重味物理理论综述

--新年代新气象新机遇新使命

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第七届中国LHC物理研讨会@南师大、清华，2021.11.26

1970 诞生 GIM→charm，KM→CPV+3rd generation

1980 起步 QCD sum rules，mixed CPV

1990 腾飞 HQET，weak Hamiltonian

2000 巅峰 QCDF, PQCD, SCET

2010 坚持 B anomalies，XYZ

2020 新机？ 理论+唯象

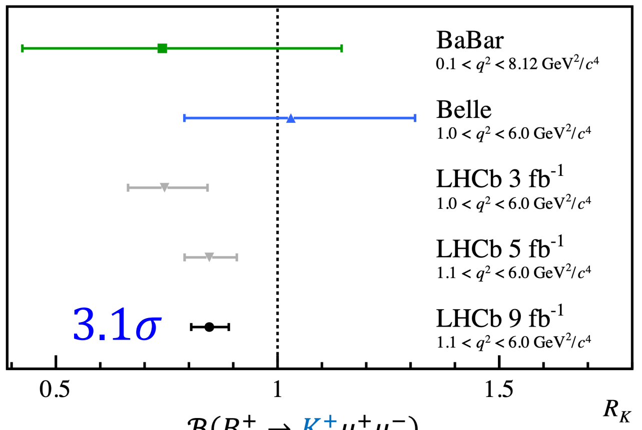
重味物理的主要任务

1980-2000，检验标准模型：KM机制

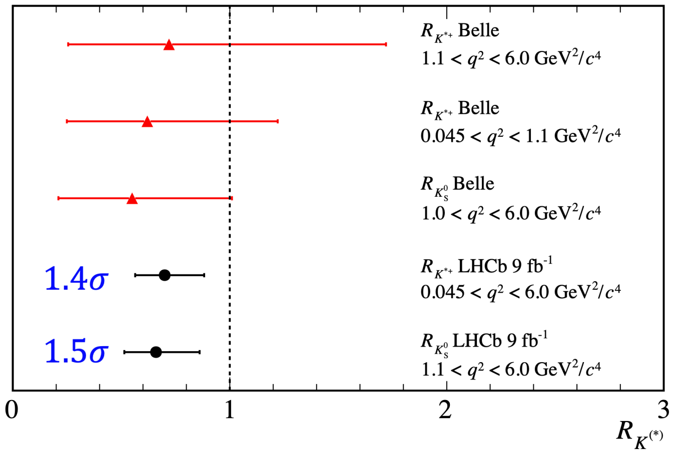
2000-至今，**精确**检验标准模型

- 寻找新物理
- 理解强相互作用

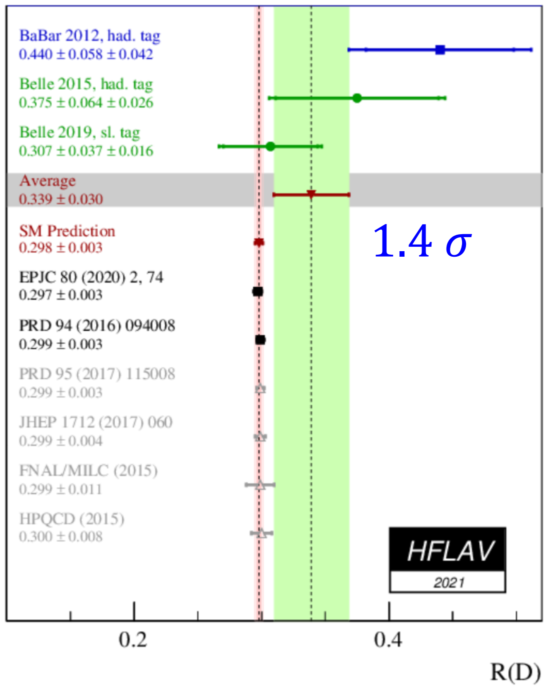
轻子味道普适性破坏

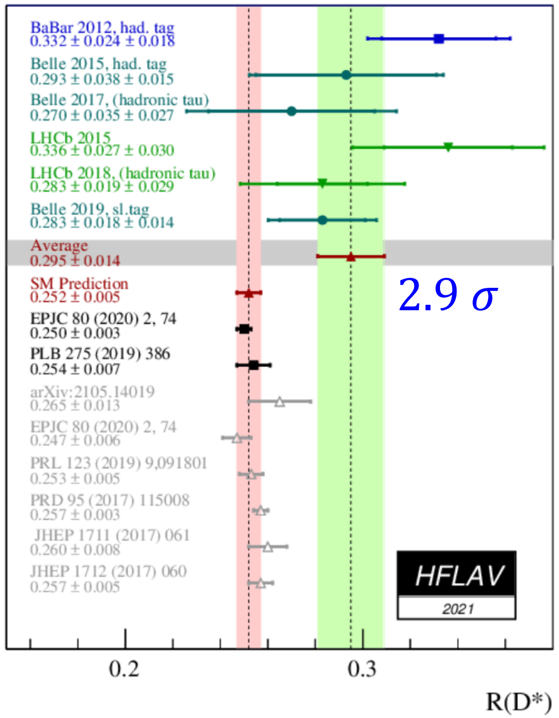


$$R_K = \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)}$$



- 实验误差大于理论误差
- 实验：除了积累数据外需测量更多衰变道和更多观测量
- 理论：趁此机会发展新的理论方法





重味物理为强子谱研究提供新思路

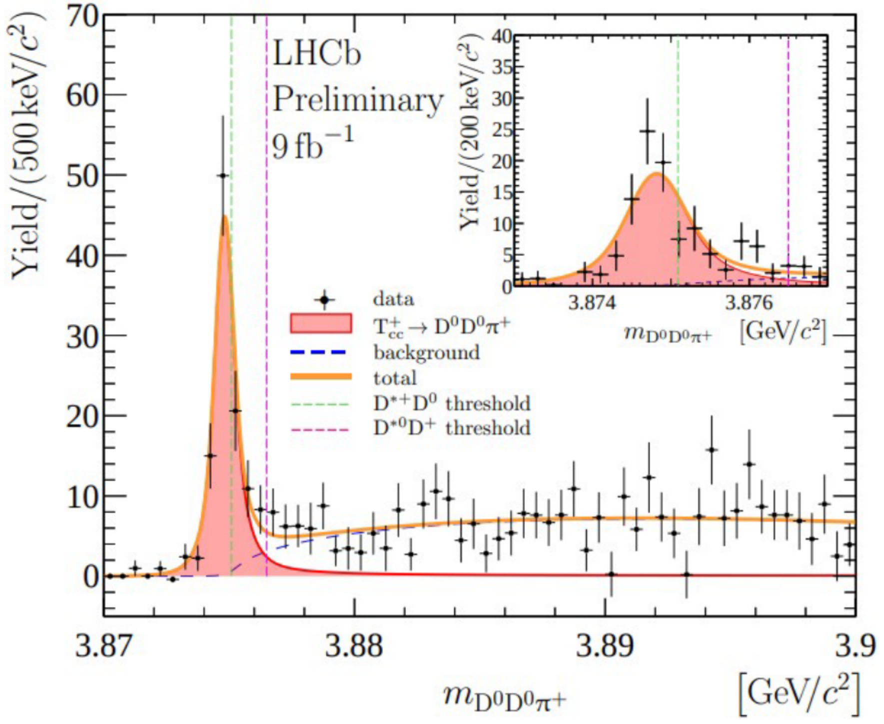
Production and decay of double-charm tetraquarks

We find that their production cross sections at the LHCb with $\sqrt{s} = 13$ TeV reach $\mathcal{O}(10^4)$ pb, which indicate that the LHCb has collected $\mathcal{O}(10^8)$ such particles. Through the decay channels of $T^{\{cc\}}_{[\bar{u}\bar{d}]} \to D^+K^-\pi^+$ or $D^0D^+\gamma$ (if stable) or $T^{\{cc\}}_{[\bar{u}\bar{d}]} \to D^0D^{*+}$ (if unstable), it is highly hopeful that they get discovered at the LHCb in the near future. We also discuss the productions and decays of the double-charm tetraquarks at future Tera-Z factories.

branching fractions of $T^{\{cc\}}_{[\bar{u}\bar{d}]}$ decays is the same as the observed Ξ^{++}_{cc}. Comparing with the production rates between double-charm tetraquarks and baryons, and considering around 2×10^3 events of Ξ^{++}_{cc} with the current LHCb data, the signal yields of $T^{\{cc\}}_{[\bar{u}\bar{d}]}$ would be $\mathcal{O}(10^2)$ at LHCb, and will reach $\mathcal{O}(10^3)$ at LHCb Run III. Thus it is hopefully expected that the double-charm tetraquark will be observed in the near future. Although the production rates are smaller at the future Z factories, it is also expected to be observed at the Tera-Z factories due to the smaller backgrounds.

[Q.Qin, Y.F.Shen, F.S.Yu, 2008.08026]

$N_s = 117 \pm 16$



[LHCb,2109.01038;2109.01056]

Fully reconstructed: $T_{cc}^{+} \to D^0 D^{*+}$

Compared with $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ $u \leftrightarrow \bar{u}\bar{d}$

Production

final prod $f_{\Lambda_b} / f_{B_u} \sim 0.5$

primarily prod $f_{\Sigma_b^{(*)}} / f_{\Lambda_b} \sim 1$

$\frac{f_{T_{cc}}}{f_{\Xi_{cc}}} \sim \frac{1}{4}$

Decay $Br(\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+)\ Br(\Lambda_c^+ \to p K^- \pi^+)$ one track more

10% 6% 1/3

$\sim Br(T_{cc} \to D^0 D^{*+})\ Br(D^{*+} \to D^0 \pi^+)\ Br(D^0 \to K^- \pi^+)^2$

1/2 2/3 $(4\%)^2$

$\longrightarrow Br(T_{cc})/Br(\Xi_{cc}^{++}) \sim 1/4$

1500 events of $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$ $\longrightarrow$ 100 events of Tcc

Qin, Shen, **FSY**, 2008.08026

重味物理的主要任务

1980-2000，检验标准模型：KM机制

Outline

2000-至今，**精确**检验标准模型

寻找新物理

理解强相互作用

理论新进展：格点、反问题

唯象新方向：重子、极化

Based on personal bias

近期理论发展：格点QCD

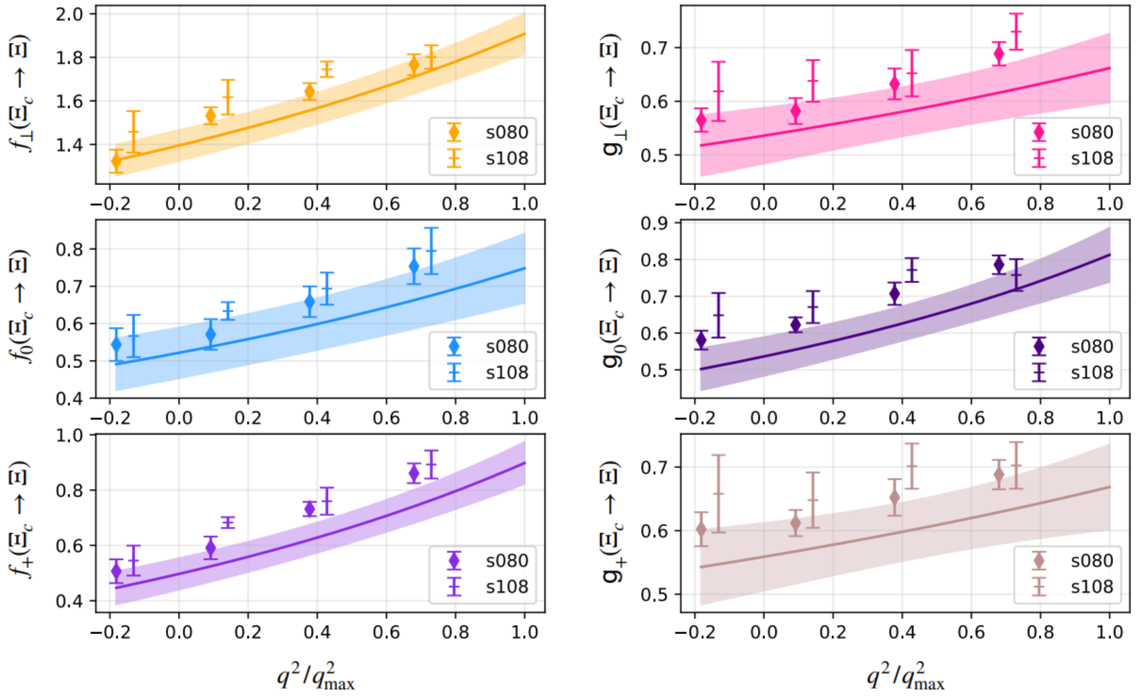
Lattice QCD

- 理解和计算非微扰物理量成为标准模型内的最后一个前沿问题
- 对于寻找新物理，非微扰物理量的计算也往往至关重要，例如muon g-2中的真空极化，半轻衰变中的形状因子等
- 格点是目前基于第一性原理计算非微扰物理量最可靠的理论方法
- 重味物理过程连接微扰与非微扰QCD，涉及很多非微扰物理量需要用格点QCD进行计算
- 目前国内很多大型计算资源可以利用，为我们格点发展提供了可能
- 最重要的是有了国产组态，解决了卡脖子问题。

Lattice QCD: charmed baryons

$\Xi_c \to \Xi$ Form Factors and $\Xi_c \to \Xi \ell^+ \nu_\ell$ Decay Rates From Lattice QCD

Qi-An Zhang,[1] Jun Hua,[2] Fei Huang,[2] Renbo Li,[3] Yuanyuan Li,[3]
Cai-Dian Lü,[4,5] Peng Sun,[3,*] Wei Sun,[4] Wei Wang,[2,†] and Yi-Bo Yang[6,7,8,‡]



最重要的是使用

国产对称格子clover组态

的第一篇文章

解决了卡脖子问题

Q.A.Zhang, et al, 2103.07064, to be appeared in Chinese Physics C

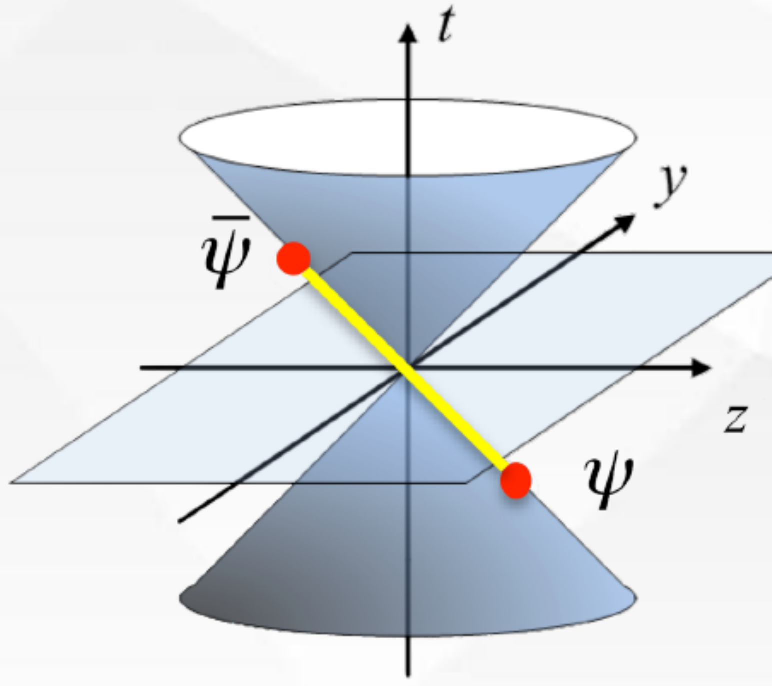
Large-Momentum effective theory

Define a new matrix element with an equal-time correlator, named quasi-PDF/DA:

$$\tilde{q}(x,P^z,\mu)=\int\frac{dz}{4\pi}e^{ixP^z z}\langle P|\overline{\psi}(z)\gamma^z\exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

Can be calculated on lattice directly!

X. Ji. Parton Physics on a Euclidean Lattice, Phys.Rev.Lett. 110, 262002 (2013).



For large P^z, the leading power of quasi-PDF/DA under the expansion of $\Lambda^2, M^2/(P^z)^2$ can be factorized into PDF/DA:

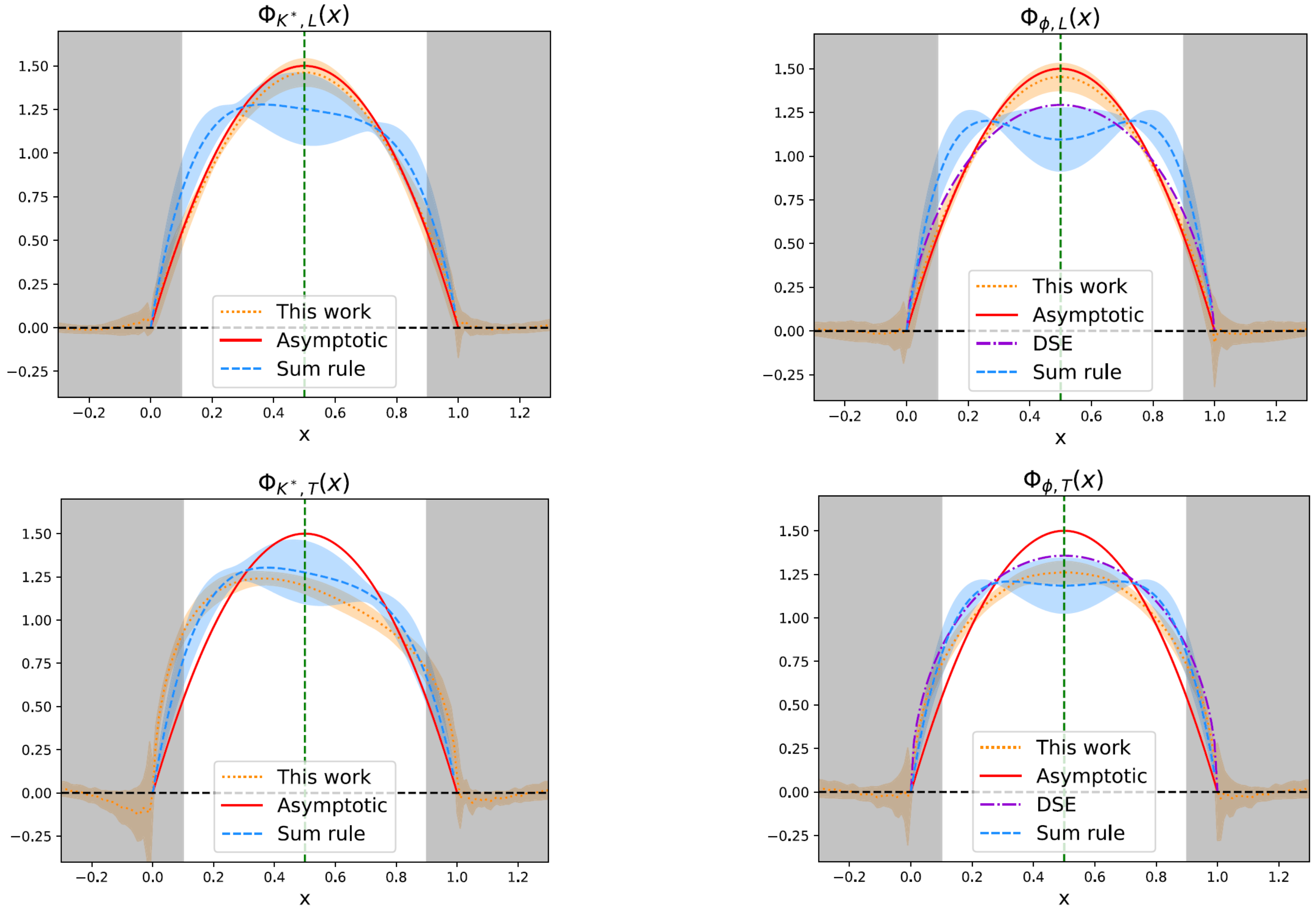
$$\tilde{q}(x,P^z,\mu)=\int\frac{dy}{|y|}\boxed{C\left(\frac{x}{y},yP^z,\mu\right)}q(y,\mu)+\mathcal{O}\left(\frac{\Lambda^2,M^2}{(P^z)^2}\right)$$

Jun Hua' s talk at China Lattice 202

Distribution Amplitudes of K^* and ϕ at the Physical Pion Mass from Lattice QCD

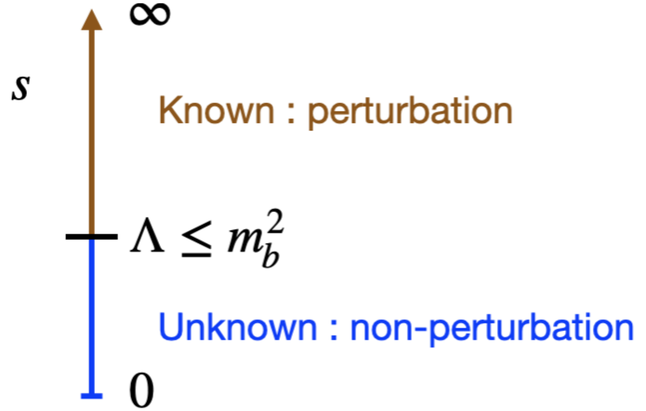
Jun Hua,[1] Min-Huan Chu,[1,2] Peng Sun,[3,*] Wei Wang,[1,†] Ji Xu,[1,4] Yi-Bo Yang,[5,6,7]
Jian-Hui Zhang,[8] and Qi-An Zhang[2]

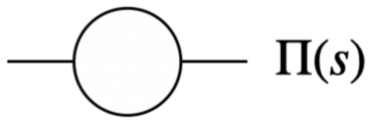
(Lattice Parton Collaboration)



近期理论发展：反问题方法

反问题方法





Dispersion Relation:

$$\mathcal{R}e[\Pi(s)] = \frac{1}{\pi}\mathcal{P}\int_0^\infty \frac{\mathcal{I}m[\Pi(s')]}{s-s'}ds'$$

If $s > \Lambda$,

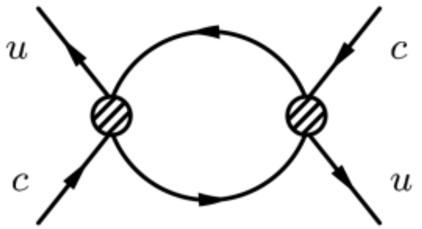
$$\mathcal{P}\int_0^\Lambda \frac{\mathcal{I}m[\Pi(s')]}{s-s'}ds' = \pi\mathcal{R}e[\Pi(s)] - \mathcal{P}\int_\Lambda^\infty \frac{\mathcal{I}m[\Pi(s')]}{s-s'}ds'$$

To be solved

calculable

Proposed in solving the $D^0 - \overline{D}^0$ mixing problem [H.n.Li, Umeeda, F.R.Xu, F.S.Yu, 2001.04079]

Inclusive Approach



quark level

Short-distance

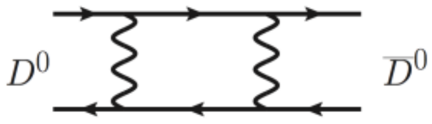
Theory / Exp. comparison (for inclusive)

D meson	B_s meson	B_d meson
Hagelin 1981, Cheng 1982 Buras, Slominski and Steger 1984 **NLO QCD** Golowich and Petrov 2005	Artuso, Borissov and Lenz, 2016	Artuso, Borissov and Lenz, 2016
SM $\begin{cases} x \simeq 6 \times 10^{-7} \\ y \simeq 6 \times 10^{-7} \end{cases}$ Suppressed by GIM	SM $\begin{cases} \Delta M_s = (18.3 \pm 2.7)\mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.088 \pm 0.020)\mathrm{ps}^{-1} \end{cases}$	SM $\begin{cases} \Delta M_d = (0.528 \pm 0.078)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = (2.61 \pm 0.59) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$
HFLAV at Moriond2019 Exp. $\begin{cases} x = (3.9^{+1.1}_{-1.2}) \times 10^{-3} \\ y = (6.51^{+0.63}_{-0.69}) \times 10^{-3} \end{cases}$	HFLAV Exp. $\begin{cases} \Delta M_s = (17.757 \pm 0.021)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_s = (0.082 \pm 0.006)\ \mathrm{ps}^{-1} \end{cases}$	HFLAV Exp. $\begin{cases} \Delta M_d = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1} \\ \Delta\Gamma_d = 0.66(1 \pm 10) \cdot 10^{-3}\ \mathrm{ps}^{-1} \end{cases}$

- For B_s, B_d mesons, the data are reproduced within 1σ.
- For D meson, the order of magnitude is not reproduced within leading-power.

Inverse Problem

$D^0 - \overline{D}^0$ mixing

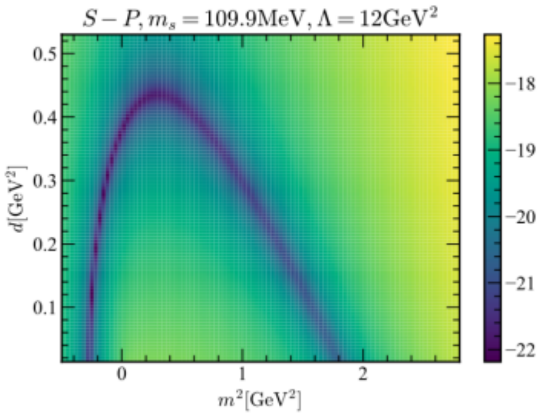


$$\int_0^\Lambda ds' \frac{y(s')}{s-s'} = \pi x(s) - \int_\Lambda^\infty ds' \frac{y(s')}{s-s'} \equiv \omega(s)$$

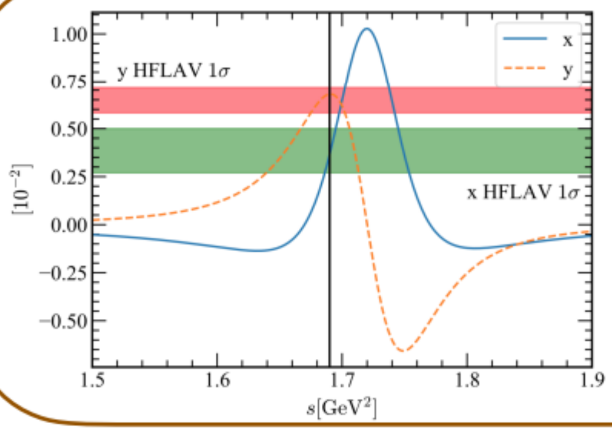
parametrization:

$$y(s) = \frac{Ns[b_0 + b_1(s-m^2) + b_2(s-m^2)^2]}{[(s-m^2)^2 + d^2]^2}$$

Li, Umeeda, Xu, **FSY**, PLB(2020)



unstable solutions



Additional conditions:
data of x and y as inputs

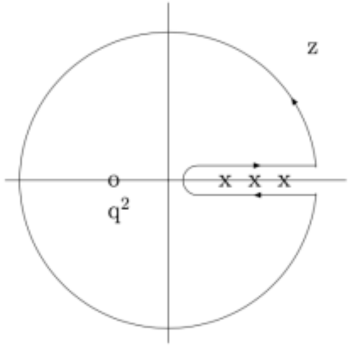
Predict indirect CPV

$q/p = 1.0002e^{i0.006^\circ}$

consistent with data

$q/p = (0.969^{+0.050}_{-0.045})e^{i(-3.9^{+4.5}_{-4.6})^\circ}$

Applications of the Inverse Problem: QCD sum rules



- Conventional QCD sum rules $\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu(0)]|0\rangle$

Dispersion relation: $\Pi(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \frac{\mathrm{Im}\,\Pi(s)}{s-q^2-i\epsilon}$

$$\mathrm{Im}\Pi(q^2) = \pi f_V^2\delta(q^2-m_V^2) + \pi\rho^h(q^2)\theta(q^2-s_h)$$

Quark-hadron duality: $\rho^h(s) = \frac{1}{\pi}\mathrm{Im}\Pi^{\mathrm{pert}}(s)\theta(s-s_0)$

$$\int_{s_h}^{\infty} ds \frac{\rho^h(s)}{s-q^2} = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\mathrm{Im}\Pi^{\mathrm{pert}}(s)}{s-q^2}$$

Uncertainty sources: quark-hadron duality and Borel transformation

Applications of the Inverse Problem: QCD sum rules

- **Inverse-Problem QCD sum rules**

$$\frac{1}{2\pi i}\oint ds\frac{\Pi(s)}{s-q^2}=\frac{1}{\pi}\int_{s_i}^{\Lambda}ds\frac{\mathrm{Im}\Pi(s)}{s-q^2}+\frac{1}{\pi}\int_{\Lambda}^{R}ds\frac{\mathrm{Im}\Pi^{\mathrm{pert}}(s)}{s-q^2}+\frac{1}{2\pi i}\int_C ds\frac{\Pi^{\mathrm{pert}}(s)}{s-q^2}$$

Involving excited states and parameterization:

$$\begin{aligned}\mathrm{Im}\Pi(q^2) &= \pi f_\rho^2\delta(q^2-m_\rho^2)+\pi f_{\rho(1450)}^2\delta(q^2-m_{\rho(1450)}^2)+\pi f_{\rho(1700)}^2\delta(q^2-m_{\rho(1700)}^2)\\ &\quad+\pi f_V^2\delta(q^2-m_V^2)+\pi\rho^h(q^2),\end{aligned}$$

$$\rho^h(y)=b_0P_0(2y-1)+b_1P_1(2y-1)+b_2P_2(2y-1)+b_3P_3(2y-1)+\cdots$$

$$\boxed{\begin{aligned}&m_{\rho(770)}(m_{\rho(1450)},m_{\rho(1700)},m_{\rho(1900)})\approx 0.78\ (1.46,\ 1.70,\ 1.90)\ \mathrm{GeV}\\ &f_{\rho(770)}(f_{\rho(1450)},f_{\rho(1700)},f_{\rho(1900)})\approx 0.22\ (0.19,\ 0.14,\ 0.14)\ \mathrm{GeV}\end{aligned}}$$

H.n.Li, Umeeda, '20

近期理论发展：重子物理

Baryon physics: introduction

- The visible matter of the Universe is mainly made of baryons
- CPV are well established in K, B and D mesons, but not in baryons yet.
- Helicities of baryons provide fruitful phenomenological observables.
- BESIII and Belle have fruitful results on charmed baryons and hyperons.
- LHCb is a baryon factory, $f_{\Lambda_b}/f_{u,d} \sim 1/3$

Machine	CEPC (10^{12} Z)	Belle II (50 ab^{-1} + 5 ab^{-1} at $\Upsilon(5S)$)	LHCb (50 fb^{-1})
Data taking	2030-2040	$\rightarrow$ 2025	$\rightarrow$ 2030
B^+	6×10^{10}	3×10^{10}	3×10^{13}
B^0	6×10^{10}	3×10^{10}	3×10^{13}
B_s	2×10^{10}	3×10^{8}	8×10^{12}
B_c	6×10^{7}	–	6×10^{10}
b baryons	10^{10}	–	10^{13}

Baryon physics: QCD

- QCD in baryons is different from mesons, but not systematically studied.
- Scalar diquark in $\Lambda_{b(c)}$ is simpler than spin ½ quark in B mesons in the heavy quark limit. Much less free parameters in $\Lambda_b \to \Lambda_c$ than $B \to D^{(*)}$ transitions [Bernlochner, Ligeti, Robinson, Sutcliffe, 2019]

Decay	N_{IW} at $\mathcal{O}(1)$	N_{SIW} at $\mathcal{O}(\Lambda_{\mathrm{QCD}}/m_{b,c})$	N_{SIW} at $\mathcal{O}(\Lambda^2_{\mathrm{QCD}}/m^2_c)$
$\Lambda_b \to \Lambda_c \ell^- \bar{\nu}_\ell$	1	0	2
$B \to D^* l \nu$	1	3	6

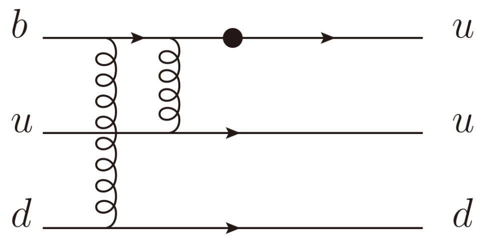
- Heavy-to-light form factor is factorizable at leading power in SCET, which is however one order of magnitude smaller than the results from sum rules , due to two hard gluons [W.Wang, 1112.0237] $\xi_\Lambda = f_{\Lambda_b}\Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_\Lambda \Phi_\Lambda(y_i)$

Leading power: $\xi_\Lambda(q^2 = 0) = -0.012^{+0.009}_{-0.023}$

Sum Rules [Feldman, Yip, 2011]: $\xi_\Lambda(q^2 = 0) = 0.38$

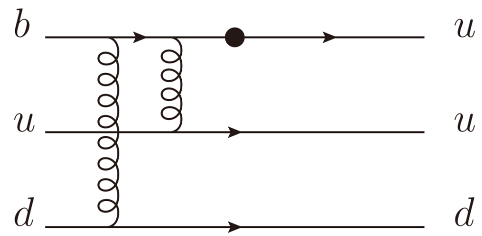
Baryon physics: PQCD

- The current predictions on b-baryon CPV are based on generalized factorization approach [Y.K.Hsiao, C.Q.Geng, 2015; C.W.Liu, C.Q.Geng, 2109.09524, 2111.02091]
- QCD-inspired methods are required to be developed to predict baryon CPV.
- PQCD successfully predicted correct CPV in B meson decays [Y.Y.Keum, H.n.Li, Sanda, 2000; C.D.Lu, K.Ukai, M.Z.Yang, 2000].
- It is hopeful to predict CPV of b-baryons.
- The only prediction of CPV of $\Lambda_b \to p\pi, pK$ by PQCD is given in [C.D.Lu, Y.M.Wang, H.Zou, A.Ali, G.Kramer, 2009].
- However, the form factors are two orders of magnitude smaller than Lattice or LCSRs [H.n.Li, 1999].



Lattice [35]	0.22 ± 0.08
PQCD [67]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$

Baryon physics: PQCD



- The problem is that only leading twist LCDAs were considered.
- Now revisiting the $\Lambda_b \to p$ form factors in PQCD. Considering higher twist LCDAs, the form factors are significantly enhanced and consistent with other results. [J.J.Han, Y.Li, Y.L.Shen, Z.J.Xiao, F.S.Yu, 2111.xxxxx]

Lattice [35]	0.22 ± 0.08
PQCD [67]	$2.2^{+0.8}_{-0.5} \times 10^{-3}$

$\longrightarrow$ Same order as Lattice

- Leading power is suppressed by $O(\alpha_s^2)$. Sub-leading power dominated.
- It is expected that PQCD can be used to predict CPV of b-baryons. [J.J.Han, Y.Li, Y.L.Shen, Z.J.Xiao, F.S.Yu, 2112.xxxxx]

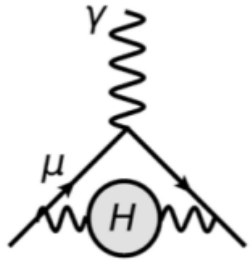
Many other beautiful works

- ✓ Charmed baryon decays [F.R.Xu, H.Y.Cheng, 2020, 2021][Y.K.Hsiao, 2020, 2021] [C.Q.Geng, 2020, 2021]
- ✓ Heavy Diquark Effective Theory [Q.Qin, Y.J.Shi, W.Wang, G.H.Yang, F.S.Yu, R.L.Zhu, 2108.06716]
- ✓ Production and decays of double-charm tetraquarks [Q.Qin, Shen, F.S.Yu, 2008.08026]
- ✓ Unified framework of topological diagrams and SU(3) irreducible representation [X.G.He, W.Wang, 2018][D.Wang, C.P.Jia, F.S.Yu, 2020]
- ✓ D_s^* weak decays [S.Cheng, Q.Qin, F.S.Yu, 2021]
- ✓ Hyperon decays under the SU(3) symmetry [R.M.Wang, X.D.Cheng, Y.G.Xu, 2020, 2021]
- ✓ And so on…

总结

- 21世纪20年代刚刚开启
- 理论新进展：格点QCD、反问题方法
- 唯象新方向：重子物理
- 更多新机遇

Applications of the Inverse Problem: muon g-2



- Muon g-2: 4.2σ deviation from the SM — Muon g-2, PRL(2021)
- Dominate uncertainty of the SM prediction: hadronic vacuum polarization (HVP)

Aoyama, et al, Phys.Rept(2020)

- Inverse Problem:

$$\int_{\lambda_r}^{\Lambda_r} ds' \frac{\mathrm{Im}\Pi_r(s')}{s'(s'+s)} - \pi\frac{\Pi_r(0)}{s} = -\pi\frac{\Pi_r(-s)}{s} - \int_{\Lambda_r}^{\infty} ds' \frac{\mathrm{Im}\Pi_r(s')}{s'(s'+s)} \qquad r = \rho,\ \omega,\ \phi$$

- Result: Inverse problem: $a_\mu^{\mathrm{HVP}} = (641^{+65}_{-63}) \times 10^{-10}$ — H.n.Li, Umeeda, '20

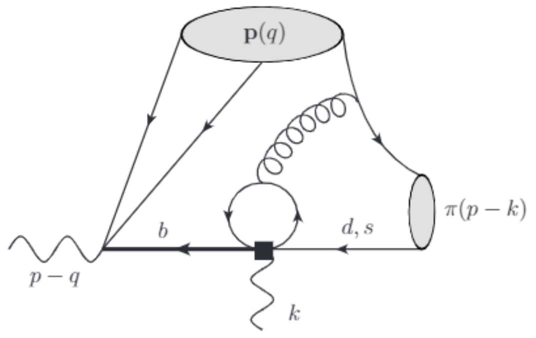
Data driven: $a_\mu^{\mathrm{HVP}} = (693.9 \pm 4.0) \times 10^{-10}$ — Davier, Hoecker, Malaescu, Zhang, '20

Lattice QCD: $a_\mu^{\mathrm{HVP}} = (654 \pm 32^{+21}_{-23}) \times 10^{-10}$ — Della Morte et al, '17

Non-perturbative properties can be revealed from asymptotic QCD by solving an inverse problem.

Baryon physics: LCSRs

- Heavy-to-light form factors have been systematically studied in the light-cone QCD sum rules
  - ✓ Next-to-leading order corrections [Y.M.Wang, Y.L.Shen, 2016]
  - ✓ LCDAs of heavy baryons [Bell, Feldman, Y.M.Wang, Yip, 2013]
  - ✓ $\Lambda_b \to N^*$ transitions [K.S.Huang, W.Liu, Y.L.Shen, F.S.Yu, 2112.xxxxx]
- Two-body hadronic decays of $\Lambda_b \to p\pi, pK$ are studied firstly in LCSRs [H.Y.Jiang, Khodjamirian, F.S.Yu, S.Cheng, 2112.xxxxx]
- It has been studied in $B \to \pi\pi$ and $D \to \pi\pi$ [Khodjamirian, 2002, 2004, 2017]
- It overcome the difficulty of calculation on W-exchange diagrams in QCDF

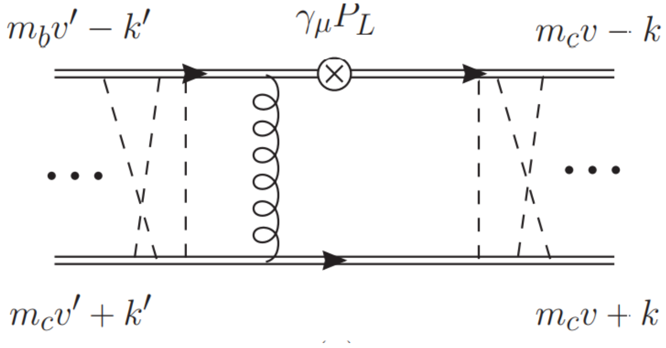


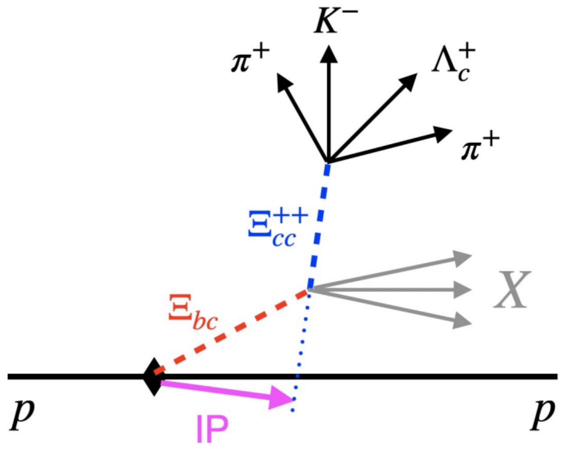
Heavy Diquark Effective Theory

- The Heavy Diquark Effective Theory is proposed to study the transitions of $bb \to bc$ and $bc \to cc$ [Y.J.Shi, W.Wang, Z.X.Zhao, Meissner, '20]
- Two heavy quarks → point-like diquark particle.
- Matching at small recoil region: HQET
- Matching at large reoil region: NRQCD
- Decay rate of inclusive processes:

$$\Gamma(\Xi_{bc} \to H_{cc} + X) = \sum_{f,f'} \Gamma(\mathcal{X}_{bc} \to \mathcal{X}_{cc} \bar{f} f') + \mathcal{O}\left(\frac{1}{M_{\mathcal{X}}}\right)$$

- Propose the inclusive approach to search for Ξ_{bc} via $\Xi_{bc} \to \Xi_{cc}^{++} + X$ with displaced vertex [Q.Qin, Y.J.Shi, W.Wang, G.H.Yang, F.S.Yu, R.L.Zhu, 2108.06716]

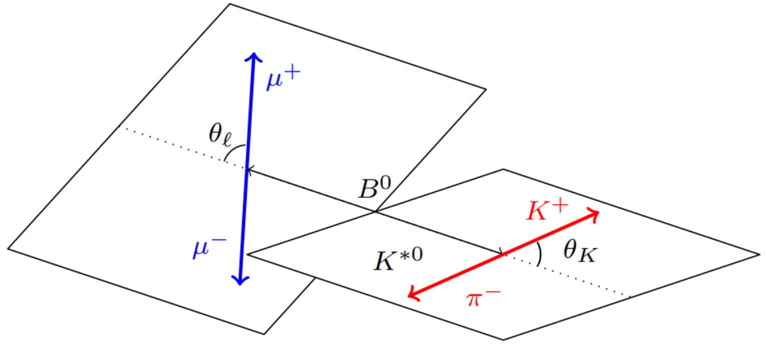




近期理论发展：极化角分布

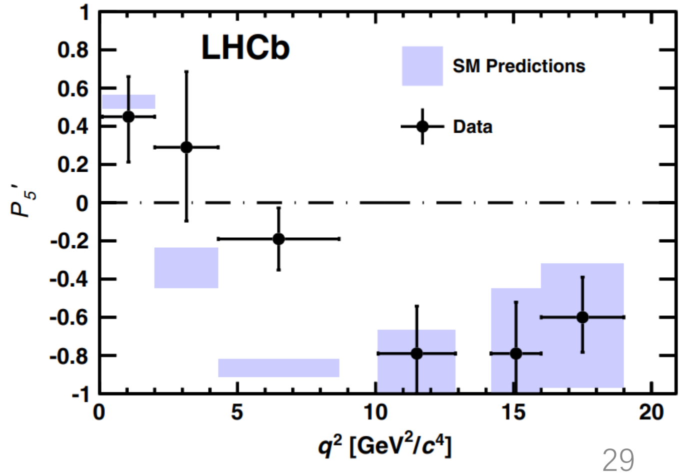
Angular distributions: BSM

- Large data samples collected by BESIII, Belle II and LHCb, much more observables to distinguish the new physics operators
- For example, the P_5' anomaly in $B \to K^*\mu^+\mu^-$ [LHCb, 2013, 2016]



$$
\frac{1}{d\Gamma/dq^2}\frac{d^4\Gamma}{d\cos\theta_\ell d\cos\theta_K d\phi dq^2}
= \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta_K + \frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell
- F_L\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi
+ S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi
+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big],
$$

$$
P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}
$$



Angular distributions: BSM

- Recently, a lot of works on angular distribution for BSM.
- Measurable angular distributions of

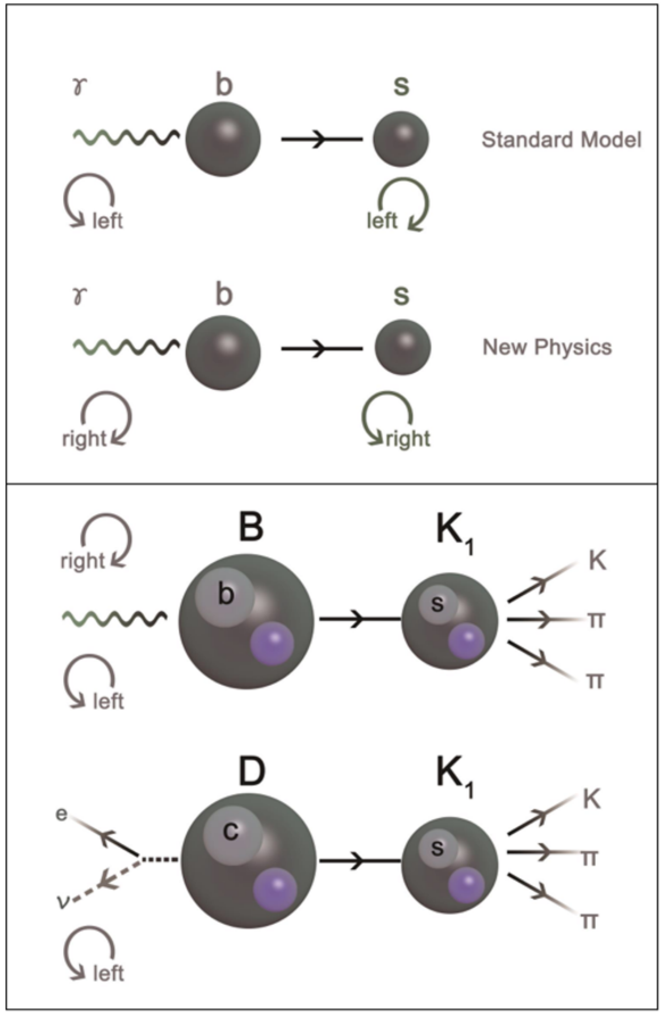
$B_c^- \to J/\psi(\to \mu^+\mu^-)\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ Q.Y.Hu, X.Q.Li, X.L.Mu, Y.D.Yang, D.H.Zheng, 2104.04942

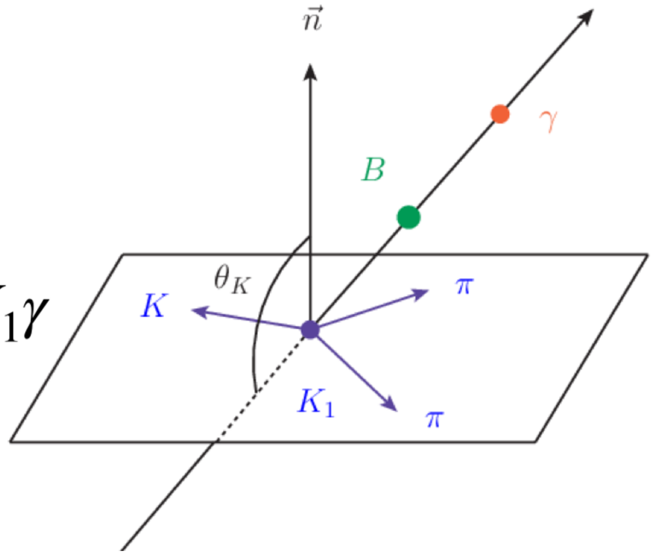
$\Lambda_b^0 \to \Lambda_c^+(\to \Lambda^0\pi^+)\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ Q.Y.Hu, X.Q.Li, Y.D.Yang, D.H.Zheng, 2011.05912
X.L.Mu, Y.Li, Z.T.Zou, B.Zhu, 1909.10769

- $\tau \to K_S\pi\nu$ F.Z.Chen, X.Q.Li, Y.D.Yang, 2003.05735
- $B \to D^*\ell\nu$ Z.R.Huang, E.Kou, C.D.Lu, R.Y.Tang, 2106.13855
- $B \to S\ell^+\ell^-$, $B \to P\ell^+\ell^-$, $B \to V\ell^+\ell^-$, $B \to A\ell^+\ell^-$, $\Lambda_b \to \Lambda\ell^+\ell^-$

Bhutta, Z.R.Huang, C.D.Lu, Ali Paracha, W.Y.Wang, 2009.03588

Angular distributions: photon polarization





$$B \to K_1\gamma$$

$$\lambda_\gamma = \frac{|C'_{7\gamma}|^2 - |C_{7\gamma}|^2}{|C'_{7\gamma}|^2 + |C_{7\gamma}|^2}$$

$$\mathcal{A}(K_1 \to K\pi\pi) = \vec{\epsilon}_{K_1} \cdot \vec{J}$$

Up-down asymmetry: $$\mathcal{A}_{\rm UD} \equiv \frac{\Gamma_{K_1\gamma}[\cos\theta_K > 0] - \Gamma_{K_1\gamma}[\cos\theta_K < 0]}{\Gamma_{K_1\gamma}[\cos\theta_K > 0] + \Gamma_{K_1\gamma}[\cos\theta_K < 0]}$$

$$= \lambda_\gamma \frac{3}{4} \boxed{\frac{\mathrm{Im}[\vec{n} \cdot (\vec{J} \times \vec{J}^*)]}{|\vec{J}|^2}}$$

measured

$A_{\rm UD} = (6.9 \pm 1.7) \times 10^{-2}$

at $m_{K\pi\pi} = [1.1,\ 1.3]$ GeV

LHCb, '14

desired

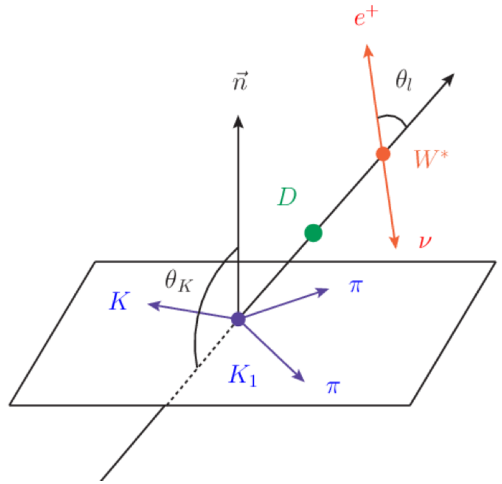
Unknown hadronic input

non-perturbative quantity

Ratio of up-down asymmetry

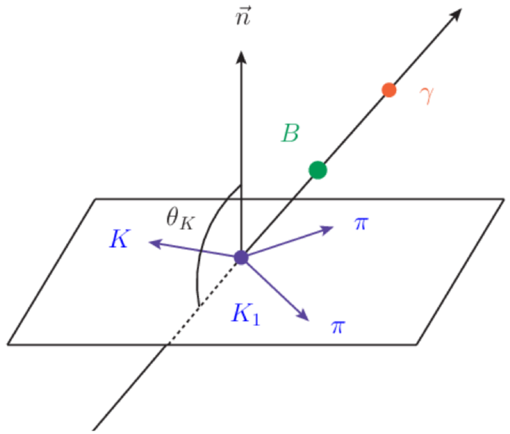
$D \to K_1 e\nu$

$$\mathcal{A}'_{\rm UD} \equiv \frac{\Gamma_{K_1 e\nu_e}[\cos\theta_K > 0] - \Gamma_{K_1 e\nu_e}[\cos\theta_K < 0]}{\Gamma_{K_1 e\nu_e}[\cos\theta_l > 0] - \Gamma_{K_1 e\nu_e}[\cos\theta_l < 0]} = \frac{{\rm Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)]}{|\vec{J}|^2}$$



$B \to K_1\gamma$

$$\mathcal{A}_{\rm UD} \equiv \frac{\Gamma_{K_1\gamma}[\cos\theta_K > 0] - \Gamma_{K_1\gamma}[\cos\theta_K < 0]}{\Gamma_{K_1\gamma}[\cos\theta_K > 0] + \Gamma_{K_1\gamma}[\cos\theta_K < 0]} = \lambda_\gamma \frac{3}{4}\frac{{\rm Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)]}{|\vec{J}|^2}$$



Photon polarization can be **model-independently and reliably** obtained by

$$\boxed{\lambda_\gamma = \frac{4}{3}\frac{\mathcal{A}_{\rm UD}}{\mathcal{A}'_{\rm UD}}}$$

if $A'_{\rm UD}$ in $D \to K_1 e\nu$ is measured

Wang, Yu, Zhao, PRL125, 051802 (2020)

Angular distributions: CPV

- Rich resonance structures in multi-body decays of b hadrons.
- Except for individual resonances, interference between different resonances might also induce large CP violation, due to possible large strong phases.
- Angular analysis can help to isolate the interference effect between different resonances: Partial-Wave CPV [Z.H.Zhang, X.H.Guo, 2102.12263, 2103.11335]
- Interference of $\rho^0(770)$ and $f_0(500)$ in $B^- \to \pi^+\pi^-\pi^-$

$$\mathcal{A} = a_S + a_P c_\theta$$

$$A^{FB}_{H\to h_1 h_2 h_3 \cdots h_n} = \frac{\Gamma_H(c_{\theta_1^*} > 0) - \Gamma_H(c_{\theta_1^*} < 0)}{\Gamma_H(c_{\theta_1^*} > 0) + \Gamma_H(c_{\theta_1^*} < 0)}$$

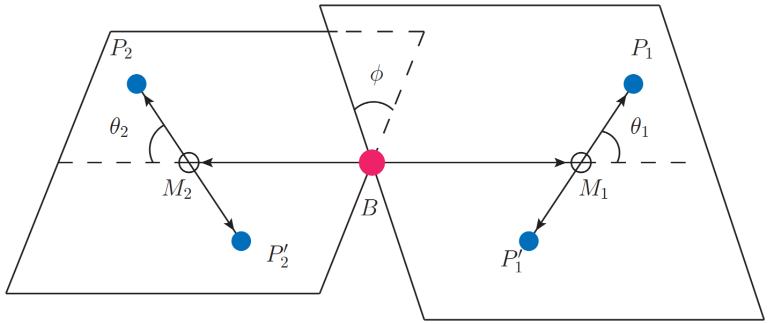
$$A^{FB}_{CP} = \frac{\Re(\langle a_P a_S^* \rangle)}{|\langle a_P \rangle|^2/3 + |\langle a_S \rangle|^2} - \frac{\Re(\langle \bar{a}_P \bar{a}_S^* \rangle)}{|\langle \bar{a}_P \rangle|^2/3 + |\langle \bar{a}_S \rangle|^2}$$

- A more general case

$$d\Gamma \propto \overline{|\mathcal{M}|^2} dc_{\theta_1^*} \qquad \overline{|\mathcal{M}|^2} \propto \sum_{j=0}^{\infty} w^{(j)} P_j(c_{\theta_1^*}) \qquad w^{(j)} \propto \int_{-1}^{1} \overline{|\mathcal{M}|^2} P_j dc_{\theta_1^*} \qquad A^{(j)}_{CP} \equiv \frac{w^{(j)} - \bar{w}^{(j)}}{w^{(j)} + \bar{w}^{(j)}}$$

Angular distributions: TPA

- Four-body decays of $B \to (P_1 P_1')(P_2 P_2')$
- Triple-product: $TP = \vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$ with $\vec{v}_i$ as a spin or a momentum. TP is odd under time reversal. [Valencia, 1989; Datta, London, 2003]



$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}$$

- Triple-product asymmetry indued CP violation: $\mathcal{A}_T \equiv \frac{1}{2}(A_T + \bar{A}_T)$
- Complementary to direct CPV: $A_{CP}^{dir} \propto \sin\phi \sin\delta, \quad A_{TP} \propto \sin\phi \cos\delta$
- TPAs are recently predicted firstly based on the QCD-inspired method: PQCD with two-meson distribution amplitudes [Z.Rui, Y.Li, H.n.Li, 2103.00642; Y.Li, D.C.Yan, Z.Rui, H.n.Li, 2107.10684]

$$A \propto \Phi_B \otimes H \otimes \Phi_{P_1 P_1'} \otimes \Phi_{P_2 P_2'}$$