilde{T}_1^B = \begin{cases} \left[-2(M'^2 - M''^2) + (M'^2 - M''^2 + \mathbf{q}_\perp^2)\frac{m_1'+m_1''}{D''_{V,\mathrm{con}}}\right]\frac{1}{M''^2} B_1^{(2)}, & \lambda = 0 \\ \left[2(M'^2 - M''^2) - \mathbf{q}_\perp^2 \frac{m_1'+m_1''}{D''_{V,\mathrm{con}}}\right]\frac{2}{M'^2 - M''^2 + \mathbf{q}_\perp^2} B_1^{(2)}, & \lambda = + \\ \left[1 + \frac{m_1'+m_1''}{D''_{V,\mathrm{con}}}\right]\frac{2\mathbf{q}_\perp^2}{M'^2 - M''^2 + \mathbf{q}_\perp^2} B_1^{(2)}. & \lambda = - \end{cases} \tag{37}$$

Table: $B_c \to D^*$过程的张量形状因子$T_1(\mathbf{q}_\perp^2)$ 在$\mathbf{q}_\perp^2$取不同值时的数值结果

$B_c \to D^*$		$[T_1]^{\text{SLF}}$	$[T_1]^{\text{val.}}$	$[T_1]^{\text{CLF}}$	$[T_1]^{\text{full}}_{\lambda=0}$	$[T_1]^{\text{full}}_{\lambda=+}$	$[T_1]^{\text{full}}_{\lambda=-}$
$\mathbf{q}_\perp^2 = 0\,\text{GeV}^2$	type-I	0.106	0.106	0.094	0.118	0.081	0.094
	type-II	0.106	0.106	0.106	0.106	0.106	0.106
$\mathbf{q}_\perp^2 = 4\,\text{GeV}^2$	type-I	0.072	0.072	0.063	0.079	0.055	0.062
	type-II	0.073	0.073	0.073	0.073	0.073	0.073
$\mathbf{q}_\perp^2 = 9\,\text{GeV}^2$	type-I	0.046	0.045	0.040	0.049	0.035	0.038
	type-II	0.047	0.047	0.047	0.047	0.047	0.047

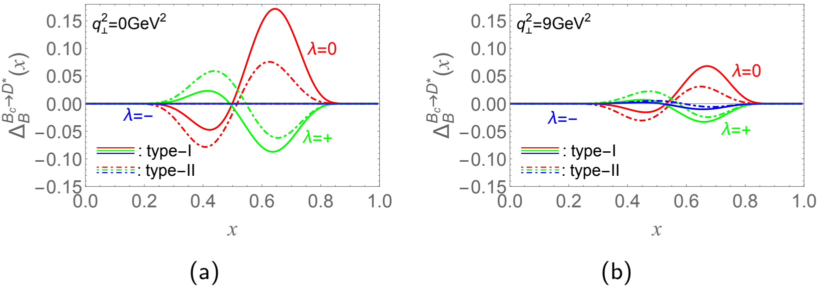


Figure: $B_c \to D^*$跃迁过程的$\Delta_{\rm B}(x)$-x关系图

定义：$\Delta_B(x) \equiv \frac{\mathrm{d}[\mathcal{F}]^{\rm B}_\lambda}{\mathrm{d}x} = N_c \int \frac{\mathrm{d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_V \chi''_V}{\bar{x}} \widetilde{\mathcal{F}}^B_\lambda$

对于$P \to V, A$过程张量形状因子$\mathcal{O} = T_{1,2,3}(q^2)$，$T^{(1,2)}_{1,2,3}(q^2)$:

- 在type-I方案中：$N_c \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_V\chi''_V}{\bar{x}} \widetilde{\mathcal{B}}_B \neq 0$

$$\Longrightarrow [\mathcal{Q}]^{\rm full}_{\lambda=0} \neq [\mathcal{Q}]^{\rm full}_{\lambda=+} \neq [\mathcal{Q}]^{\rm full}_{\lambda=-} , \qquad (\text{type-I}) \tag{38}$$

- 在type-II方案中：$N_c \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}'_\perp}{2(2\pi)^3} \frac{\chi'_V\chi''_V}{\bar{x}} \widetilde{\mathcal{B}}_B = 0$

$$\Longrightarrow [\mathcal{Q}]^{\rm full}_{\lambda=0} \doteq [\mathcal{Q}]^{\rm full}_{\lambda=+} \doteq [\mathcal{Q}]^{\rm full}_{\lambda=-} \qquad (\text{type-II}) \tag{39}$$

CLF QM的协变性：

$P \to V$张量形状因子$T_{2,3}$的B函数相关贡献：

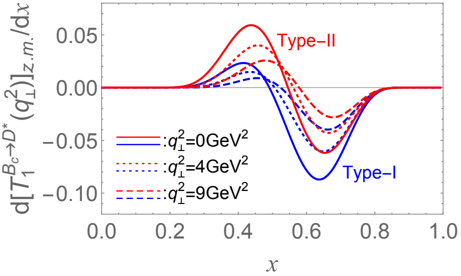
$$\begin{aligned}
\widetilde{\mathcal{B}}_B^{\mu}(\Gamma = \sigma^{\mu\nu}\gamma_5 q_\nu) =& 4i\frac{B_1^{(2)}}{\omega \cdot P}\Bigg\{ -P^{\mu}(\omega\cdot\epsilon^*)q^2\left(1+\frac{m_1'-m_1''-2m_2}{D''_{V,\mathrm{con}}}\right) \\
&+ q^{\mu}(\omega\cdot\epsilon^*)\left(M'^2-M''^2\right)\left(1+\frac{m_1'-m_1''-2m_2}{D''_{V,\mathrm{con}}}\right) \\
&-\omega^{\mu}(q\cdot\epsilon^*)\Bigg[q^2\left(1+\frac{m_1'-m_1''-2m_2}{D''_{V,\mathrm{con}}}\right) \\
&+\left(M'^2-M''^2\right)\left(1+\frac{m_1'-m_1''}{D''_{V,\mathrm{con}}}\right)\Bigg]\Bigg\}. \qquad (40)
\end{aligned}$$

- **type-I**：张量算符矩阵元的协变性被破坏。
- **type-II**：协变性问题得以解决。

zero-mode贡献：

$P \to V, A$过程的张量形状因子受到zero-mode贡献的影响：

$$[\mathcal{F}]^{\rm CLF} = [\mathcal{F}]^{\rm val.} + [\mathcal{F}]^{\rm z.m.} \tag{41}$$



$$[\mathcal{Q}]^{\rm z.m.} \neq 0 \quad (\text{type-I})\,, \qquad [\mathcal{Q}]^{\rm z.m.} \doteq 0 \quad (\text{type-II}) \tag{42}$$

CLF QM“新”的自洽性问题：

H. Y Cheng and C. K. Chua, Phys. Rev. D 81 (2010) 114006;

$P\to V$过程张量形状因子：$T_1^{\rm our}=T_1^{\rm CC}$，$T_{2,3}^{\rm our}\neq T_{2,3}^{\rm CC}$ (type-I)

原因：对矩阵元$\mathcal{B}_{\rm CLF}^{P\to V}[\Gamma=\sigma^{\mu\nu}\gamma_5]$中的夸克圈求迹部分$S^{\mu\nu\lambda}$的处理方式不同。

以其中一项为例：$[S_\lambda^{\mu\nu}]_{\text{last term}}=-2i\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\sigma\lambda\alpha\beta}\,k'_{1\rho}k_1'^{\alpha}(P+q)^\beta$

恒等式：

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\sigma\lambda\alpha\beta}=[g_\lambda^\mu(g_\alpha^\nu g_\beta^\rho-g_\alpha^\rho g_\beta^\nu)+g_\lambda^\nu(g_\alpha^\rho g_\beta^\mu-g_\alpha^\mu g_\beta^\rho)+g_\lambda^\rho(g_\alpha^\mu g_\beta^\nu-g_\alpha^\nu g_\beta^\mu)]\,,$$

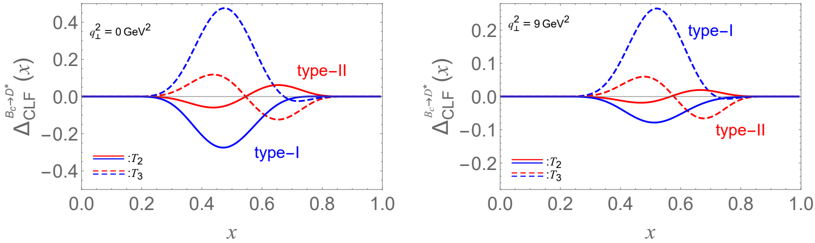
$$\begin{aligned}[\hat{S}_\lambda^{\mu\nu}]_{\text{last term}}^{\rm CC}&=2ig_\lambda^\nu g_\alpha^\mu g_\beta^\rho[g_\rho^\alpha A_1^{(2)}+P^\alpha P_\rho A_2^{(2)}+(P_\rho q^\alpha+q_\rho P^\alpha)A_3^{(2)}\\&\quad+q^\alpha q_\rho A_4^{(2)}](P+q)^\beta\ +\ \ldots\end{aligned}\tag{43}$$

$$\begin{aligned}[S_\lambda^{\mu\nu}]_{\text{last term}}^{\rm ours}&=-2ig_\lambda^\nu k_1'^\mu k_1'\cdot(P+q)+\ldots\\&=-2ig_\lambda^\nu k_1'^\mu\left(M'^2+m_1'^2+m_1'^2-m_2^2-N_2+N_1'\right)+\ldots\,,\end{aligned}\tag{44}$$

$$[\hat{S}^{\mu\nu}_{\lambda}]^{\rm CC}_{\rm last\ term} = 2i g^{\nu}_{\lambda} \Big\{ P^{\mu} \left[A_1^{(2)} + (3M'^2 - M''^2 - q^2) A_2^{(2)} + (M'^2 - M''^2 + q^2) A_3^{(2)} \right] + q^{\mu} \left[A_1^{(2)} + \left(3M'^2 - M''^2 - q^2 \right) A_3^{(2)} + \left(M'^2 - M''^2 + q^2 \right) A_4^{(2)} \right] \Big\} + \ldots, \tag{45}$$

$$[\hat{S}^{\mu\nu}_{\lambda}]^{\rm ours}_{\rm last\ term} = 2i g^{\nu}_{\lambda} \Big\{ P^{\mu} \left(M'^2 + m_1'^2 - m_2^2 + \hat{N}_1' \right) A_1^{(1)} + q^{\mu} \left[\left(M'^2 + m_1'^2 - m_2^2 + \hat{N}_1' - Z_2 \right) A_2^{(1)} - \frac{M'^2 - M''^2}{q^2} A_1^{(2)} \right] \Big\} + \ldots. \tag{46}$$

定义：$\Delta^{\mathcal{F}}_{\rm CLF}(x, \mathbf{q}_\perp^2) \equiv \frac{\mathrm{d}[\mathcal{F}]^{\rm CLF}_{\rm ours}}{\mathrm{d}x} - \frac{\mathrm{d}[\mathcal{F}]^{\rm CLF}_{\rm CC}}{\mathrm{d}x}$, $\mathcal{F} = T_2$ and T_3



(a) (b)

Figure: $B_c \to D^*$过程的$\Delta^{T_{2,3}}_{\rm CLF}(x)$-$x$关系图

- $[T_{2,3}]^{\mathrm{CLF}}_{\mathrm{ours}} - [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{CC}} = \int_0^1 \mathrm{d}x\, \Delta^{T_{2,3}}_{\mathrm{CLF}}(x) \neq 0 \qquad \text{(type-I)}$

  $\Longrightarrow [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{ours}} \neq [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{CC}}$ “新”的自洽性问题

- $[T_{2,3}]^{\mathrm{CLF}}_{\mathrm{ours}} - [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{CC}} = \int_0^1 \mathrm{d}x\, \Delta^{T_{2,3}}_{\mathrm{CLF}}(x) = 0 \qquad \text{(type-II)}$

  $\Longrightarrow [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{ours}} \doteq [T_{2,3}]^{\mathrm{CLF}}_{\mathrm{CC}}$ 数值结果自洽

- $P \to A$跃迁的张量形状因子也存在与之类似的“新”的自洽性问题。

小结：

- 传统CLF QM的自洽性和协变性问题有相同根源，均可以通过type-II方案得以解决。
- 在type-II方案中，zero-mode对介子衰变的张量形状因子的数值结果没有贡献。
- $[\mathcal{Q}]^{\rm SLF} = [\mathcal{Q}]^{\rm val.} \doteq [\mathcal{Q}]^{\rm CLF}$　　(type-II)；
- 在传统CLF QM中，对于$\mathcal{B}(\sigma^{\mu\nu}\gamma_5)$，在计算中处理夸克圈trace的方案不同也会导致一个“新的”自洽性问题。该问题同样可以通过type-II得以解决。

B^*介子的性质：

Table: B和B^*介子的质量(GeV)

介子	质量	介子	质量
B^-	5.279	B^{*-}	5.325
B^0	5.280	B^{*0}	5.325
B_s^0	5.367	B_s^{*0}	5.415
B_c^-	6.275	B_c^{*-}	5.332

- $B^*_{u,d,s,c}$的量子数：$n^{2s+1}L_J = 1^3S_1$和$J^P = 1^-$；
- B^*_q和B_q的夸克组分完全相同；
- $\triangle_m = m_{B^*_q} - m_{B_q} \lesssim 50\mathrm{MeV} < \mathrm{m}_\pi$
- B^*_q介子衰变以$B^*_q \to B_q\gamma$过程为主

$\Upsilon(nS)$介子的性质：

- $\Upsilon(nS)$介子是位于$B\bar{B}$阈值下的底夸克偶素粒子。
- $\Upsilon(nS)$介子的量子数：$I^G J^{PC} = 0^- 1^{--}$。
- $\Upsilon(nS)$介子主要是通过强相互作用和电磁相互作用的过程发生衰变。

$$\mathcal{B}(\Upsilon(1S) \to ggg) = (81.7 \pm 0.7)\%$$

Belle-II

1. 年积分亮度的目标~ $13ab^{-1}$。
2. 生成截面：$\sigma(e^+e^- \to \Upsilon(5S)) = 0.301nb$。
3. 每年将会有$4 \times 10^9(B^*_{u,d} + \bar{B}^*_{u,d})$，$2 \times 10^9(B^*_s + \bar{B}^*_s)$。

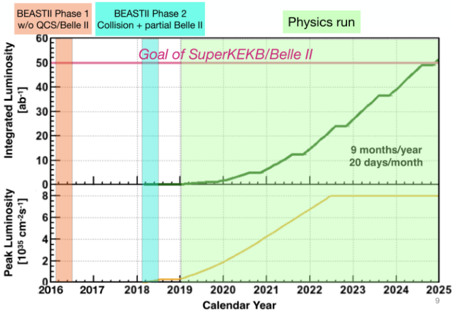


Figure: Bell-II 的运行状态

LHC

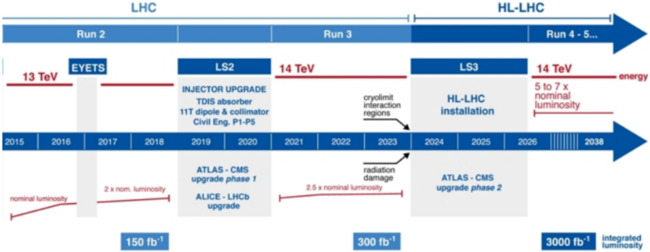


Figure: LHC 的运行状态

相比于e^+e^-的对撞，pp的对撞截面要大很多，相应地，在LHC上收集到的$b\bar{b}$数据要更大。

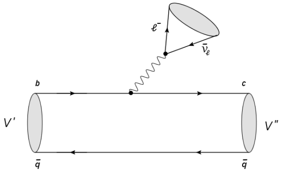


Figure: 费曼图

在SM中，通过$b \to c\ell^-\bar{\nu}_\ell$过程引起的$V' \to V''\ell^-\bar{\nu}_\ell$ 的低能有效哈密顿量为：

$$\mathcal{H}_{\rm eff}(b \to c\ell^-\bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma_\mu(1-\gamma_5)b][\bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell]\,, \tag{47}$$

$V' \to V''\ell^-\bar{\nu}_\ell$半轻衰变振幅的模方：

$$\begin{aligned} \mid \mathcal{M}(V' \to V''\ell^-\bar{\nu}_\ell) \mid^2 &= \frac{G_F^2 \mid V_{cb} \mid^2}{2} \mid \langle V'' \mid \bar{c}\gamma_\mu(1-\gamma_5)b \mid V' \rangle \bar{\ell}\gamma^\mu(1-\gamma_5)\nu_\ell \mid^2 \\ &\equiv \frac{G_F^2 \mid V_{cb} \mid^2}{2} L_{\mu\nu} H^{\mu\nu}\,, \end{aligned} \tag{48}$$

插入完备性关系：

$$\sum_{m,n} \bar{\epsilon}_\mu(m)\bar{\epsilon}^*_\nu(n) g_{mn} = g_{\mu\nu} \,, \tag{49}$$

振幅的模方可以进一步表示为：

$$| \mathcal{M}(V' \to V''\ell^-\bar{\nu}_\ell) |^2 = \frac{G_F^2 V_{cb}^2}{2} \sum_{m,m',n,n'} L(m,n) H(m',n') g_{mm'} g_{nn'} \,, \tag{50}$$

螺旋度展开的算法

- 在初态矢量介子的质心系中对强子部分完成计算
- 在末态轻子$\ell - \bar{\nu}_\ell$的质心系中对轻子部分完成计算
- 采用Wigner's d^J函数合并强子部分和轻子部分的螺旋度振幅，最终得到$L_{\mu\nu}H^{\mu\nu}$

强子螺旋度振幅：

$V' \to V''\ell^- \bar{\nu}_\ell$半轻衰变的强子螺旋度振幅$H_{\lambda_{W^*}\lambda_{V'}\lambda_{V''}}$，其定义如下：

$$H_{\lambda_{W^*}\lambda_{V'}\lambda_{V''}} = H_\mu(\lambda_{V'}\lambda_{V''})\bar{\epsilon}^{*\mu}(\lambda_{W^*}) \tag{51}$$

其中，$V' \to V''$的强子矩阵元可以被参数化为：

$$\begin{aligned}\langle V''(\epsilon_2, p_{V''})|\bar{c}\gamma_\mu b|\bar{V}'(\epsilon_1, p_{V'})\rangle = & \quad (\epsilon_1 \cdot \epsilon_2^*)[-P_\mu V_1(q^2) + q_\mu V_2(q^2)] \\ & + \frac{(\epsilon_1 \cdot q)(\epsilon_2^* \cdot q)}{m_{V'}^2 - m_{V''}^2}[P_\mu V_3(q^2) - q_\mu V_4(q^2)] \\ & - (\epsilon_1 \cdot q)\epsilon_{2\mu}^* V_5(q^2) + (\epsilon_2^* \cdot q)\epsilon_{1\mu} V_6(q^2), \end{aligned} \tag{52}$$

$$\begin{aligned}\langle V''(\epsilon_2, p_{V''})|\bar{c}\gamma_\mu\gamma_5 b|\bar{V}'(\epsilon_1, p_{V'})\rangle = & - i\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\alpha \epsilon_2^{*\beta}[P^\nu A_1(q^2) - q^\nu A_2(q^2)] \\ & - \frac{i\epsilon_2^* \cdot q}{m_{V'}^2 - m_{V''}^2}\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\nu P^\alpha q^\beta A_3(q^2) \\ & + \frac{i\epsilon_1 \cdot q}{m_{V'}^2 - m_{V''}^2}\varepsilon_{\mu\nu\alpha\beta}\epsilon_2^{*\nu} P^\alpha q^\beta A_4(q^2), \end{aligned} \tag{53}$$

我们可以得到10个非零的强子螺旋度振幅$H_{\lambda_{W^*}\lambda_{V'}\lambda_{V''}}$ 为：

$$\begin{aligned}
H_{0++}(q^2) &= -\frac{m_{V'}^2 - m_{V''}^2}{\sqrt{q^2}} A_1(q^2) + \sqrt{q^2} A_2(q^2) + \frac{2m_{V'}|\vec{p}|}{\sqrt{q^2}} V_1(q^2), \\
H_{t++}(q^2) &= -\frac{2m_{V'}|\vec{p}|}{\sqrt{q^2}} A_1(q^2) + \frac{m_{V'}^2 - m_{V''}^2}{\sqrt{q^2}} V_1(q^2) - \sqrt{q^2} V_2(q^2), \\
H_{-+0}(q^2) &= -\frac{m_{V'}^2 + 3m_{V''}^2 - q^2}{2m_{V''}} A_1(q^2) + \frac{m_{V'}^2 - m_{V''}^2 - q^2}{2m_{V''}} A_2(q^2) + \ldots \\
& \ldots \\
& \ldots \\
H_{t00}(q^2) &= \frac{(m_{V'}^2 + m_{V''}^2 - q^2)(m_{V'}^2 - m_{V''}^2)}{2m_{V''} m_{V''} \sqrt{q^2}} V_1(q^2) - \ldots \qquad (54)
\end{aligned}$$

事实上，尽管$H_{\lambda_{W^*}\lambda_{V'}\lambda_{V''}}$有$4 \times 3 \times 3 = 36$种不同的组合态，但当且仅当$\lambda_{V'} = \lambda_{V''} - \lambda_{W^*}$ 时，强子螺旋度振幅不为零

$\boxed{L_{\mu\nu}H^{\mu\nu}:}$

将轻子张量按照Wigner's d^J函数展开，$V' \to V''\ell^-\bar{\nu}_\ell$半轻衰变中$L_{\mu\nu}H^{\mu\nu}$可以写为：

$$\begin{aligned} L_{\mu\nu}H^{\mu\nu} &= \frac{1}{8}\sum_{\substack{\lambda_\ell,\lambda_{\bar{\nu}_\ell},J,J' \\ \lambda_{W^*},\lambda'_{W^*}}} (-1)^{J+J'} |\ h_{\lambda_\ell,\lambda_{\bar{\nu}_\ell}}\ |^2 \delta_{\lambda_{V'},\lambda_{V''}-\lambda_{W^*}} \delta_{\lambda_{V'},\lambda_{V''}-\lambda'_{W^*}} \\ &\quad \times d^J_{\lambda_{W^*},\lambda_\ell-\frac{1}{2}} d^{J'}_{\lambda'_{W^*},\lambda_\ell-\frac{1}{2}} H_{\lambda_{W^*}\lambda_{V'}\lambda_{V''}} H_{\lambda'_{W^*}\lambda_{V'}\lambda_{V''}}\ , \end{aligned} \tag{55}$$

1. 主角动量指标J和J'遍取1和0。
2. $\lambda'_{W^*,V',V''}$遍取$+1,-1$和0。
3. λ_ℓ指标遍取$+\frac{1}{2}$和$-\frac{1}{2}$。
4. $\lambda_{\bar{\nu}_\ell}$指标仅有$+\frac{1}{2}$。

1. 定义：$h_{\lambda_\ell,\lambda_{\bar{\nu}_\ell}} = \bar{u}_\ell(\lambda_\ell)\gamma^\mu(1-\gamma_5)v_{\bar{\nu}}(\frac{1}{2})\bar{\epsilon}_\mu(\lambda_{W^*})$
2. $|\ h_{-\frac{1}{2},\frac{1}{2}}\ |^2 = 8(q^2-m_\ell^2)$
3. $|\ h_{\frac{1}{2},\frac{1}{2}}\ |^2 = 8\frac{m_\ell^2}{2q^2}(q^2-m_\ell^2)$

双重微分宽度:

不同轻子螺旋态($\lambda_\ell = \pm\frac{1}{2}$)的$V' \to V''\ell^- \bar{\nu}_\ell$ 半轻衰变的双重微分宽度:

$$
\begin{aligned}
\frac{d^2\Gamma[\lambda\ell = -1/2]}{dq^2 d\cos\theta} &= \frac{G_F^2 \mid V_{cb} \mid^2 \mid \vec{p} \mid}{256\pi^3 m_{V'}^2} \frac{1}{3} q^2 (1 - \frac{m_\ell^2}{q^2})^2 \\
&\times [(1-\cos\theta)^2 (H_{+0+}^2 + H_{+-0}^2) + (1+\cos\theta)^2 (H_{-0-}^2 + H_{-+0}^2) \\
&+ 2\sin^2\theta (H_{0++}^2 + H_{0--}^2 + H_{000}^2)] , \\
\frac{d^2\Gamma[\lambda\ell = 1/2]}{dq^2 d\cos\theta} &= \frac{G_F^2 \mid V_{cb} \mid^2 \mid \vec{p} \mid}{256\pi^3 m_{V'}^2} \frac{1}{3} q^2 (1 - \frac{m_\ell^2}{q^2})^2 \frac{m_\ell^2}{q^2} \\
&\times [\sin^2\theta (H_{+0+}^2 + H_{+-0}^2 + H_{-0-}^2 + H_{-+0}^2) \\
&+ 2(H_{t++} - \cos\theta H_{0++})^2 + 2(H_{t--} - \cos\theta H_{0--})^2 \\
&+ 2(H_{t00} - \cos\theta H_{000})^2] , \qquad (56)
\end{aligned}
$$

式中$\frac{1}{3}$因子是对初态矢量介子的自旋求平均引入的。

$\boxed{\text{(I) } \bar{B}^* \to V\ell\bar{\nu}_\ell}$

$\Gamma_{\text{tot}}(\mathbf{B}^*)$:

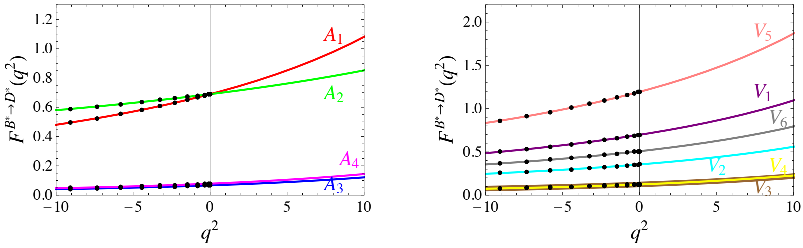
在光前夸克模型中，$B^* \to B\gamma$衰变过程的宽度可以写为：

$$\begin{aligned}
\Gamma(B^* \to B\gamma) &= \frac{\alpha}{3}\left[e_1 I(m_1, m_2, 0) + e_2 I(m_2, m_1, 0)\right]^2 \kappa_\gamma^3\,, \\
I(m_1, m_2, q^2) &= \int_0^1 \frac{dx}{8\pi^3} \int d^2\mathbf{k}_\perp \frac{\psi(x, \mathbf{k}'_\perp)\psi(x, \mathbf{k}_\perp)}{x\tilde{M}_0\tilde{M}'_0} \\
&\quad \times \left\{\mathcal{A} + \frac{2}{\mathcal{M}_0}\left[\mathbf{k}_\perp^2 - \frac{(\mathbf{k}_\perp \cdot \mathbf{q}_\perp)^2}{\mathbf{q}_\perp^2}\right]\right\}\,,
\end{aligned} \tag{57}$$

其中α是精细结构常数，$\kappa_\gamma = (m_{B^*}^2 - m_B^2)^2/2m_{B^*}$ 是发射光子的能量。

$$\begin{aligned}
\Gamma_{\text{tot}}(B^{*+}) &\simeq \Gamma(B^{*+} \to B^+\gamma) = (349 \pm 18)\text{eV}\,, \\
\Gamma_{\text{tot}}(B^{*0}) &\simeq \Gamma(B^{*0} \to B^0\gamma) = (116 \pm 6)\text{eV}\,, \\
\Gamma_{\text{tot}}(B_s^{*0}) &\simeq \Gamma(B_s^{*0} \to B_s^0\gamma) = (84^{+11}_{-9})\text{eV}\,, \\
\Gamma_{\text{tot}}(B_c^{*+}) &\simeq \Gamma(B_c^{*+} \to B_c^+\gamma) = (49^{+28}_{-21})\text{eV}\,.
\end{aligned} \tag{58}$$

形状因子：



(a) (b)

$$F_i(q^2) = \frac{F_i(0)}{1 - a(q^2/M_{B^*}^2) + b(q^2/M_{B^*}^2)^2}, \tag{59}$$

上式中，F_i表示的是形状因子A_{1-4}和V_{1-6}

$\boxed{\mathcal{B}(\bar{B}^* \to V\ell^-\bar{\nu}_\ell)}$:

Decay mode	This Work	BS[1]	HQS[2]
$\bar{B}^{*-} \to D^{*0}\ell'^-\bar{\nu}_{\ell'}$	$8.42^{+0.23+0.11+0.45}_{-0.24-0.24-0.42} \times 10^{-8}$	1.26×10^{-7}	6.41×10^{-8}
$\bar{B}^{*-} \to D^{*0}\tau^-\bar{\nu}_\tau$	$2.26^{+0.08+0.03+0.12}_{-0.08-0.06-0.11} \times 10^{-8}$	2.74×10^{-8}	1.29×10^{-8}
$\bar{B}^{*0} \to D^{*+}\ell'^-\bar{\nu}_{\ell'}$	$2.51^{+0.08+0.03+0.13}_{-0.07-0.07-0.12} \times 10^{-7}$	–	1.92×10^{-7}
$\bar{B}^{*0} \to D^{*+}\tau^-\bar{\nu}_\tau$	$6.73^{+0.24+0.09+0.35}_{-0.25-0.19-0.32} \times 10^{-8}$	–	3.88×10^{-8}
$\bar{B}_s^{*0} \to D_s^{*+}\ell'^-\bar{\nu}_{\ell'}$	$3.46^{+0.17+0.05+0.41}_{-0.17-0.10-0.41} \times 10^{-7}$	4.63×10^{-7}	2.53×10^{-7}
$\bar{B}_s^{*0} \to D_s^{*+}\tau^-\bar{\nu}_\tau$	$9.10^{+0.60+0.12+1.07}_{-0.59-0.26-1.07} \times 10^{-8}$	1.05×10^{-7}	5.05×10^{-8}
$\bar{B}_c^{*-} \to J/\psi\ell'^-\bar{\nu}_{\ell'}$	$5.44^{+0.33+0.07+4.06}_{-0.33-0.16-2.00} \times 10^{-7}$	5.37×10^{-7}	2.91×10^{-7}
$\bar{B}_c^{*-} \to J/\psi\tau^-\bar{\nu}_\tau$	$1.43^{+0.13+0.02+1.07}_{-0.12-0.04-0.52} \times 10^{-7}$	1.49×10^{-7}	5.65×10^{-8}

[1].semi-leptonic decays of B^*, B_s^*, B_c^* with the Bethe-Salpeter method

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[2].Semileptonic decays of $B^{(*)}$, $D^{(*)}$ into $\nu\ell$ and pseudoscalar or vector mesons

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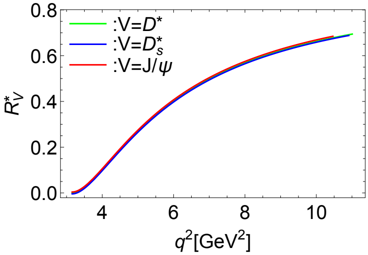
物理可观测量R_V^*和F_L^{*V}：

Observable	Prediction	Observable	Prediction
$R_{D^*}^*$	$0.269^{+0.003}_{-0.003}$	$F_L^{*D^*}$	$0.304^{+0.003}_{-0.004}$
$R_{D_s^*}^*$	$0.263^{+0.005}_{-0.005}$	$F_L^{*D_s^*}$	$0.306^{+0.005}_{-0.006}$
$R_{J/\psi}^*$	$0.262^{+0.009}_{-0.007}$	$F_L^{*J/\psi}$	$0.303^{+0.006}_{-0.007}$

$$\begin{aligned} &R_{D^*}^* = 0.217\,, \quad R_{D_s^*}^* = 0.227\,, \quad R_{J/\psi}^* = 0.277\,, \quad \text{BS method} \\ &R_{D^*}^* = 0.202\,, \quad R_{D_s^*}^* = 0.200\,, \quad R_{J/\psi}^* = 0.194\,, \quad \text{HQS} \end{aligned} \tag{60}$$

$\bar{B}^* \to V\ell^-\bar{\nu}_\ell$半轻衰变的纵向极化分数$F_L^{*(V)}$：

$$F_L^{*V} = \frac{\Gamma_{\lambda_V=0}(\bar{B}^* \to V\tau^-\bar{\nu}_\tau)}{\Gamma(\bar{B}^* \to V\tau^-\bar{\nu}_\tau)}\,, \tag{61}$$



- R_V^*：$R_{D^*}^* \simeq R_{D_s^*}^* \simeq R_{J/\psi}^*$
- $F_L^{*D^*}$：$F_L^{*D^*} \simeq F_L^{*D_s^*} \simeq F_L^{*J/\psi} \simeq 30\%$，

$\Longrightarrow \bar{B}^* \to V\tau^-\bar{\nu}_\tau$半轻衰变是以横向极化态为主的。显然其与$\bar{B} \to V\tau^-\bar{\nu}_\tau$的半轻衰变过程不同，后者是以纵向极化态为主。

$\boxed{(\mathrm{II})\Upsilon(1S,2S) \to B_c^* \ell \bar{\nu}_\ell}$

$\mathcal{B}(\Upsilon(nS) \to B_c^* \ell \bar{\nu}_\ell)$:

衰变过程	分支比
$\Upsilon(1S) \to B_c^* \ell' \bar{\nu}_{\ell'}$	$5.48^{+0.07+0.13+0.59}_{-0.16-0.12-0.62} \times 10^{-10}$
$\Upsilon(1S) \to B_c^* \tau \bar{\nu}_\tau$	$1.17^{+0.02+0.03+0.15}_{-0.03-0.03-0.13} \times 10^{-10}$
$\Upsilon(2S) \to B_c^* \ell' \bar{\nu}_{\ell'}$	$1.86^{+0.03+0.17+0.17}_{-0.05-0.14-0.21} \times 10^{-9}$
$\Upsilon(2S) \to B_c^* \tau \bar{\nu}_\tau$	$6.60^{+0.09+0.59+0.59}_{-0.19-0.50-0.90} \times 10^{-10}$

小结:

- 所有通过$b \to c\ell^- \bar{\nu}_\ell$过程的$\bar{B}^* \to V\ell^- \bar{\nu}_\ell$半轻衰变的分支比均$\mathcal{O} > 10^{-9}$，在LHC和Bell-II的实验探测范围内。
- $\Upsilon(1S, 2S) \to B_c^* \ell^- \bar{\nu}_\ell$半轻衰变也有着相对较大的分支比，达到了$\mathcal{O}(10^{-10} \sim 10^{-9})$。
- 与$B \to D^* \tau^- \bar{\nu}_\tau$不同，通过$b \to c\ell^- \bar{\nu}_\ell$过程引起的矢量介子半轻衰变均以横向极化态为主，而前者是以纵向极化为主。

总结与展望:

(1) 在LFQM中，通过$P \to P, S, V, A$过程的张量形状因子，详细地分析了CLF QM的自洽性和协变性，以及zero-mode效应，首次发现传统CLF QM“新”的自洽性问题。传统CLF QM的自洽性和协变性问题，以及首次发现的“新”自洽性问题均可以通过type-II方案得以解决。

(2) 采用CLF方法对$b \to c\ell^- \bar{\nu}_\ell$过程的$b$味强子半轻衰变进行唯象研究。

(3) 以上研究为将来关于非微扰物理量计算方法的改进以及未来高能物理实验中关于b味强子半轻衰变的探测提供了理论参考。

感谢各位专家、老师批评指正！