Lattice QCD and high-intensity frontier

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Frontiers in high energy physics

Discovery of Higgs boson ⇒ Nobel prize to Englert & Higgs

⇒ The final particle in the Standard Model is found!

Three frontiers to search for Physics Beyond Standard Model

- **Cosmic frontier**
  ⇒ detect dark matter, energy and cosmically-produced new particles
- **High-energy frontier**
  ⇒ increase collision energy, directly produce new particles
- **High-intensity frontier**
  ⇒ precisely measure rare processes, look for discrepancies with SM

This requires the precise prediction from Standard Model

Short slab of cask: non-perturabtive QCD obstructs the predication

QCD: different strategies

QCD is the fundamental theory

$\Rightarrow$ describing strong interaction between quarks and gluons

- High-Q ($>$ few GeV) $\leftrightarrow$ short distance (< 0.1 fm)
  $\Rightarrow$ Theory of weakly interacting quarks and gluons
  $\Rightarrow$ (Perturbative QCD: Gross, Politzer, Wilczek for asymptotic freedom)

- Low-Q ($\ll 1$ GeV) $\leftrightarrow$ long distance (> 1 fm)
  $\Rightarrow$ Spontaneous chiral symmetry breaking
  $\Rightarrow$ EFT of weakly interacting Nambu-Goldstone bosons
  $\Rightarrow$ EFT treats hadrons as dynamical degree of freedom (no quarks, gluons)

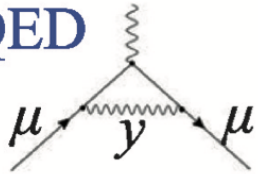
- Lattice QCD
  $\Rightarrow$ Large-scale supercomputer simulation on Euclidean spacetime lattice
  - Provide most accurate α_s for pQCD
  - Provide LECs for EFT

Application of LQCD: Muon g-2

Contributions from the Standard Model

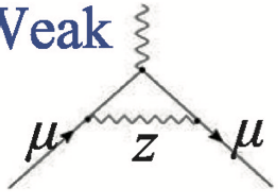
$$a_\mu(SM)=a_\mu(QED)+a_\mu(Weak)+a_\mu(Hadronic)$$

QED



+… $116584718.9(1)\times10^{-11}$ 0.001 ppm

Weak

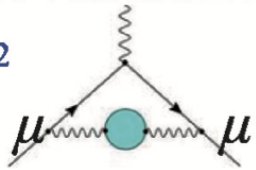


+… $153.6(1.0)\times10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

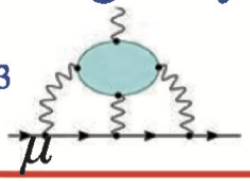
α^2



+… $6845(40)\times10^{-11}$ [0.6%] 0.37 ppm

…Light-by-Light (HLbL)

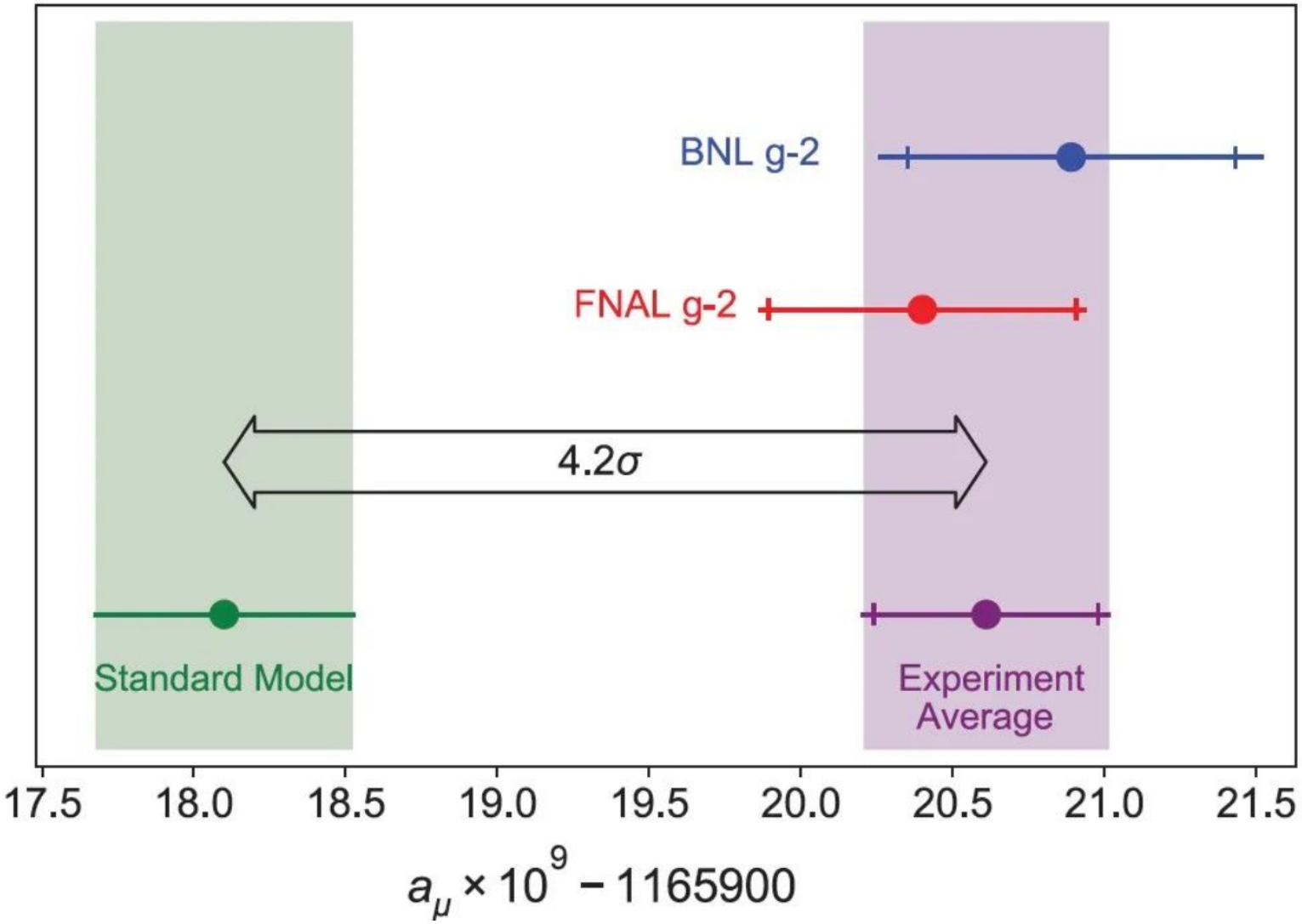
α^3



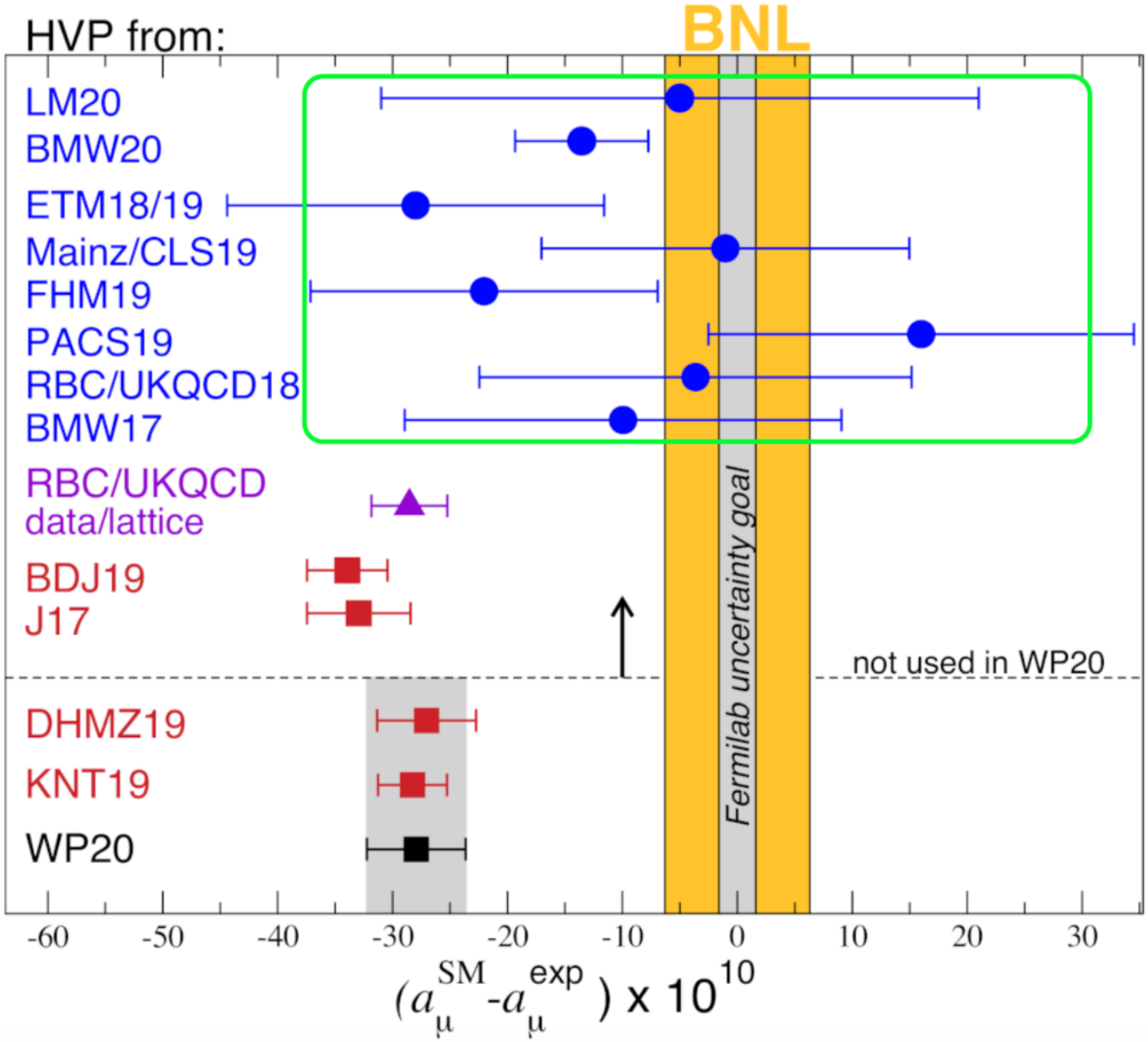
+… $92(18)\times10^{-11}$ [20%] 0.15 ppm

Uncertainty dominated by hadronic contributions

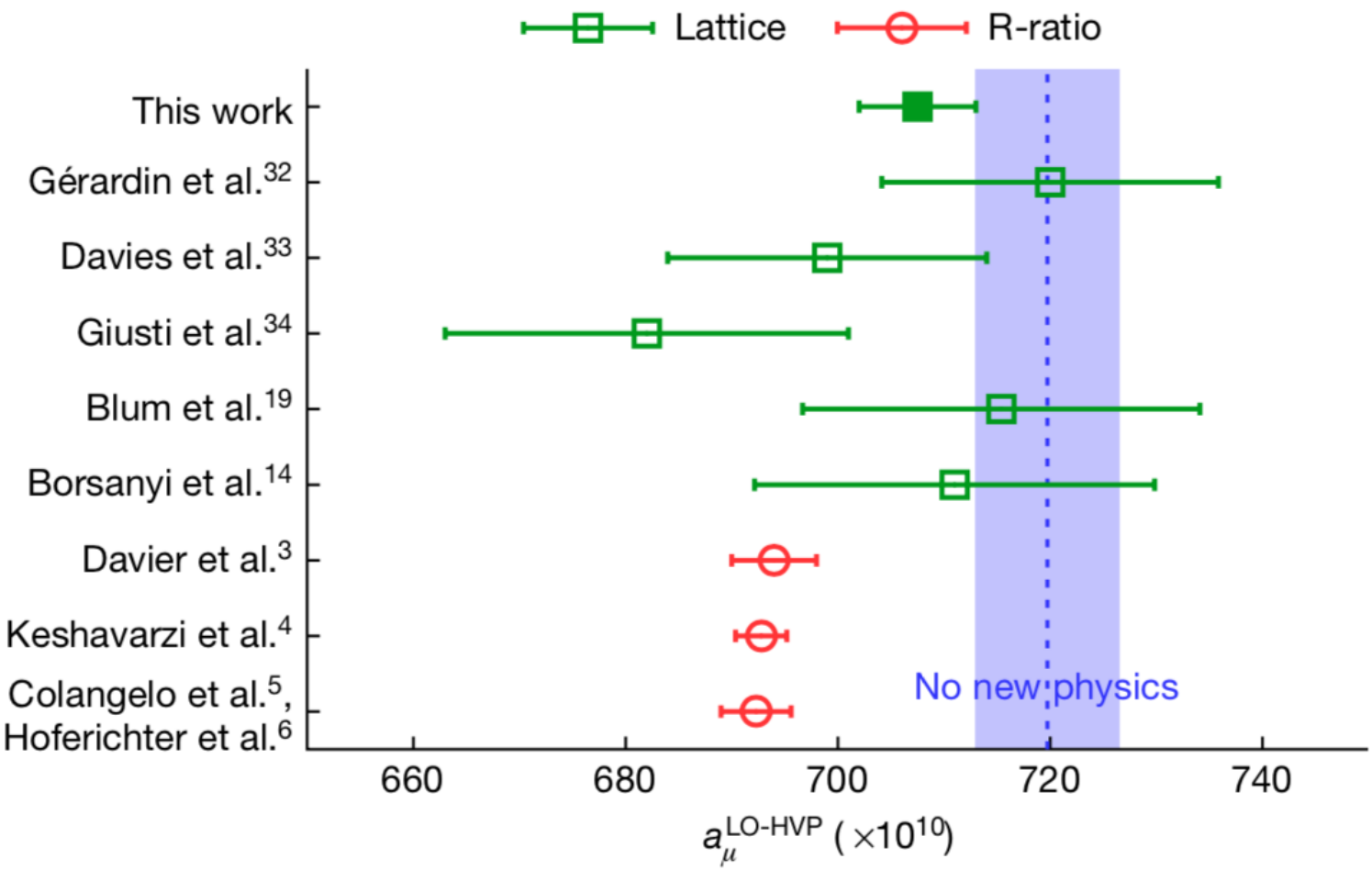
Discrepancy between theory and experiment



Hadronic vacuum polarization contributions



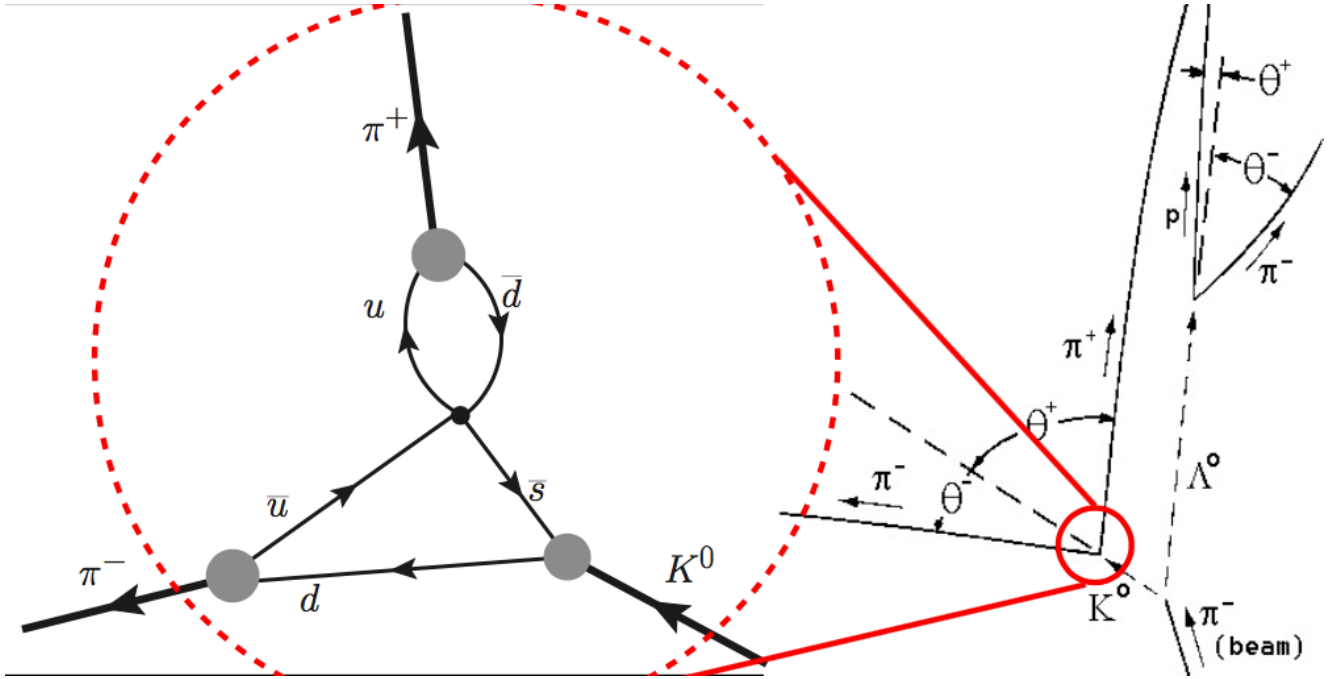
Results from BMW Collaboration



"Our lattice result shows some tension with the R-ratio determinations of refs.3– 6. Obviously, our findings should be confirmed - or refuted - by other studies using different discretizations of QCD. Those investigations are underway." - quoted from BMW's paper - Nature (2021)

Application of LQCD: Kaon decays and CP violation

[RBC-UKQCD, latest results, arXiv:2004.09440]



- *CP* violation: $\mathrm{Re}[\epsilon'/\epsilon]$

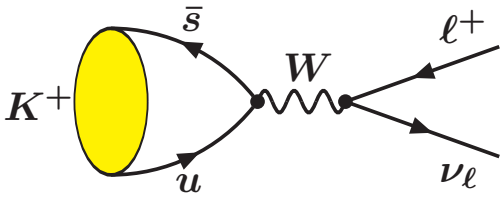
$$\mathrm{Re}[\epsilon'/\epsilon] = 21.7(2.6)_{\mathrm{stat}}(8.0)_{\mathrm{syst}} \times 10^{-4} \quad \text{Lattice}$$
$$\mathrm{Re}[\epsilon'/\epsilon] = 16.6(2.3) \times 10^{-4} \quad \text{Experiment}$$

theoretical uncertainty$\sim$ 40%, experimental uncertainty$\sim$ 14%, theory consistent with experiment

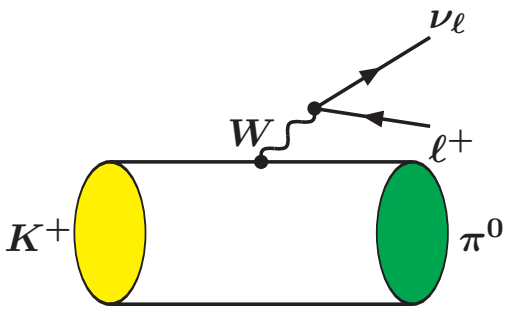
Application of LQCD: Flavor physics

Evaluate the hadronic matrix elements for electroweak processes

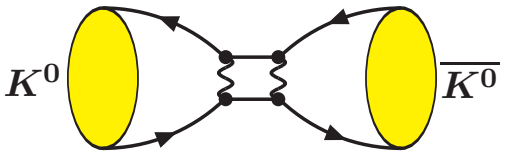
- Lattice QCD is powerful for "standard" hadronic matrix elements with



$K_{\ell 2} \to f_K$



$K_{\ell 3} \to f_+(0)$



K^0-$\overline{K^0}$ mixing $\to B_K$

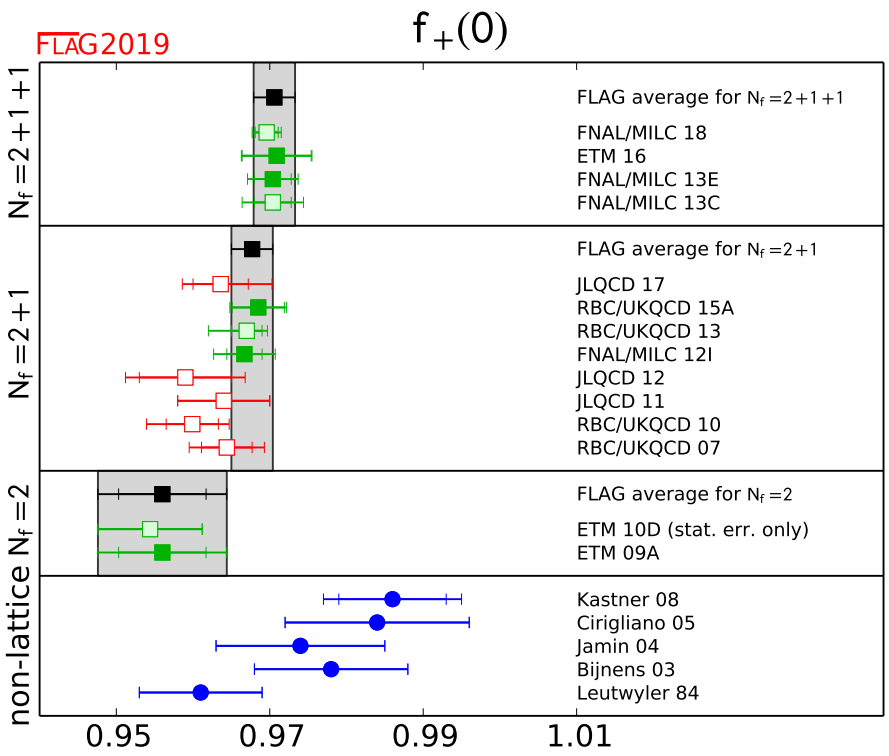
- single local operator insertion
- only single stable hadron or vacuum in the initial/final state
- Requires only two- or three-point correlation functions

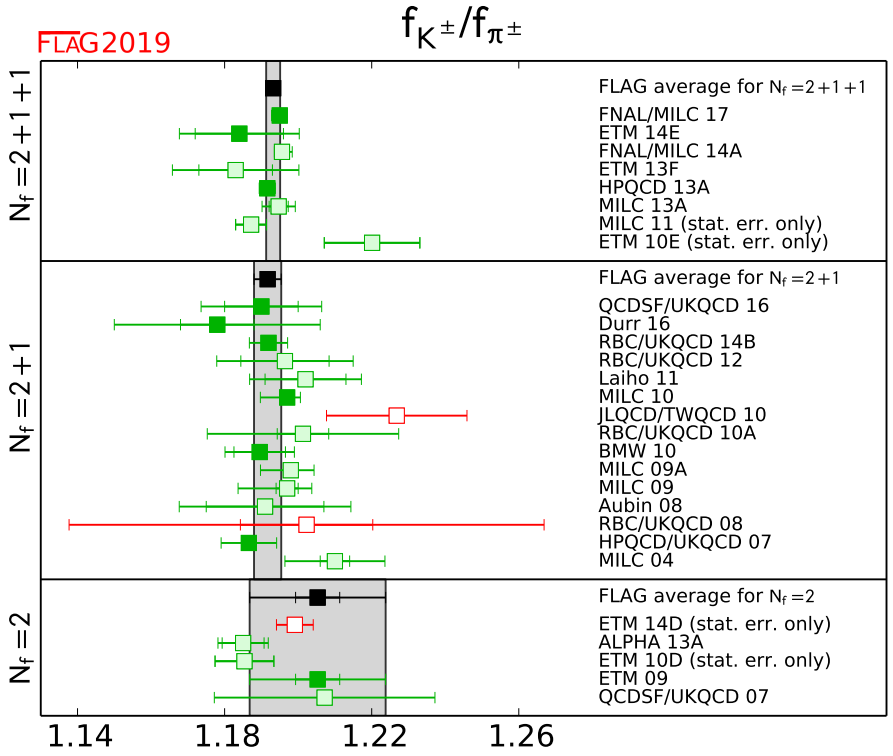
Precision era for lattice QCD

Flavor Lattice Averaging Group (FLAG) average 2021 [arXiv:2111.09849]

$$f_+^{K\pi}(0) = 0.9698(17) \quad \Rightarrow \quad 0.18\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1932(21) \quad \Rightarrow \quad 0.18\% \text{ error}$$





Experimental information [arXiv:1411.5252, 1509.02220]

$$K_{\ell 3} \quad \Rightarrow \quad |V_{us}| f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2232(6)$$

$$K_{\mu 2}/\pi_{\mu 2} \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2320(5)$$

Flag average 2021

Error $< 1\%$

	N_f	FLAG average	Frac. Err.
f_K/f_π	$2+1+1$	1.1932(21)	0.18%
$f_+(0)$	$2+1+1$	0.9698(17)	0.18%
f_D	$2+1+1$	212.0(7) MeV	0.33%
f_{D_s}	$2+1+1$	249.9(5) MeV	0.20%
f_{D_s}/f_D	$2+1+1$	1.1783(16)	0.13%
$f_+^{DK}(0)$	$2+1+1$	0.7385(44)	0.60%
f_B	$2+1+1$	190.0(1.3) MeV	0.68%
f_{B_s}	$2+1+1$	230.3(1.3) MeV	0.56%
f_{B_s}/f_B	$2+1+1$	1.209(5)	0.41%

Error $< 5\%$

	N_f	FLAG average	Frac. Err.
$\hat{B}_K$	$2+1$	0.7625(97)	1.3%
$f_+^{D\pi}(0)$	$2+1$	0.666(29)	4.4%
$\hat{B}_{B_s}$	$2+1$	1.35(6)	4.4%
B_{B_s}/B_{B_d}	$2+1$	1.032(28)	3.7%
...			

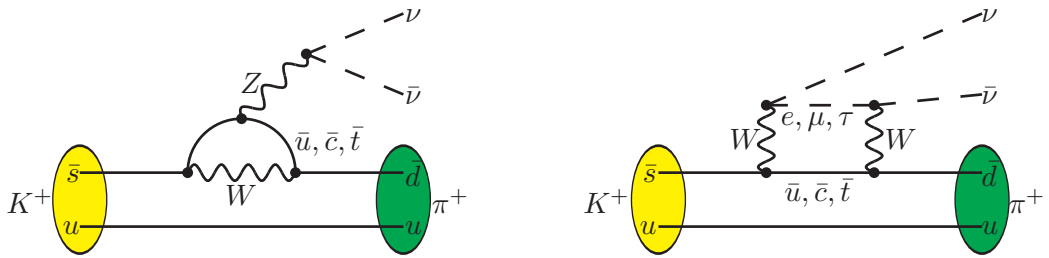
Time to go beyond leading-order electroweak transitions

Exploration at new frontiers

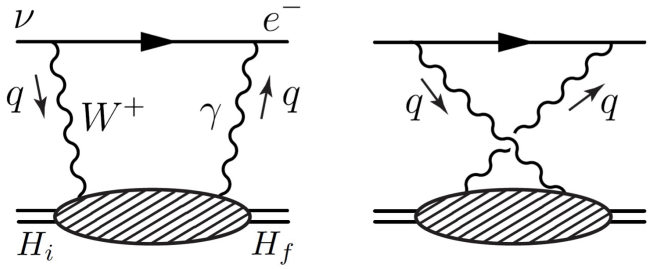
Go for higher-order electroweak processes – opportunities

Opportunities in flavor physics

- Rare decays, e.g. $\mathrm{Br}[K^+ \to \pi^+ \nu\bar{\nu}] = 1.73^{+1.15}_{-1.05} \times 10^{-10}$



- Electroweak radative corrections to hadronic decays
  $\Rightarrow$ superallowed nuclear β decay half-life time with precision 10^{-6}

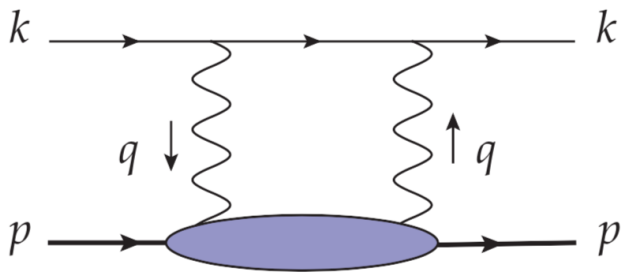


- Proton's weak charge $Q_W^p = 1 - 4\sin^2\theta_W$
  $\Rightarrow$ 0.3%-precision measurement of $\sin^2\theta_W$ by Q-weak at JLab
  - Parity-violating e-p scattering, $\Box^V_{\gamma Z}$ contribution

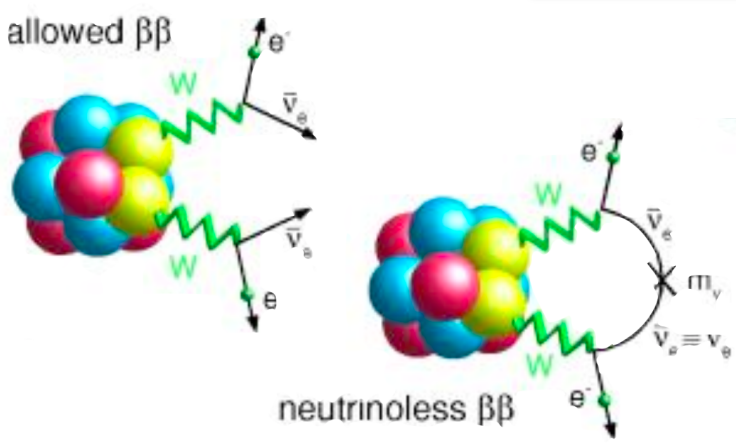
Go for higher-order electroweak processes – opportunities

Opportunities in nuclear physics

- Muonic hydrogen spectrum $\rightarrow$ proton charge radius $r_p = 0.84087(39)$ fm
  $\Rightarrow$ 10 times more accurate than e-p scattering



- Neutrinoless double beta decays

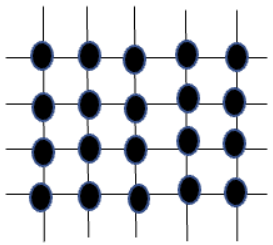


- Hadron electromagnetic polarizability

Go for higher-order electroweak processes – challenges

Computational demanding

- Three-point function



$$\sum_{x \in L^3} \square$$

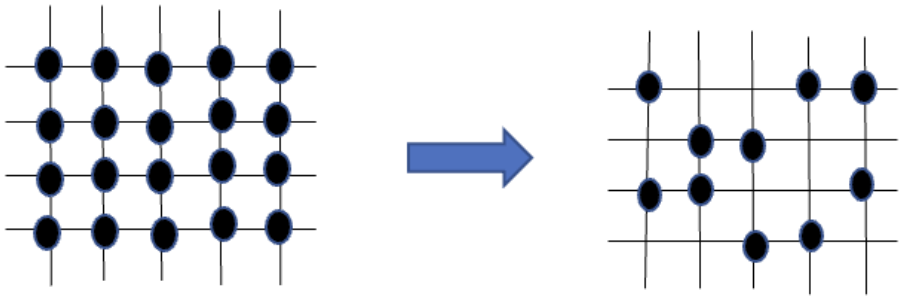
$$\langle H_f(x_f) O(0) H_i^\dagger(x_i) \rangle \quad \Rightarrow \quad \int d^3\vec{x}_i \int d^3\vec{x}_f \quad \Rightarrow \quad \sum_{\vec{x}_i} \sum_{\vec{x}_f} \sim L^6$$

- Four-point function

$$\langle H_f(x_f) O_1(x) O_2(0) H_i^\dagger(x_i) \rangle \rangle \quad \Rightarrow \quad \int d^3\vec{x}_i \int d^3\vec{x}_f \int d^3\vec{x} \quad \Rightarrow \quad \sum_{\vec{x}_i} \sum_{\vec{x}_f} \sum_{\vec{x}} \sim L^9$$

with $L = 24, 32, 48, 64, 96, \cdots$

Field sparsening technique



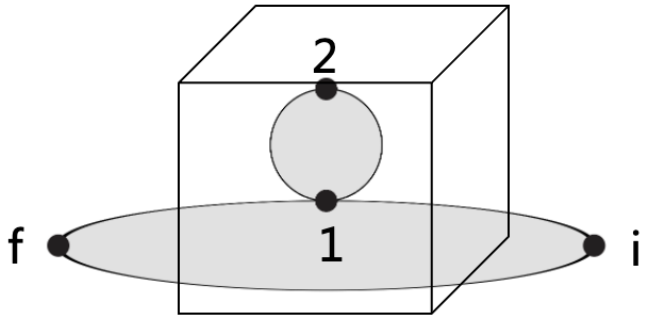
Y. Li, S. Xia, XF, L. Jin, C. Liu, PRD 103 (2021) 014514

Go for higher-order electroweak processes – challenges

Divergence due to intermediate multi-particle states

$$\int dt \, \langle H_f | O_1(t) O_2(0) | H_i \rangle = \sum_n \frac{\langle H_f | O_1(0) | n \rangle \langle n | O_2(0) | H_i \rangle}{E_n - E_i}$$

- In finite volume, state $|n\rangle$ is discrete $\Rightarrow$ Divergence at $E_n \approx E_i$
- In infinite volume, summation $\sum_n$ replaced by $\int dE$ $\Rightarrow$ No divergence



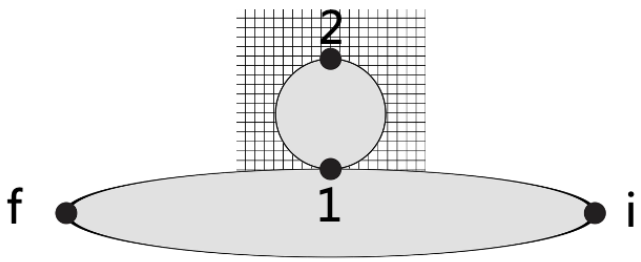
Development of finite-volume correction formula

$$\Delta_{FV} = \frac{k}{16\pi E} \cot(\phi + \delta) \left| \langle H_f | O_1(0) | n \rangle_\infty \, {}_\infty\langle n | O_2 | H_i \rangle \right|$$

N. Christ, XF, G. Martinelli, C. Sachrajda, PRD 91 (2015) 11, 114510

Go for higher-order electroweak processes – challenges

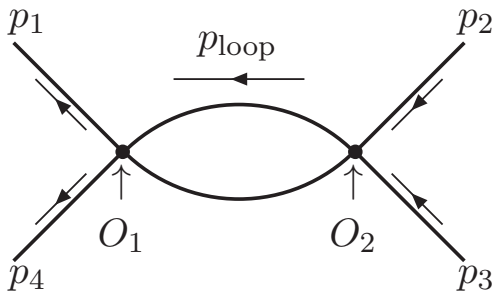
Short-distance divergence in $O_1(x)O_2(0)$ when $x \to 0$



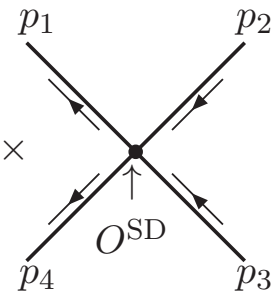
- With lattice spacing $a \to 0$, lattice cutoff effects $\sim O(a^{-2})$ or $O(\ln a^2)$
  $\Rightarrow$ No continuum limit!

Define renormalized bilocal operator

$\langle\{O_1 O_2\}^{\rm RI}\rangle\Big|_{p_i^2=\mu_{RI}^2} =$



$- X(\mu_{RI}, a) \times$



$= 0$

- Subtract $X(\mu_{RI}, a) O^{\rm SD}$ to remove the lattice cutoff effects
- Add the physical short-distance contribution from perturbation theory

N. Christ, XF, A. Portelli, C. Sachrajda, PRD 93 (2016) 114517

Our recent work on higer-order EW processes (meson sector)

- QCD+QED & pion mass splitting

  [XF, L. Jin, PRD100 (2019) 094509]
  [XF, L. Jin, M. Riberdy, arXiv:2108.05311]

- Rare kaon decays $K^+ \to \pi^+ \nu \bar{\nu}$

  [Z. Bai, XF, N. Christ, et.al. PRL118 (2017) 252001]

- Electroweak box contribution to $\pi_{\ell 3}$ and $K_{\ell 3}$ decay

  [XF, M. Gorchtein, L. Jin, P. Ma, C. Seng, PRL124 (2020) 192002]
  [P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, PRD103 (2021) 114503]

- Neutrinoless double beta decays

  [XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001]
  [X. Tuo, XF, L. Jin, PRD100 (2019) 094511]

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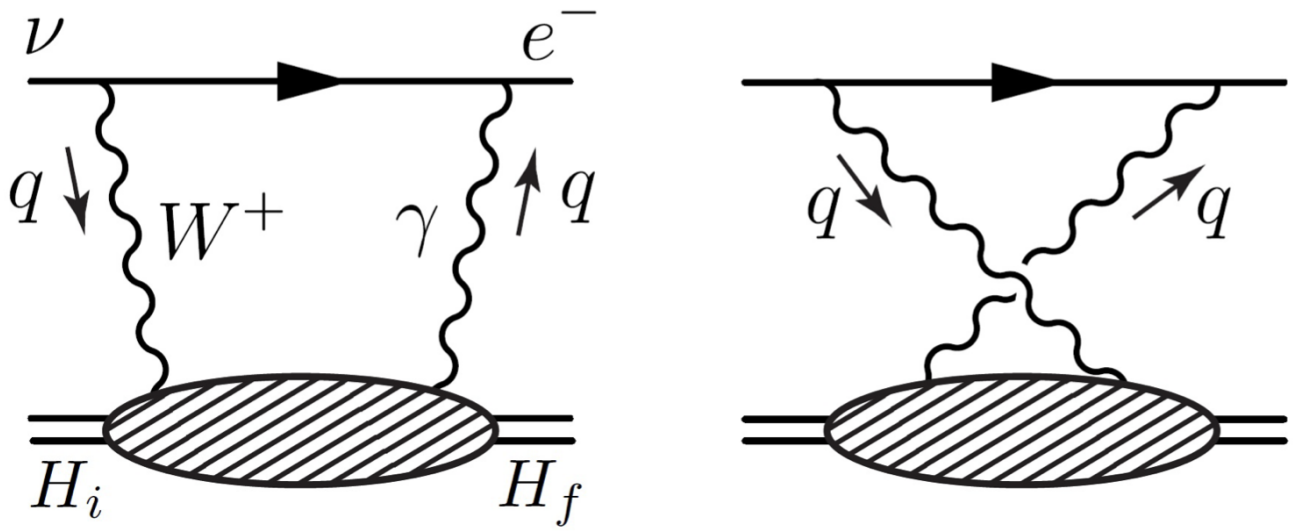
Electroweak box diagram



CKM unitarity – a constraint from Standard Model

First-row CKM unitarity

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

PDG 2019 $\Rightarrow$ PDG 2021

	PDG 2019	PDG 2021
$\lvert V_{ud}\rvert$	0.97420(21)	0.97370(14)
$\lvert V_{us}\rvert$	0.2243(5)	0.2245(8)
$\lvert V_{ub}\rvert$	0.00394(36)	0.00382(24)
Δ_{CKM}	$-0.00061(47)$	$-0.00149(45)$

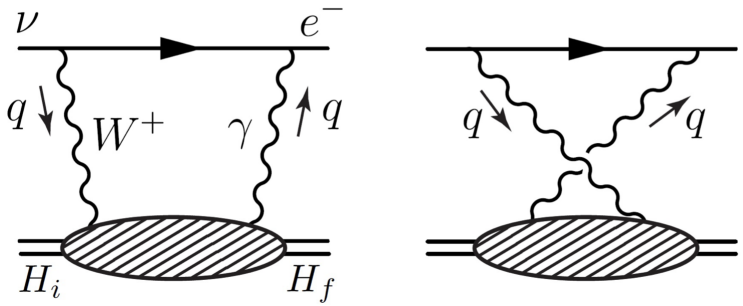
- Main update from $|V_{ud}| \Rightarrow 3.3\ \sigma$ deviation from CKM unitarity
- $|V_{ud}|$ is from superallowed $0^+ \to 0^+$ nuclear beta decay
  - Pure vector transitions at leading order
  - Uncertainty is dominated by electroweak radiative correction

[J. Hardy, I. Towner, PRC 91 (2015) 025501]

Axial γW-box diagram

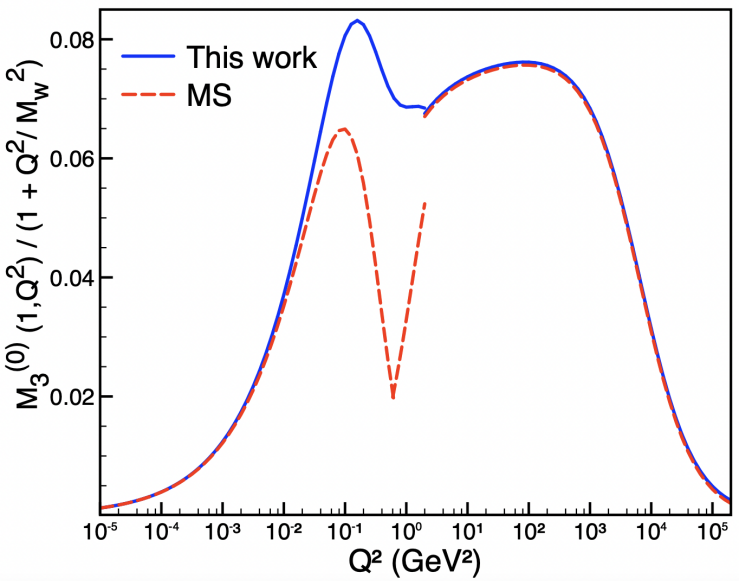
Based on current algebra, only axial γW-box diagram sensitive to hadronic scale

[A. Sirlin, Rev. Mod. Phys. 07 (1978) 573]



$$T^{VA}_{\mu\nu} = \frac{1}{2}\int d^4x\, e^{iqx} \langle H_f(p)| T\left[J^{em}_\mu(x) J^{W,A}_\nu(0)\right] |H_i(p)\rangle$$

Re-evaluation of the γW-box diagram



$|V_{ud}| = 0.97420(18)_{\rm RC}(10)_{\mathcal{F}t}$

Using VMD model

[Marciano & Sirlin, PRL 2006]

$\Downarrow$

$|V_{ud}| = 0.97370(10)_{\rm RC}(10)_{\mathcal{F}t}$

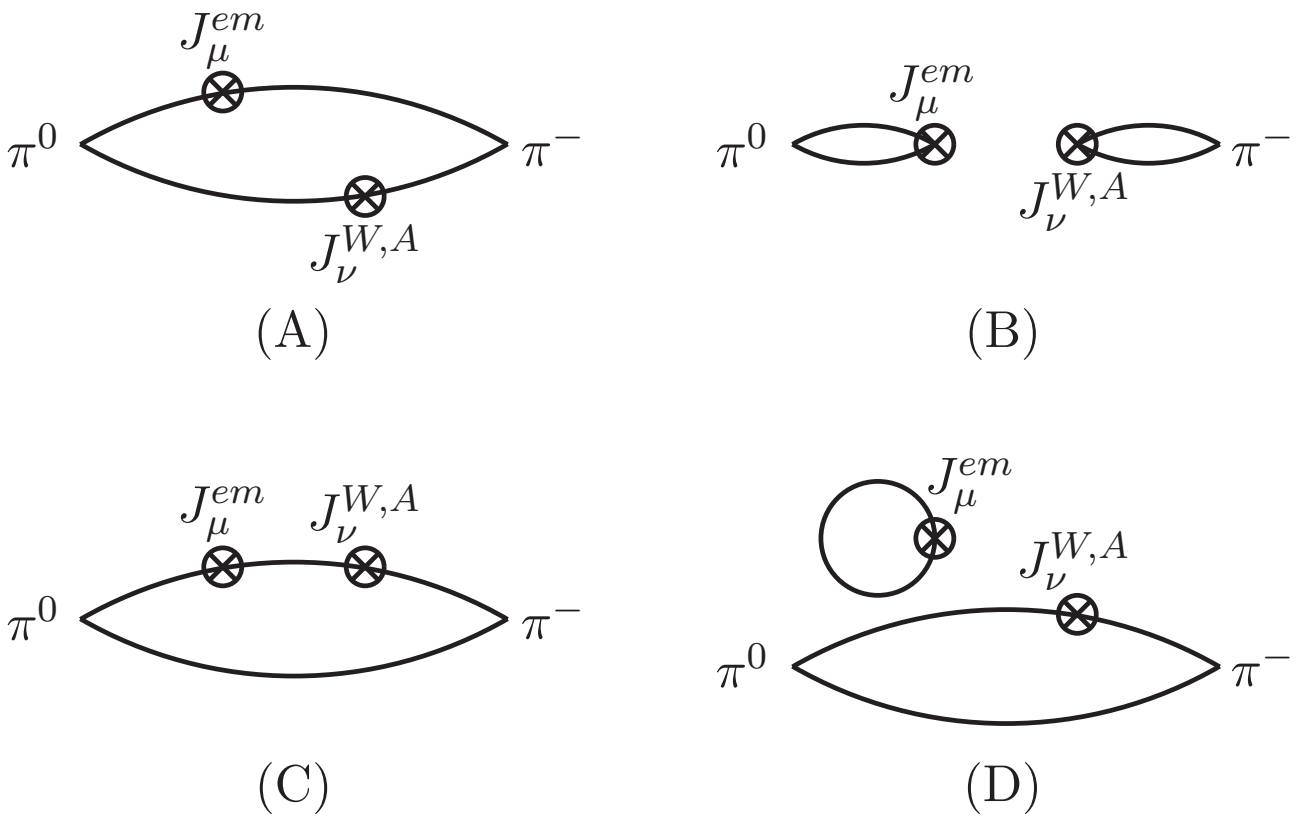
Using dispsive & data-driven analysis

[Seng et.al. PRL 2018]

$> 3\sigma$ violation of CKM unitarity $\Rightarrow$ first-principle calculation

Quark contractions for the γW-box diagrams

$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p)| T\left[J^{em}_\mu(x) J^{W,A}_\nu(0)\right] |\pi^-(p)\rangle$$

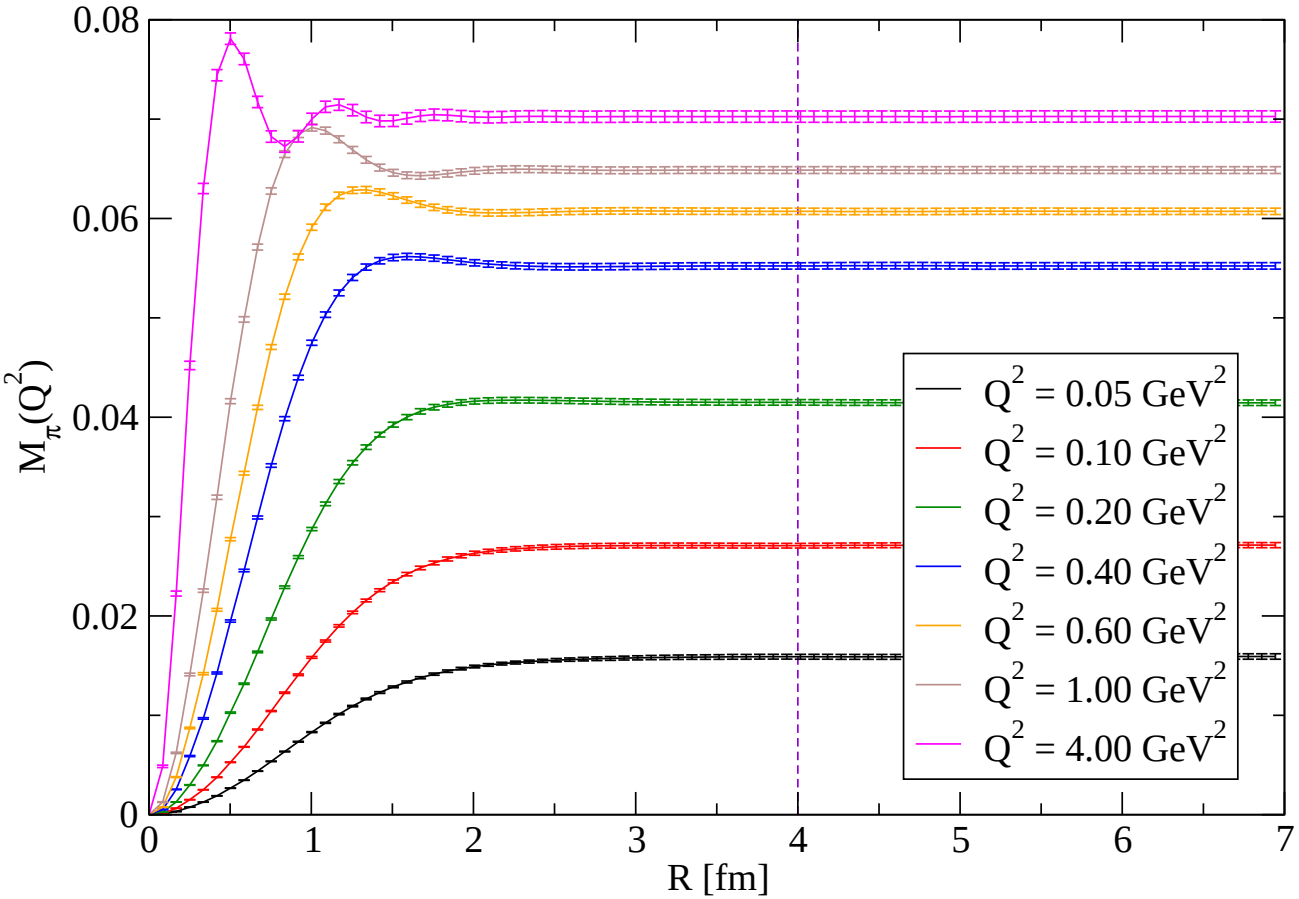


- Coulomb gauge fixed wall source is used for the pion interpolating field
- $J^{W,A}_\nu(0)$ is treated as a source and $J^{em}_\mu(x)$ is a sink
- Calculate $\mathcal{H}^{VA}_{\mu\nu}(x)$ as a function of x

Lattice results for the hadronic functions

Construct the Lorentz scalar function $M_\pi(Q^2)$ from $\mathcal{H}^{VA}_{\mu\nu}(x)$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}}\frac{\sqrt{Q^2}}{m_\pi}\int d^4x\,\omega(Q,x)\epsilon_{\mu\nu\alpha 0}x_\alpha \mathcal{H}^{VA}_{\mu\nu}(x)$$

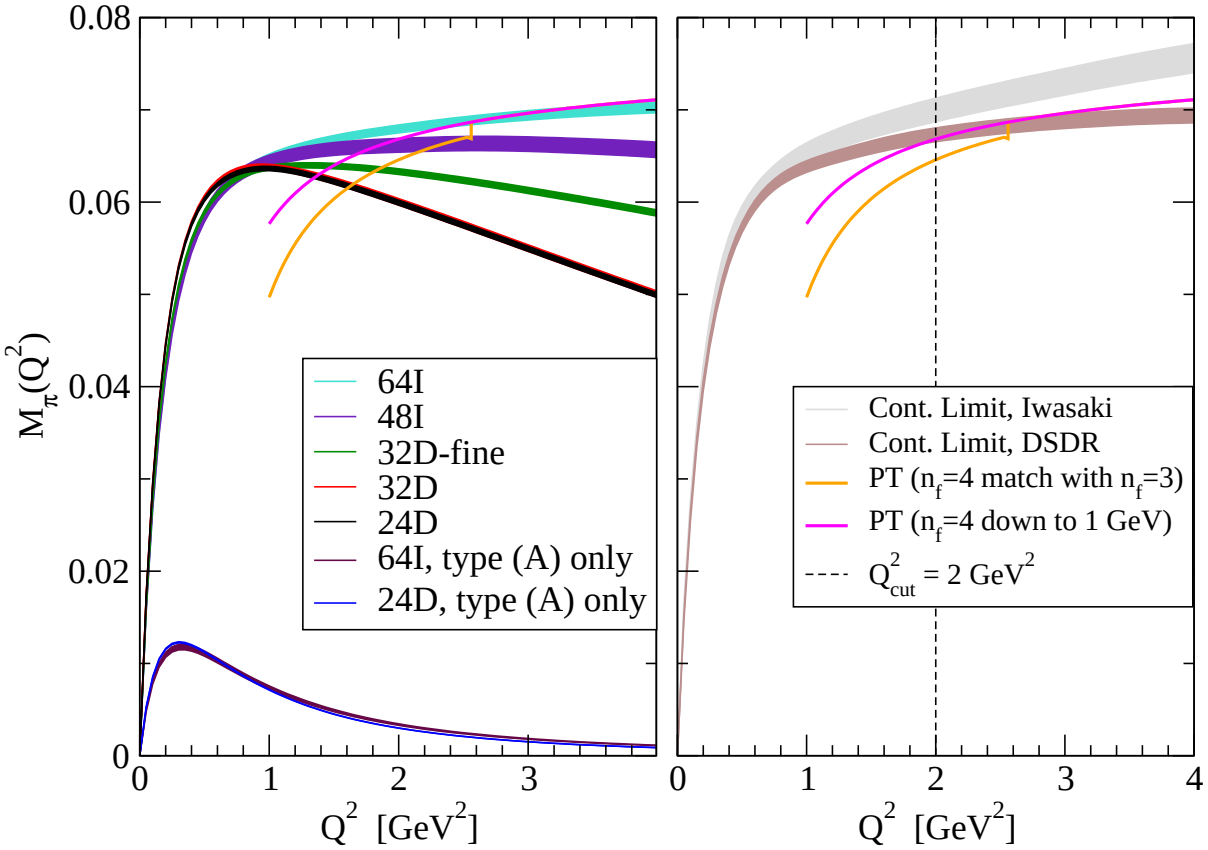


Combine lattice results with pQCD

Radiative correction requires the momentum integral from $0 < Q^2 < \infty$

$$\Box^{VA}_{\gamma W} = \frac{3\alpha_e}{2\pi} \int \frac{dQ^2}{Q^2} \frac{m_W^2}{m_W^2 + Q^2} M_\pi(Q^2)$$

- Lattice data used for low-Q^2 region
- OPE and perturbative Wilson coefficients used for high-Q^2 region



Error analysis

Use the momentum scale $Q^2_{\rm cut}$ to separate the LD and SD contributions

$$\Box^{VA}_{\gamma W} = \begin{cases} 2.816(9)_{\rm stat}(24)_{\rm PT}(18)_{\rm a}(3)_{\rm FV} \times 10^{-3} & \text{using } Q^2_{\rm cut} = 1 \text{ GeV}^2 \\ 2.830(11)_{\rm stat}(9)_{\rm PT}(24)_{\rm a}(3)_{\rm FV} \times 10^{-3} & \text{using } Q^2_{\rm cut} = 2 \text{ GeV}^2 \\ 2.835(12)_{\rm stat}(5)_{\rm PT}(30)_{\rm a}(3)_{\rm FV} \times 10^{-3} & \text{using } Q^2_{\rm cut} = 3 \text{ GeV}^2 \end{cases}$$

- When $Q^2_{\rm cut}$ increase, the lattice artifacts become larger
- When $Q^2_{\rm cut}$ decrease, systematic effects in pQCD become larger
- For 1 GeV$^2 \leq Q^2_{\rm cut} \leq$ 3 GeV^2, all results are consistent within uncertainties

Pion semileptonic β decay

Decay width measured by PIBETA experiment

$$\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta) I_\pi$$

- ChPT [Cirigliano et.al. (2002), Czarnecki, Marciano, Sirlin (2019)]

$$\delta = 0.0334(10)_{\rm LEC}(3)_{\rm HO}$$

- Sirlin's presentation [A. Sirlin, Rev. Mod. Phys. 07 (1978) 573]

$$\begin{aligned}\delta &= \frac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\frac{m_Z}{m_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right] + \delta_{\rm HO}^{\rm QED} + 2\Box_{\gamma W}^{VA} \\ &= 0.0332(1)_{\gamma W}(3)_{\rm HO}\end{aligned}$$

where $\frac{\alpha_e}{2\pi}\bar{g} = 1.051\times 10^{-2}$, $\frac{\alpha_e}{2\pi}\tilde{a}_g = -9.6\times 10^{-5}$, $\delta_{\rm HO}^{\rm QED} = 0.0010(3)$

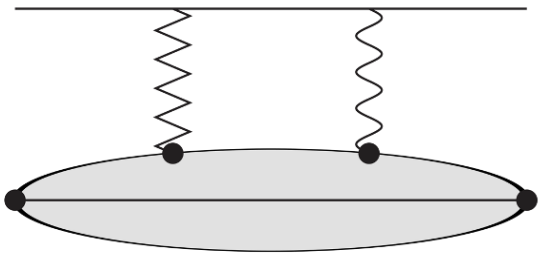
- Hadronic uncertainty reduced by a factor of 10, which results in

$$|V_{ud}| = 0.9739(28)_{\rm exp}(5)_{\rm th} \quad \Rightarrow \quad |V_{ud}| = 0.9739(28)_{\rm exp}(1)_{\rm th}$$

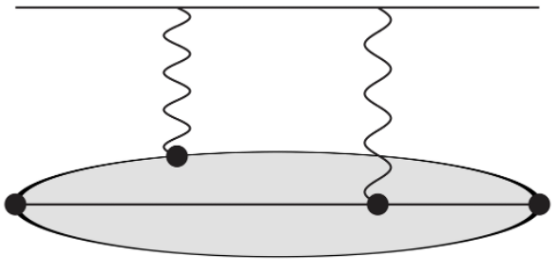
[XF, Gorchtein, Jin, Ma, Seng, PRL124 (2020) 192002]

First time to calculate γW box diagram $\Rightarrow$ method set up for nucleon decay

Move on to nucleon system

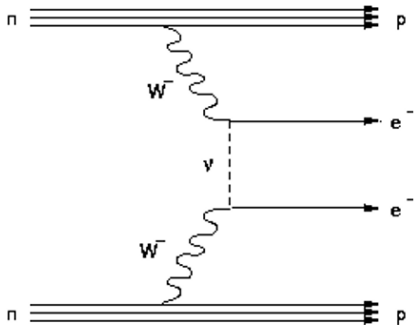


γW-box for neutron beta decay

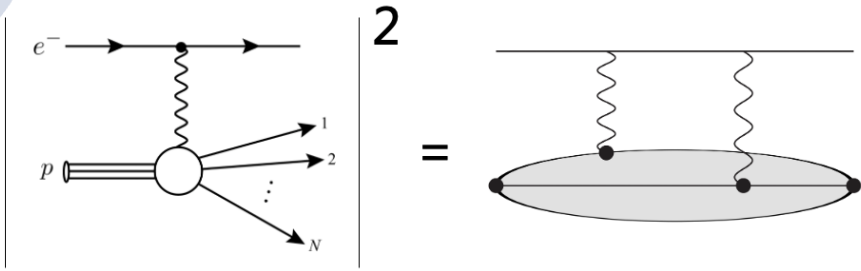


Puzzle of proton charge radius

Nucleon System



Neutrinoless double beta decay for dibaryon system

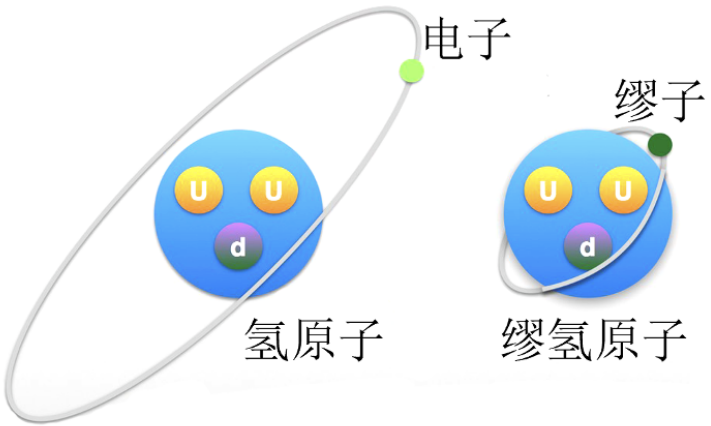


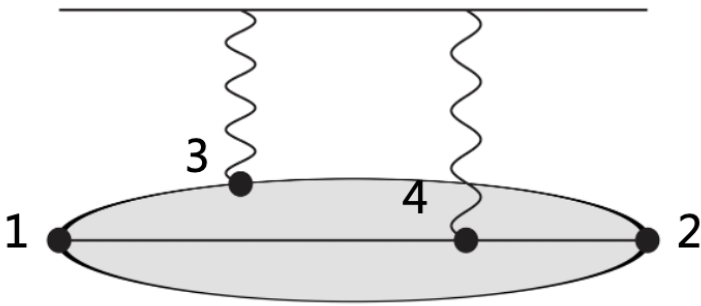
Lepton nucleon scattering

Puzzle of proton size

A decade puzzle since 2010

- Proton charge radius from μH spectroscopy differs from e-p scattering & H spectroscopy by 4%, $\sim$5 σ deviation

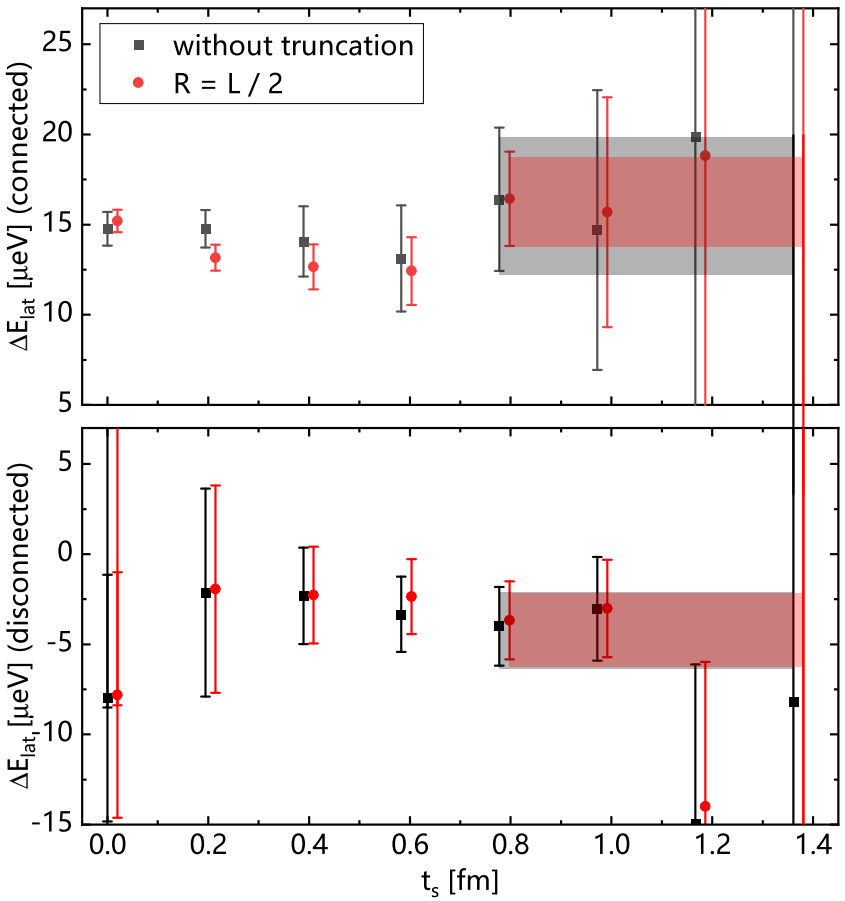




- Measurements from μH spectroscopy is 10 times more accurate
- Dominant theoretical uncertainty from two-photon exchange diagram

Two-photon exchange contribution to μH Lamb shift

Preliminary results @ $m_\pi = 142$ MeV



- To explain the puzzle, one needs $\Delta E_{\mathrm{TPE}} \sim 300\ \mu\mathrm{eV}$
- Recommended phenomenological value: $\Delta E_{\mathrm{TPE}} = 33.2(2.0)\ \mu\mathrm{eV}$
  [Science 339 (2013) 417. Ann. of Phy. 331 (2013), 127]
- Our lattice result: $\Delta E_{\mathrm{TPE}} = 54.7(3.2)\ \mu\mathrm{eV}$, statistical error only

Outlook