ICTP-AP
International Centre for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

Lattice QCD and PDF

Lattice Parton Collaboration

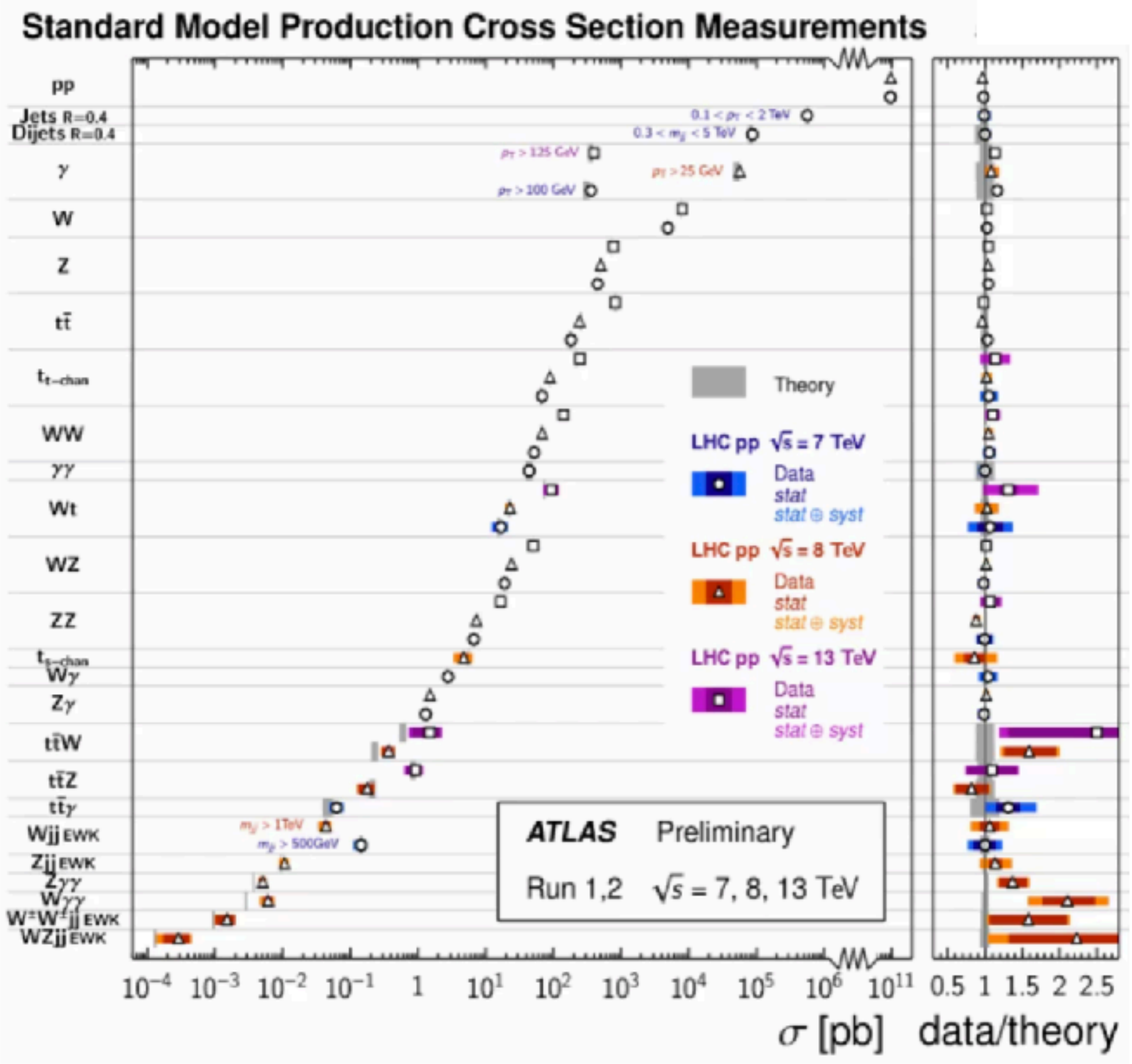
Yi-Bo Yang

2021/11/27

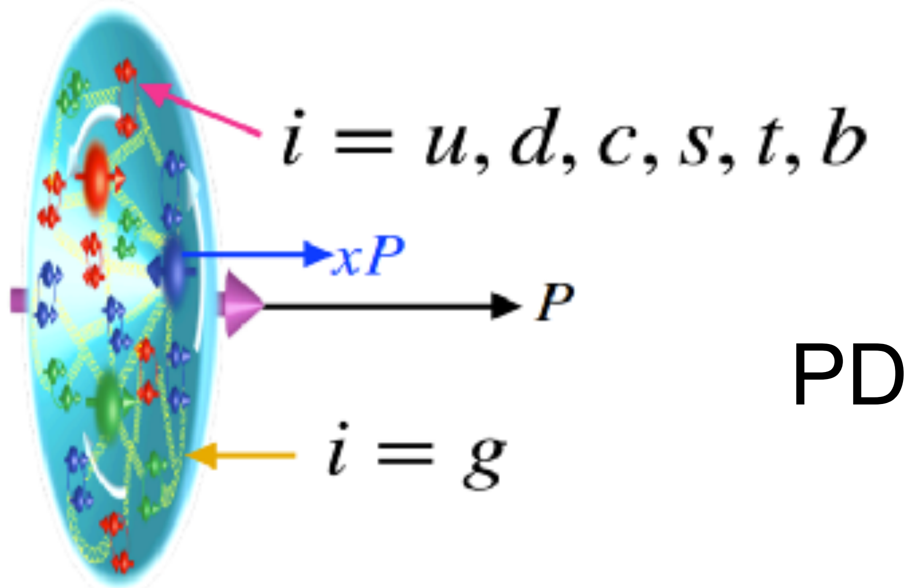
CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China
School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

Strong interaction frontier:

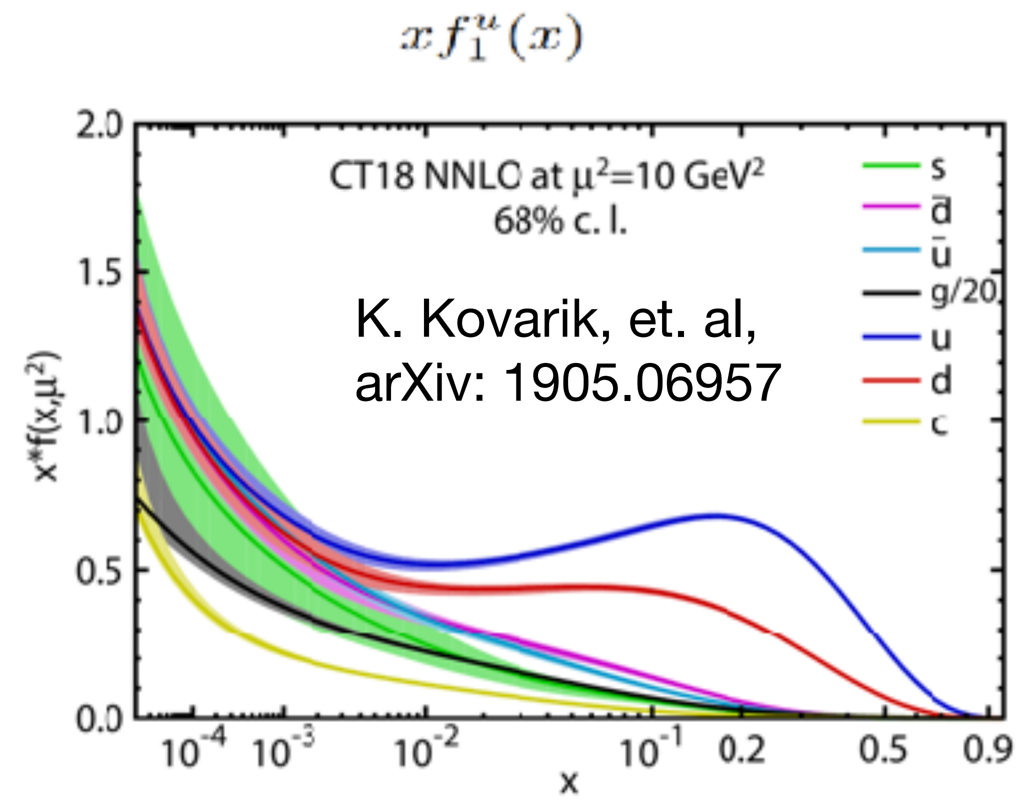
from 1D to 3D nucleon structure

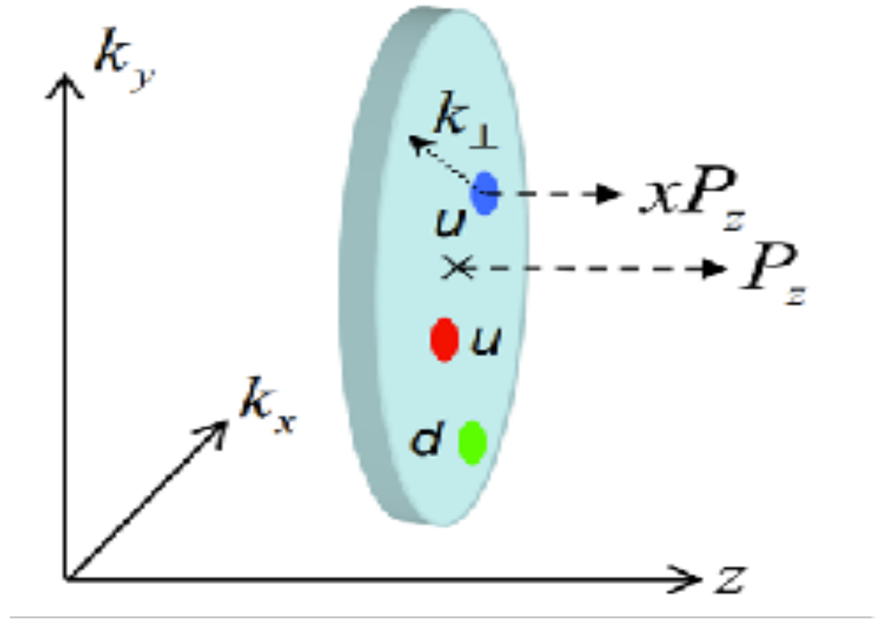


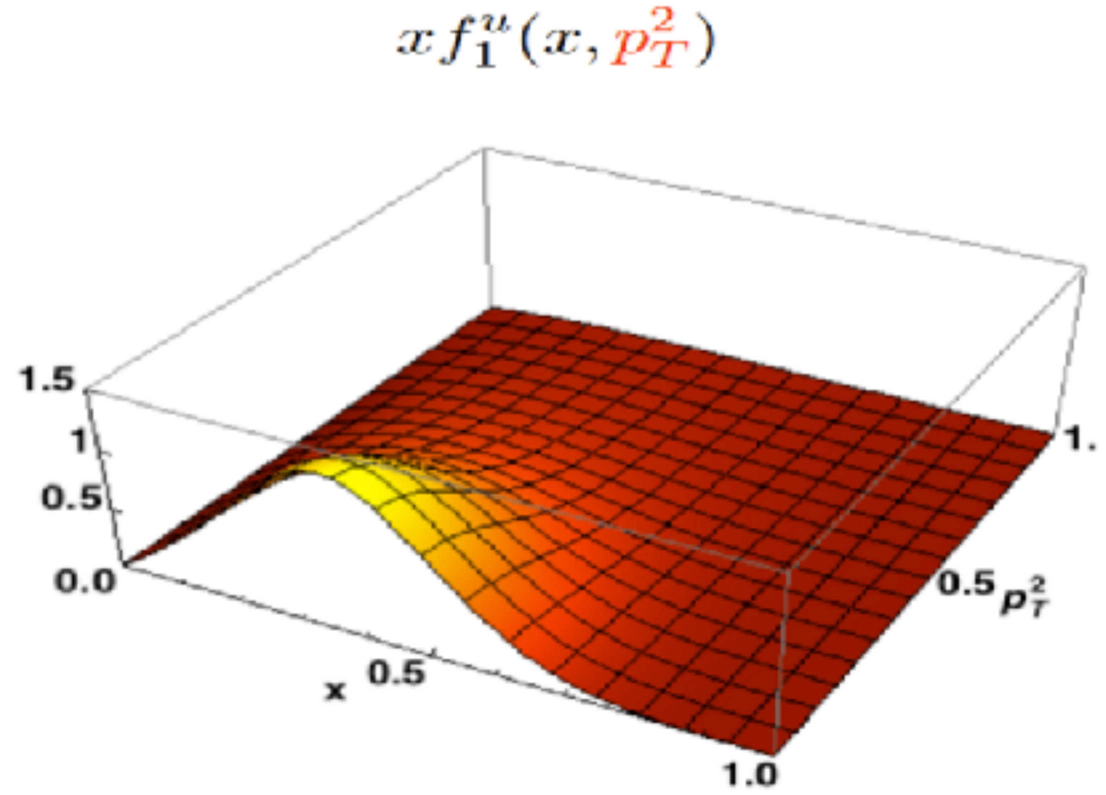
- SM has been very successful, while the strong interaction inputs are still imprecise.
- The experimental constraints on parton distribution function keep improving;
- While the transverse momentum dependent (TMD) PDF is still less known.



PDF







TMD

Figure from A. Bacchetta, F. Conti, M. Radici, PRD78 (2008) 074010, arXiv:0807.0323

Reconstruct PDF from the lattice QCD observables

- Large momentum effective theory and quasi-PDF;

  X. Ji, PRL 110 (2013) 262002, arXiv: 1305.1539

  X. Ji, et.al., RMP 93 (2021) 035005, arXiv: 2004.03543

- Lattice cross section;

  Y.Q. Ma and J.-W. Qiu, PRD 98 (2018) 074021, arXiv:1404.6860

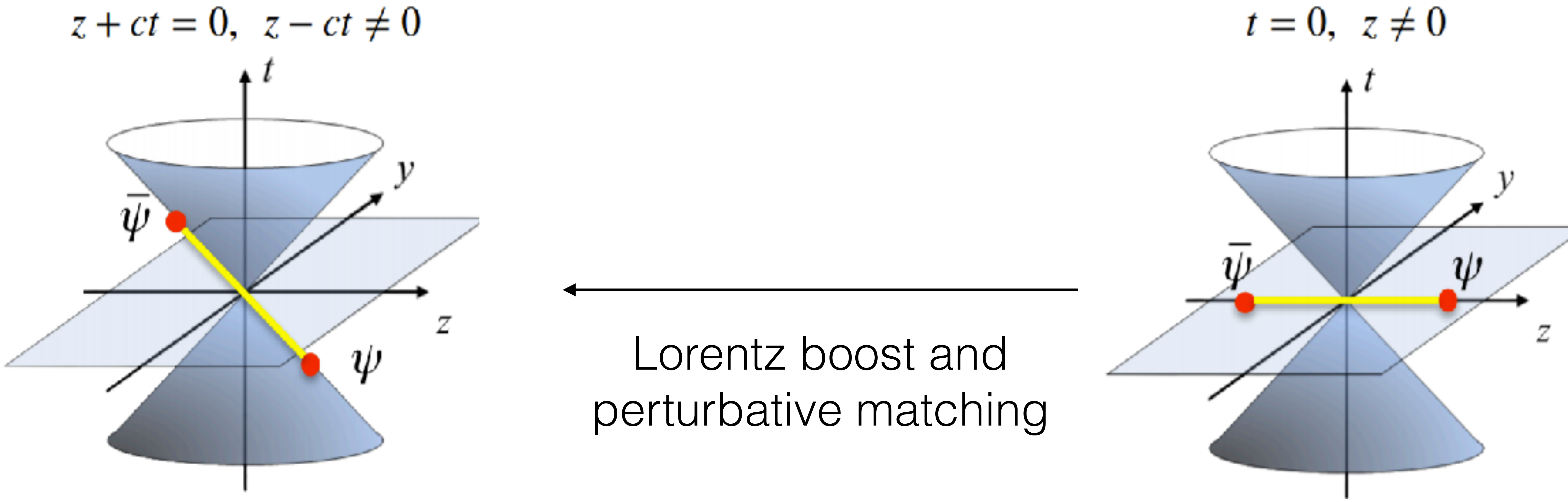
- Hadronic tensor;

  J. Liang, et.al., χQCD, PRD 101 (2020) 114503, arXiv:1906.05312

- Operator product expansion…

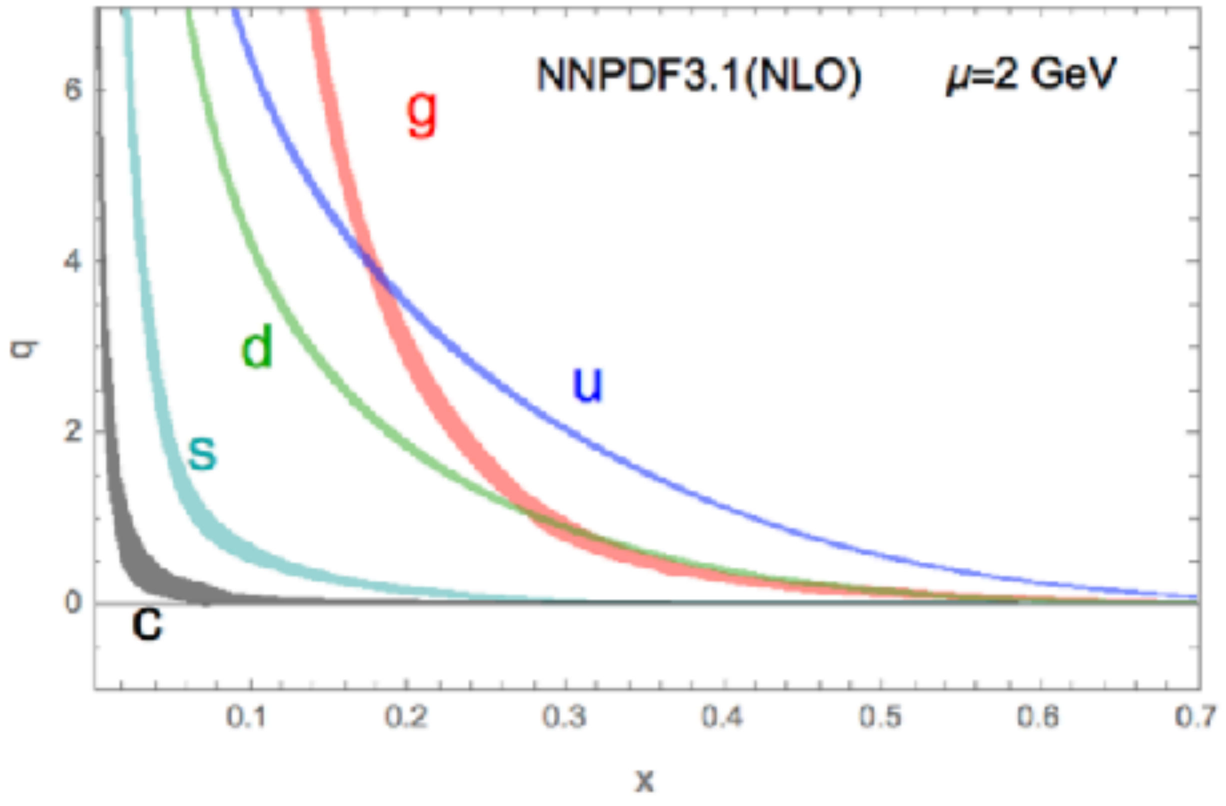
  QCDSF, PRL118 (2017) 242001, arXiv:1703.01153

  W. Detmold, et.al, arXiv:1909.15241

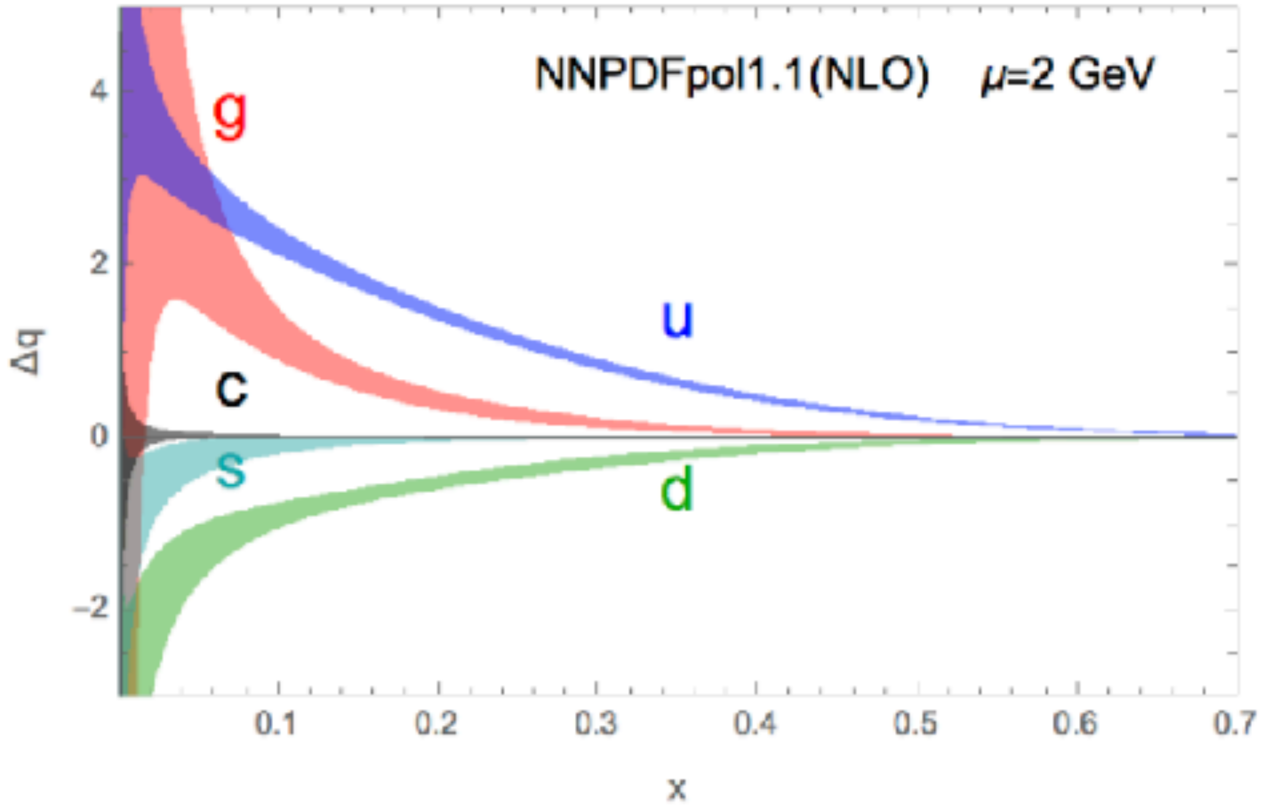


Outline

R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428

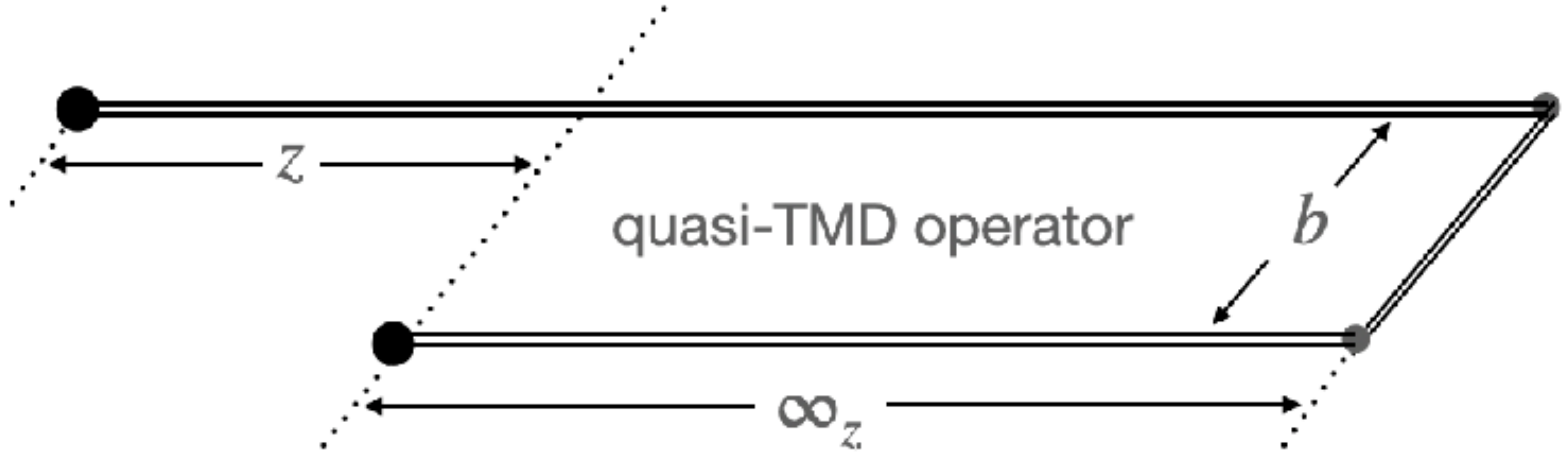


E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539



- **Colinear PDF**

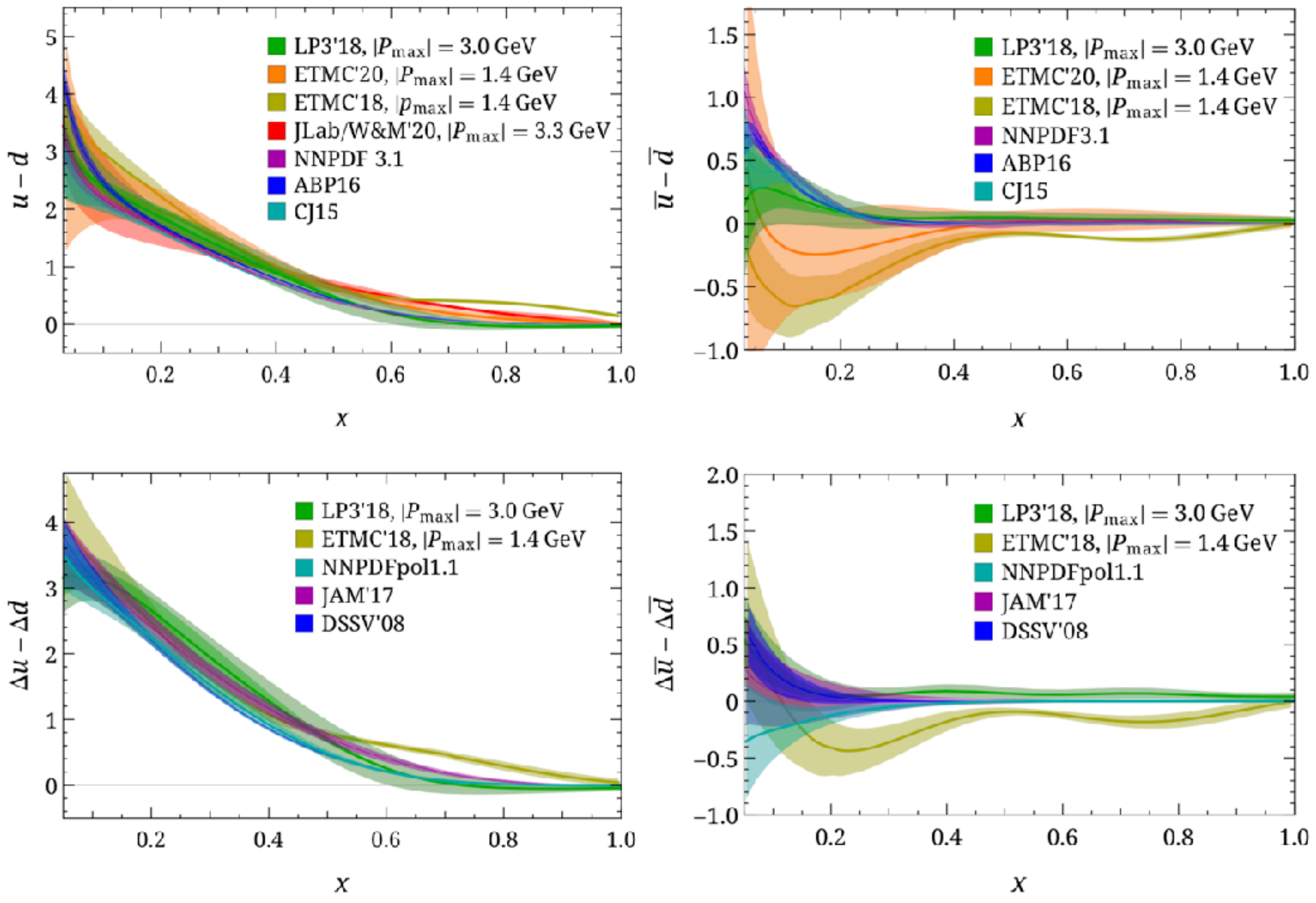
- Transverse Momentum dependent PDF



Valence quark PDF

$$f(x,\mu) \equiv \int dy K(\frac{x}{y}, \frac{\mu_R}{yP_z}, \frac{\mu}{yP_z}) \int dz e^{-iyzP_z} \langle P | \bar{s}(z) \Gamma U(z,0) s(0) | P \rangle^R + \mathcal{O}(\frac{1}{P_z^2}) + \mathcal{O}(\alpha_s^2)$$

- Summary plot for the iso-vector part of the valence quark unpolarized/polarized PDF;
- The values with $|P_{\max}| = \mathrm{x.x\ GeV}$ are the Lattice QCD predictions using the LaMET approach.
- It is still very challenging to access the anti-quark distribution precisely and accurately.

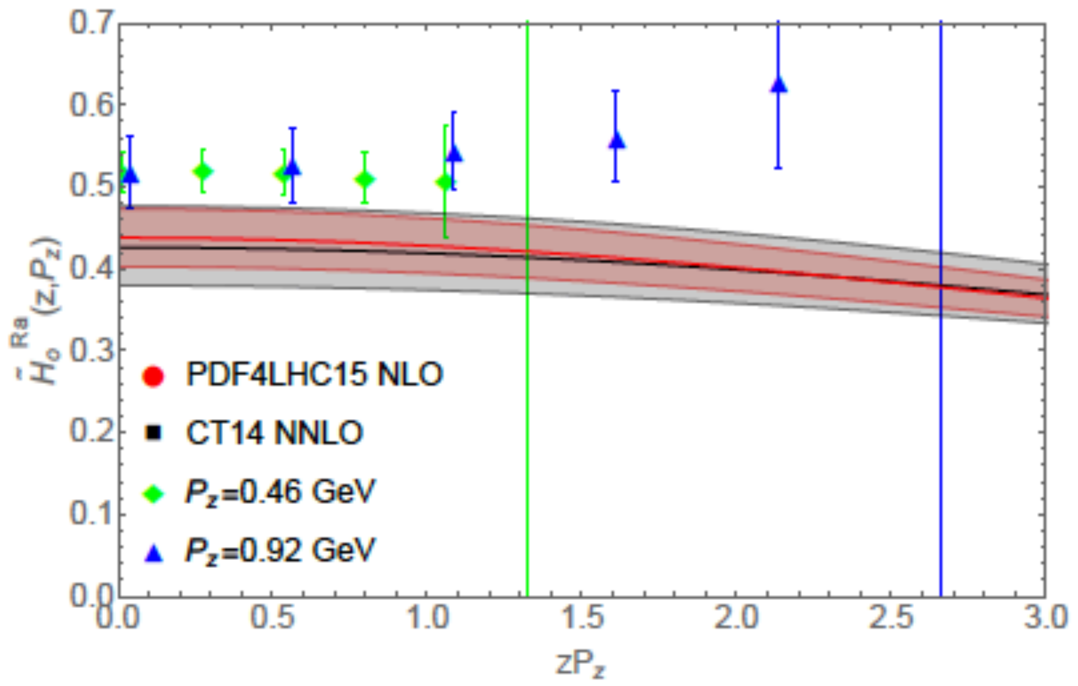


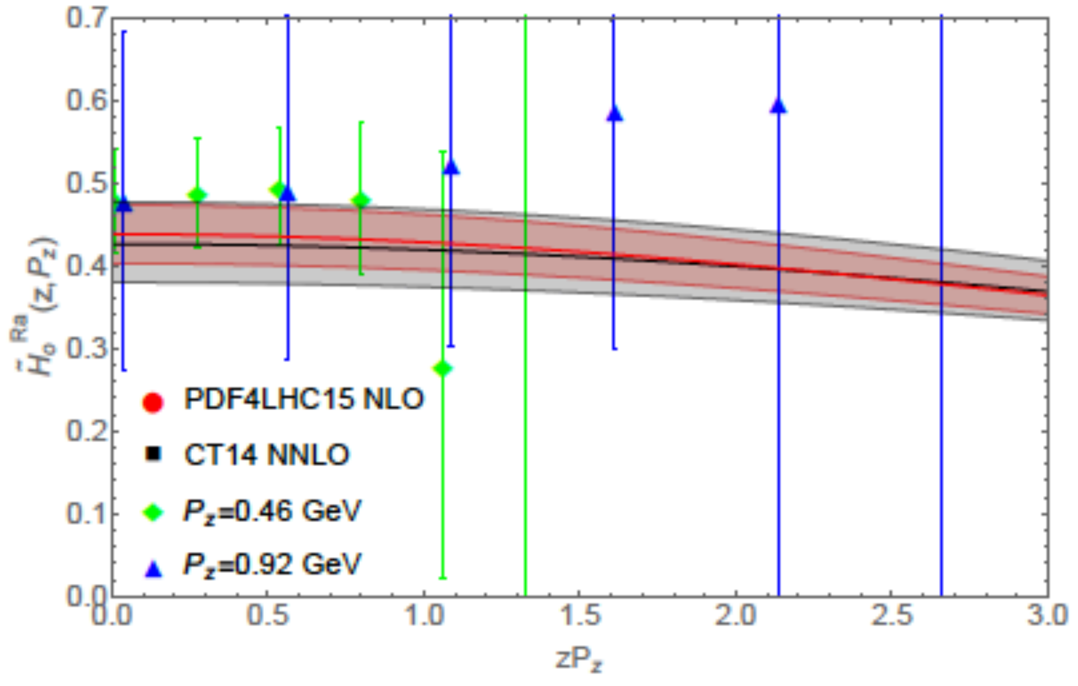
Martha Constantinou, et.al., PPNP 121 (2021) 103908, arXiv:2006.08636

Gluon PDF

$$M_{\mu\alpha,\lambda\beta}(z,P_z) \equiv \langle P | G_{\mu\alpha}(z) U^{adj}(z,0) G_{\lambda\beta}(0) | P \rangle$$

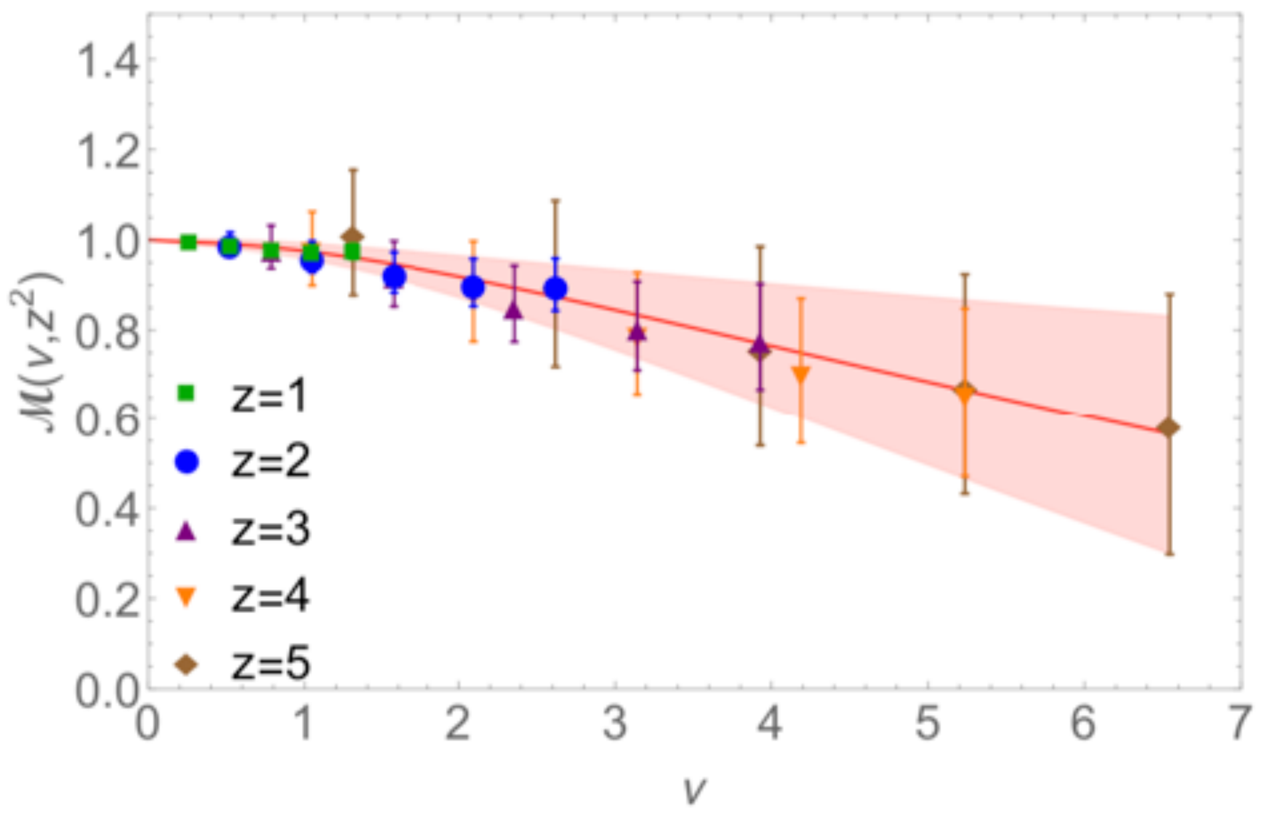
$$\propto \int_{-1}^{1} e^{-ix\nu} x g(x) \mathrm{d}x + \mathcal{O}(z^2),\ \nu \equiv z P_z$$

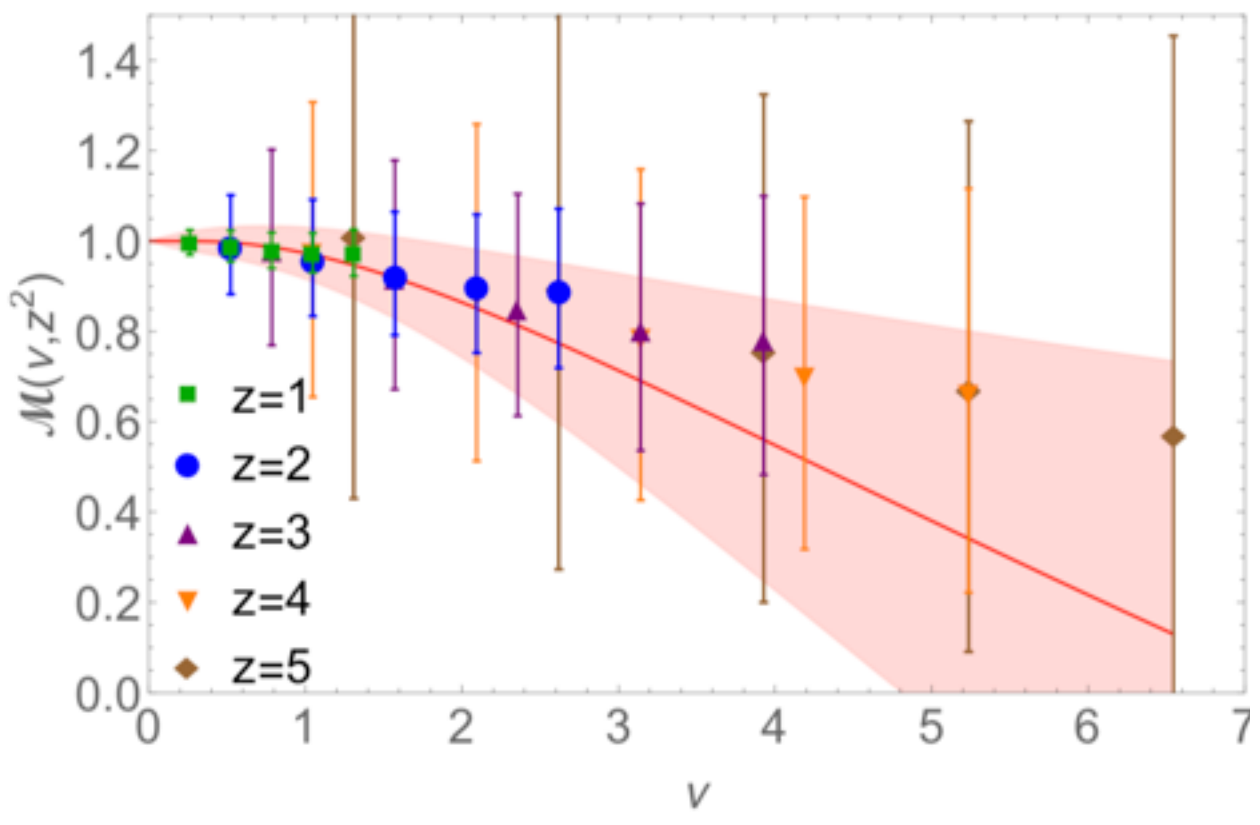




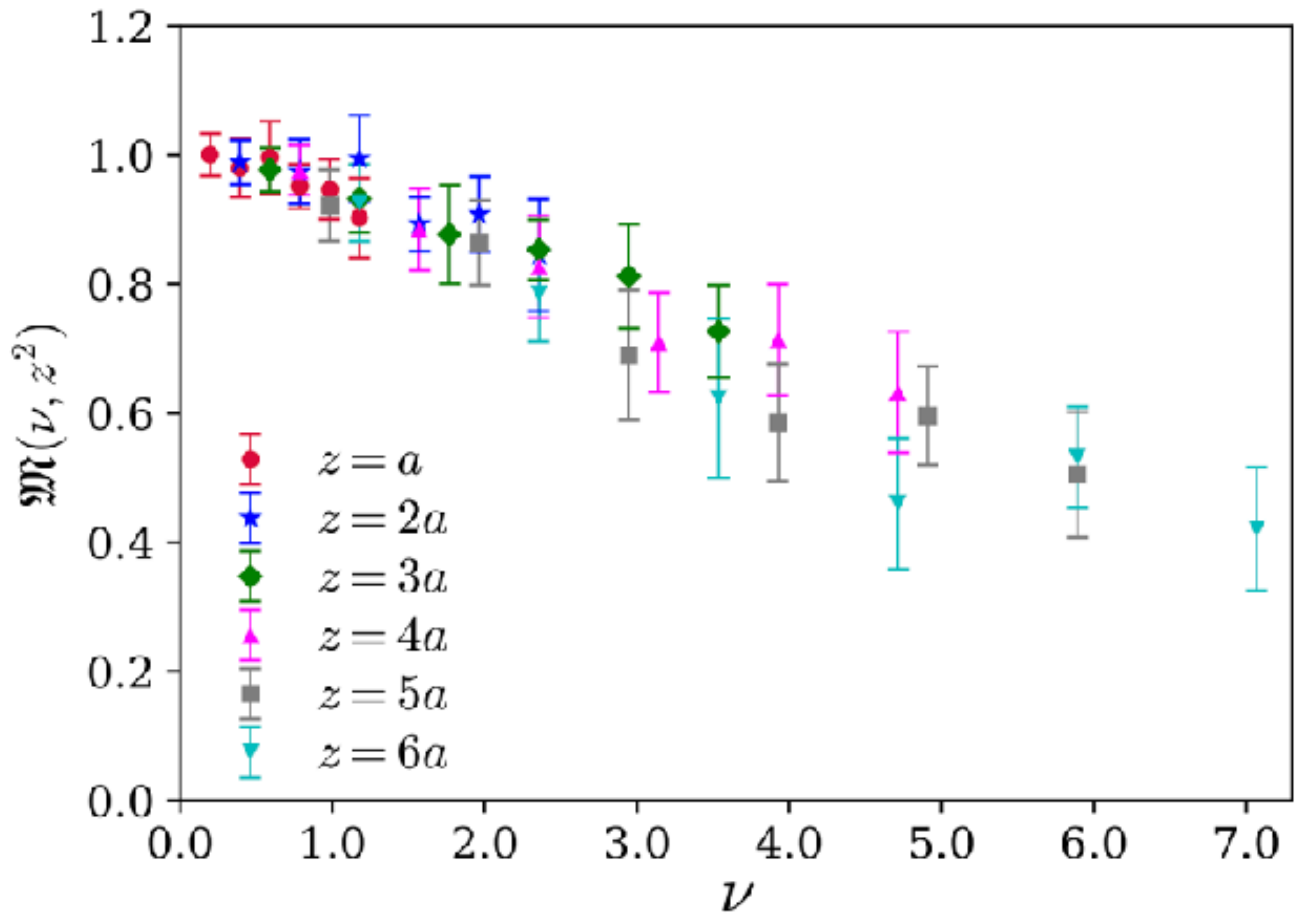
Z. Fan, **YBY**, et.al, PRL121, 242001 (2018), arXiv:1808.02077

- The Fourier transform of the gluon PDF can be extracted from LQCD with proper matching in the large momentum (small z) limit;
- First LQCD attempt was made at 2018;
- With kinds of the improvements, it would be possible to access the entire gluon PDF in the near future.





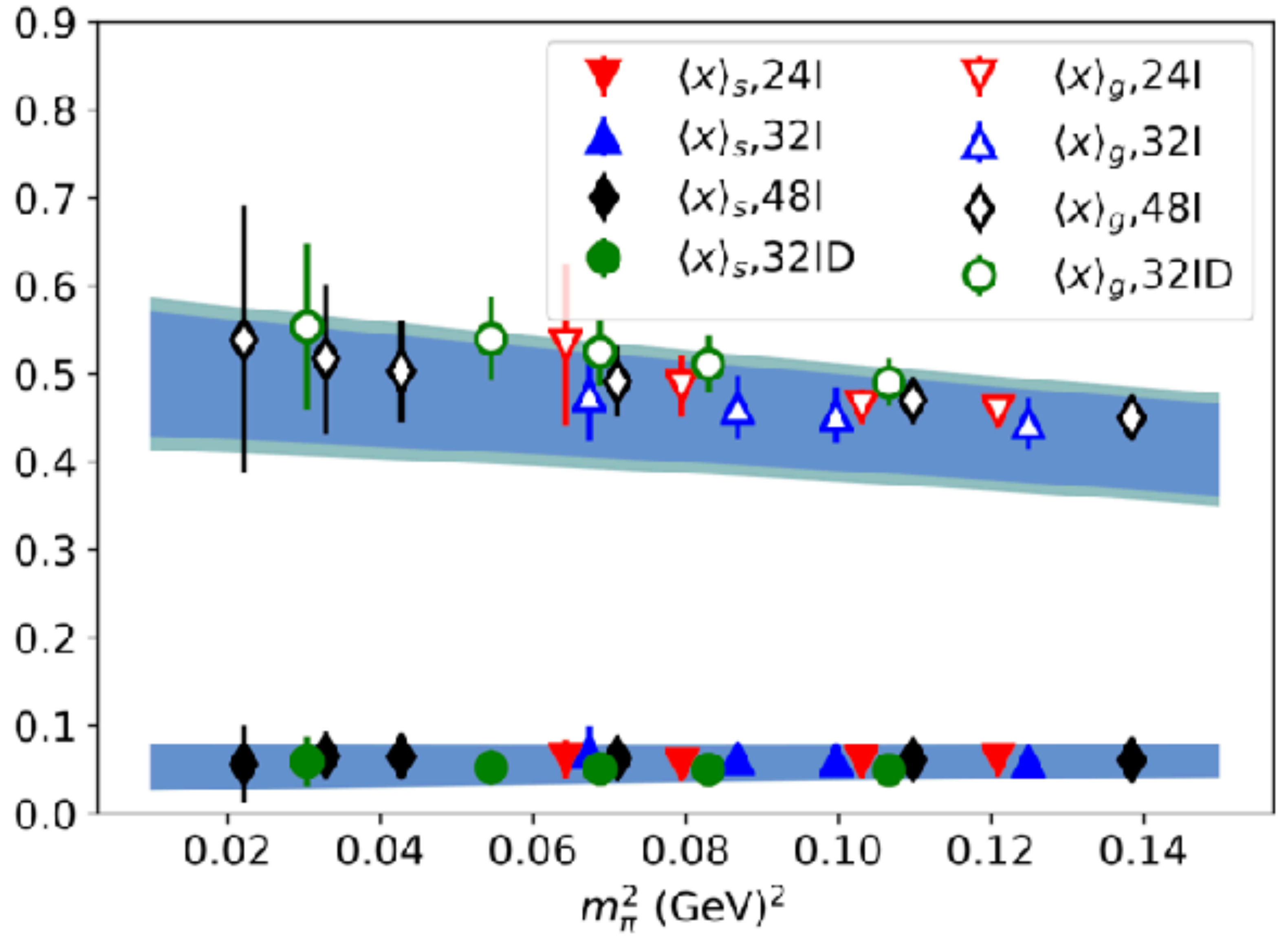
Z. Fan, R. Zhang, H.W. Lin, IJMPA 36(2021)2150080, arXiv:2007.16113



Tanjib Khan, et.al, HadStruc, arXiv: 2107.08960

Strange quark PDF

$$\langle x\rangle_s = \int_0^1 xs(x)\mathrm{d}x,\ \langle x\rangle_g = \int_0^1 xg(x)\mathrm{d}x\,.$$

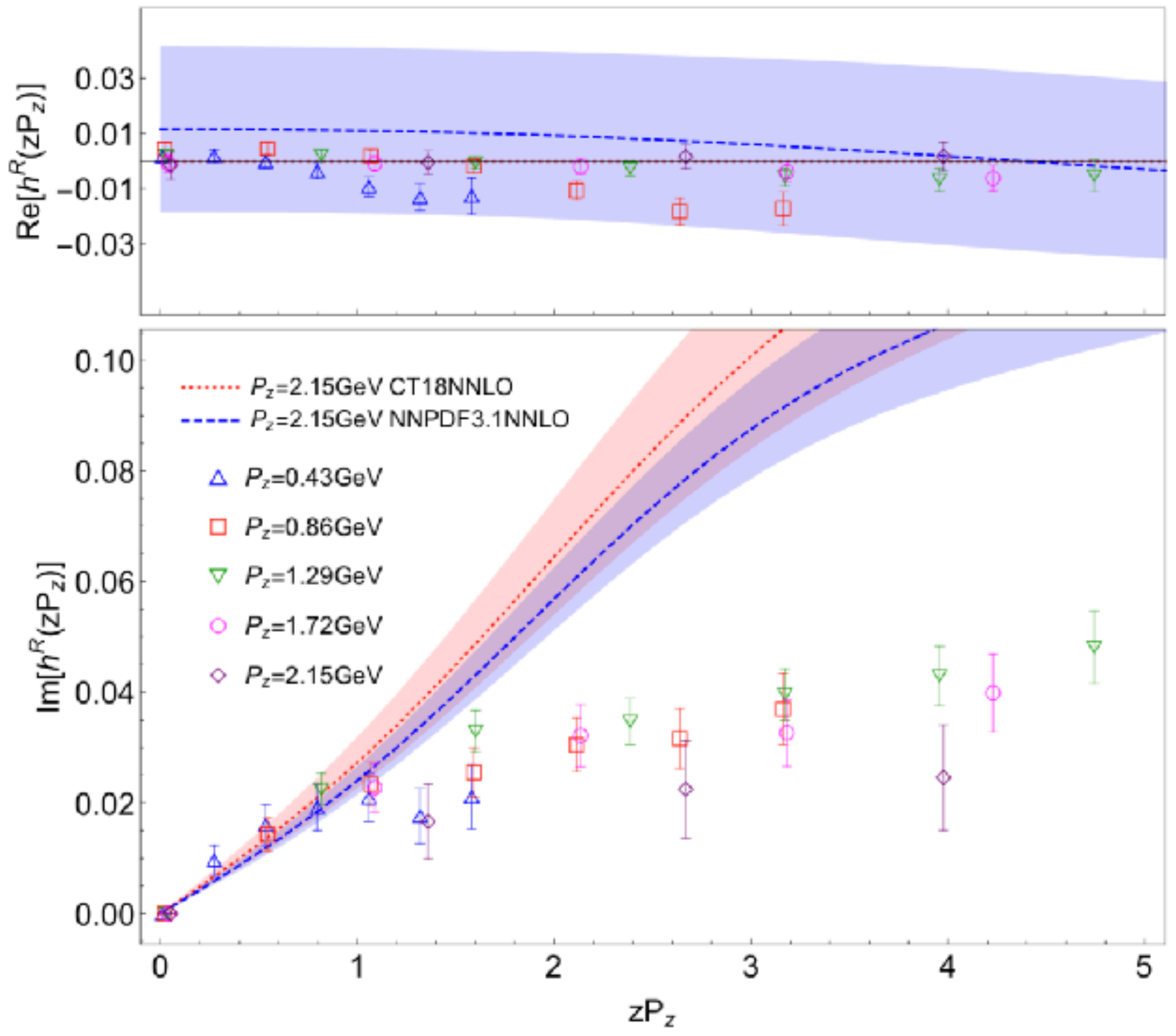


YBY, et.al., χQCD, PRL121 (2018) 212001

Editor's suggestion and Viewpoint

- The first attempt on the distribution is made last year.
- Strange and anti-strange quark seem to be symmetric (vanishing real part);
- The second-moment from the imaginary part is consistent with the direct moment calculation;
- Information beyond second moment is still absent.

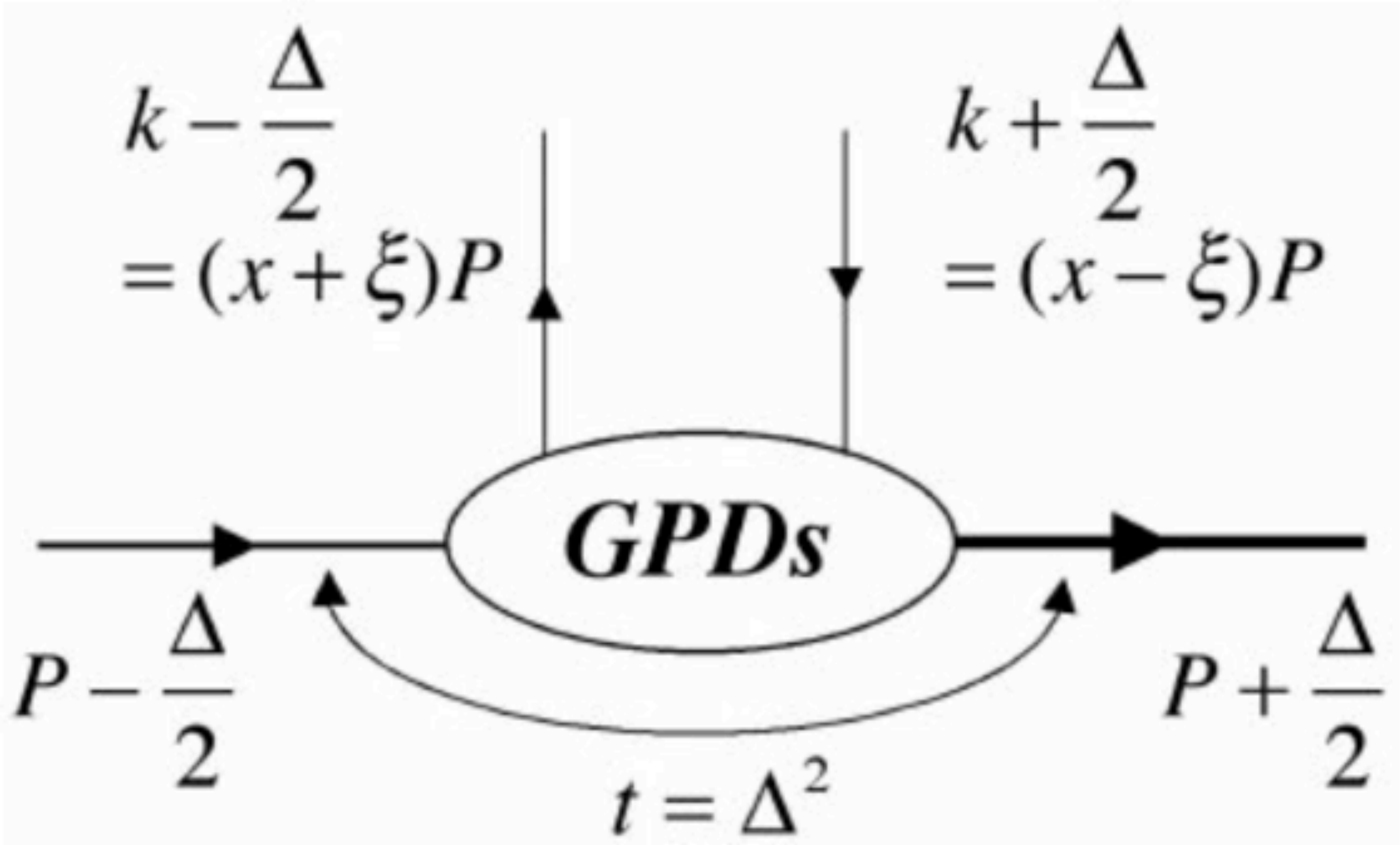
$$h^R(z,P_z) \equiv \langle P|\bar{s}(z)\gamma_t U(z,0)s(0)|P\rangle \propto \int_{-1}^{1} e^{-ixv}s(x)\mathrm{d}x + \mathcal{O}(\frac{1}{P_z^2})$$

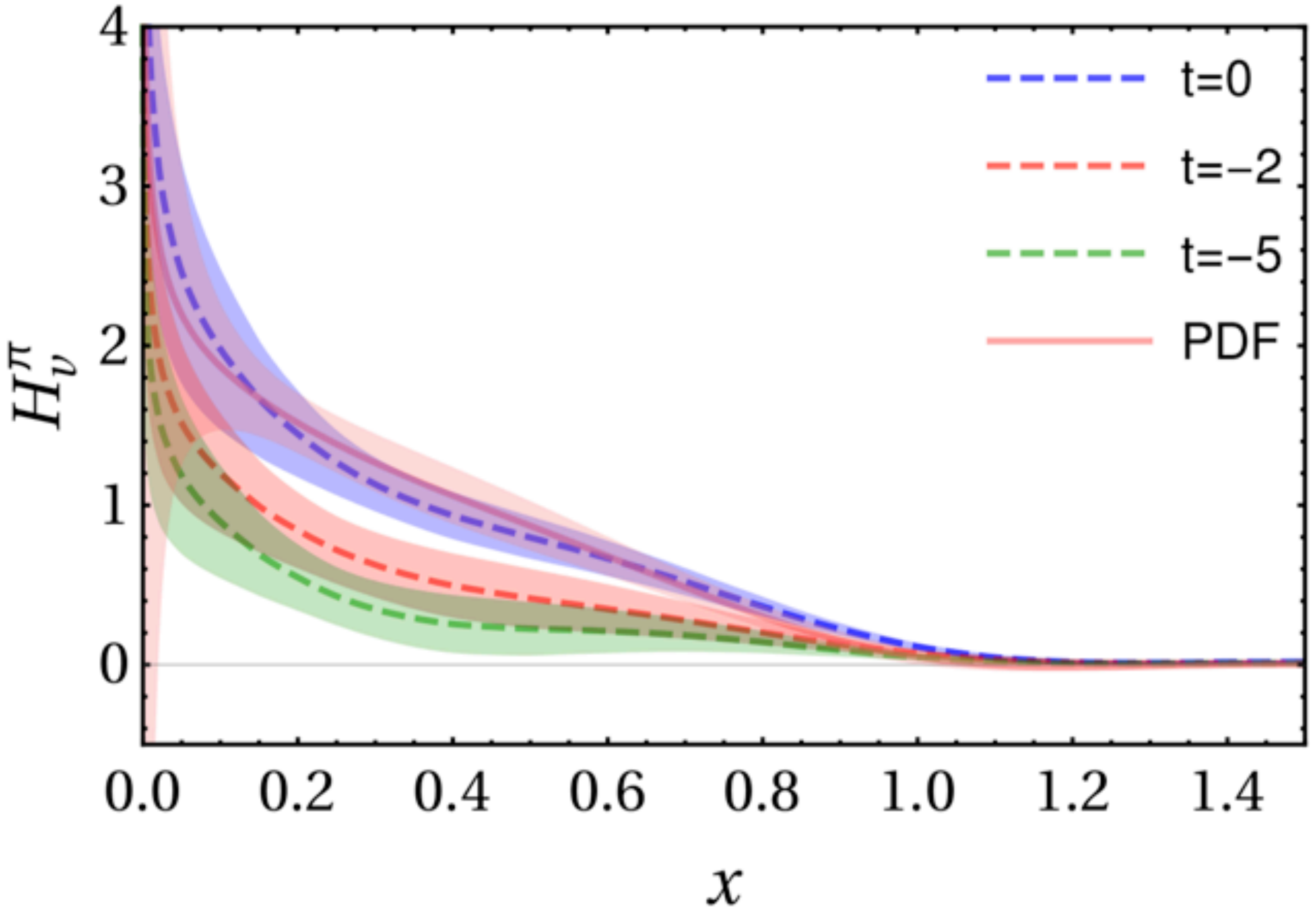


R. Zhang, et,al, PRD 102 (2020) 094519, arXiv:2005.13955

H. Lin, arXiv:2110.06779

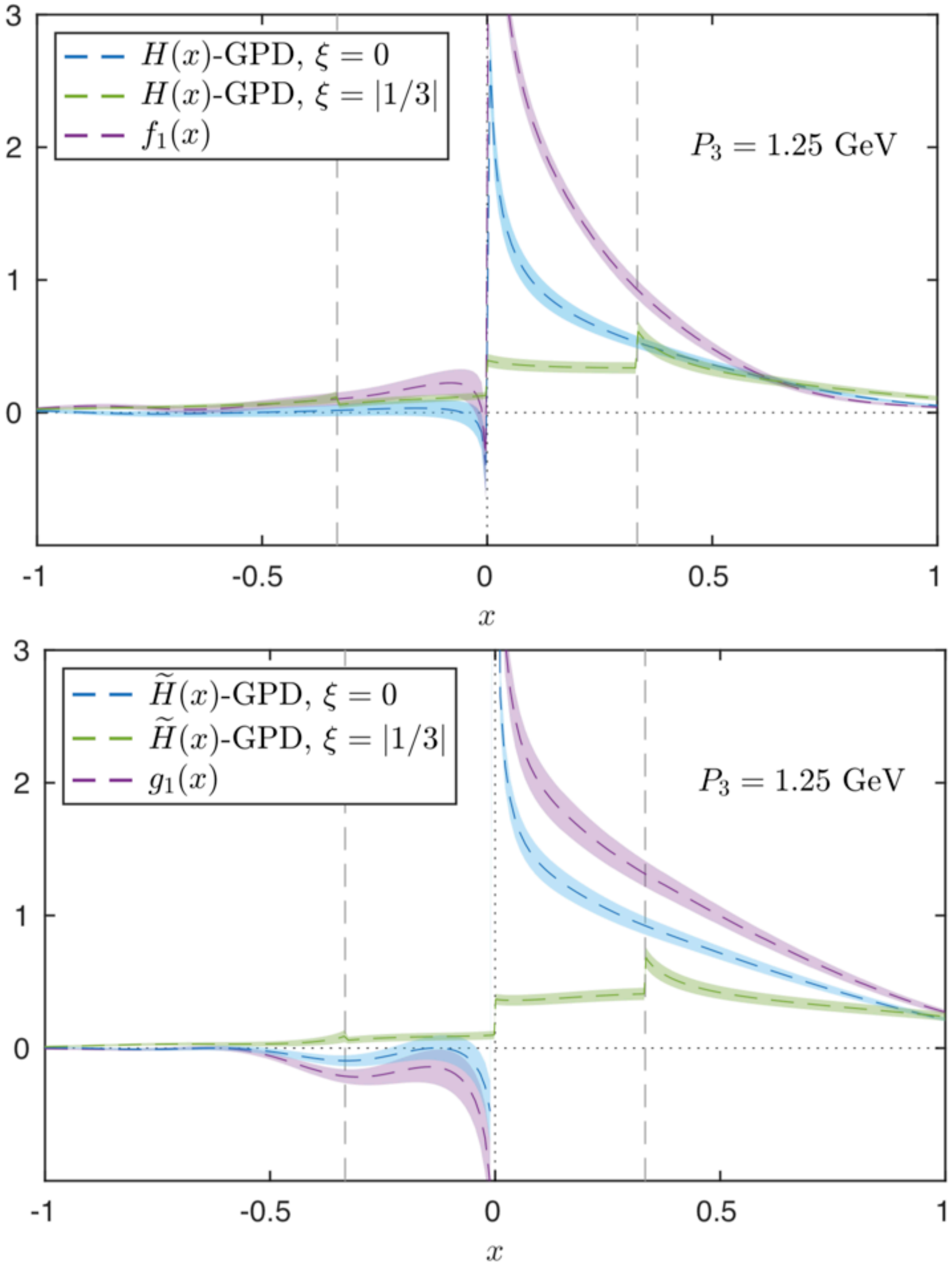
GPD





J. Chen, et,al, NPB 952 (2020) 114940, 1904.12376

- Generalized parton distribution (GPD) describes the parton distribution with momentum transfer;
- First LQCD investigation is done at 2019 for the zero-Skewness pion valence quark GPD at -t={0, 0.3, 0.8} GeV^2;
- ETMC managed to calculate the nucleon GPD with non-zero Skewness with -t~0.7 GeV^2.



C. Alexandrou, et,al, ETMC, PRL 125 (2020) 262001, 2008.10573

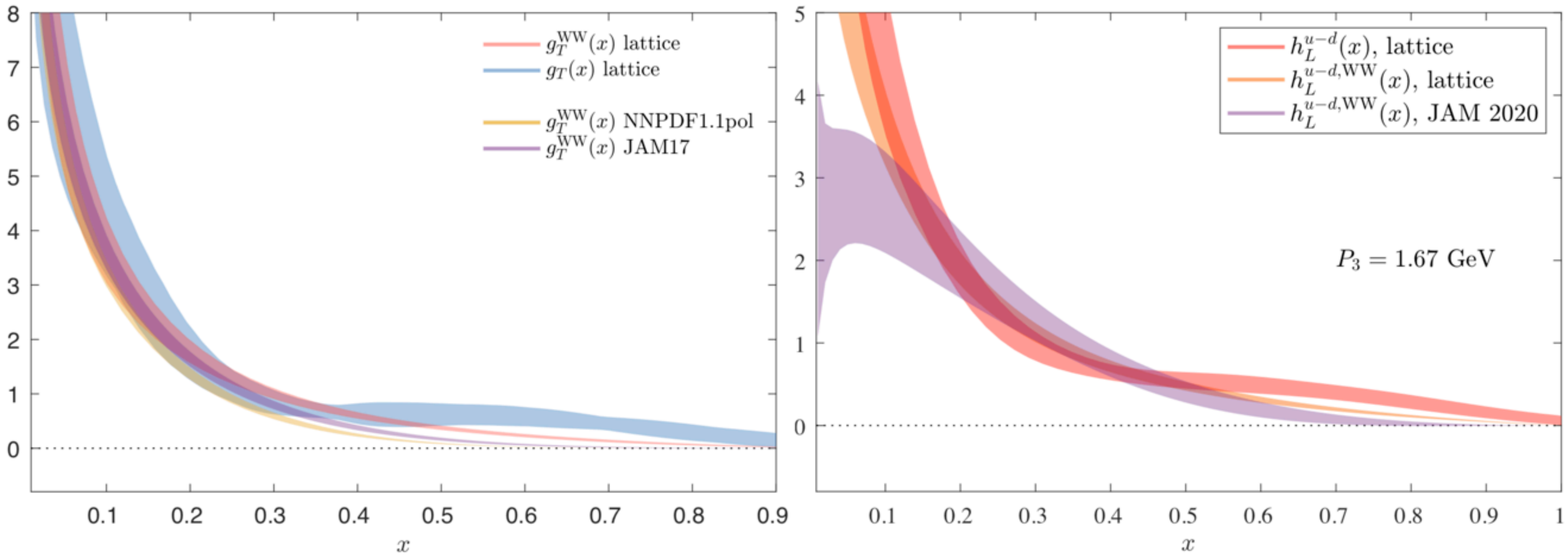
Twist-3 PDF

- First twist-3 nucleon PDF result from ETMC, as a parallel talk at Lattice2021;
- The results are consistent with the Wandzura-Wilczek approximation using the twist-2 PDF in most of the x-region.

$$M_\Gamma(z, P_z) \equiv \langle P \,|\, \bar{q}(z)\Gamma U(z,0) q(0) \,|\, P\rangle_{u/d}$$

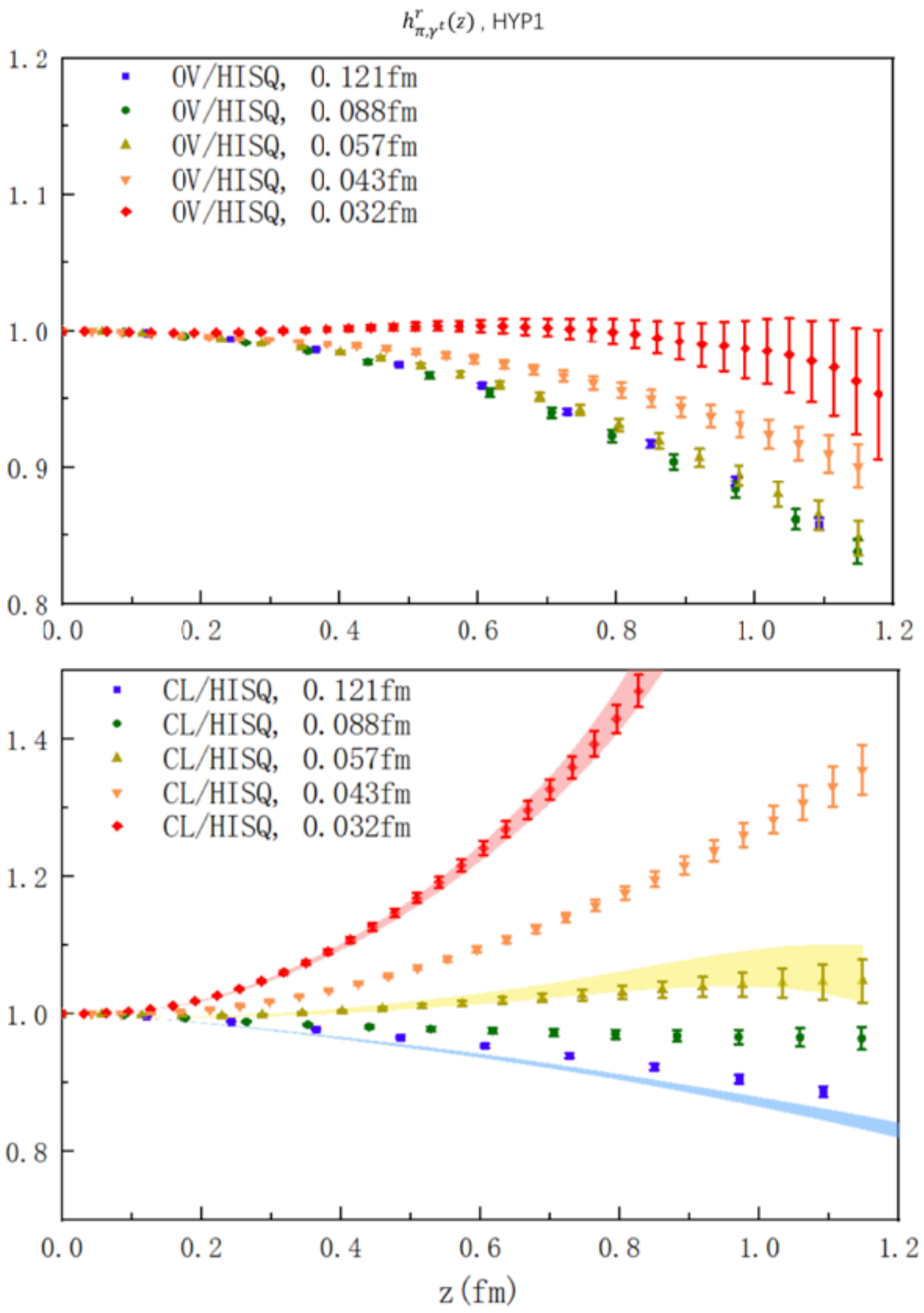
$$F_{g_T}(z, P_3, \mu) = -i\frac{E}{m} Z_{\gamma^5\gamma^j}(z,\mu)\, \mathcal{M}_{\gamma^5\gamma^j}(P_3, z)\,,$$

$$F_{h_L}(z, P_3\mu) = -i\,\epsilon_{jk30}\,\frac{E}{m} Z_{\sigma_{jk}}(z,\mu)\, M_{\sigma_{jk}}(P_3, z)\,.$$



C. Alexandrou, et,al, ETMC, 2111.03226

Renormalization

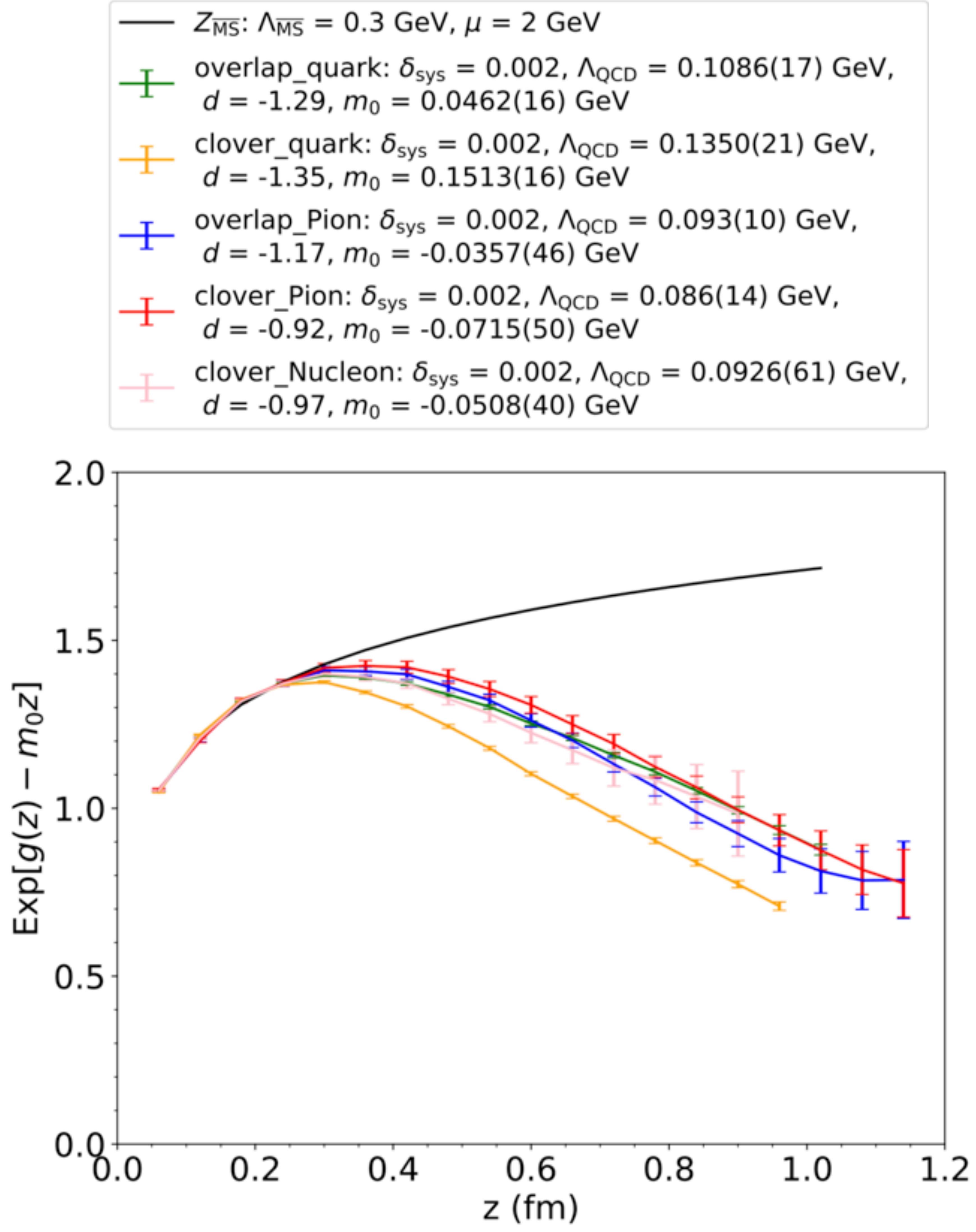


- Despite the success, the current renormalization scheme fails at small lattice spacing, especially the Wilson-like fermion used in the above calculations.

K. Zhang, et,al, χQCD, PRD 104 (2021) 074501, arXiv: 2012.05448

- A new renormalization strategy has been proposed to understand the linear divergence and renormalize the lattice result properly.

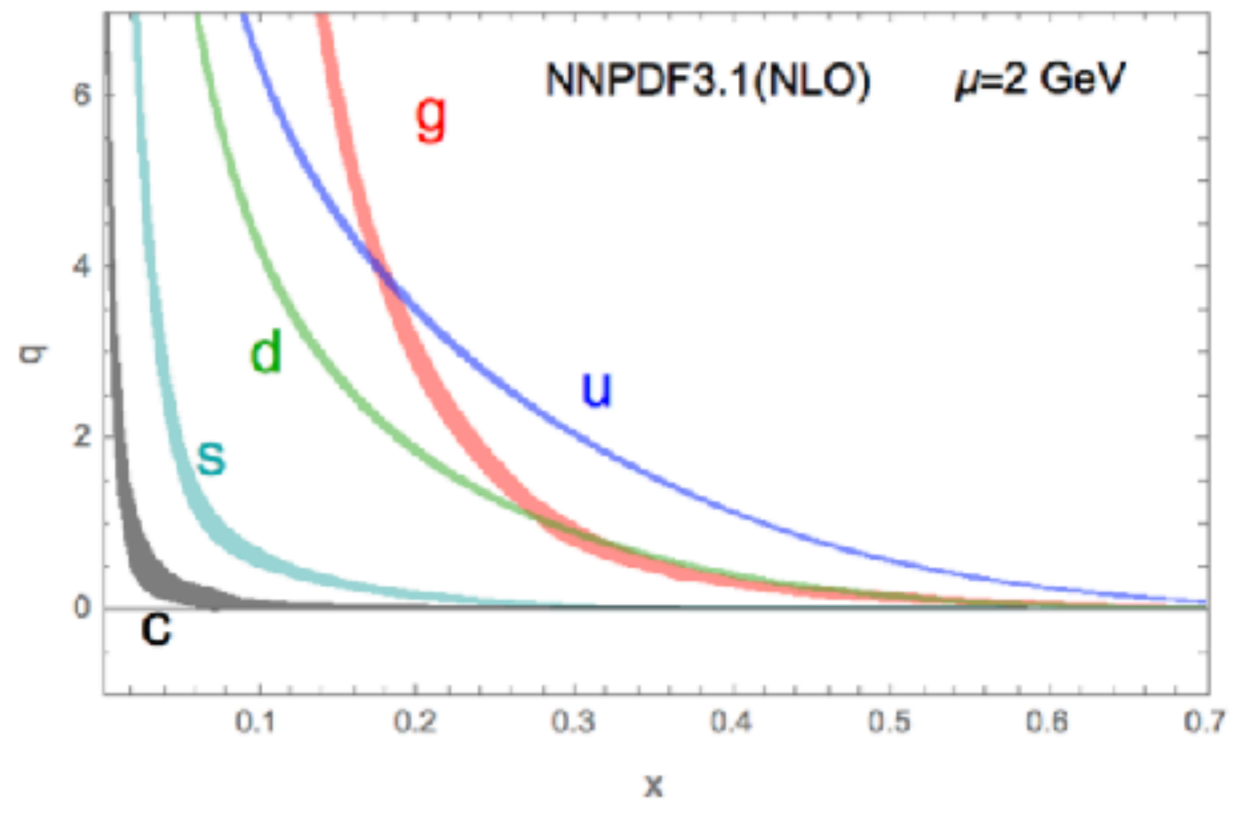
Y. Huo, et,al, LPC, NPE (2021) 115443, arXiv: 2103.02965



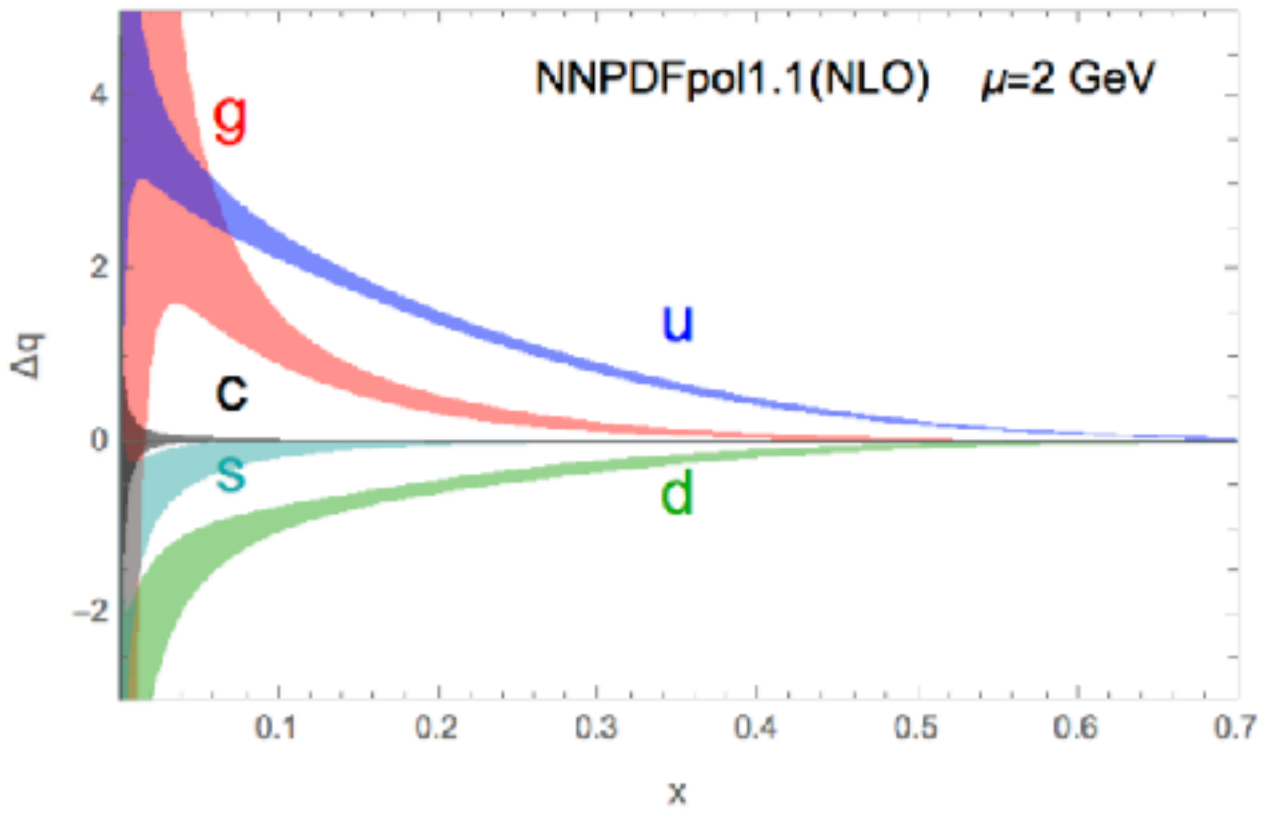
$$\ln \mathcal{M}(z,a) = \frac{kz}{a\ln[a\Lambda_{QCD}]} + g(z) + f_{1,2}(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{QCD})]}{\ln[\mu/\Lambda_{QCD}]}\right] + \ln\left[1+\frac{d}{\ln(a\Lambda_{QCD})}\right]$$

Outline

R. D. Ball, et al. (NNPDF), EPJC77, 663 (2017), 1706.00428

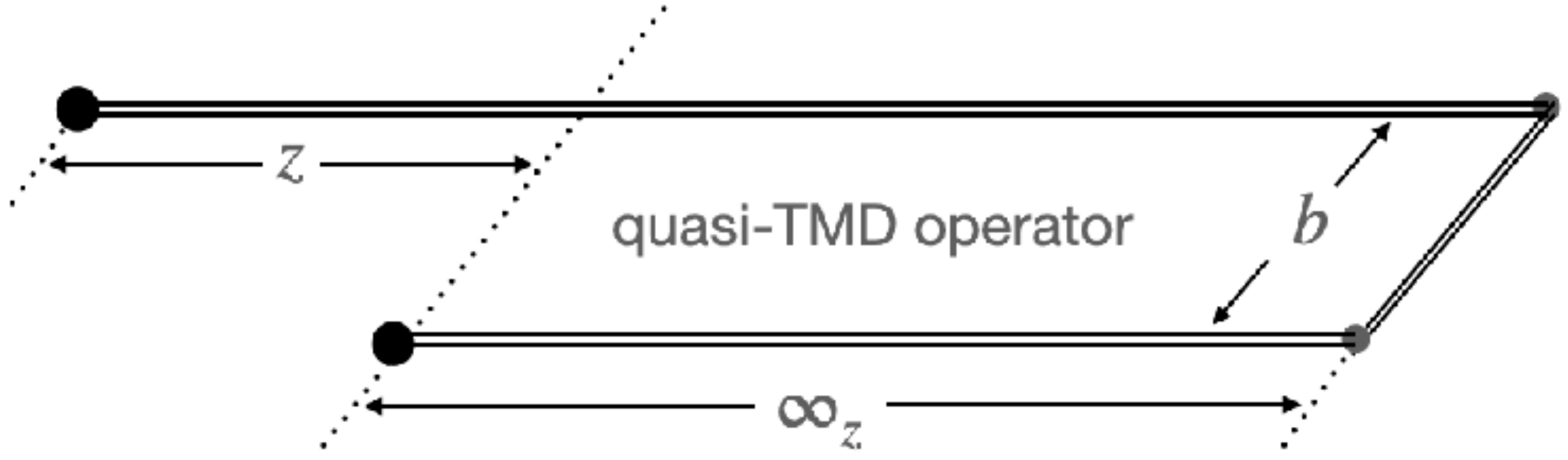


E. R. Nocera, et.al. (NNPDF), NPB887, 276 (2014), 1406.5539



- Colinear PDF

- **Transverse Momentum dependent PDF**



Lattice Parton Collaboration

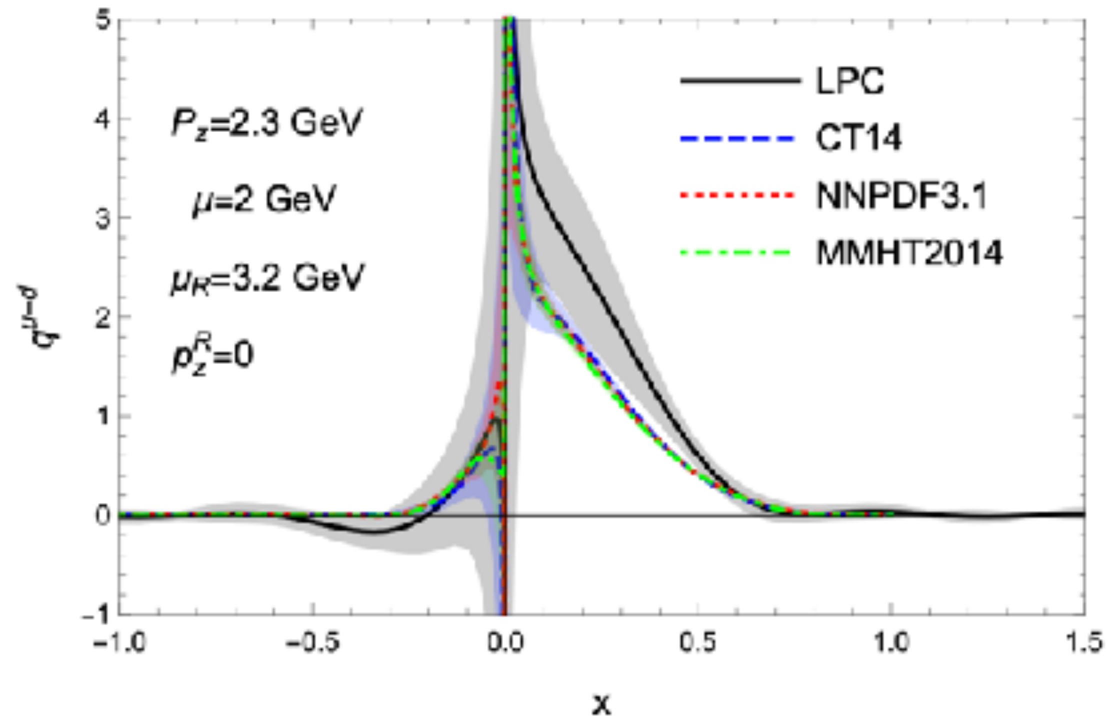
- Understanding partonic structure of hadrons from lattice QCD

- Like first principle predictions on PDF, Form factor, **TMD-PDF**……

- Permanent Members:
  - Xiangdong Ji (SJTU)
  - Peng Sun (NJNU)
  - Andreas Schaefer (U. Reg.)
  - Wei Wang (SJTU)
  - Yi-Bo Yang (ITP/CAS, spokesperson)
  - Jian-hui Zhang (BNU)

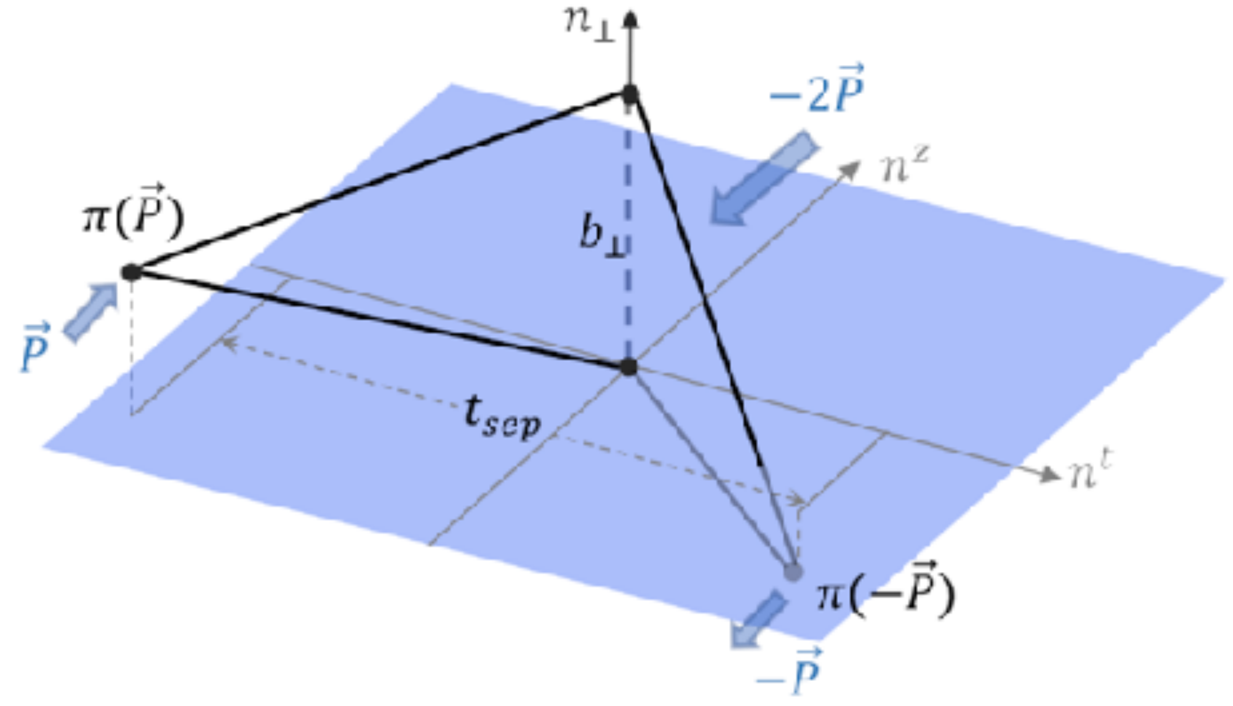
- New Members last year:
  - Long-Cheng Gui (HNNU)
  - Jian liang (SCNU)
  - Liuming Liu (IMP/CAS)
  - Xiao-Nu Xiong (CSU)
  - Jun Hua (SCNU)
  - Qi-An Zhang (BHU)

- Post-doc and students
  - Min-Huan Chu (SJTU)
  - Yuan-Yuan Li (NJNU)
  - Yu-Jie Pan (NJNU)
  - Maximilian Schlemmer (U. Reg.)
  - Hai-Tao Shu (U. Reg.)
  - Yu-Shan Su (UMA)
  - Lisa Walter (U. Reg.)
  - Fei Yao (BNU)
  - Kuan Zhang (ITP/CAS)
  - ……



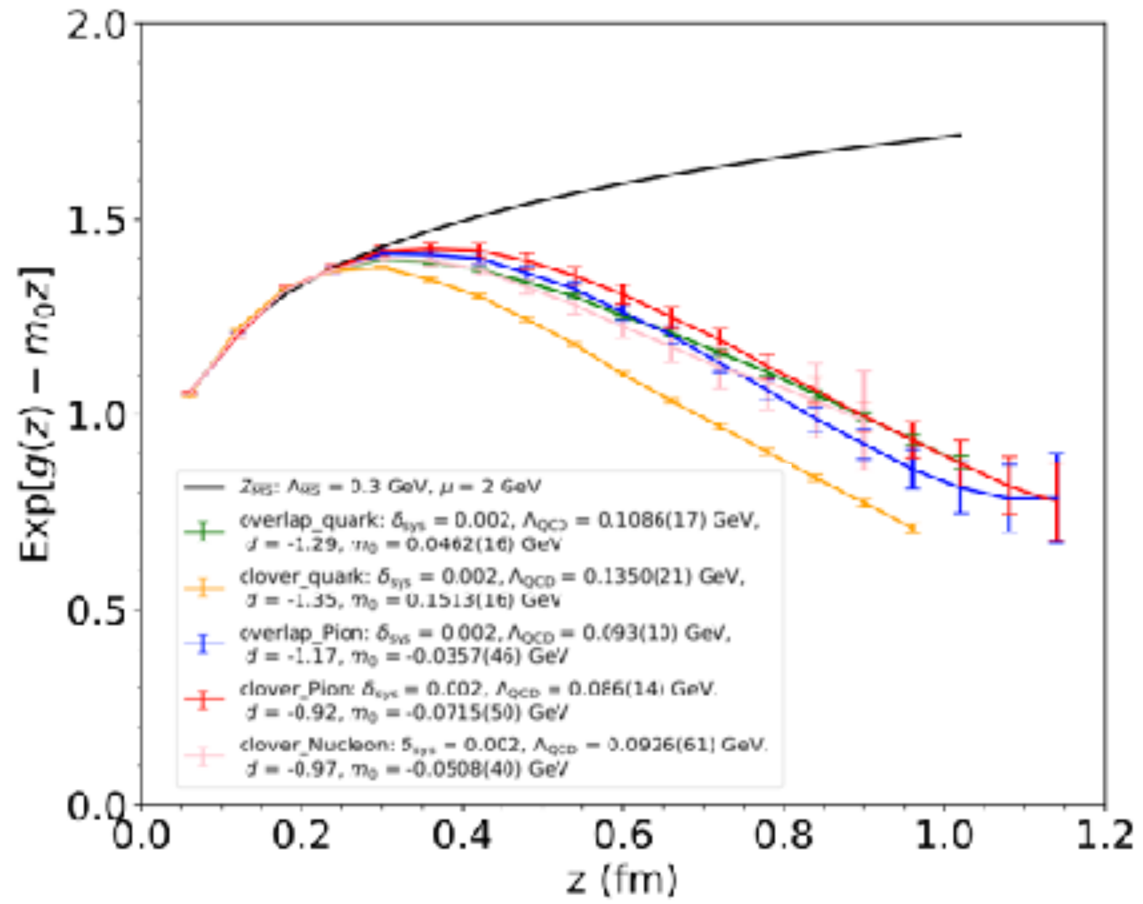
Unpolarized nucleon PDF

Y. Liu, et.al., LPC, PRD101(2020)034020



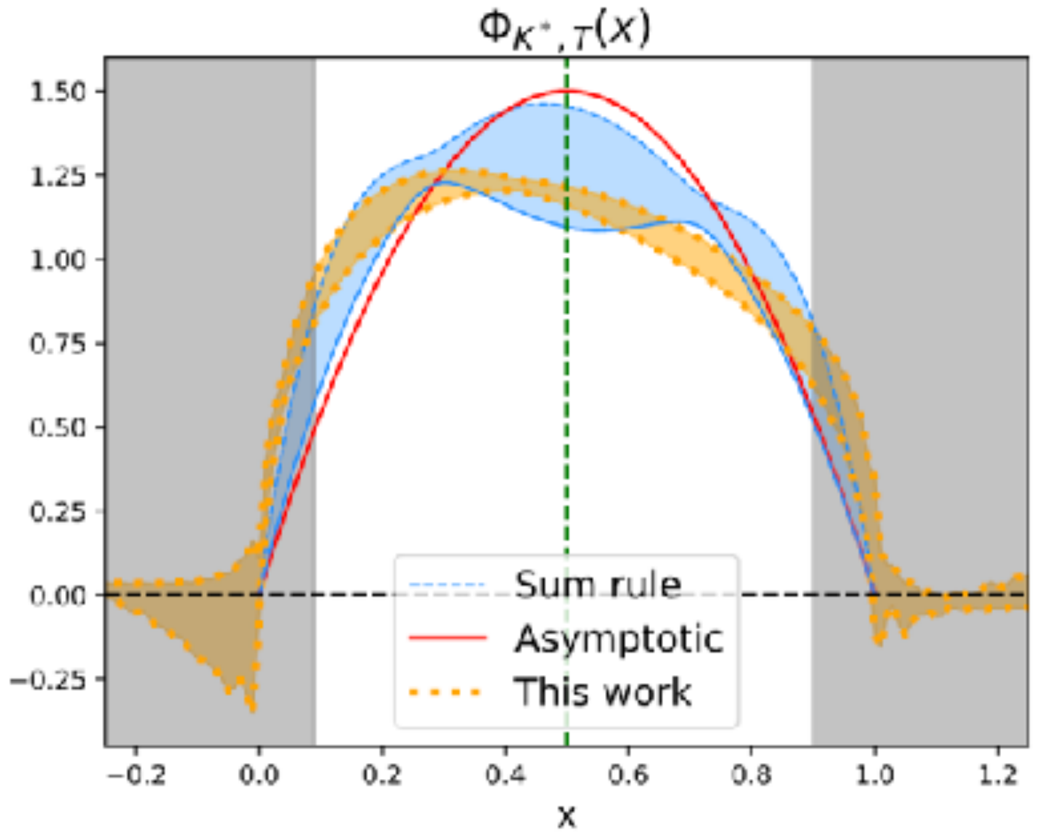
Soft function

Q.-A. Zhang. et.al., LPC, PRL125(2020)192001



Self renormalization

Y.-K. Huo. et.al., LPC, NPB969(2021)115443



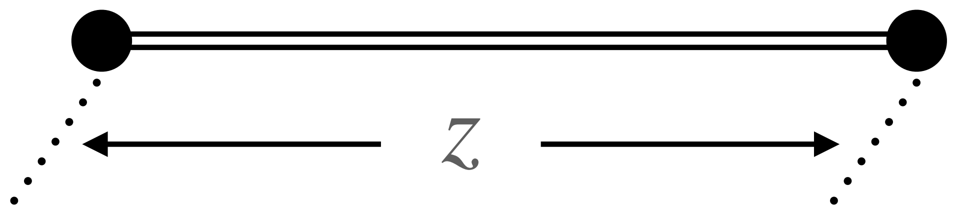
Vector meson DA

J. Hua. et.al., LPC, PRL127(2021)062002

Linear divergence

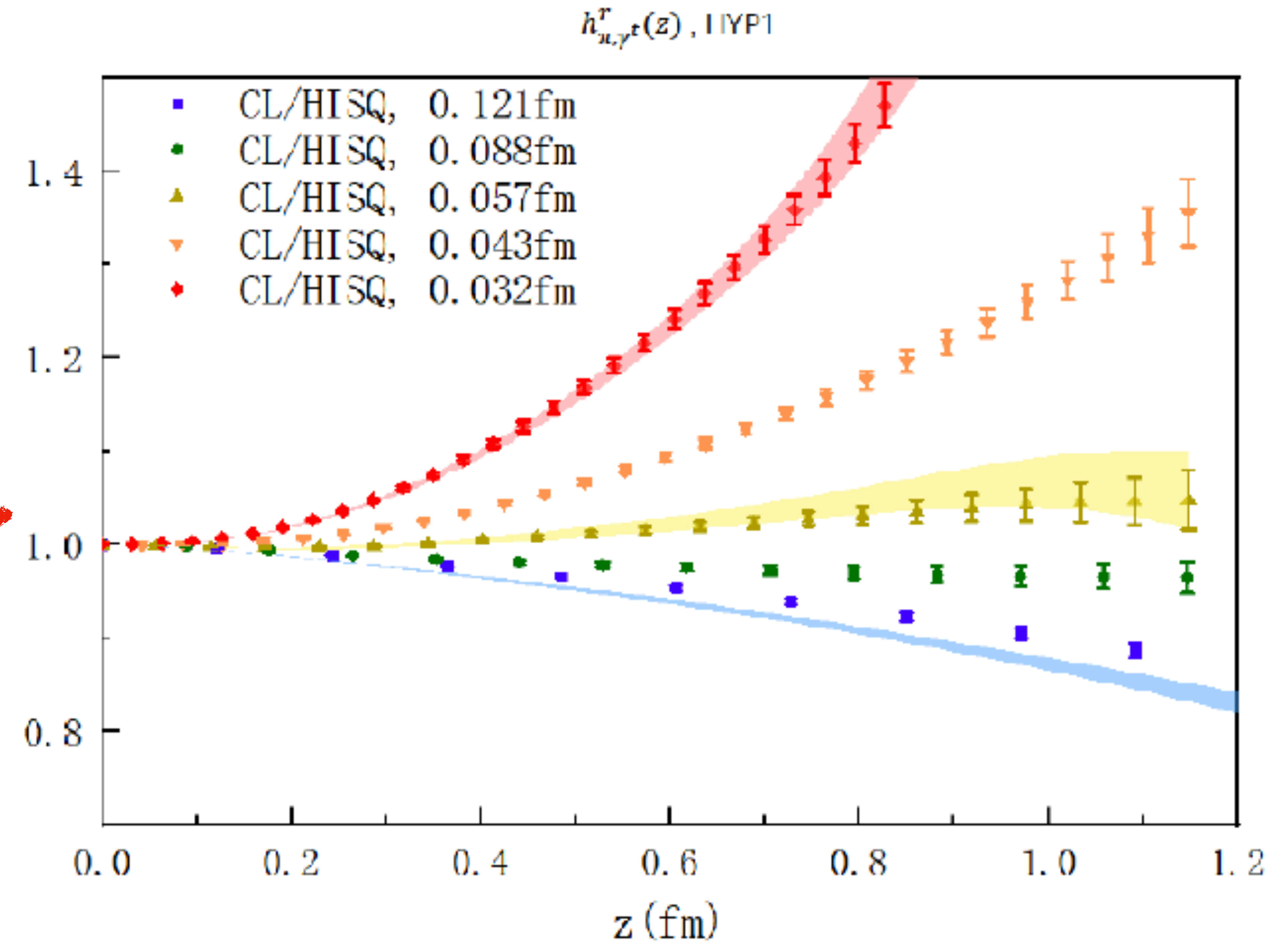
straight link case

$$O_{\gamma_t}(z) = \bar{q}(0)\gamma_t U_z(0,z) q(z)$$



quasi-PDF operator

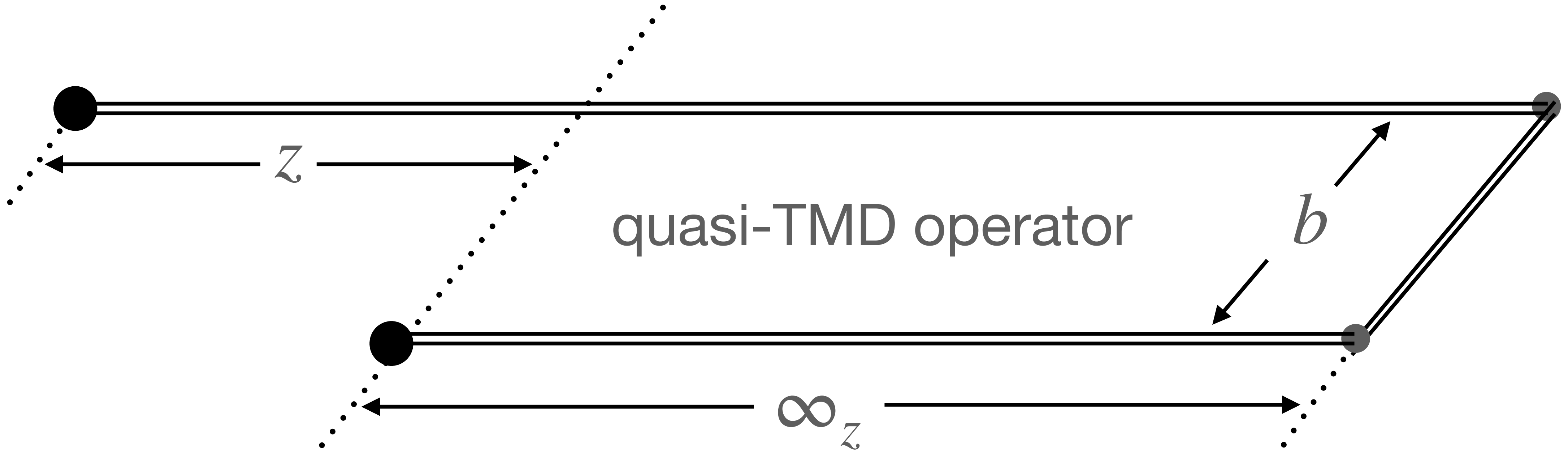
RI/MOM
renormalization



$$\langle \pi | O_{\gamma_t}(z) | \pi \rangle^{\text{RI/MOM}} \equiv \frac{\langle \pi | O_{\gamma_t}(z) | \pi \rangle}{\langle q | O_{\gamma_t}(z) | q \rangle} = \gamma_t \left(1 + \alpha_s \frac{\delta m z}{a} + \ldots \right)$$

K. Zhang, et.al, χQCD, PRD104(2021)074501

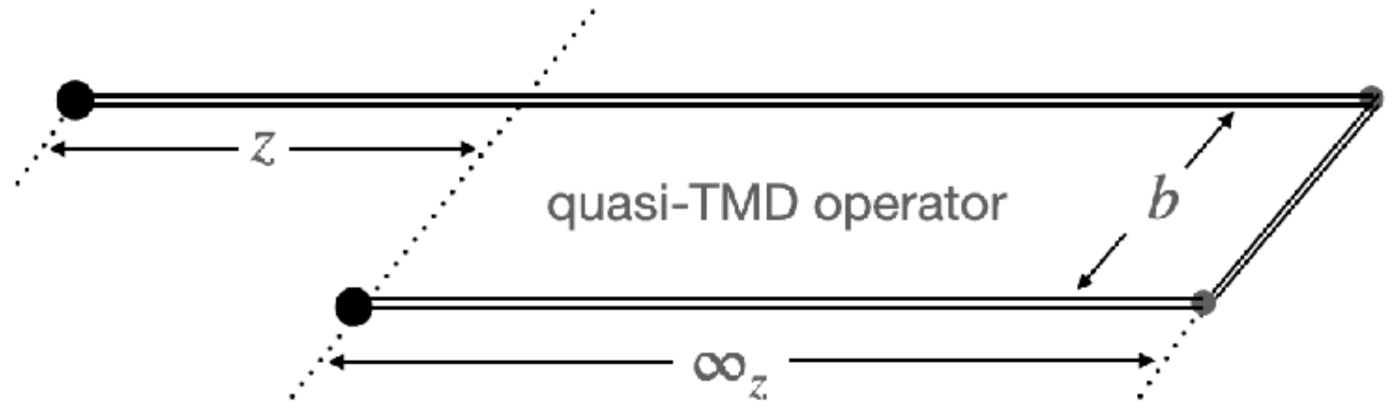
$$O^{\text{TMD}}_{\gamma_t}(z,b) = \bar{q}(0)\gamma_t U_z(0,\infty_z) U_b(\infty_z, \infty_z + b) U_{-z}(\infty_z + b, z) q(z)$$

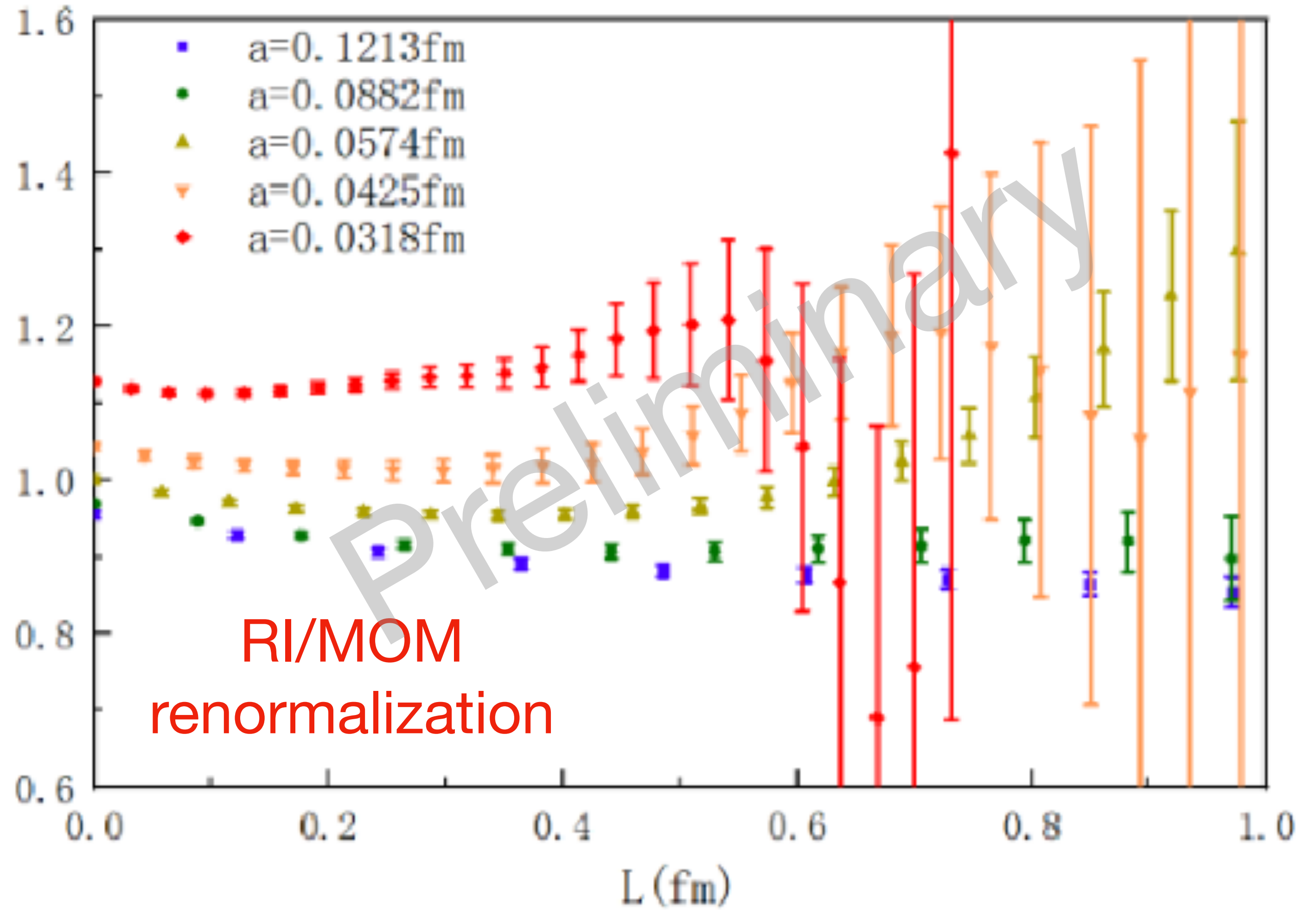


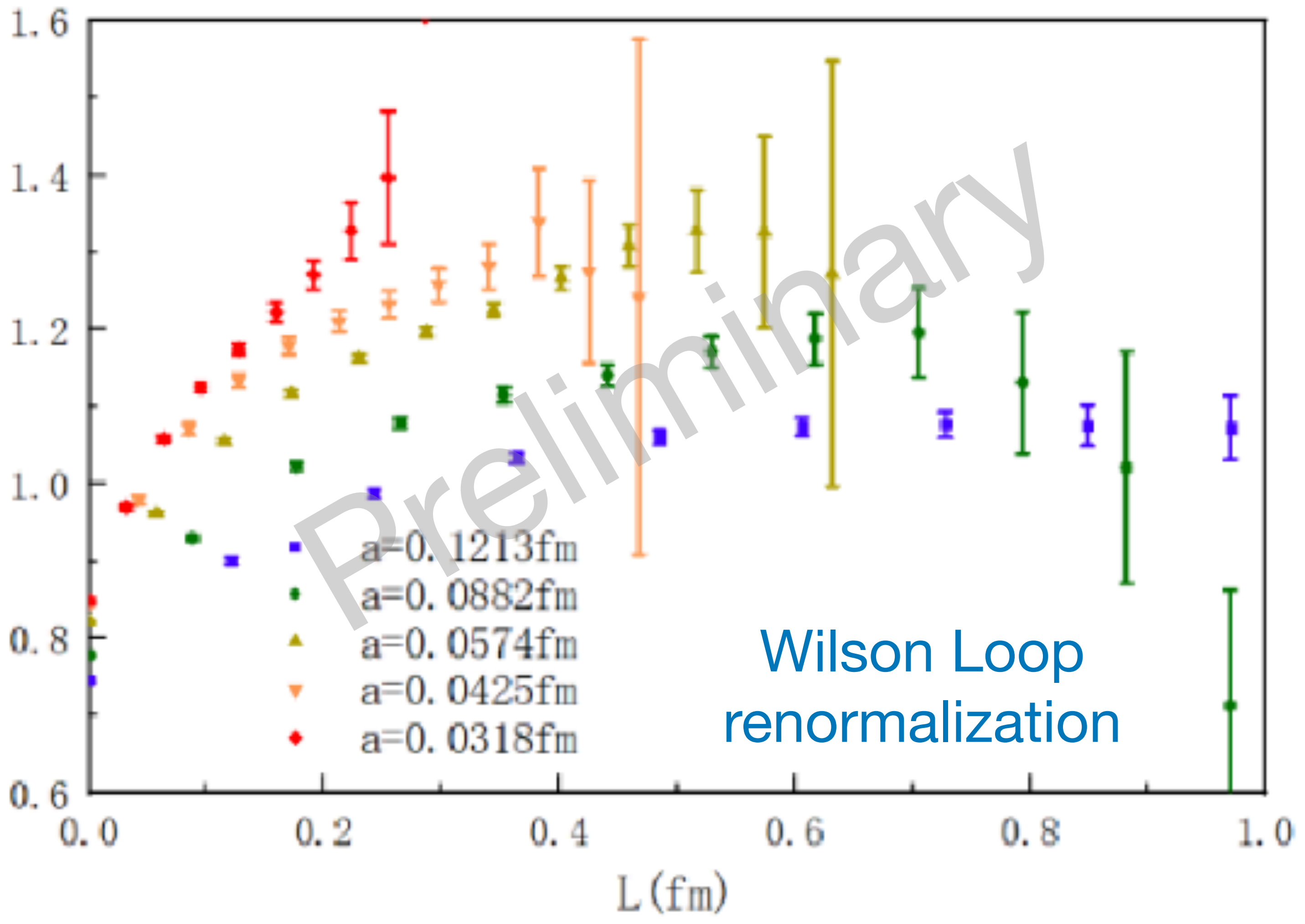
How to renormalize the staple link operator properly?

$$\langle \pi | O^{\text{TMD}}_{\gamma_t}(z,b) | \pi \rangle^{\text{RI/MOM}} = ?$$

Wilson loop renormalization



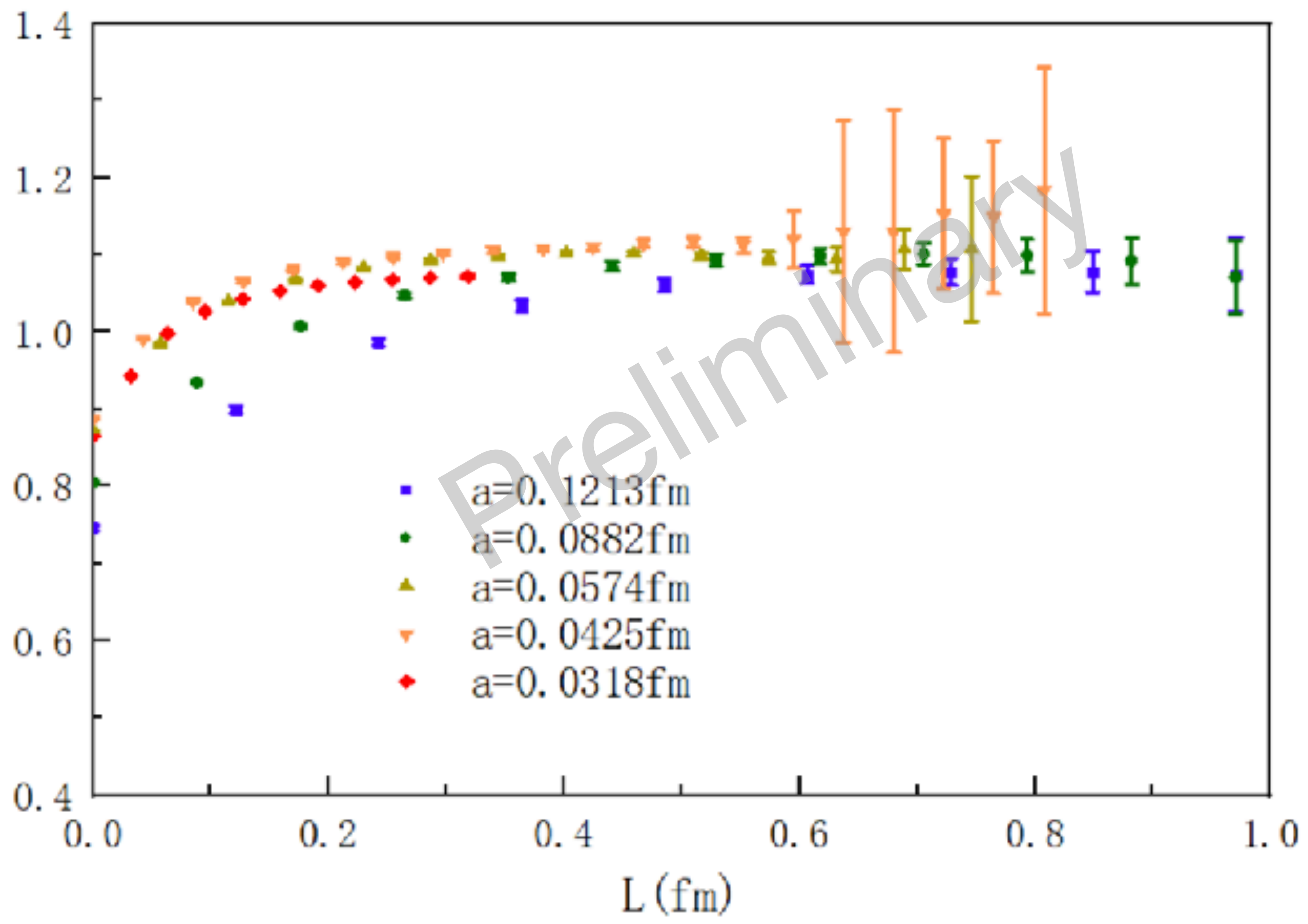




$b = 0.36$ fm, $z = 0.36$ fm

$$\langle \pi | O_{\gamma_t}(z,b) | \pi \rangle^{\text{Loop}} \equiv \frac{\langle \pi | O_{\gamma_t}(z,b) | \pi \rangle}{\sqrt{U_{\text{loop}}(2\infty_z + z, b)}}$$

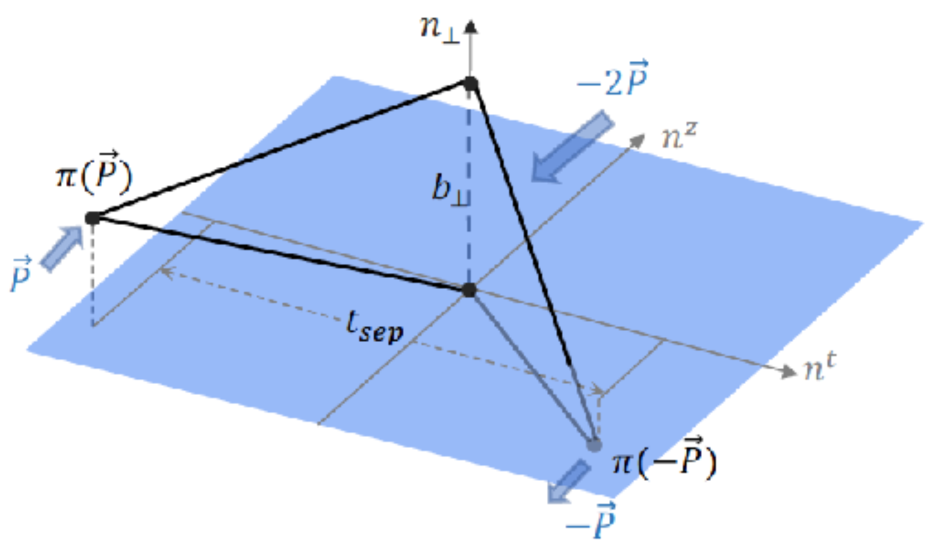
- much better continuum convergence, comparing to the RI/MOM case;
- The residual lattice spacing dependence would be the $\log(z/a)$ and $\log(b/a)$ terms, as it should be.



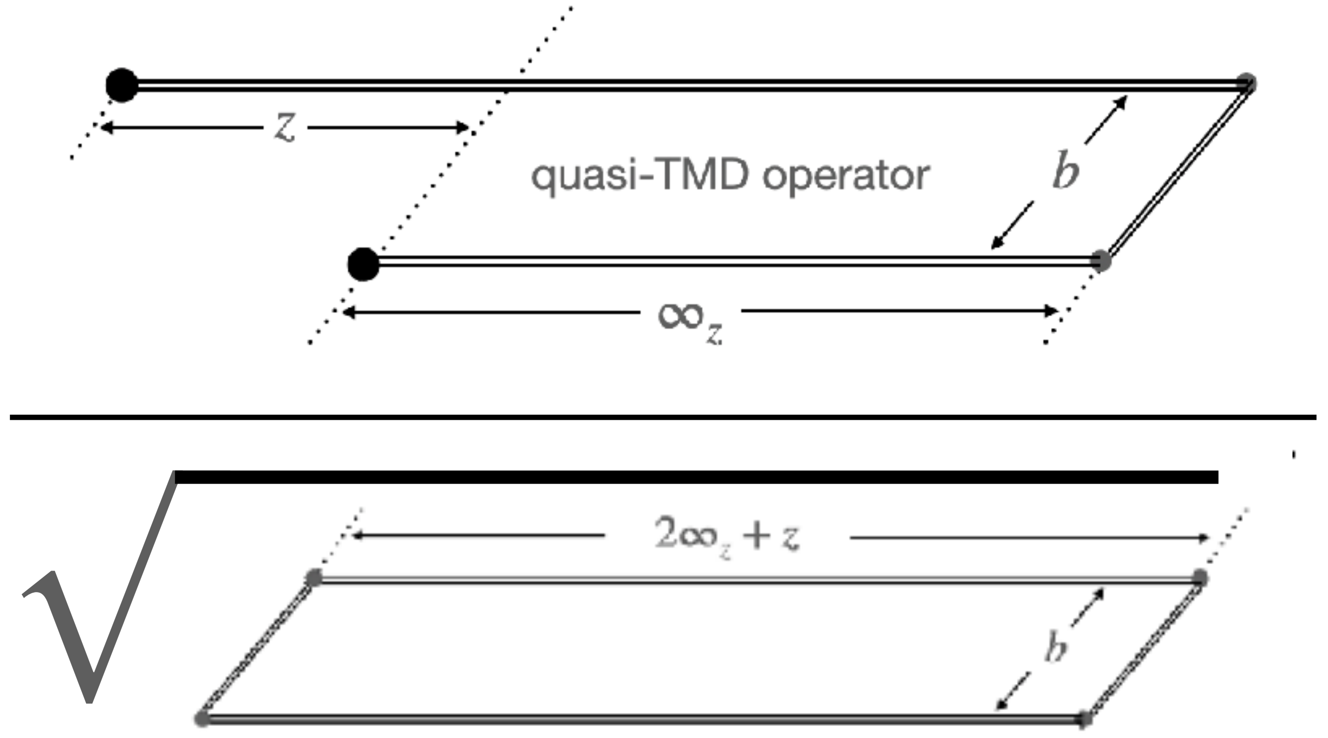
$b = 3a$, $z = 3a$

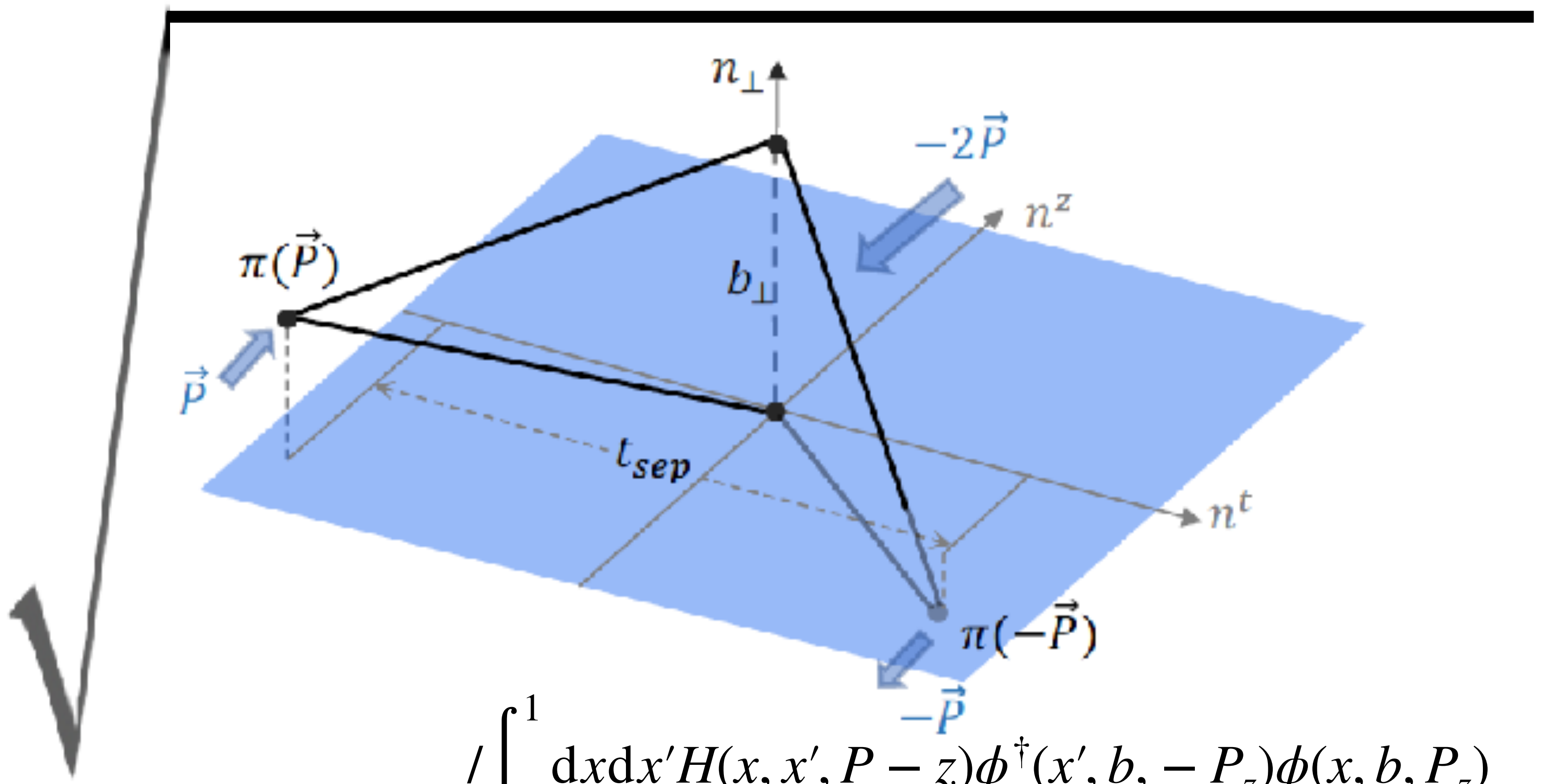
K. Zhang. et.al., LPC, In preparation

From quasi-TMD to TMD
additional soft function



- Such a problem can be fixed by introducing an additional “soft function” in the flat Wilson loop case.



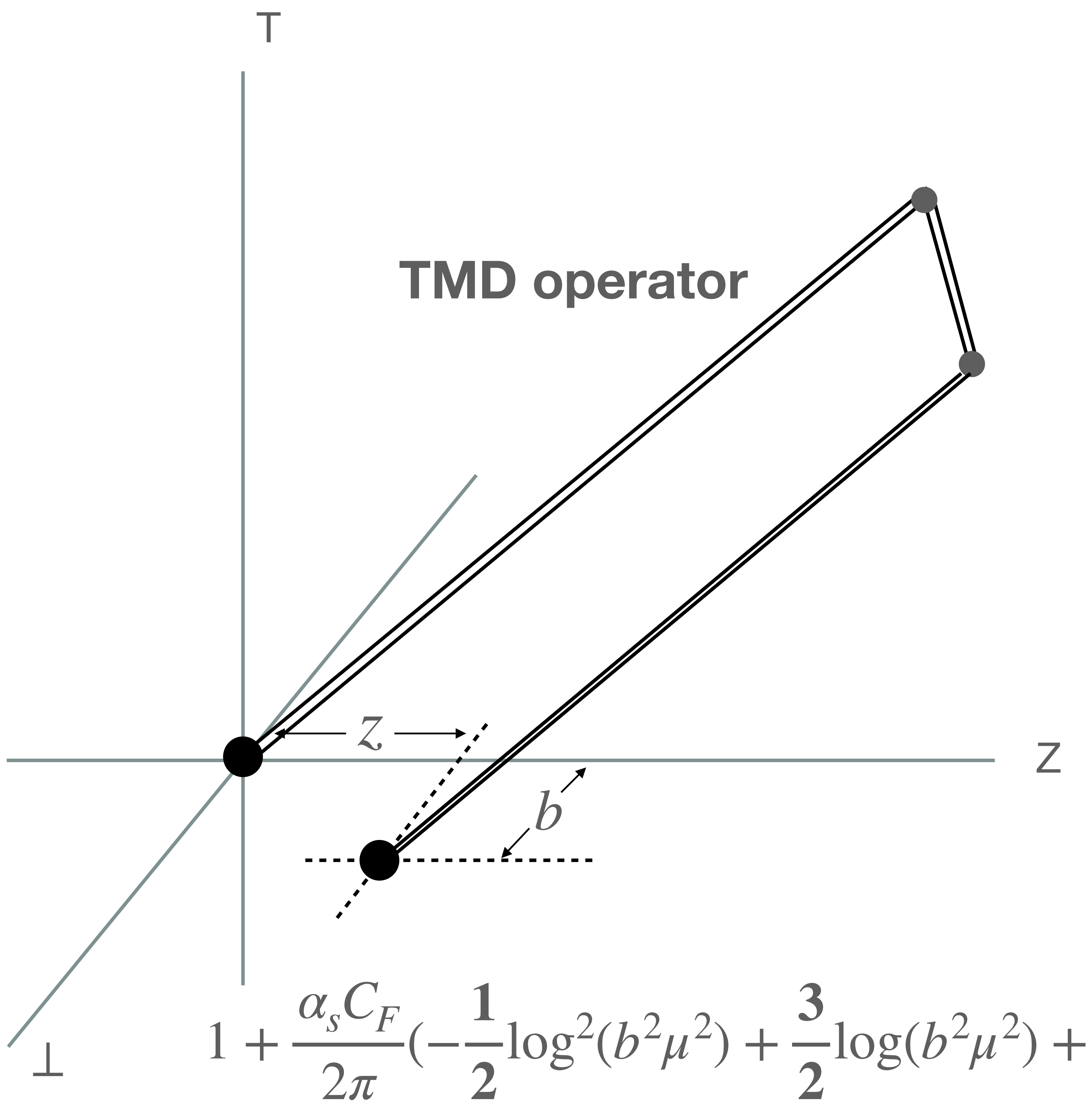


$$/\int_0^1 \mathrm{d}x\mathrm{d}x' H(x,x',P-z)\phi^\dagger(x',b,-P_z)\phi(x,b,P_z)$$

$$= \frac{1+\frac{\alpha_s C_F}{2\pi}(-\frac{1}{2}\log^2(b^2\mu^2)+\frac{9}{2}\log(b^2\mu^2)+\dots)}{1+\frac{\alpha_s C_F}{2\pi}(2\log(b^2\mu^2)+\dots)}(1+\frac{\alpha_s C_F}{2\pi}(-\log(b^2\mu^2)+\dots))$$

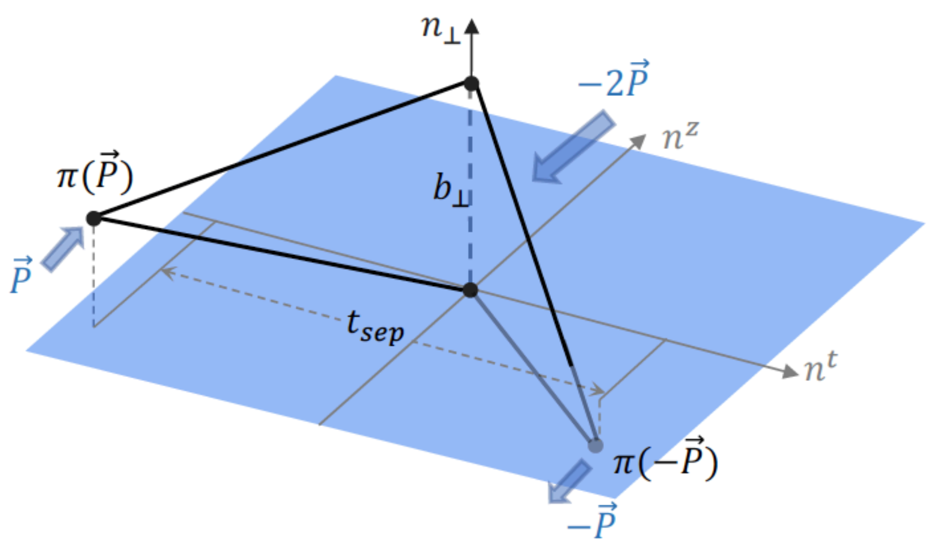
$$= 1+\frac{\alpha_s C_F}{2\pi}(-\frac{1}{2}\log^2(b^2\mu^2)+\frac{3}{2}\log(b^2\mu^2)+\dots)$$

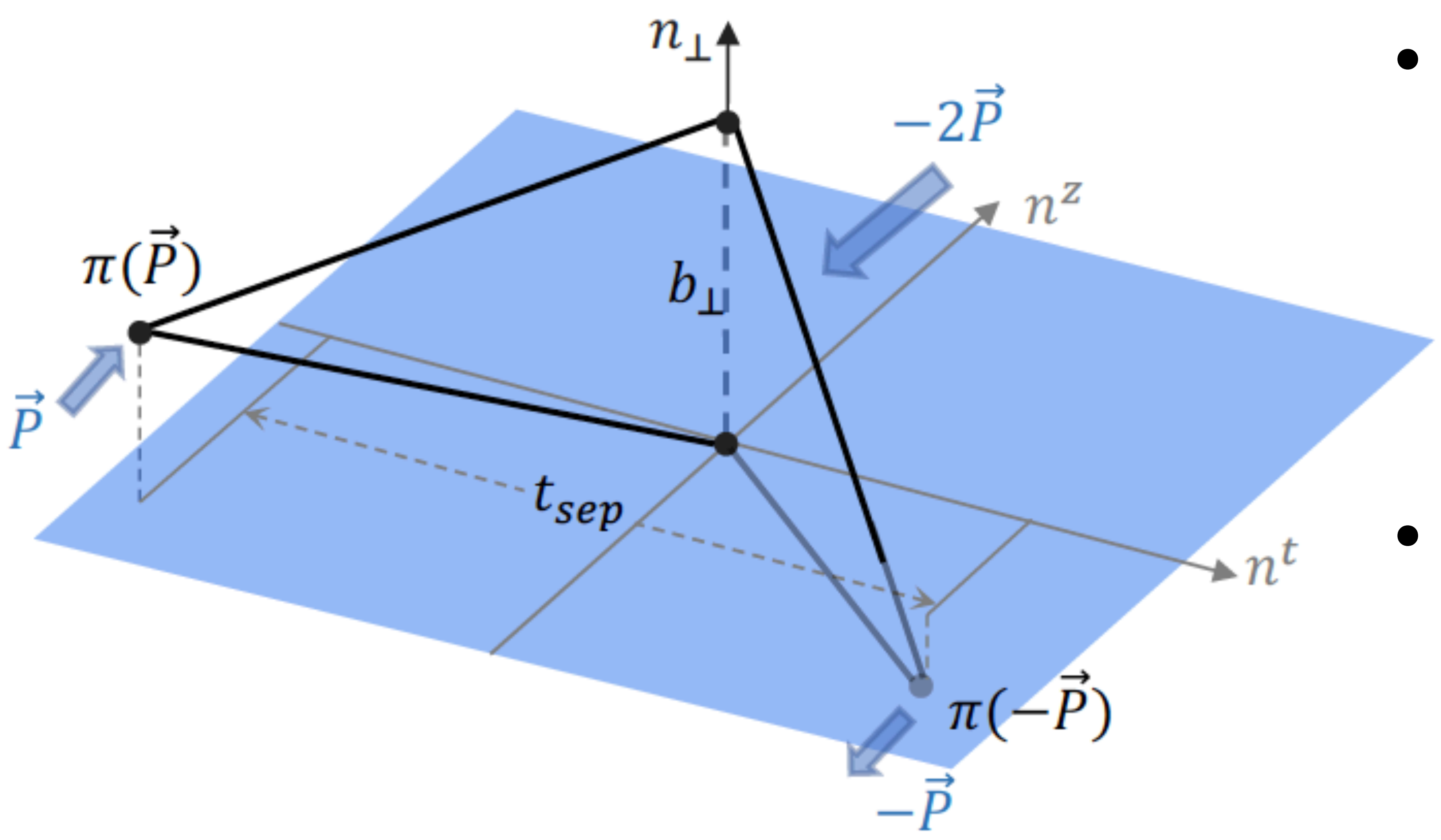
X.Ji. et.al., PLB(2020)135946



$$1+\frac{\alpha_s C_F}{2\pi}(-\frac{1}{2}\log^2(b^2\mu^2)+\frac{3}{2}\log(b^2\mu^2)+\dots)$$

Intrinsic soft function
first lattice QCD attempt





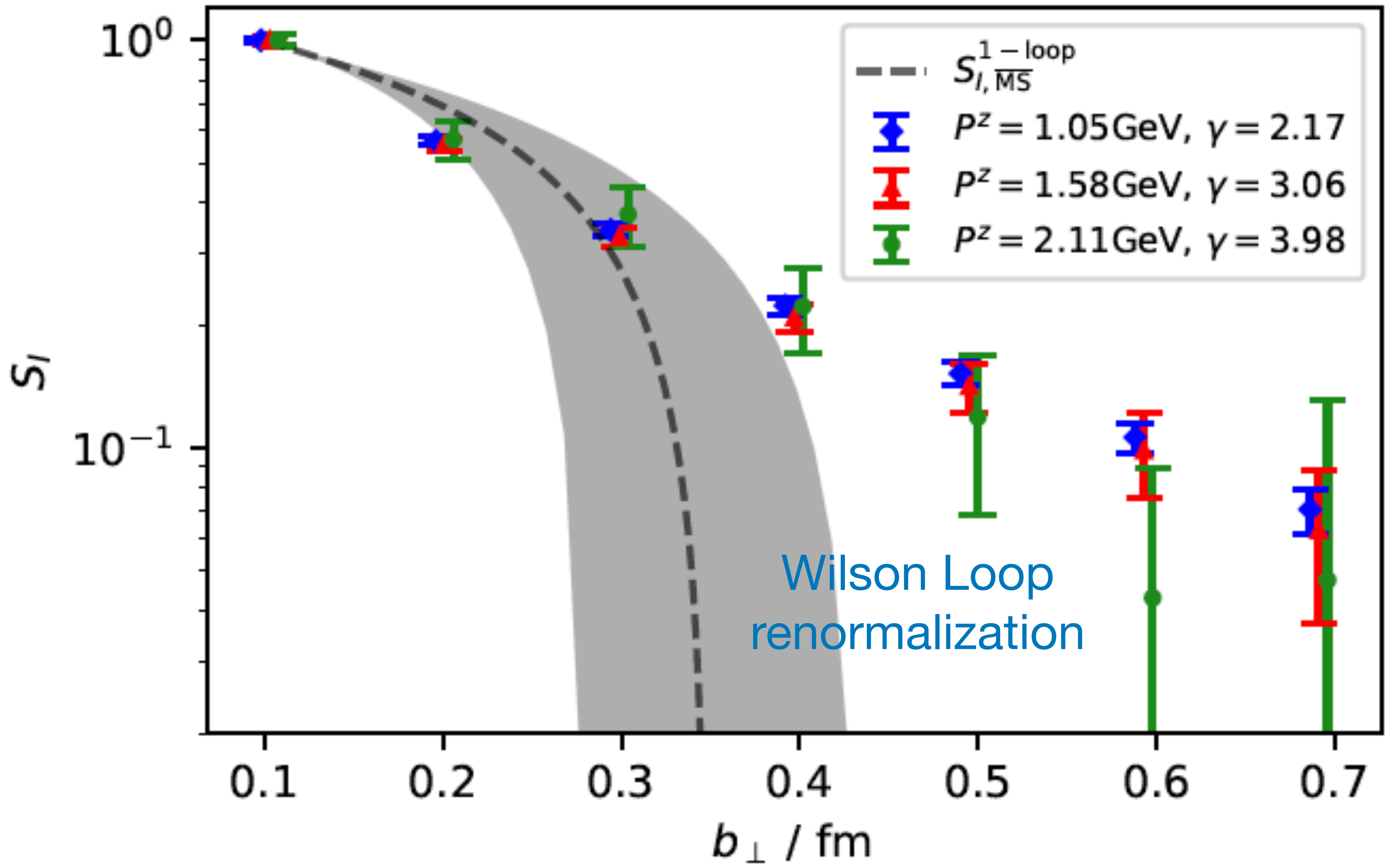
- More precisely, what we need is the rapidity-independent part of pion form factor $F(b, P_z)$:

$$S_I(b) = \frac{F(b, P_z)}{\int_0^1 dx dx' H(x, x', P - z)\phi^\dagger(x', b, -P_z)\phi(x, b, P_z)} = \frac{F(b, P_z)}{|\phi(0, b, -P_z)|^2} + \mathcal{O}(\alpha_s) + \mathcal{O}(\frac{1}{P_z^2}),$$

- where $\phi(x, b, P_z)$ is the TMD wave function which cancels the rapidity (momentum P_z) dependence.

- The form factor looks like a back elastic scattering of pion, with two currents separated by a spacial distance $b_\perp$.

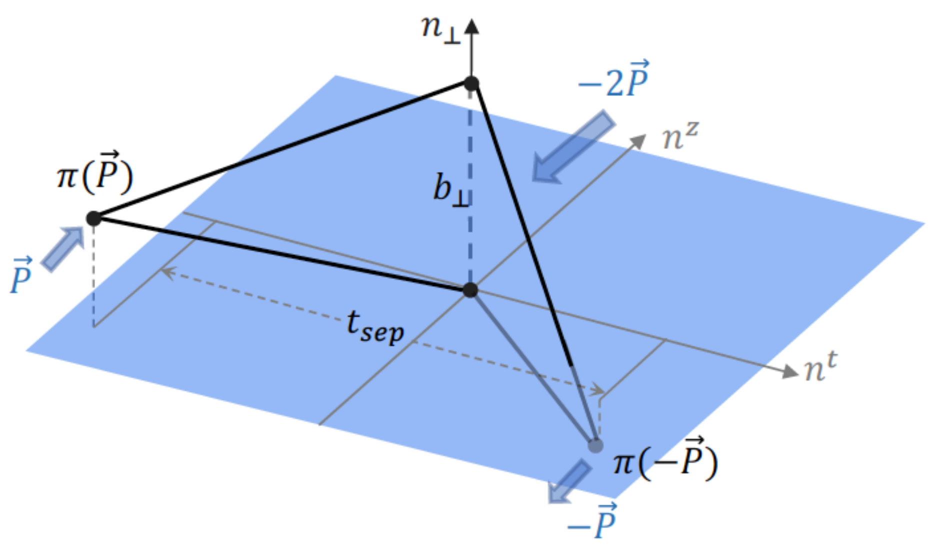
- Our lattice calculation shows that it is doable and converges with reasonable momentum.

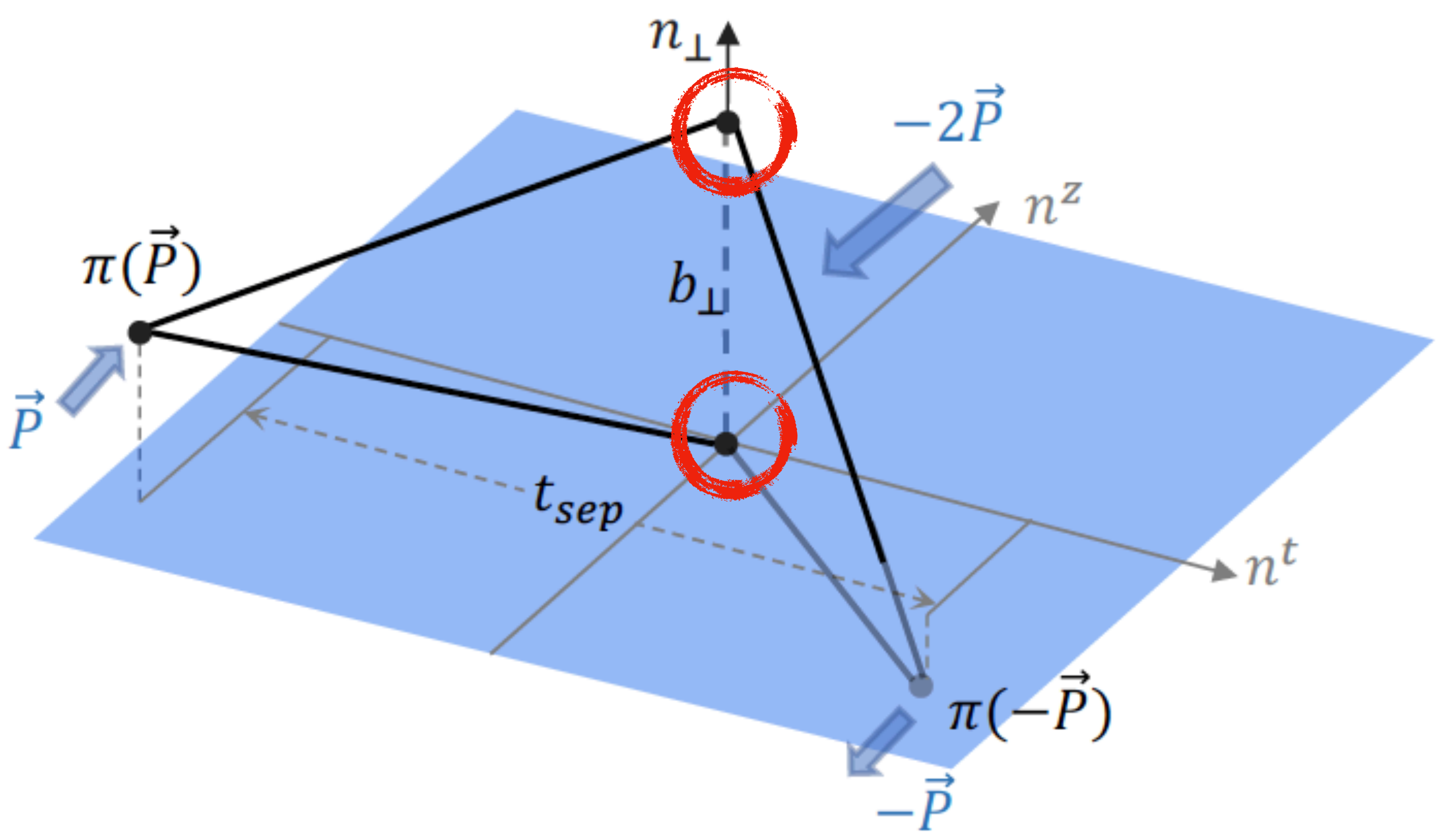


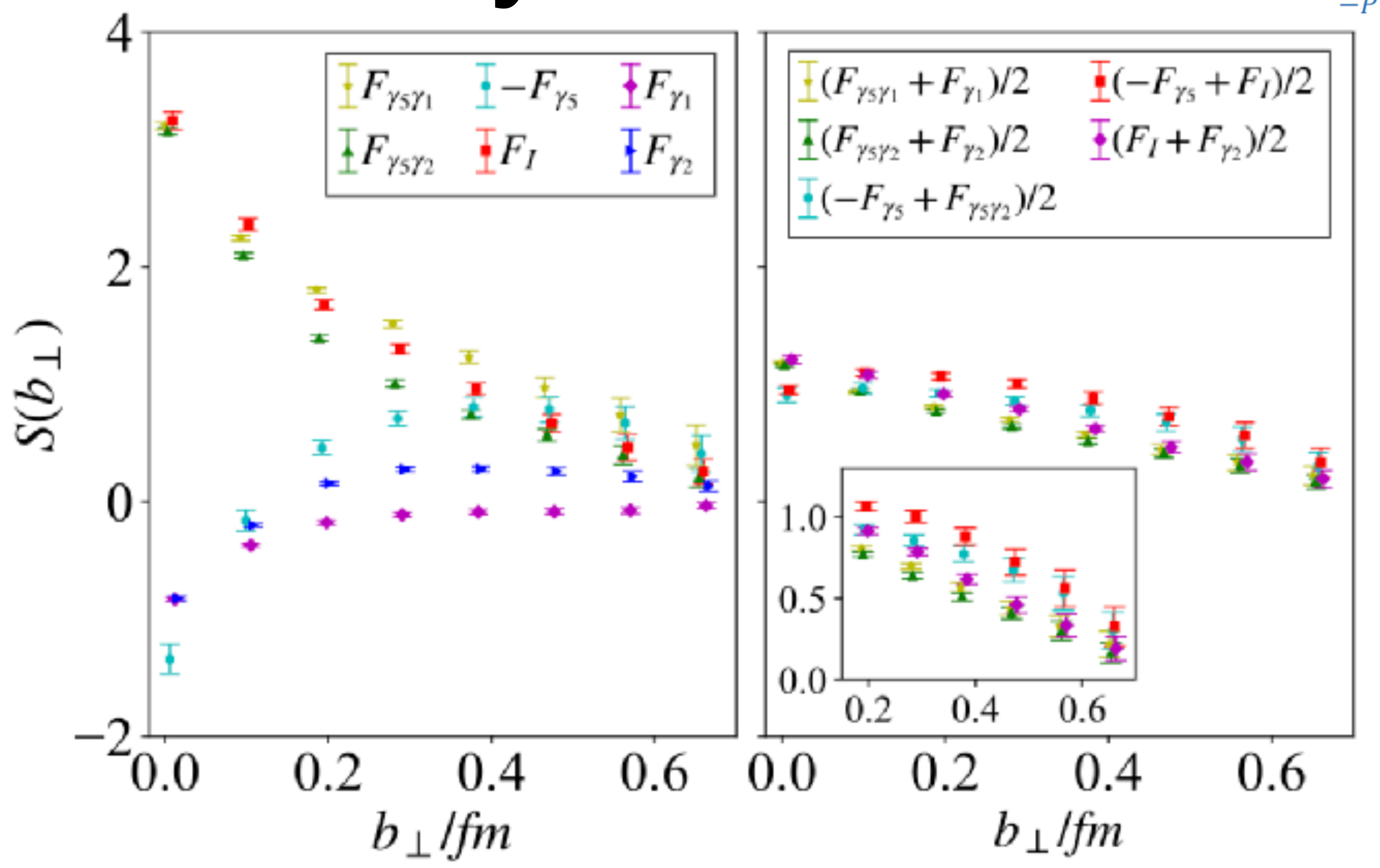
Q.-A. Zhang. et.al., LPC, PRL125(2020)192001

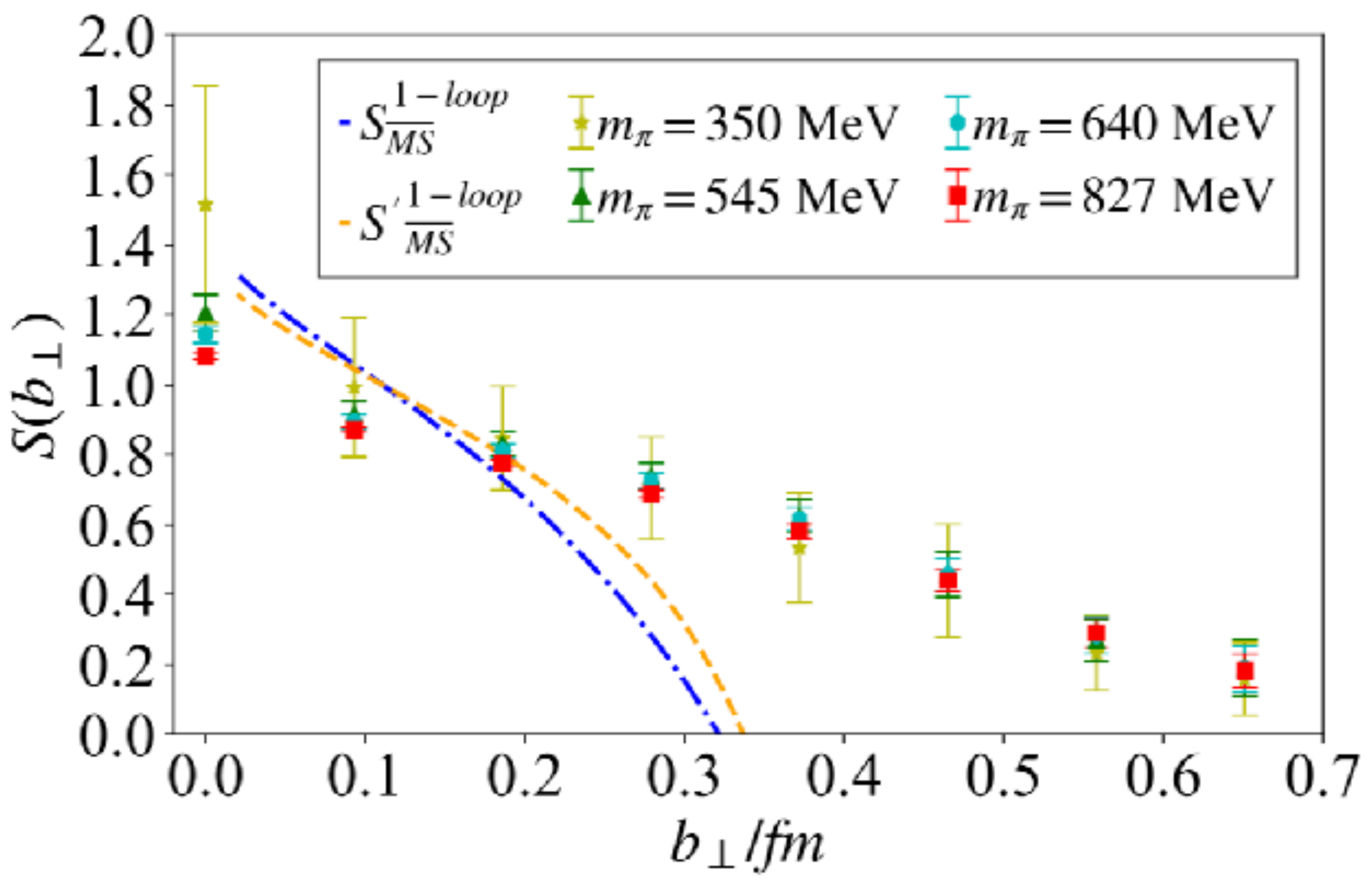
Intrinsic soft function

more systematic study









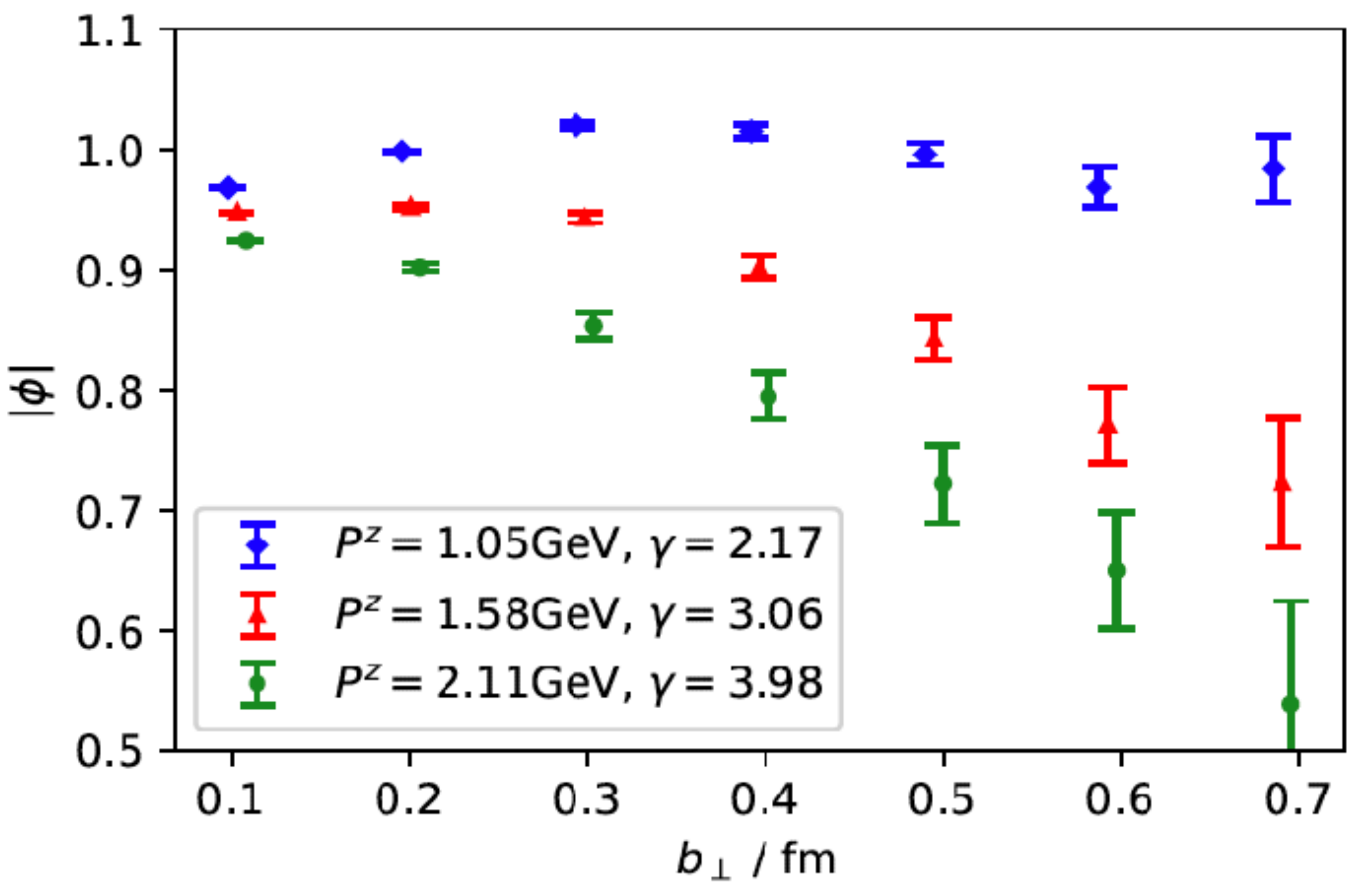
- The higher-twist effects can be very sensitive to the gamma matrices used in the currents, and proper combinations are helpful to suppress them.

- The combined results is independent to the pion mass within the statistical uncertainty.

Y. Li. et.al., arXiv: 2106.13027

Soft function

rapidity-dependent part



Q.-A. Zhang. et.al., LPC, PRL125(2020)192001

- Both the pion form factor $F(b, P_z)$ and quasi-TMD wave function $\Phi(z, b, P_z) \equiv \int \mathrm{d}x e^{ixzP_z}\phi(x, b, P_z) = \langle 0 | O_{\gamma_t\gamma_5}(z, b) | \pi(P_z)\rangle^R$ are rapidity (momentum P_z) dependent.

- Such a dependence should be universal and described by the Collins-Soper kernel $K(b)$:

$$\phi(x, b, P_z) = e^{\log\frac{P_z^2}{(P_z)'^2}K(b)}\phi(x, b, P_z').$$

- $K(b)$ can also extracted from the (quasi-)TMD-PDF.

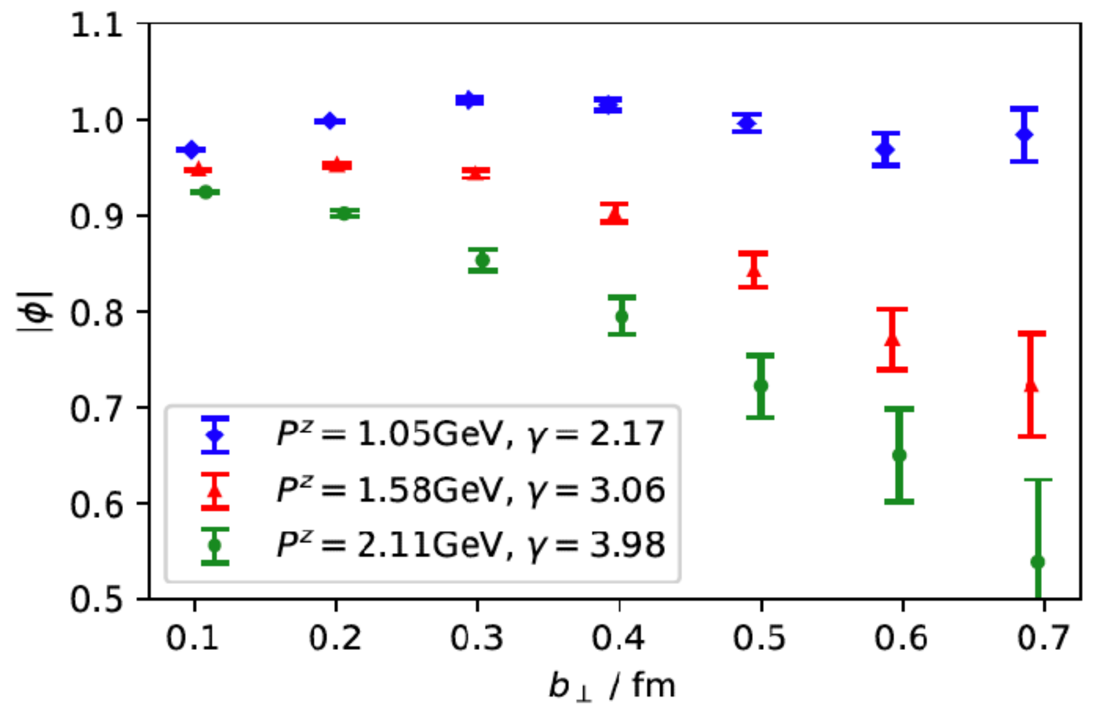
P. Shanahan. et.al., PRD102(2020)014511

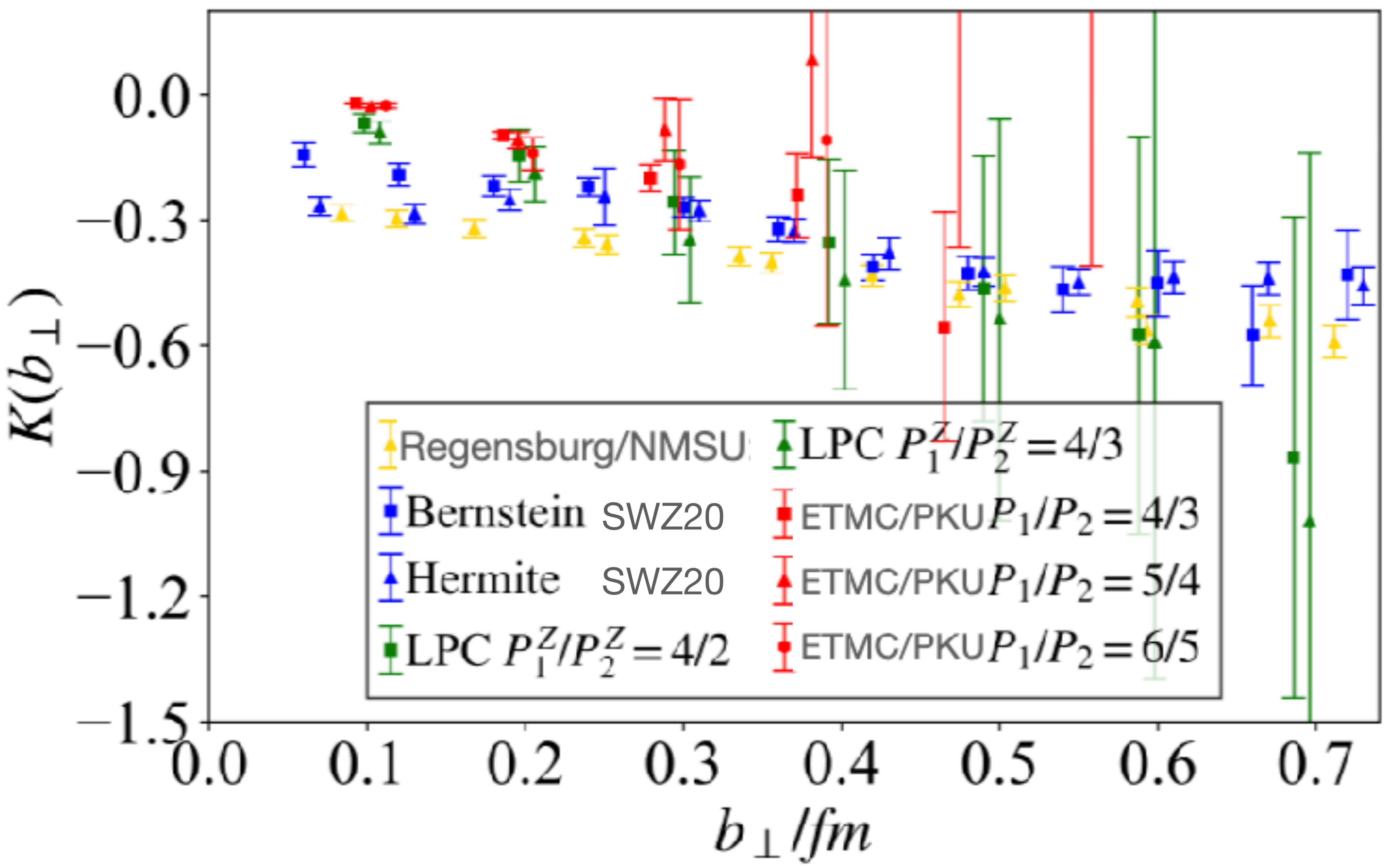
M. Schlemmer. et.al., JHEP(2021)004

P. Shanahan. et.al., arXiv:2107.11930

Collins-Soper kernel

from (quasi-)TMD wave function (WF)





SWZ 20:	P. Shanahan. et.al., PRD102(2020)014511
LPC:	Q.-A. Zhang. et.al., LPC, PRL125(2020)192001
Regensburg/NMSU:	M. Schlemmer. et.al., JHEP(2021)004
ETMC/PKU:	Y. Li. et.al., arXiv: 2106.13027

- With the tree level approximation for the perturbative correction, we can approximate the CS kernel by:

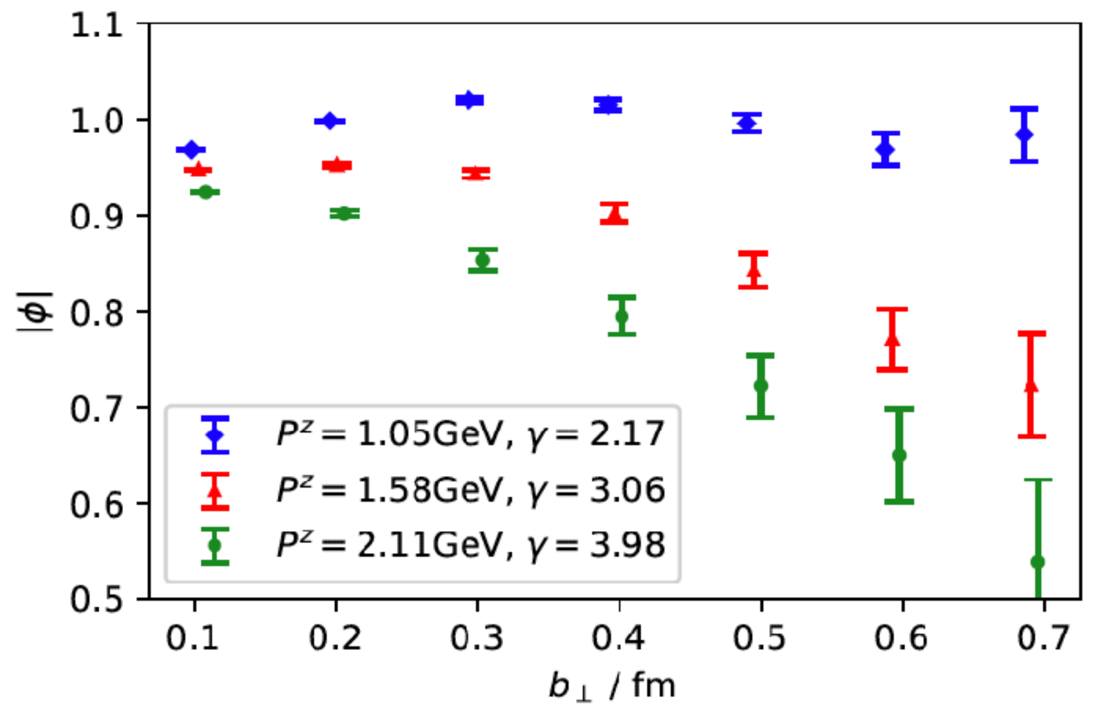
$$K(b) = \frac{1}{\log(P_z/P_z')} \log \left| \frac{\Phi(0,b,P_z)}{\Phi(0,b,P_z')} \right| + \mathcal{O}(\alpha_s).$$

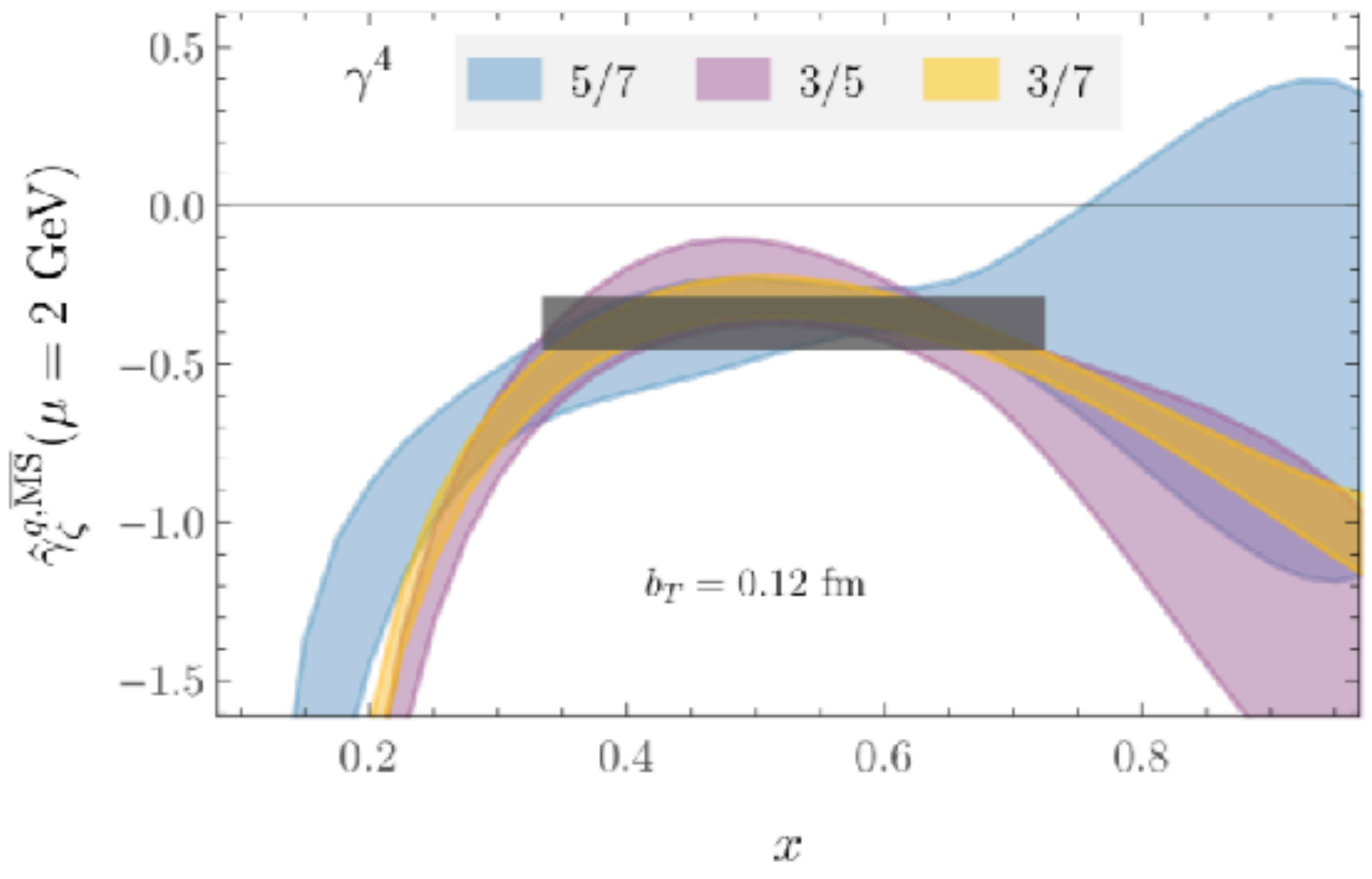
- The result from TMD WF (green and red points) seems to have significant difference comparing to that from TMD PDF (blue and yellow points) at small b, while become consistent at large b.

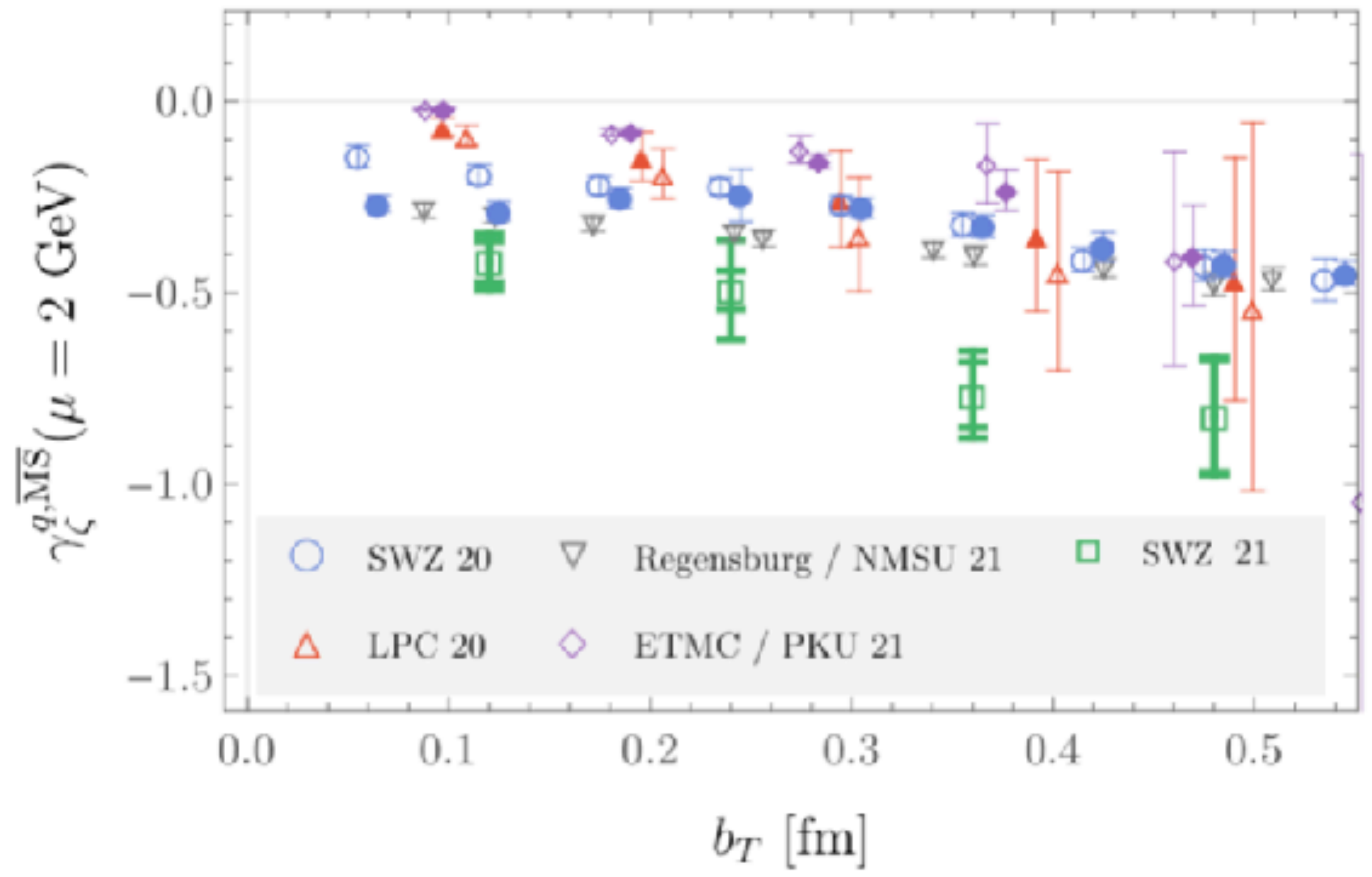
- The TMD WF results has huge uncertainty at large b due to the systematic uncertainty from the imaginary part.

Collins-Soper kernel

from (quasi-)TMD PDF





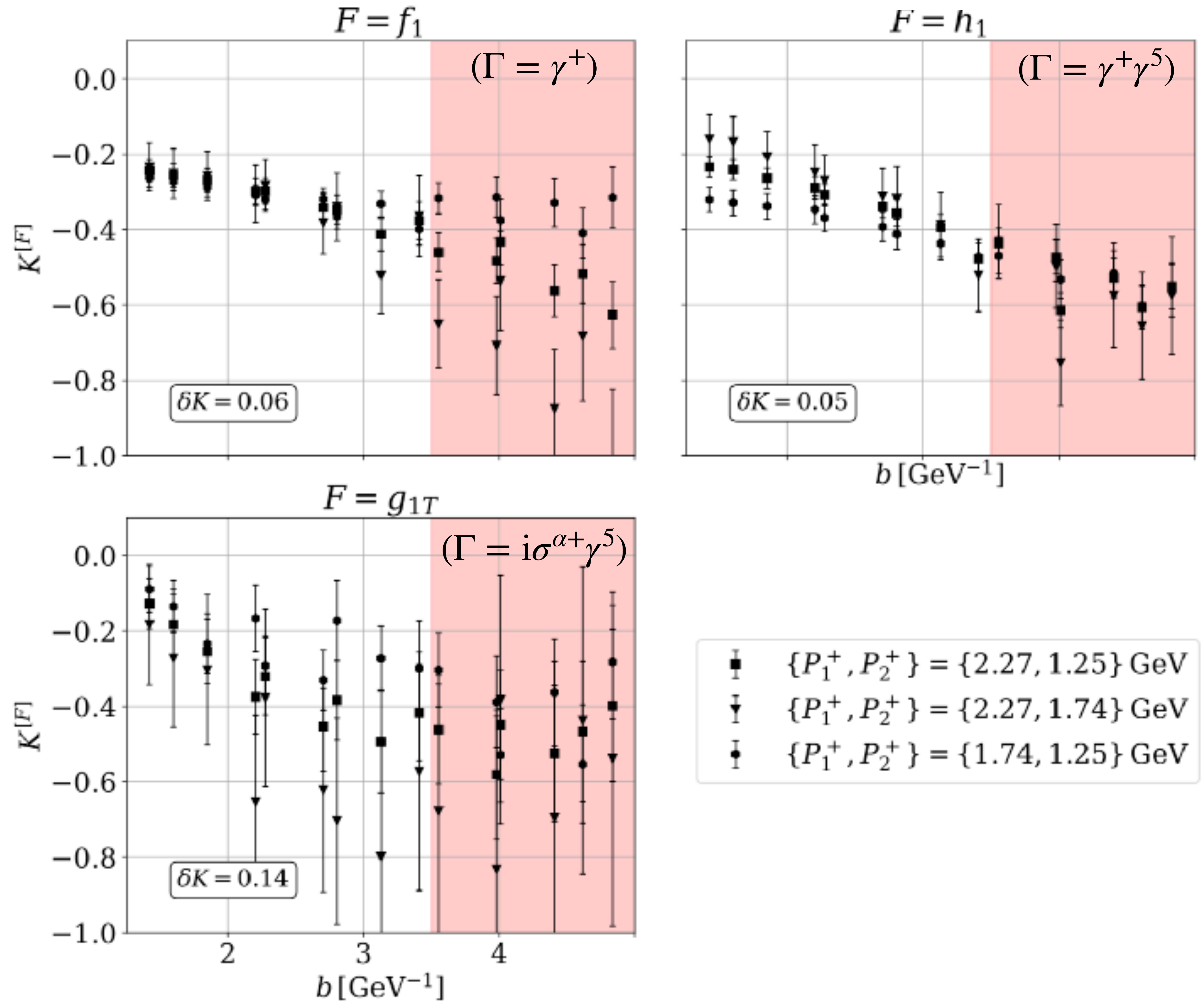


P. Shanahan. et.al., arXiv:2107.11930

Calculate the quasi-TMD PDF in the coordinate space with several momentum P_z:

- Renormalize, Fourier transform, apply the 1-loop matching and extract the CS-kernel based on the result at medium x (SWZ 20/21);

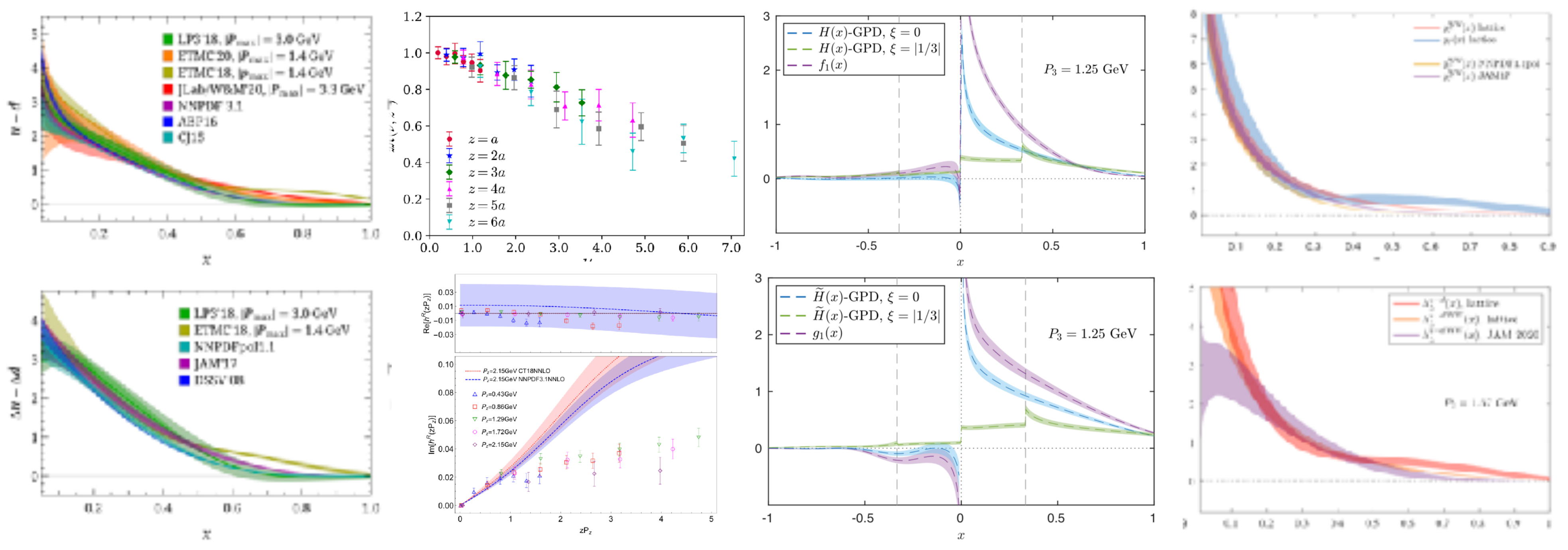
- Taking ratios on the bare matrix elements of $O_\Gamma(z, b)$ at different P_z, with different Γ(Regensburg/NMSU, the correlated systematic uncertainty δK from the normalization with the N^{3}LO resummed form).



Regensburg/NMSU 21: M. Schlemmer. et.al., JHEP(2021)004

Summary

- Using the large momentum effective theory, lattice QCD made many progresses on unpolarized/polarized light/strange/gluon PDF, GPD, twist-3 PDF and so on.



- New renormalization scheme is proposed to remove the linear divergence properly, and paves the way to reach the continuum limit correctly.

- The intrinsic soft function can be calculated on the lattice and it paves the way to the complete lattice calculation of TMD PDF; and the CS-kernel extracted from both TMD-WF and TMD-PDF are improving rapidly.