Updated Result on Nucleon Transversity PDF from Lattice QCD

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CLHCP, 2021/11/25

OUTLINE

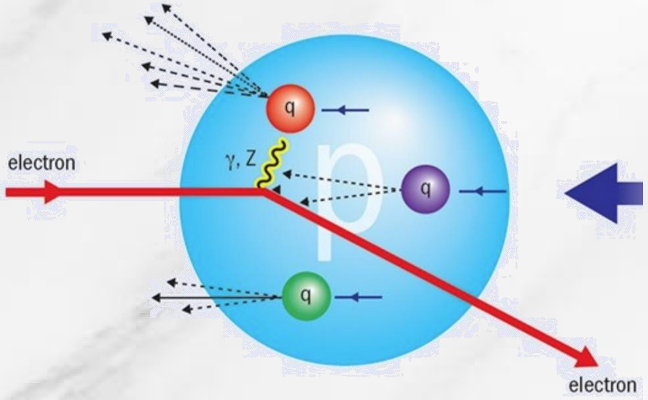
- Introduction
- Lattice calculation
- Numerical result
- Summary and outlook

PART.1

Introduction

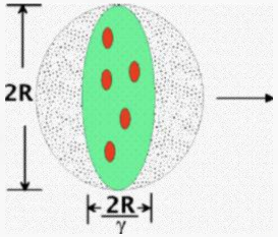
Introduction

☐ Deep inelastic scattering

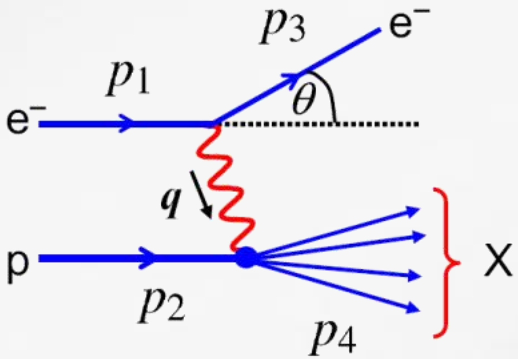


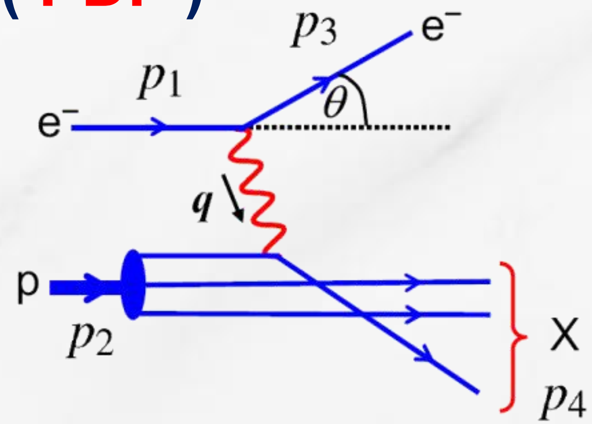
- proton contains **point-like** objects
- Feynman's parton model

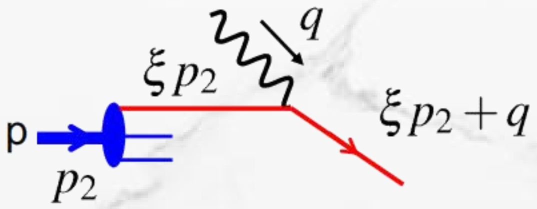
☐ Nucleons are Lorentz contracted along the direction of motion



☐ Parton distribution function (PDF)



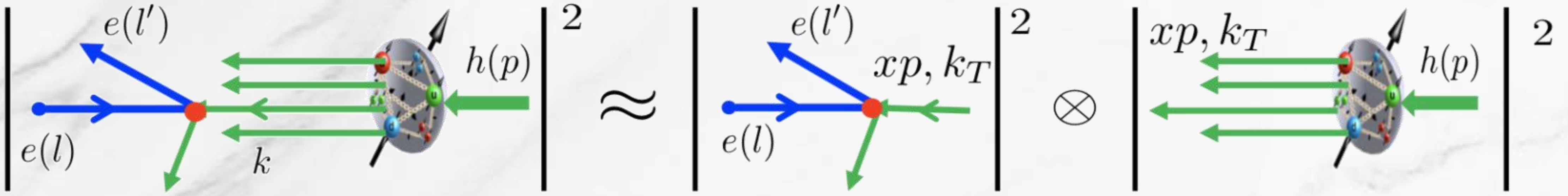




f(ξ)

Introduction

- **QCD factorization** $\sigma = \sum_a \hat{H}_{l+a\to X} \otimes f_{a/p}$

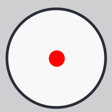
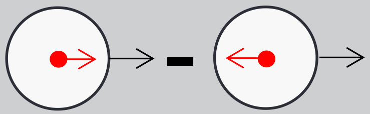
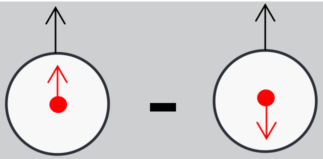


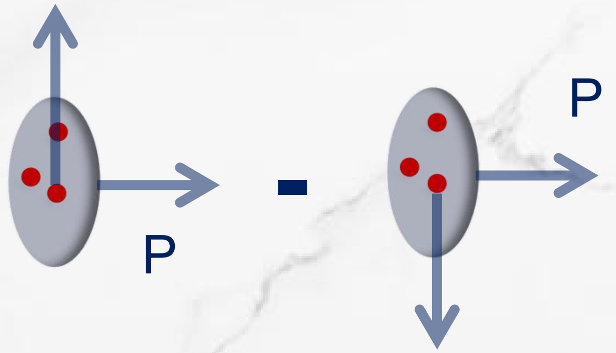
Cross section

Hard part
(Perturbative)

Parton distribution
(Nonperturbative)

- **Polarized PDF**

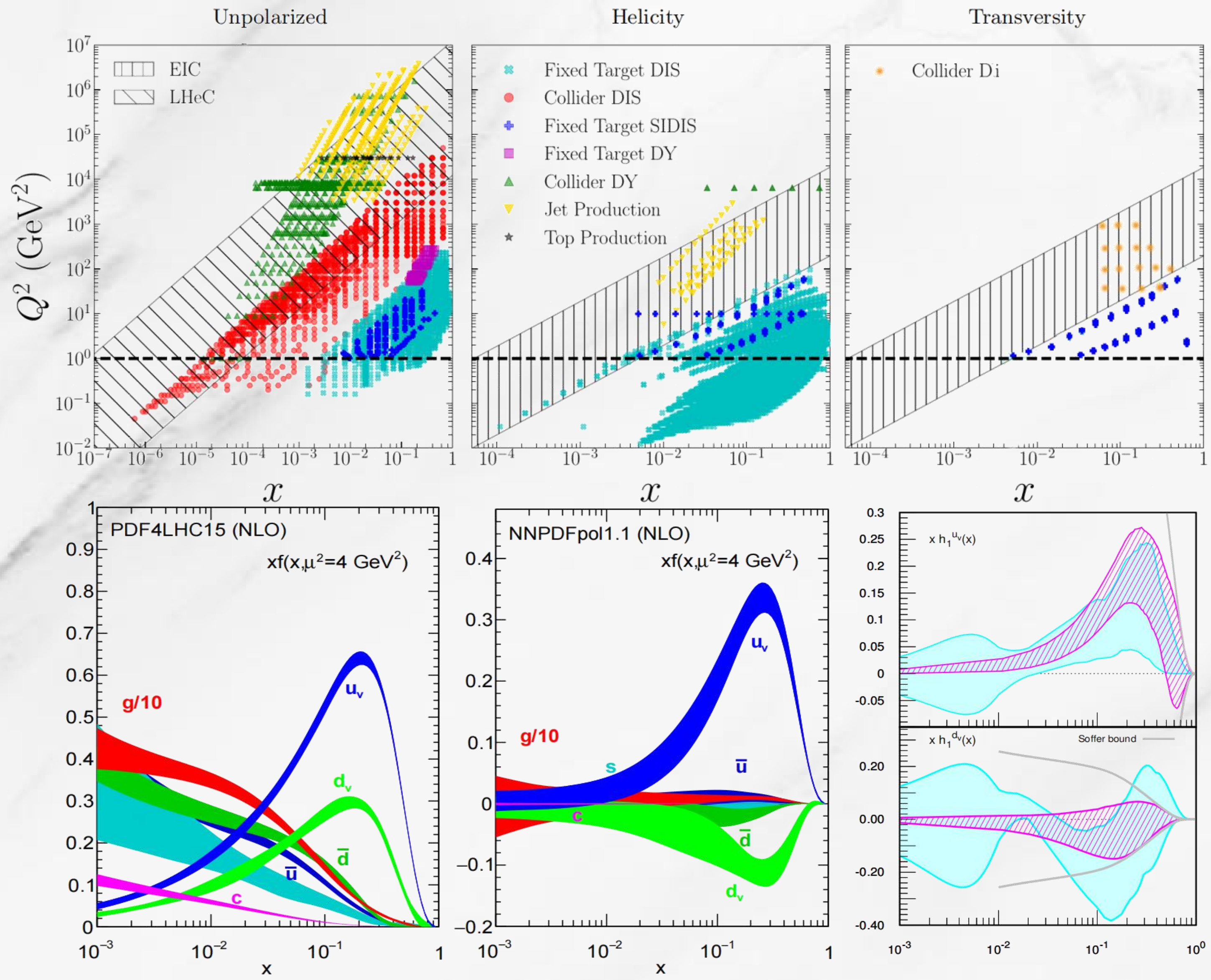
Unpolarized	Longitudinally polarized	Transversely polarized
	 helicity	 transversity



transversity PDF

Motivation

□ **Studying PDF can understand inner structure of nucleons in detail**

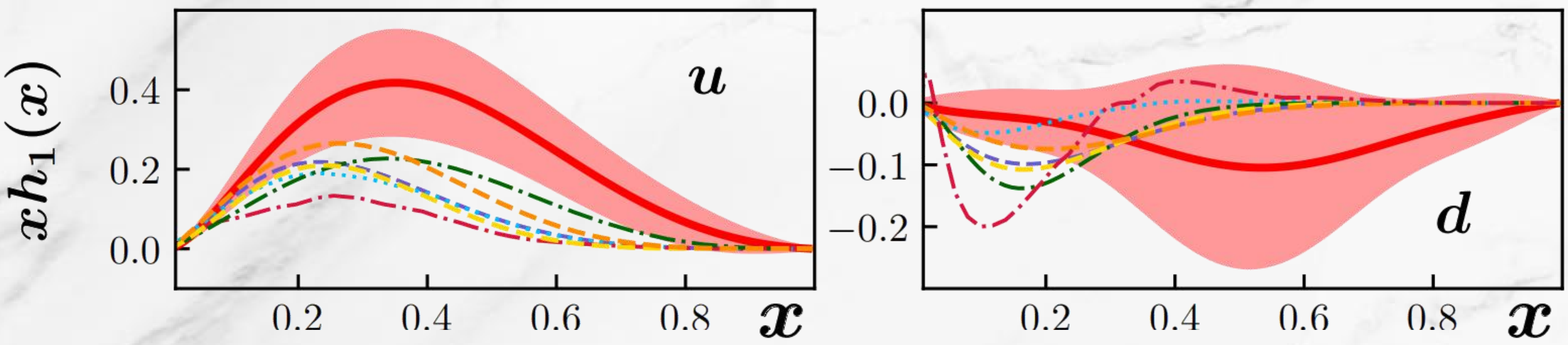


- **There are more experimental data of unpolarized and helicity PDF than transversity PDF**
- **Wider error band**

Lin et al— Prog.Part.Nucl.Phys. 121 (2021) 103908

Motivation

- Global fit



JAM Collaboration — PRD.102 (2020) 5, 054002

- Large error compared with unpolarized and helicity PDF
- The fitted antiquark transversity is zero within large uncertainties
- Lattice calculation can provide information complementary to experimental data

Theoretical framework

- The **light-cone transversity PDF** Definitions:

$$\delta q(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P, s_\perp | \bar{\psi}(\lambda n) i\gamma^x \gamma^t \gamma^5 L(\lambda n, 0) \psi(0) | P, s_\perp \rangle \qquad L(\lambda n, 0) = e^{ig \int_0^\lambda d\lambda' n \cdot A(\lambda' n)}$$

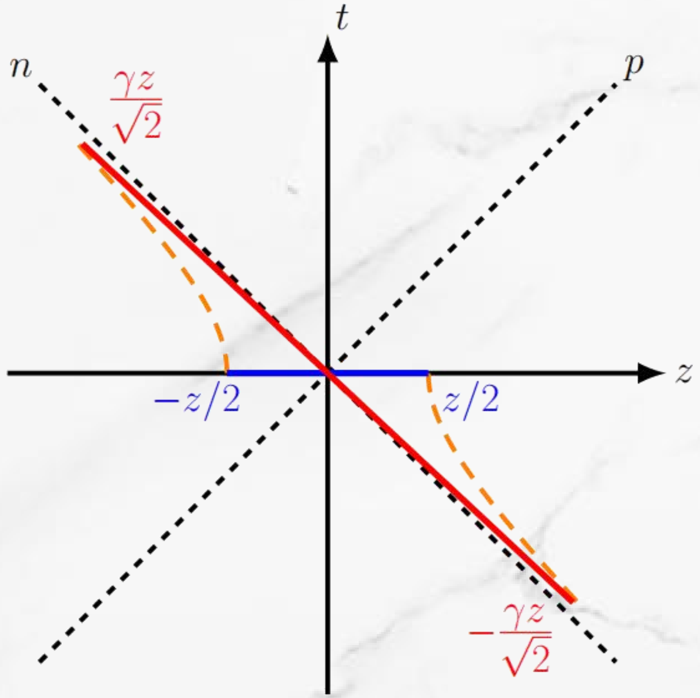
- Lattice QCD calculate physcial observable from path integral in Euclidean space

$$t \to -i\tau, e^{iS_M} \to e^{-S_E}$$

Recover physical limit

$$m_\pi \to m_\pi^{phys}, a \to 0, L \to \infty$$

- Can be used to compute Euclidean (quasi-LF) correlations at large momentum in **LaMET**



Ji, PRL 110 (2013) 262002

Theoretical framework

- **quasi-LF correlation** in Euclidean space

$$\tilde{h}(P,z) = \langle P,s|\bar{\psi}(z)i\gamma^x\gamma^t\gamma^5 L(z,0)\psi(0)|P,s\rangle$$

- The light-cone transversity PDF can be related to the quasi-PDF by the following **factorization formula**

$$\tilde{\delta q}(x,P^z) = \int_0^1 \frac{dy}{|y|}\delta C\left(\frac{x}{y},\frac{\mu}{yP_z}\right)\delta q(y,\mu) + \mathcal{O}(\Lambda_{QCD}^2/(yP^z)^2,\Lambda_{QCD}^2/((1-y)P^z)^2)$$

$$\tilde{\delta q}\left(x,\frac{\mu}{P^z}\right) = \int_{-\infty}^{\infty}\frac{d\zeta}{2\pi}e^{ix\zeta}\tilde{h}\left(\zeta,\frac{\mu^2\zeta^2}{P_z^2}\right) \quad \zeta = zp_z$$

Theoretical framework

Bare lattice matrix element — Non-perturbative

↓ Renormalization (Hybrid etc.)

Renormalized matrix element

↓ Extrapolation, Fourier transformation

Quasi-PDF

↓ Factorization (matching)

Light-cone PDF

PART.2

Lattice calculation

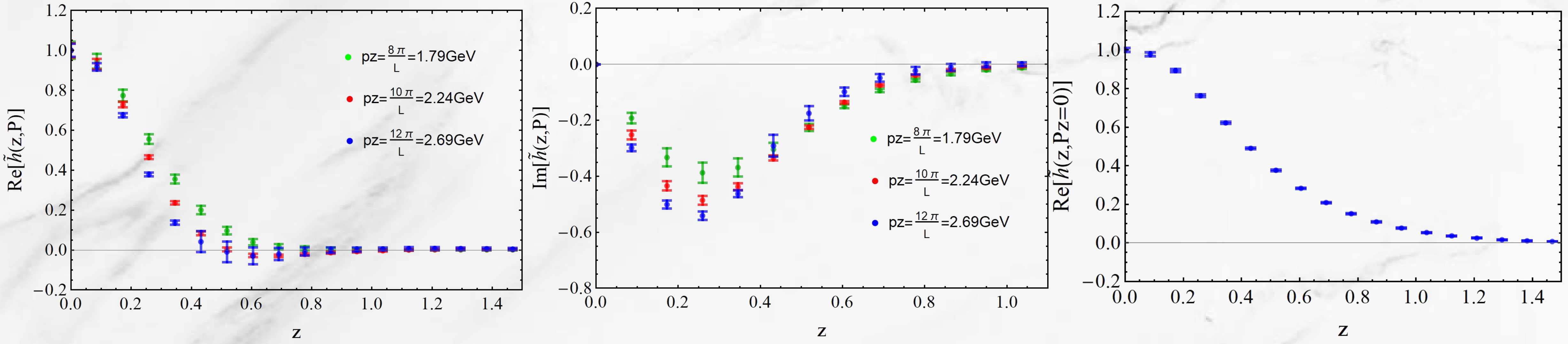
CLS ensembles

ensembles	a(fm)	$L^3 \times T$	$m_\pi(MeV)$
H102	0.08636	$32^3 \times 96$	354
N203	0.06426	$48^3 \times 128$	345
N302	0.04981	$48^3 \times 128$	346

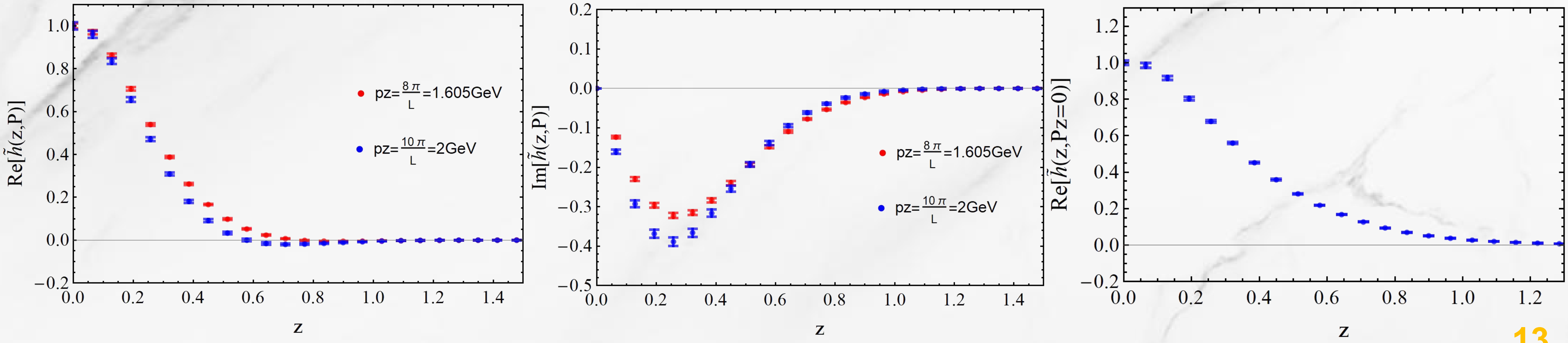
- **H102**: $\mathbf{P_z = \{4, 5, 6\} 2\pi/L \approx \{1.79,\ 2.24,\ 2.69\} GeV}$
- **N203**: $\mathbf{P_z = \{4, 5\} 2\pi/L \approx \{1.6,\ 2\} GeV}$
- **N302**: $\mathbf{P_z = \{4, 5\} 2\pi/L \approx \{2.07, 2.59\} GeV}$

Bare matrix elements

a=0.08636fm



a=0.06426fm

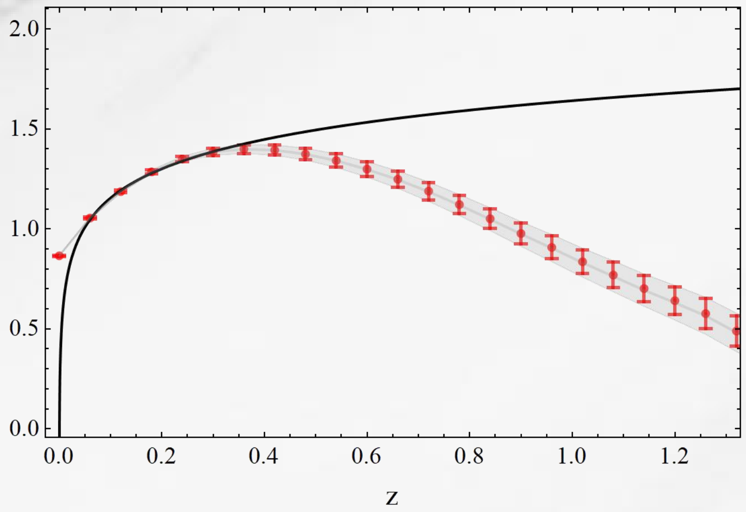


Renormalization in Hybrid scheme

- The quasi-LF correlation operator is multiplicatively renormalized

$$[\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_B = e^{\delta m|z|} Z\, [\bar{\psi}(z)\Gamma W(z,0)\psi(0)]_R$$ Ji, Zhang, Zhao, PRL 120, 112001 (2018)

- Ratio scheme : Divide by the same operator matrix element at zero momentum Radyushkin, PRD 98 (2018) 1, 014019
- Problem: introduce undesired IR effects at large distances



- Hybrid renormalization scheme

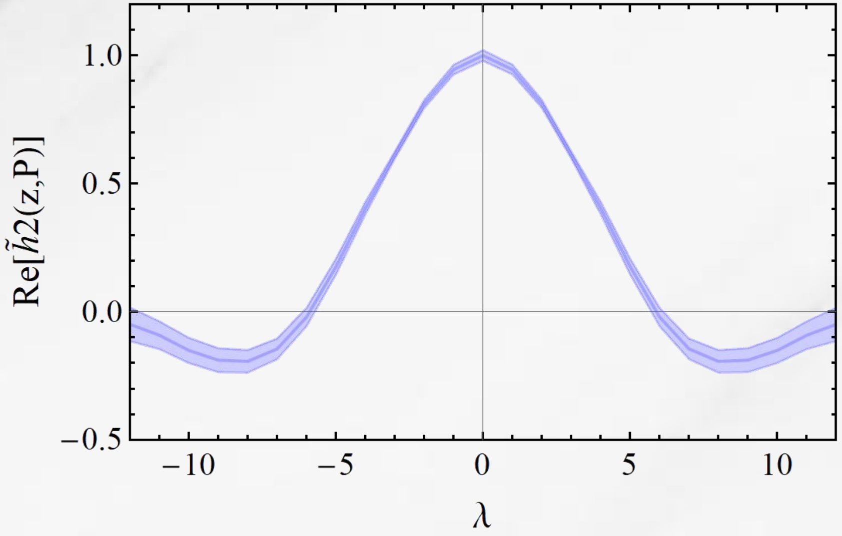
Ji, JHZ et al, Nucl.Phys.B 964 (2021) 115311

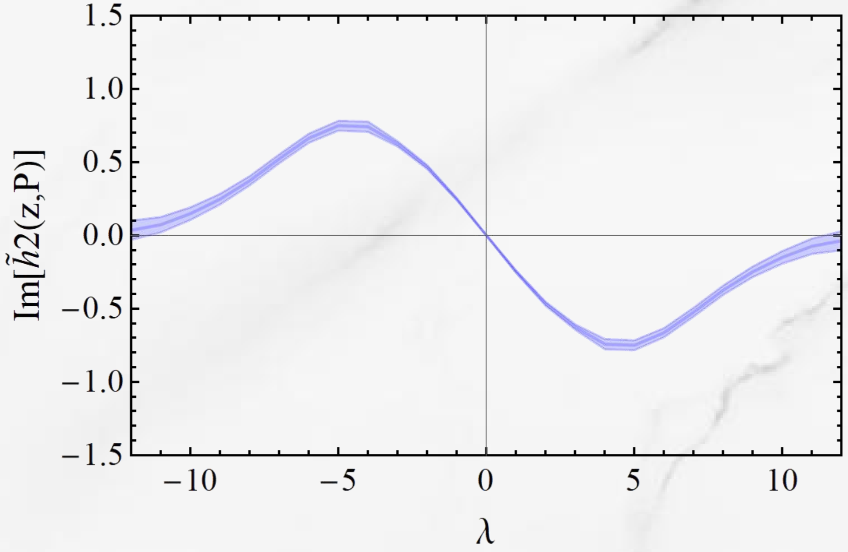
$$\tilde{h}^R(z,a,P_z) = \frac{\tilde{h}(z,a,P_z)}{Z_X(z,a)}\theta(z_S - |z|) + \tilde{h}(z,a,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S,a)\theta(|z| - z_S)$$

Renormalization in Hybrid scheme

$$\tilde{h}^R(z,a,P_z) = \frac{\tilde{h}(z,a,P_z)}{Z_X(z,a)}\theta(z_S - |z|) + \tilde{h}(z,a,P_z)e^{-\delta m|z|}Z_{\text{hybrid}}(z_S,a)\theta(|z| - z_S) \quad z_S = 0.3\,fm$$

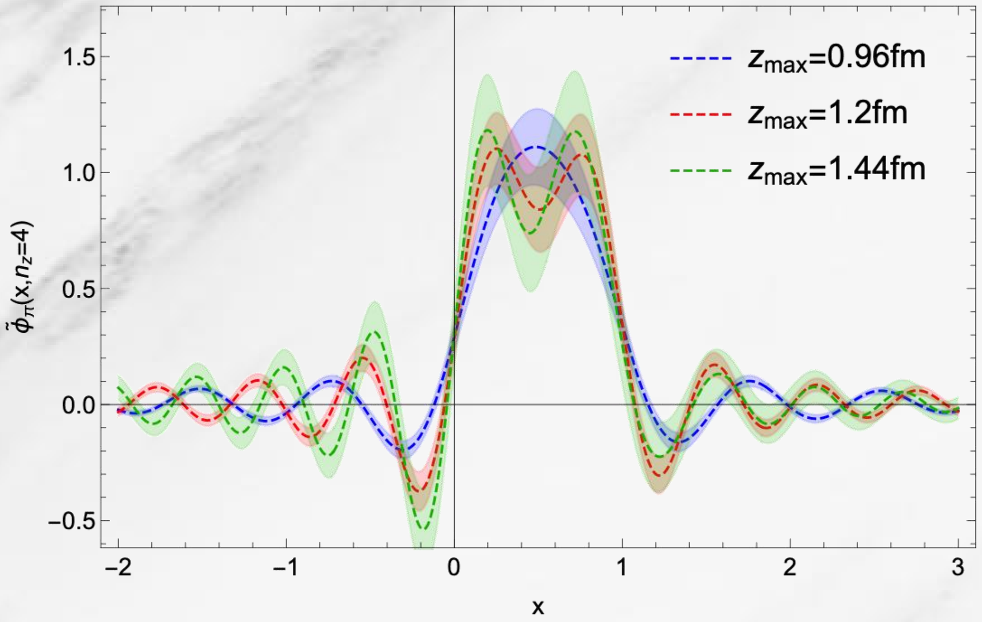
- For $|z| \le z_S$ (perturbative region) , do the renormalization in the ratio scheme
- For $|z| > z_S$, use Wilson line mass renormalization that avoids introducing extra nonperturbative effects at large distances





Extrapolation and Fourier transformation

- Missing large λ information
- Unphysical oscillation



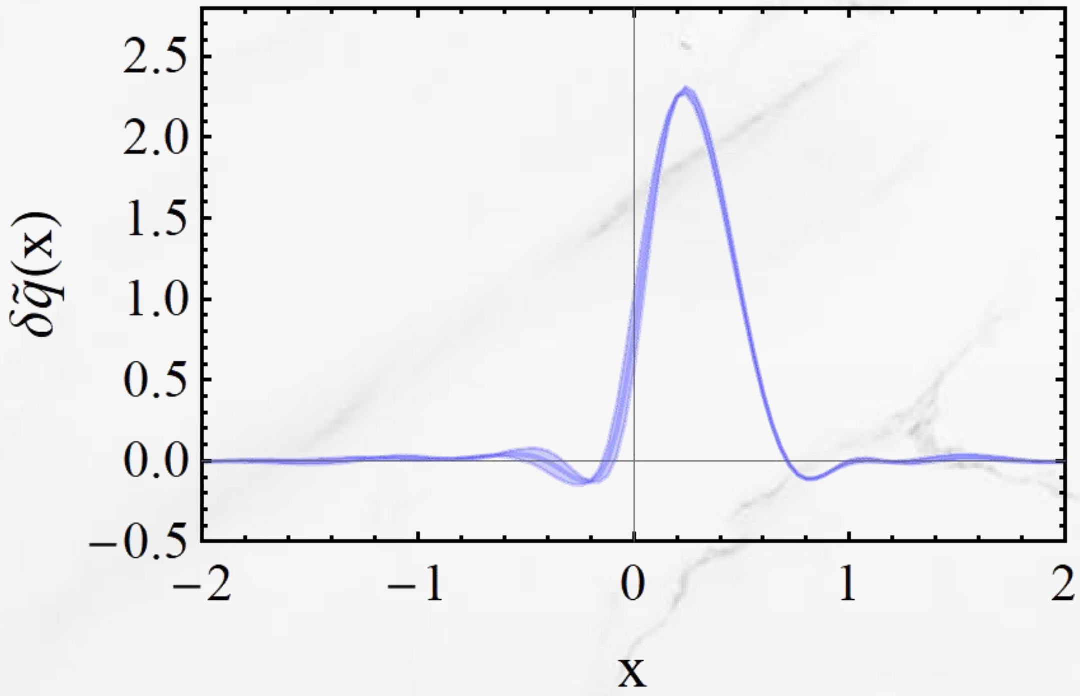
R. Zhang et al —PRD.102.094519(2020)

- Extrapolation

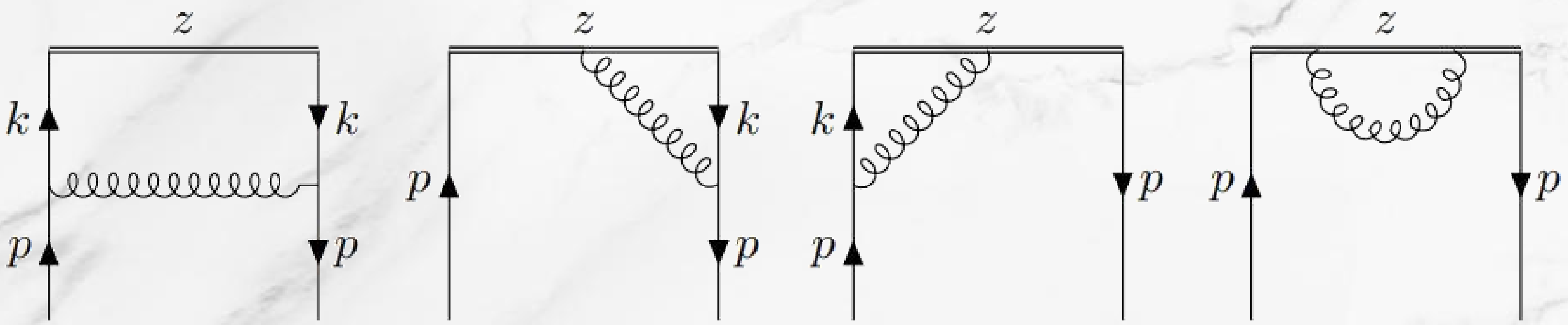
$$\tilde{h}(z,P_z) = \left[\frac{c_1}{(-i\lambda)^a} + e^{i\lambda}\frac{c_2}{(i\lambda)^b}\right]e^{-\frac{\lambda}{\lambda_0}}$$

- Fourier transformation

$$\delta\tilde{q}(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda}\tilde{h}(\lambda)$$



Matching coefficient (In momentum space)



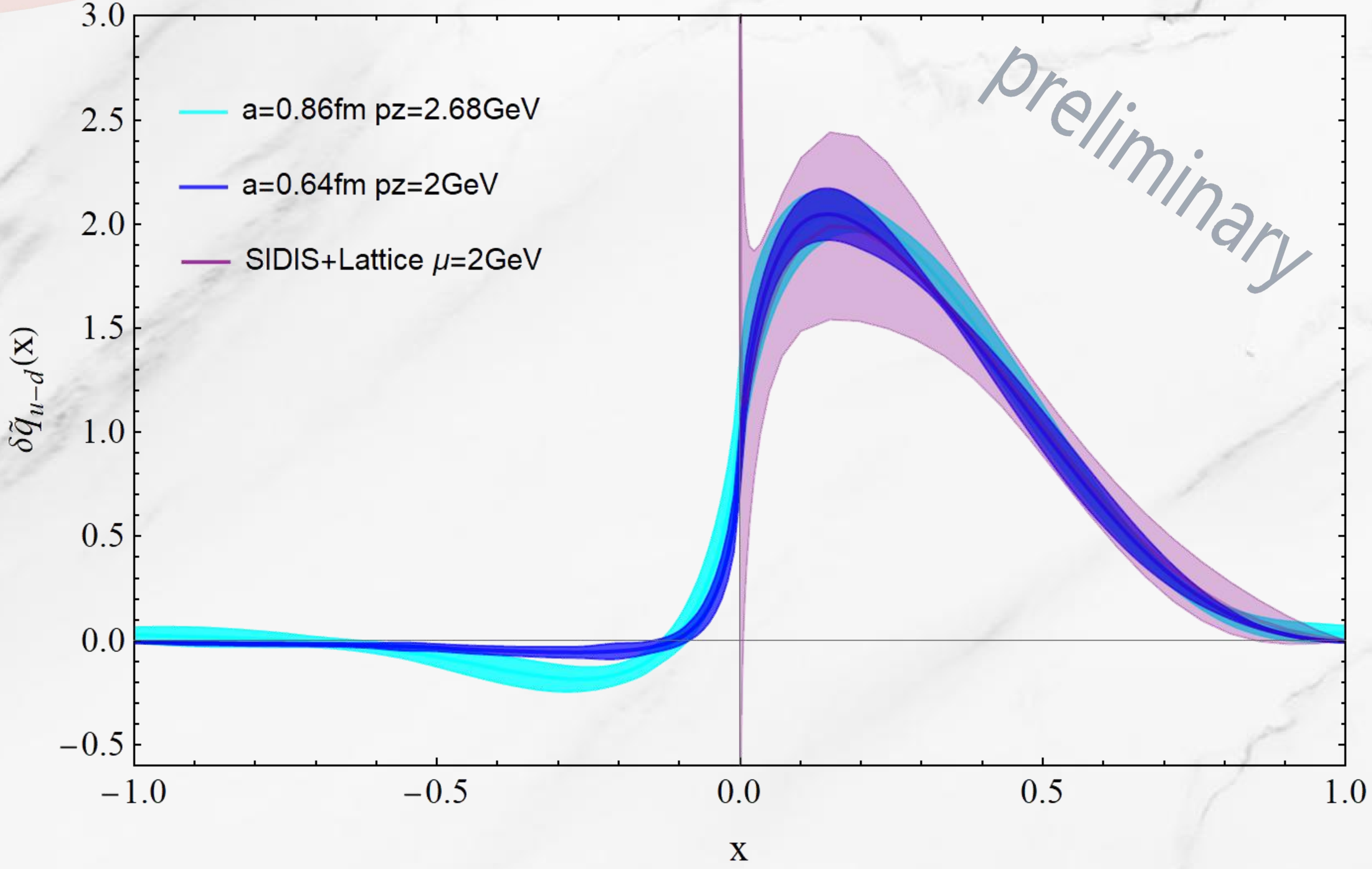
Ratio scheme

$$\delta C_r\left(x,\frac{\mu}{p_z}\right)=\delta(1-x)+\frac{\alpha_s C_F}{2\pi}\begin{cases}\left[\frac{2x}{1-x}\ln\frac{x}{x-1}-\frac{2}{1-x}\right]_+ & x>1\\ \left[\frac{2x}{1-x}\left(\ln\frac{4p_z^2}{\mu^2}+\ln x(1-x)\right)-\frac{2x}{1-x}+\frac{2}{1-x}\right]_+ & 0<x<1\\ \left[-\frac{2x}{1-x}\ln\frac{x}{x-1}+\frac{2}{1-x}\right]_+ & x<0\end{cases}$$

Hybrid scheme

$$\delta C_h\left(x,\frac{\mu}{p_z},z_s^2\mu^2\right)=\delta C_r\left(x,\frac{\mu}{p_z}\right)+\frac{\alpha_s C_F}{2\pi}2\left[-\frac{1}{|1-x|}+\frac{2Si((1-x)\lambda_s)}{\pi(1-x)}\right]_+$$

PART.3 Numerical result



Summary

- PDF plays an important role in the detailed understanding of the internal structure of nucleon
- We calculate the Nucleon Transversity PDF on lattice using LaMET
- The numerical results agree with the global fit results within the error band
- Follow-up
  - Improving error analysis
  - physical extrapolation

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Thank you for listening !