Light Nuclei Production and Critical Point

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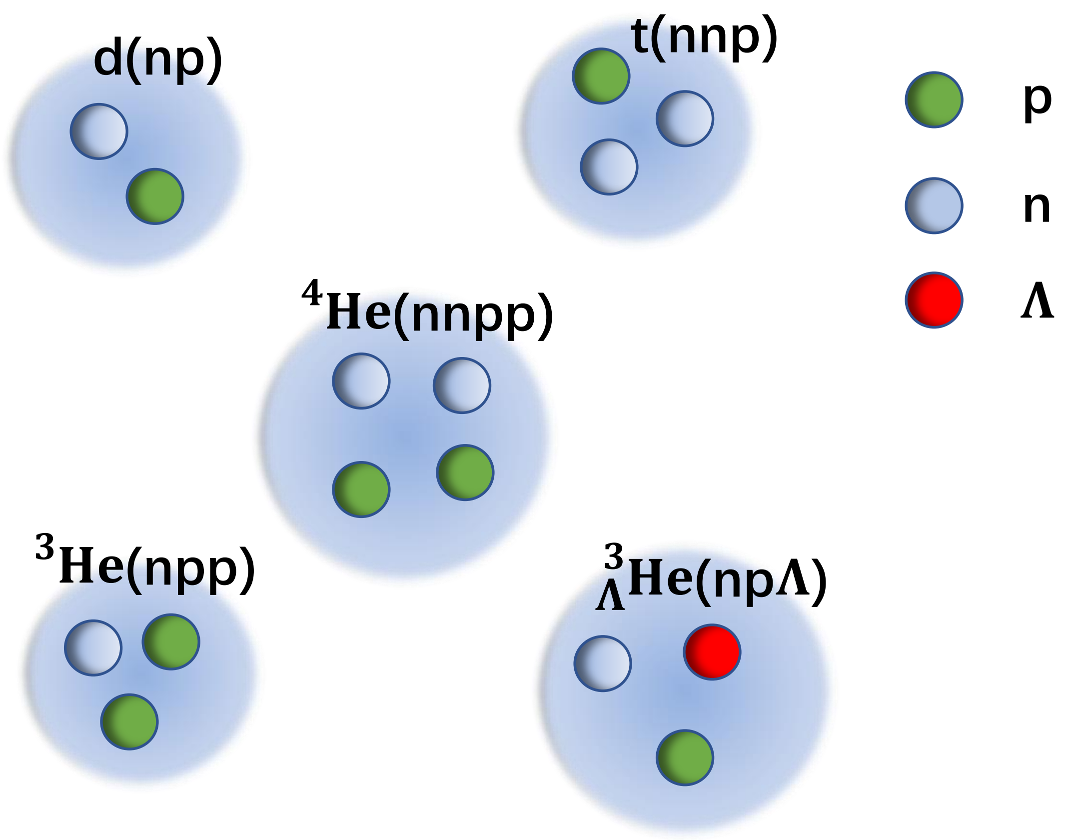
Light Nuclei Cluster

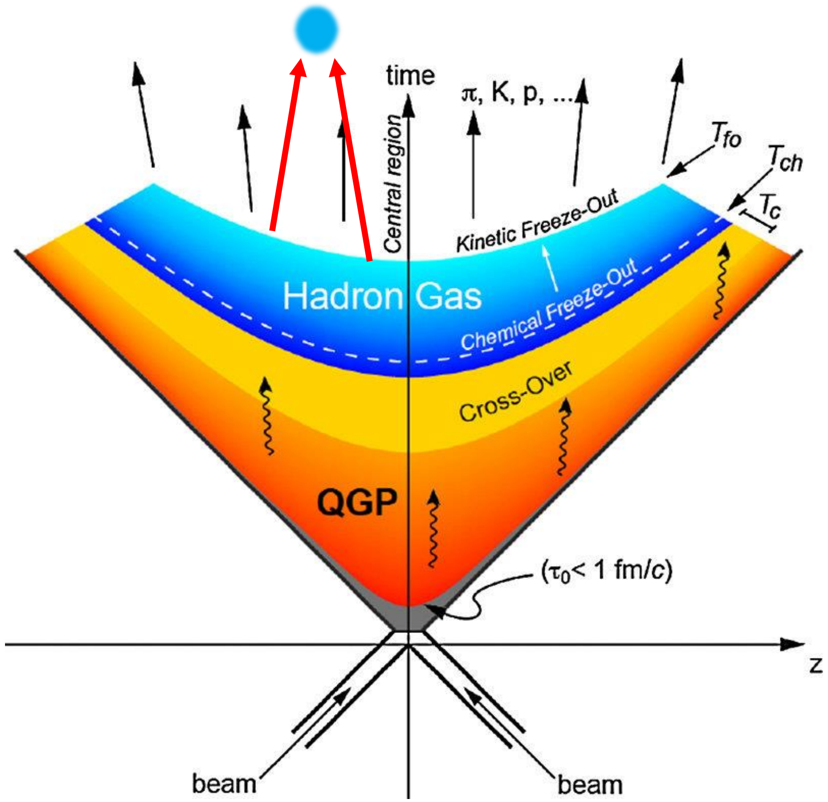


Loosely bounded objects (~MeV)

Nucleons close each other in phase-space (homogeneous):

- **Phase-space**
- nucleons interaction

Light nuclei in heavy ion collisions



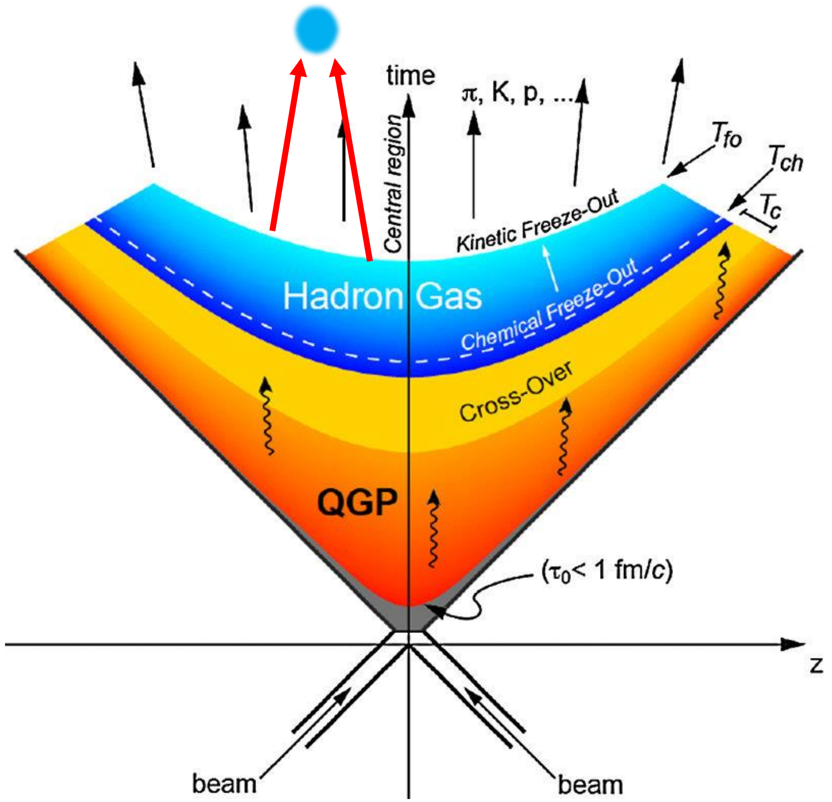
Binding energy << T_{kf}

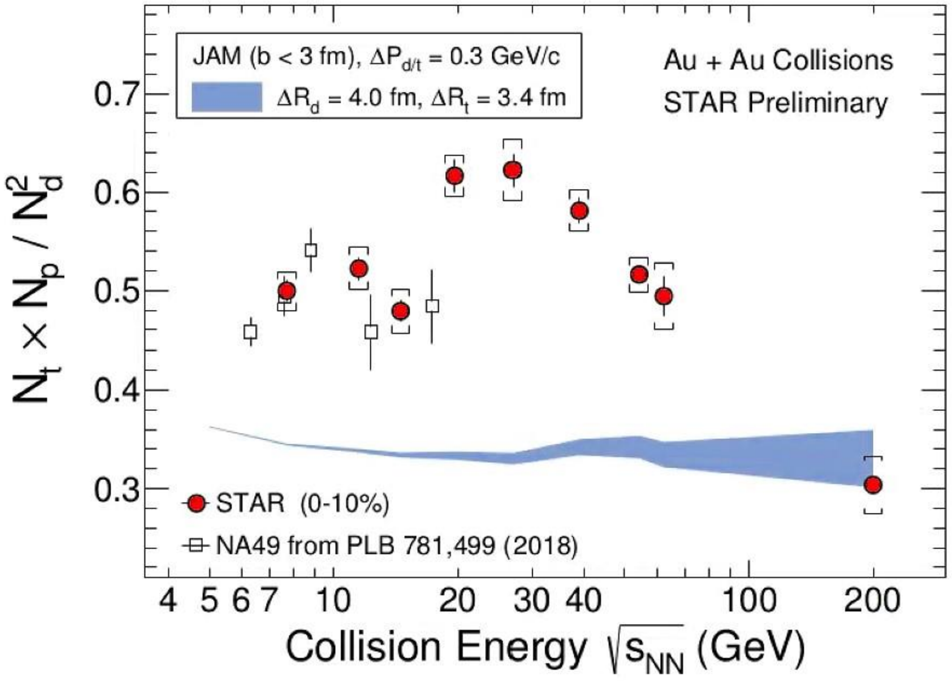
$\Rightarrow$ Form at late stages of collision

$\Rightarrow$ Detecting the phase-space distribution at freeze-out

Light nuclei in heavy ion collisions

H. Liu et al., Phys. Lett. B805, 135452 (2020)



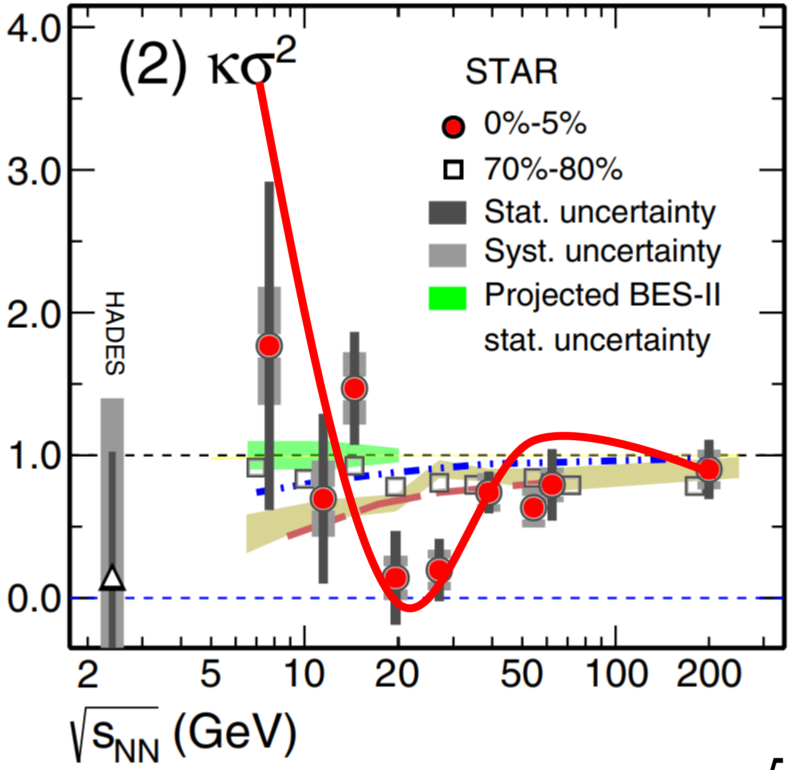


Non-monotonic v.s. $\sqrt{s_{NN}}$

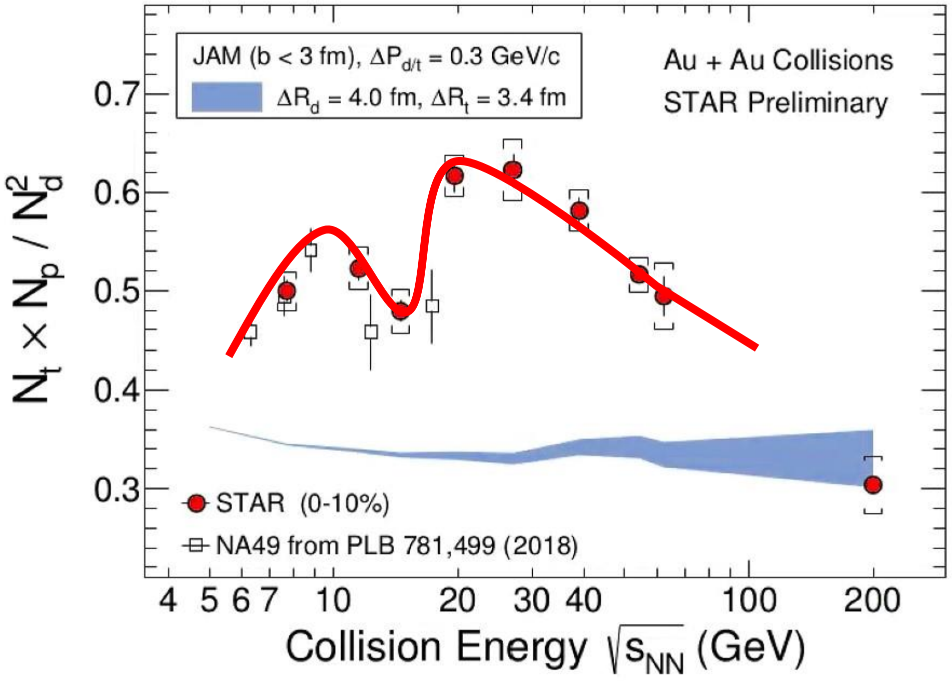
Light nuclei in heavy ion collisions

STAR, PRL 126 092301; STAR, PRC 104, 024902;

H. Liu et al., Phys. Lett. B805, 135452 (2020)



Non-monotonic v.s. $\sqrt{s_{NN}}$



Non-monotonic v.s. $\sqrt{s_{NN}}$

Phase transition?

Light nuclei: hot topic

- J. Chen et al., Phys. Rept. 760,1 (2018)
- B. Donigus, Int. J. Mod. Phys. E29, 2040001 (2020)
- D. Oliinychenko, arXiv:2003,05476(2020)
- S. Bazak et al., Mod. Phys. Lett. A3, 1850142 (2018)
- W. Zhao et al., Phys. Rev. C98,054905 (2018)
- W.Zhao et al., arXiv:2105,14204 (2021)
- F. Bellini et al., Phys. Rev. C99,054905 (2019)
- K.J.Sun and C. M. Ko, Phys. Lett. B792, 132(2019)
- D. Oliinychenko et al., Phys. Rev. C99, 044907(2019)
- X. Xu and R. Rapp, Eur.Phys.J. A55,68(2019)
- Y. Cai et al., Phys.Rev. C100 , 024911 (2019)
- V. Vovchenko et al., arXiv:2004.04411(2020)
- S. Mrowczynski, arXiv:2004.07029(2020)
- K. Blum and M. Takimoto, Phys.Rev.C99,044913(2019)
- K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)
- K. J. Sun et al., arXiv:2006.08929(2020)
- K. J. Sun et al., Phys. Lett. B 774, 103 (2017)
- K. J. Sun et al., Phys. Lett. B 781, 499 (2018)
- E. Shuryak and J. M. Torres-Rincon, arXiv:1805.04444(2018)
- E. Shuryak and J. M. Torres-Rincon, arXiv:1910.08119(2019)
- E. Shuryak and J. M. Torres-Rincon, arXiv:2005.14216(2020)
- D.DeMartini and E.Shuryak, arXiv: 2010.02785(2020)

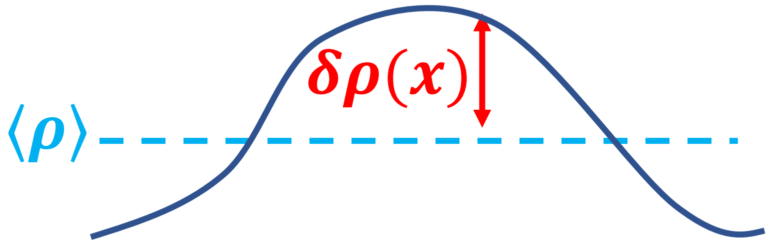
Nucleon density fluct. in coordinate space

Proton and neutron density:

$$\rho_n(x) = \langle \rho_n \rangle + \delta\rho_n(x)$$

$$\rho_p(x) = \langle \rho_p \rangle + \delta\rho_p(x)$$

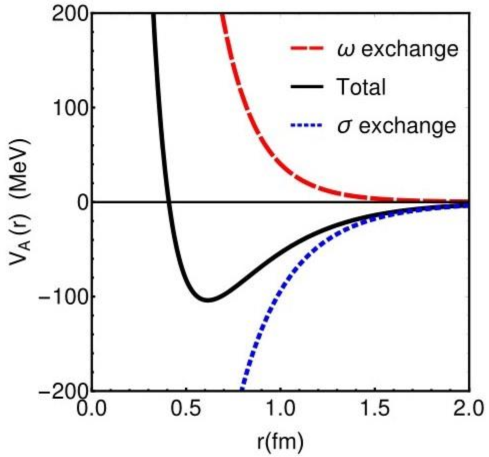
$$\frac{N_t N_p}{N_d^2} \sim \langle \delta\rho(x)^2 \rangle$$



Phase-space
(coordinate space)

K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)
K. J. Sun et al., arXiv:2006.08929(2020)
K. J. Sun et al., Phys. Lett. B 774, 103 (2017)
K. J. Sun et al., Phys. Lett. B 781, 499 (2018

NN interaction in light nuclei production



$$V_A(r) = -\frac{\alpha_\sigma}{r} e^{-m_\sigma r} + \frac{\alpha_\omega}{r} e^{-m_\omega r}$$

Nucleons interaction

Molecular Dynamics of a nucleon gas in thermal equilibrium

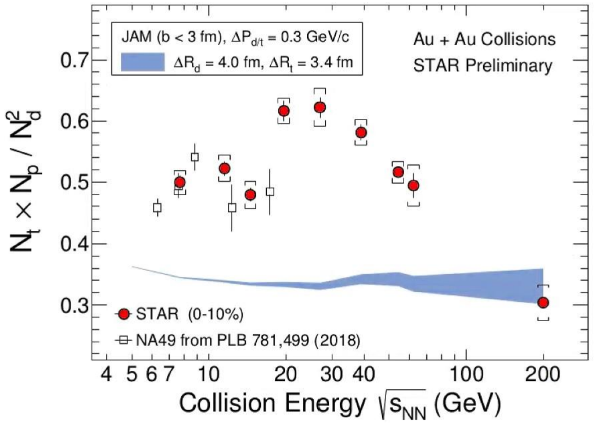
$$\frac{N_t N_p}{N_d^2} \simeq g \frac{\langle \exp\left(-\frac{3V_{NN}(r)}{T}\right)\rangle}{\langle \exp\left(-\frac{V_{NN}(r)}{T}\right)\rangle^2} \sim g \left\langle e^{-\frac{V_{NN}(r)}{T}} \right\rangle$$

$$\mathcal{O}_2 \equiv \frac{N_\alpha N_p}{N_{^3\mathrm{He}} N_d} \simeq 0.18 \frac{\langle e^{-6V(r)/T} \rangle}{\langle e^{-3V(r)/T} \rangle \langle e^{-V(r)/T} \rangle}$$

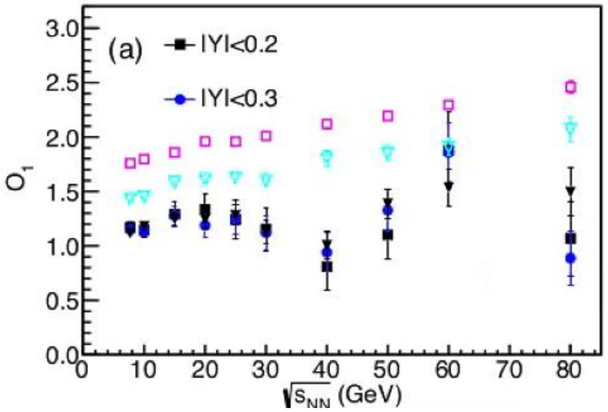
Free down effects from 4He to t, d, p near critical point

E. Shuryak and J. M. Torres-Rincon, arXiv:1805.04444(2018)
E. Shuryak and J. M. Torres-Rincon, arXiv:1910.08119(2019)
E. Shuryak and J. M. Torres-Rincon, arXiv:2005.14216(2020)

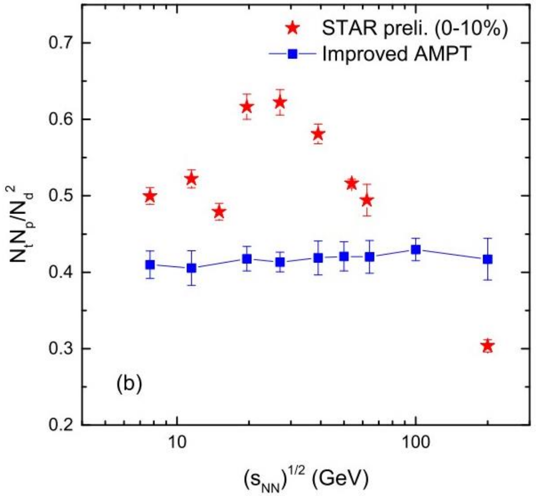
Dynamical models with light nuclei ratio



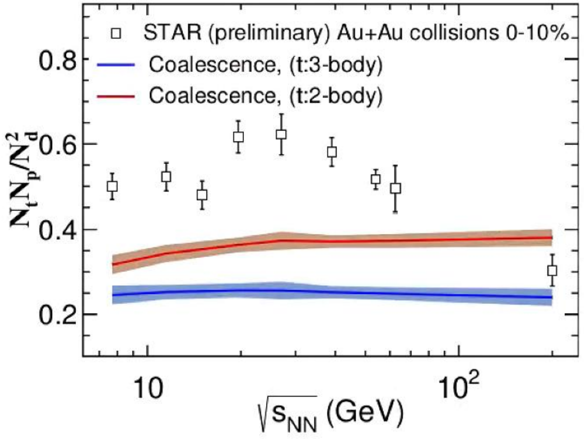
Hui Liu et al., PLB (2020)



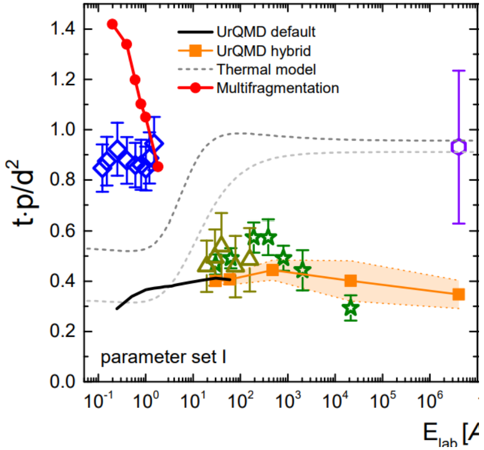
X.Deng et al., PLB (2020)



K.Sun et al., PRC (2021)



W. Zhao et al., PRC (2018)



P.Hillmann et al., 2109.05972

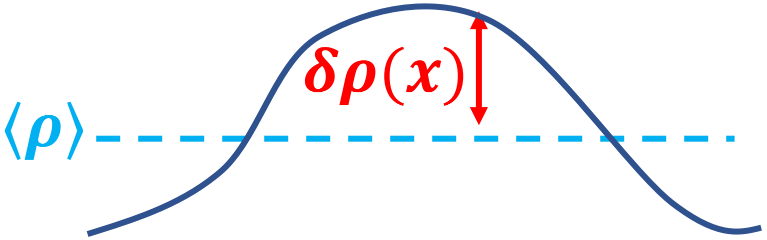
And others….

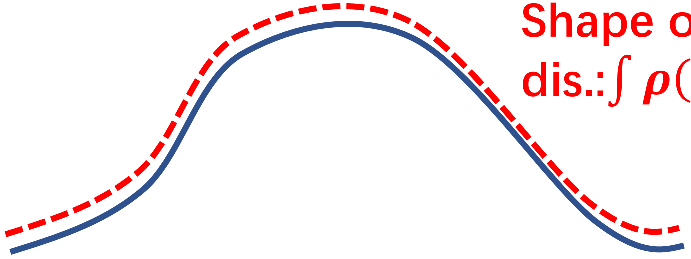
Phase-space produced in HIC

No clear non-monotonic on the model so far

Phase-Space Distribution in Light Nuclei Ratio (Background~~+Critical~~)

SW, K.Murase, S.Tang, H.Song, in preperation





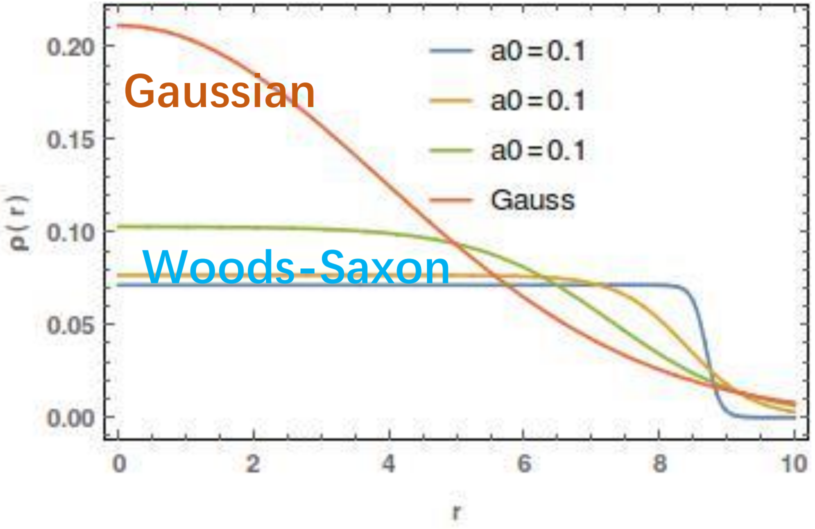
Shape of dis.: $\int \boldsymbol{\rho}(x) x^{n}\, dx$

Phase-Space Dis. in Light Nuclei Ratio: Example

SW, K.Murase, S.Tang, H.Song, in preperation

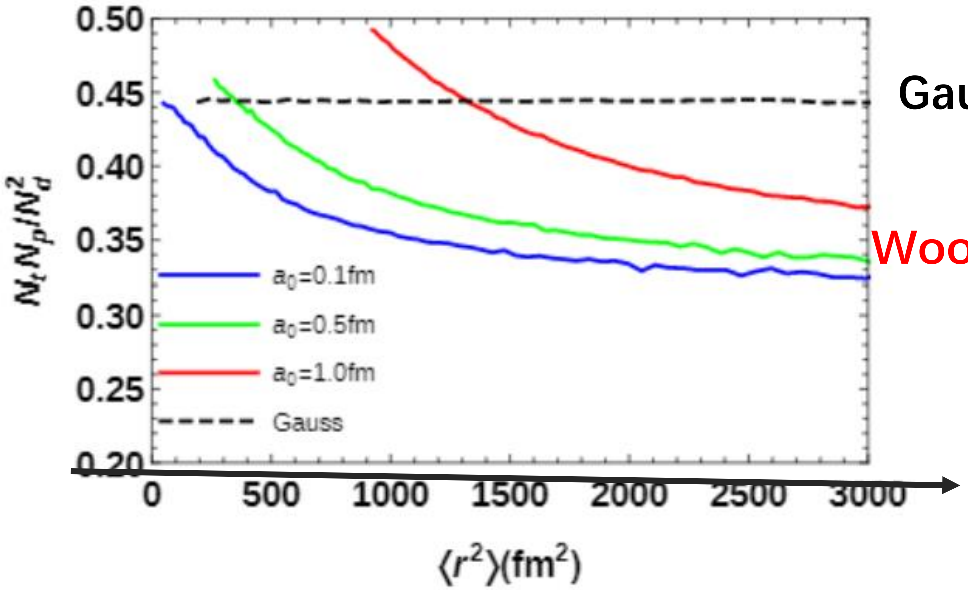
Woods-Saxon: $\rho(r)_{ws} = \frac{\rho_0}{1+Exp[\frac{r-r0}{a0}]}$

Gaussian: $\rho(r)_{Gauss} = \rho_0 Exp[-r^2/r_0^2]$



Gaussian

Woods-Saxon



Gaussian

Woods-Saxon

Broadening dis.

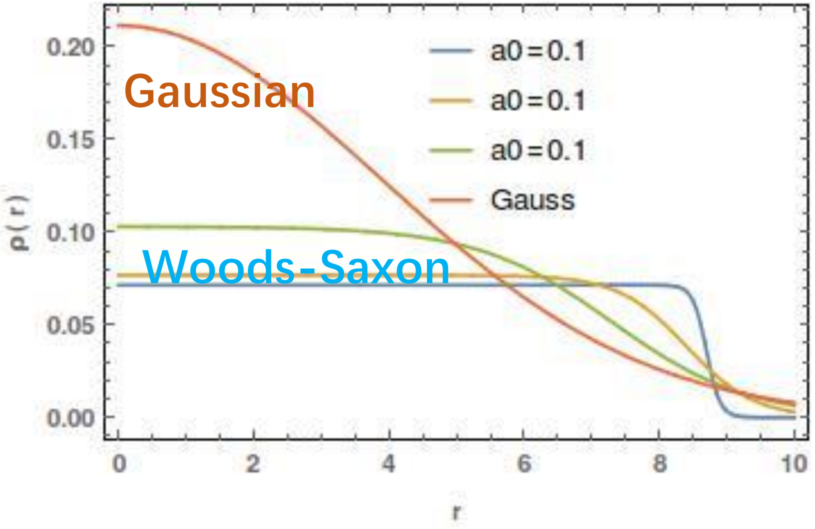
Phase-Space Dis. in Light Nuclei Ratio: Example

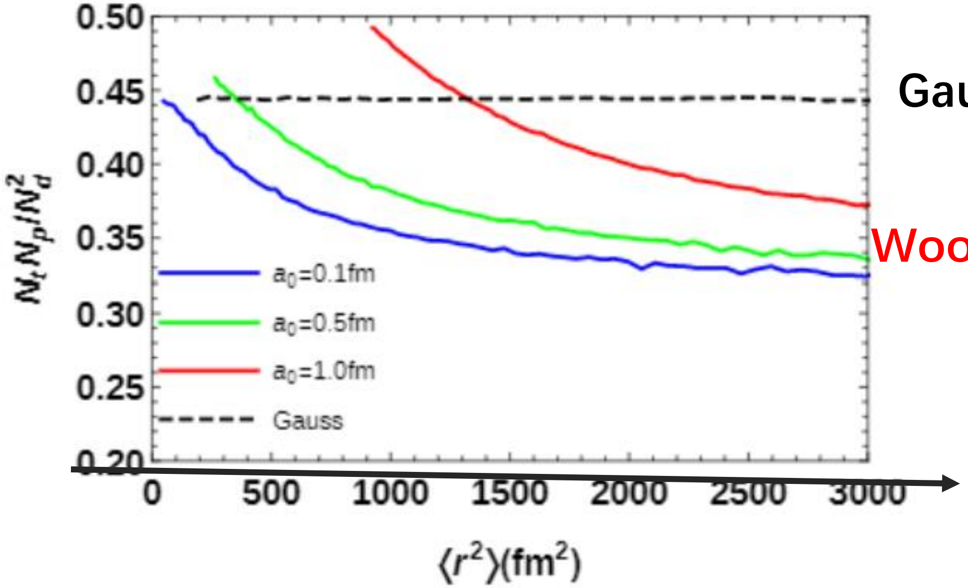
SW, K.Murase, S.Tang, H.Song, in preperation

Woods-Saxon: $\rho(r)_{ws} = \frac{\rho_0}{1+Exp[\frac{r-r0}{a0}]}$

Gaussian: $\rho(r)_{Gauss} = \rho_0 Exp[-r^2/r_0^2]$

We need decompose phase-space into Gaussian+Non-Gaussian





Gaussian

Woods-Saxon

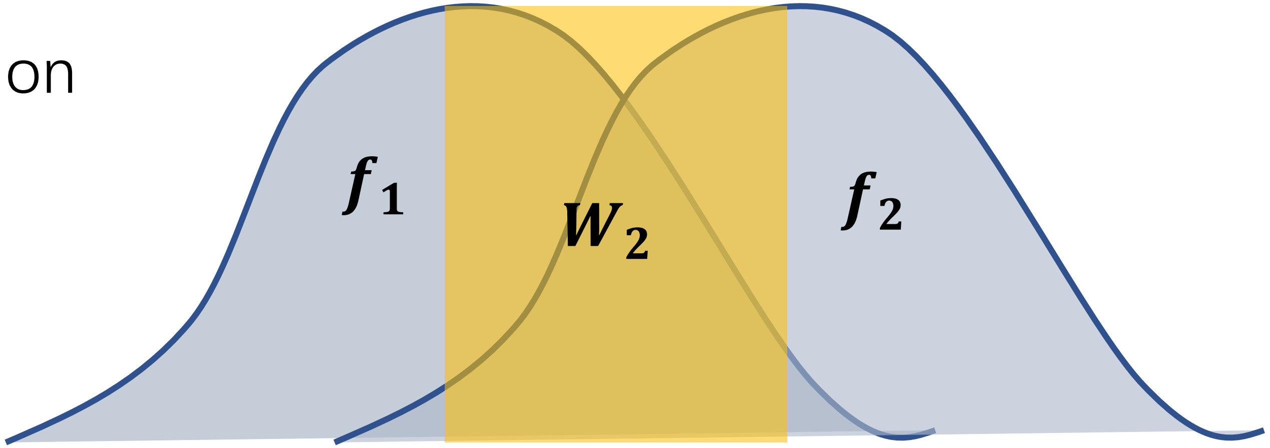
Broadening dis.

Light nuclei production

SW, K.Murase, S.Tang, H.Song, in preperation

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i)\right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Overlap of nucleus wavefunction and source size matters



Decompose phase-space distribution

SW, K.Murase, S.Tang, H.Song, in preperation

$N_A = g_A \int$

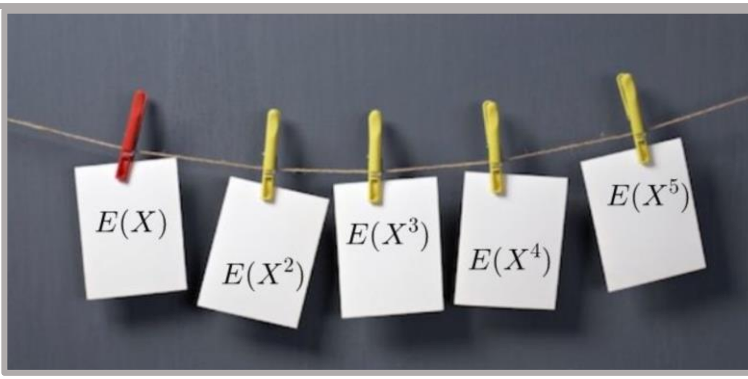
Phase space density $\boldsymbol{z_i}$

Wigner func. $\boldsymbol{Z_i}$

Characteristic func.

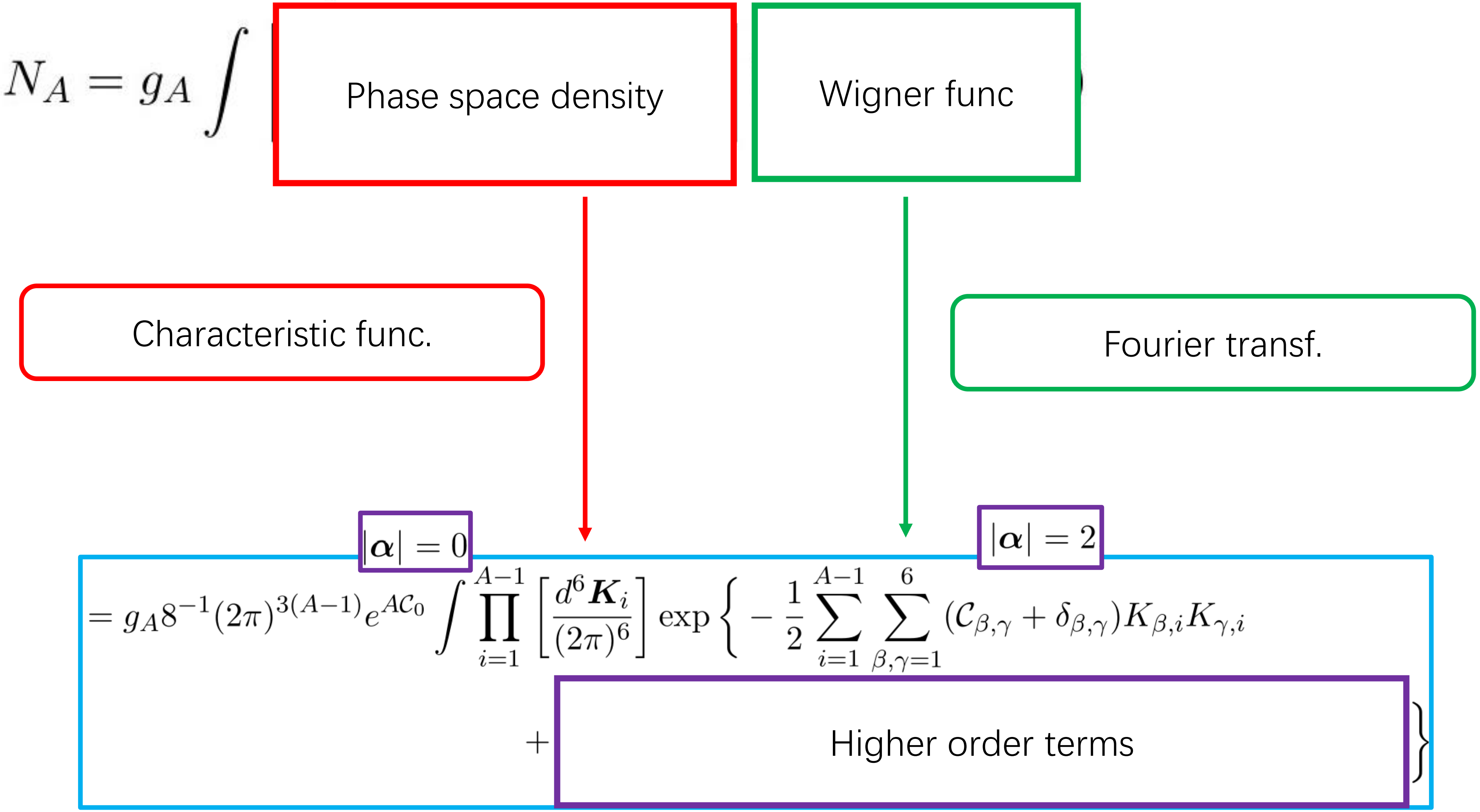
$$f(\boldsymbol{r}_i, \boldsymbol{p}_i) := f(\boldsymbol{z}_i) = \int \frac{d^6\boldsymbol{k}_i}{(2\pi)^6} e^{-\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{z}_i} \exp\left\{ \sum_{|\boldsymbol{\alpha}|\leq n} \frac{\mathcal{C}_{\boldsymbol{\alpha}}}{\boldsymbol{\alpha}!} (\mathrm{i}\boldsymbol{k}_i)^{\boldsymbol{\alpha}} \right\}$$

$$\varphi_X(t) = E[e^{itX}] = 1 + \frac{it\,\overbrace{E[X]}^{\text{一阶矩}}}{1} - \frac{t^2\,\overbrace{E[X^2]}^{\text{二阶矩}}}{2!} + \cdots + \frac{(it)^n\,\overbrace{E[X^n]}^{\text{n阶矩}}}{n!})$$



Light nuclei yield

SW, K.Murase, S.Tang, H.Song, in preperation



$$N_A = g_A \int$$

Phase space density

Wigner func

Characteristic func.

Fourier transf.

$|\boldsymbol{\alpha}| = 0$

$|\boldsymbol{\alpha}| = 2$

$$= g_A 8^{-1}(2\pi)^{3(A-1)} e^{A\mathcal{C}_0} \int \prod_{i=1}^{A-1}\left[\frac{d^6\boldsymbol{K}_i}{(2\pi)^6}\right] \exp\Big\{ -\frac{1}{2}\sum_{i=1}^{A-1}\sum_{\beta,\gamma=1}^{6}(\mathcal{C}_{\beta,\gamma}+\delta_{\beta,\gamma})K_{\beta,i}K_{\gamma,i}$$

$$+ \text{Higher order terms} \Big\}$$

Light nuclei yield: cumulants

SW, K.Murase, S.Tang, H.Song, in preperation

terms like $\langle r^2\rangle_c, \langle p^2\rangle_c, \langle rp\rangle_c \cdots$

$$= g_A 8^{-1} e^{A\mathcal{C}_0}\left[\det\left(\mathcal{C}+\mathcal{I}\right)\right]^{-(A-1)/2}$$

Factorization of $\boldsymbol{K_i K_i}$

$$= g_A 8^{-1}(2\pi)^{3(A-1)} e^{A\mathcal{C}_0}\int \prod_{i=1}^{A-1}\left[\frac{d^6\boldsymbol{K}_i}{(2\pi)^6}\right]\exp\Bigg\{-\frac{1}{2}\sum_{i=1}^{A-1}\sum_{\beta,\gamma=1}^{6}(\mathcal{C}_{\beta,\gamma}+\delta_{\beta,\gamma})K_{\beta,i}K_{\gamma,i}$$

$$+ \text{Higher order terms}\Bigg\}$$

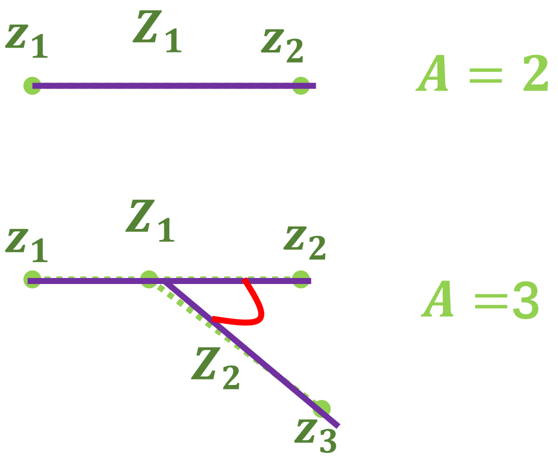
Light nuclei yield: cumulants

SW, K.Murase, S.Tang, H.Song, in preperation

terms like $\langle r^2\rangle_c, \langle p^2\rangle_c, \langle rp\rangle_c \cdots$

$$= g_A 8^{-1} e^{A\mathcal{C}_0}\left[\det\left(\mathcal{C}+\mathcal{I}\right)\right]^{-(A-1)/2}$$

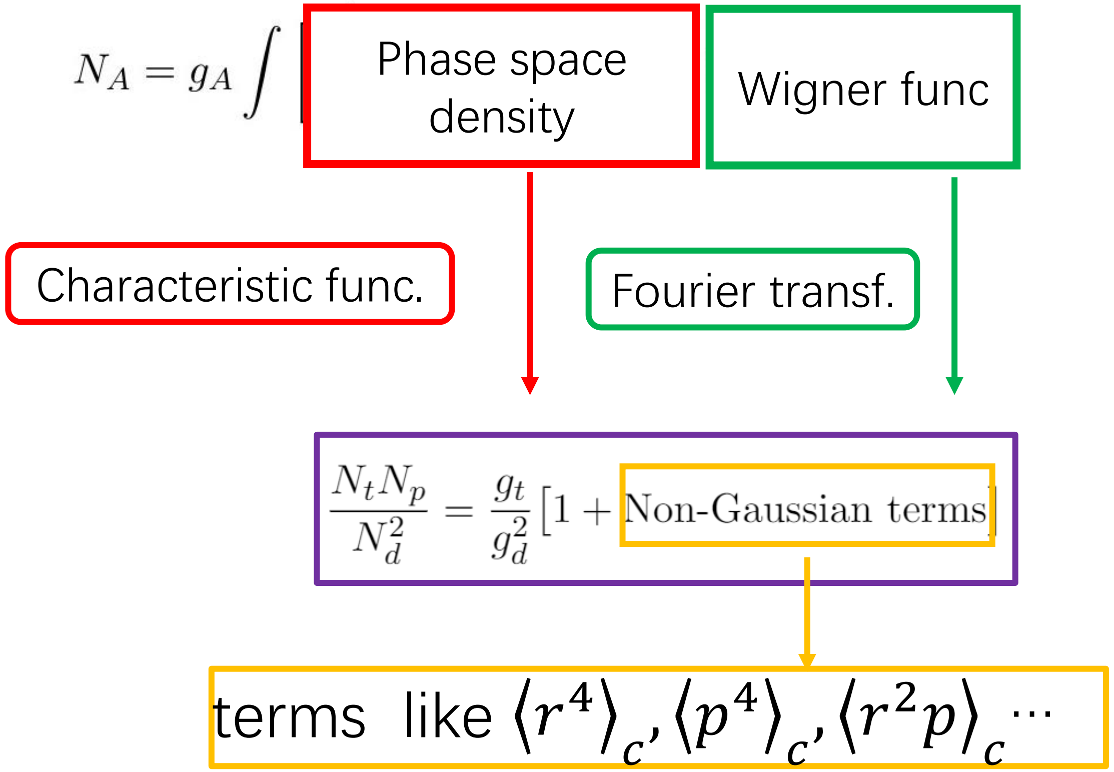
$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2}\left[1 + \text{Non-Gaussian terms}\right]$$



Non-Gaussian of phase-space is important

Summary so far: Background

- Light Nuclei~homogeneity length ~ c_s^2 ~Global EoS
- Light Nuclei Ratio ~ Finer structure ~ detail EoS $[c_s^2(t)]$ + Phase transition? Critical Point? Resonance? Other d.o.f?



$N_A = g_A \int [$ Phase space density Wigner func

Characteristic func.

Fourier transf.

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2}[1 + \text{Non-Gaussian terms}]$$

terms like $\langle r^4 \rangle_c, \langle p^4 \rangle_c, \langle r^2 p \rangle_c \cdots$

Light Nuclei Ratio Near QCD Critical Point:

(Background+Critical)

SW, K.Murase, S.Zhao, H.Song, in progress

QCD phase diagram

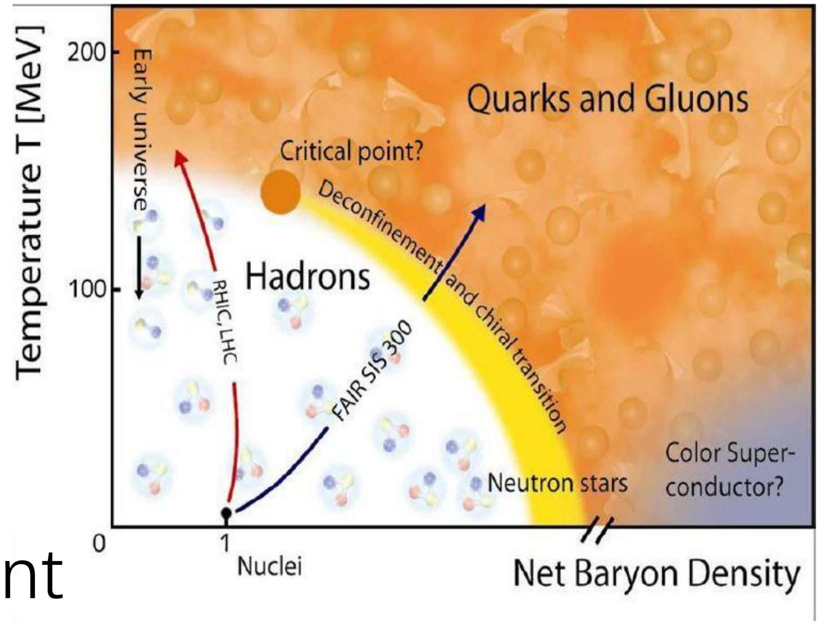
- Lattice QCD (small μ_B finite T):
  - Crossover
- Effective models(large μ_B)
  - 1st order phase trans.

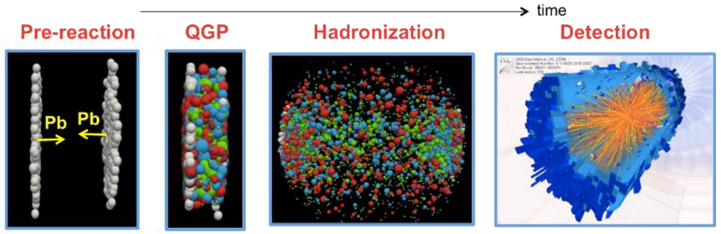
→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

→ Heavy-ion collisions :

  - changing $\sqrt{s_{NN}}$,mapping $T-\mu$: RHIC(BES),NICA,FAIR,J_PARC⋯.



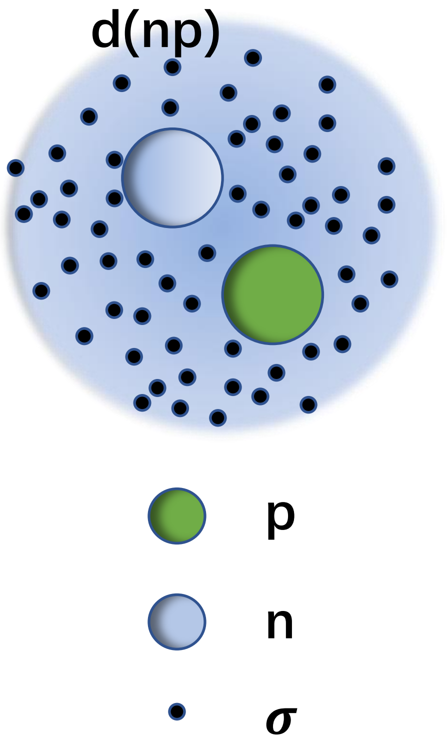


Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, in progress

$$N_A = g_A \int \boxed{\text{Phase space density}} \; \boxed{\text{Wigner func}}$$

$$= g_A \int \langle \prod_{i=1}^{A} \left[d^3 r_i d^3 p_i \left(\underbrace{f_0(\boldsymbol{r}_i; \boldsymbol{p}_i)}_{\text{Background}} + \underbrace{\delta f(\boldsymbol{r}_i; \boldsymbol{p}_i)}_{\text{Critical}} \right) \right] \rangle_\sigma W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^{A})$$



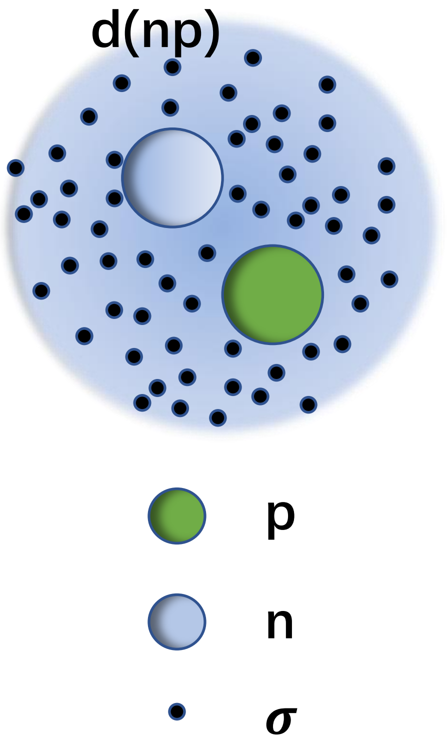
Critical δf: nucleons interact with the order parameter field $\delta f \sim g_\sigma \sigma$, which strongly fluctuates e-by-e

Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, in progress

$$N_A = g_A \int \boxed{\text{Phase space density}} \; \boxed{\text{Wigner func}}$$

$$= g_A \int \langle \prod_{i=1}^{A} \left[d^3r_i d^3p_i \left(\underbrace{f_0(\boldsymbol{r}_i;\boldsymbol{p}_i)}_{\text{Background}} + \underbrace{\delta f(\boldsymbol{r}_i;\boldsymbol{p}_i)}_{\text{Critical}} \right) \right] \rangle_\sigma W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^{A})$$

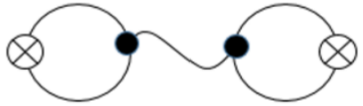


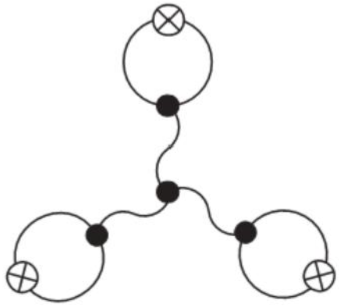
Critical δf: but we have little knowledge on the coupling constants g_σ => properly treatment of background is important.

Light nuclei yield: Critical δf

SW, K.Murase, S.Zhao, H.Song, in progress

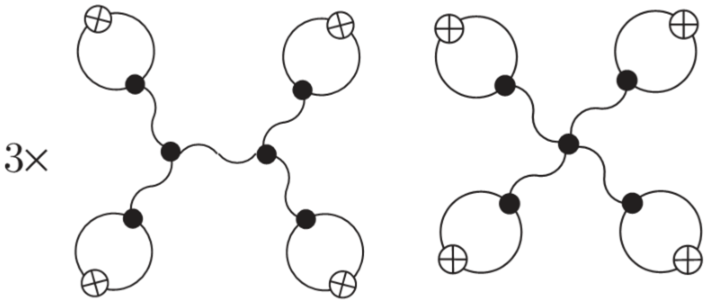
$$\delta N_d \sim \langle \delta f_1 \delta f_2 \rangle_\sigma$$





$$\delta N_t \sim \langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma$$

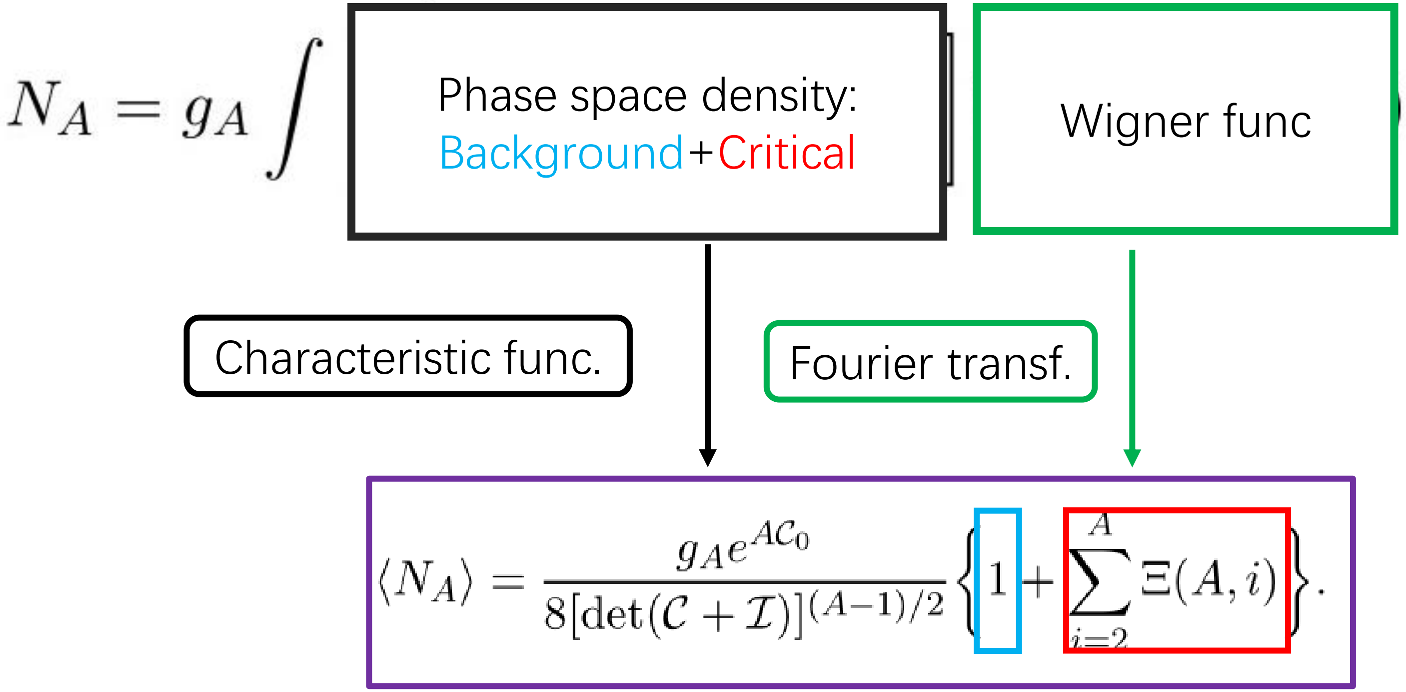
$$\delta N_{^4He} \sim \langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma$$



Critical δf: A constituent nucleons relates to A-point critical correlator

Light nuclei yield: Background+Critical

SW, K.Murase, S.Zhao, H.Song, in progress



$N_A = g_A \int$

Phase space density:
Background+Critical

Wigner func

Characteristic func.

Fourier transf.

$$\langle N_A\rangle = \frac{g_A e^{A\mathcal{C}_0}}{8[\det(\mathcal{C}+\mathcal{I})]^{(A-1)/2}}\left\{\boxed{1}+\boxed{\sum_{i=2}^{A}\Xi(A,i)}\right\}.$$

Background and Critical share the same coefficient.
Light nuclei of A constituent nucleons N_A share the similar form.

New Light Nuclei Ratio Sensitive to ξ

SW, K.Murase, S.Zhao, H.Song, in progress

$$\boxed{\langle N_A\rangle = \frac{g_A e^{A\mathcal{C}_0}}{8[\det(\mathcal{C}+\mathcal{I})]^{(A-1)/2}}\left\{\boxed{1}+\boxed{\sum_{i=2}^{A}\Xi(A,i)}\right\}.}$$

$$\downarrow$$

$$\boxed{R \equiv \left(\frac{\langle N_B\rangle}{N_p}\right)^{A-1}\left(\frac{N_p}{\langle N_A\rangle}\right)^{B-1} - \frac{g_B^{A-1}}{g_A^{B-1}}}$$

$$\boxed{= \frac{g_B^{A-1}}{g_A^{B-1}}\left\{1+\sum_{l=1}^{B-1}C^l_{(B-1)}\Big[\sum_{i=2}^{A}\Xi(A,i)\Big]^l\right\}^{-1}\boxed{\left\{\sum_{n=1}^{A-1}C^n_{(A-1)}\Big[\sum_{i=2}^{B}\Xi(B,i)\Big]^n-\sum_{m=1}^{B-1}C^m_{(m=1)}\Big[\sum_{i=2}^{A}\Xi(A,i)\Big]^m\right\}}}$$

$$\wr$$

$$\xi^{\alpha}$$

New Light Nuclei Ratio Sensitive to ξ

SW, K.Murase, S.Zhao, H.Song, in progress

$$\langle N_A\rangle = \frac{g_A e^{A\mathcal{C}_0}}{8[\det(\mathcal{C}+\mathcal{I})]^{(A-1)/2}}\left\{\boxed{1}+\boxed{\sum_{i=2}^{A}\Xi(A,i)}\right\}.$$

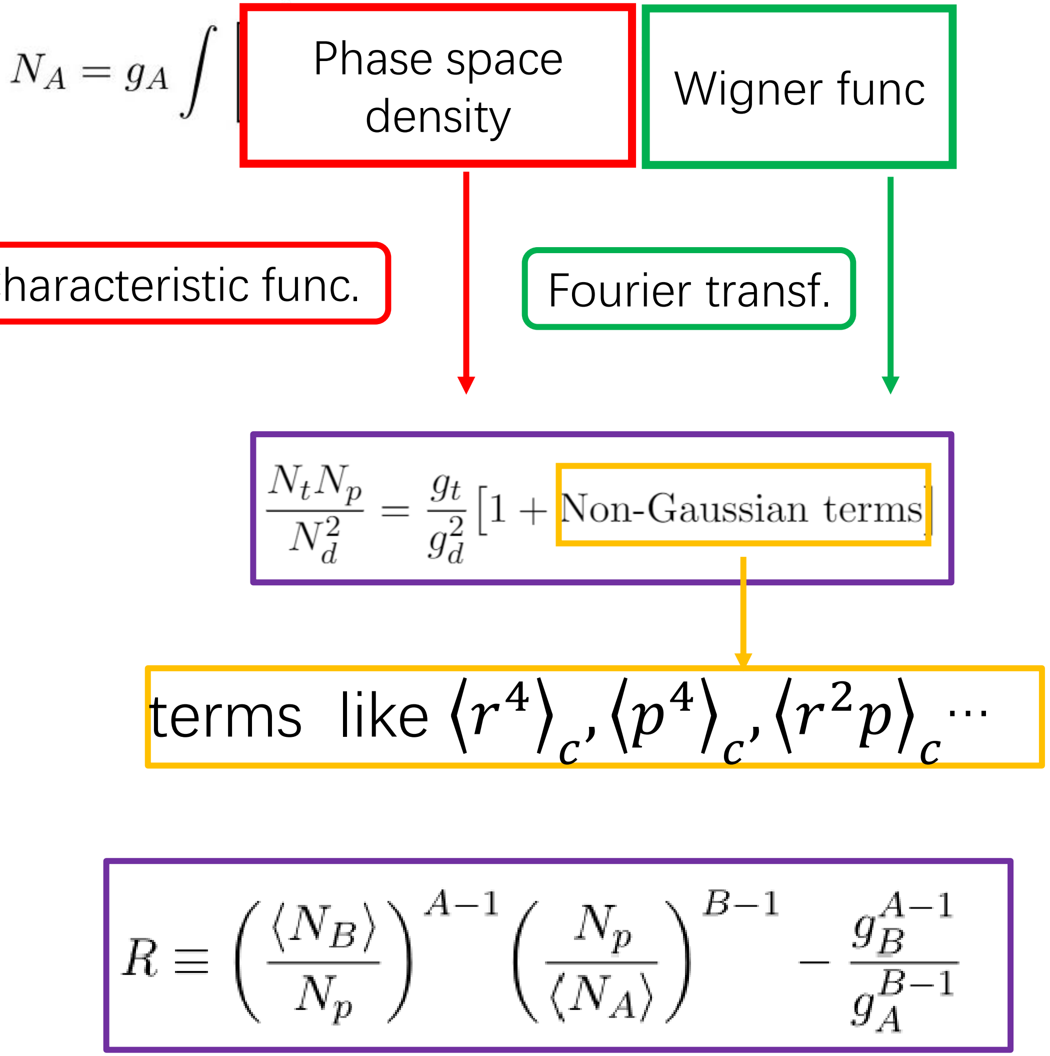
$$R \equiv \left(\frac{\langle N_B\rangle}{N_p}\right)^{A-1}\left(\frac{N_p}{\langle N_A\rangle}\right)^{B-1} - \frac{g_B^{A-1}}{g_A^{B-1}}$$

Yield Ratio

Spin, Isospin const.

Summary

- Light Nuclei Ratio (Background): Gaussian $f(x,p) \sim$ const. , Non-Gaussian $f(x,p)$ w.r.t $\sqrt{s_{NN}}$ is essential!
- New Light Nuclei Ratio R (Critical): canceling the background, sensitive to ξ
- Outlook:
  - New light nuclei ratio with Critical dynamics (in progress)
  - Long-lived Resonance decay



$N_A = g_A \int$ [Phase space density] [Wigner func]

Characteristic func.

Fourier transf.

$$\frac{N_t N_p}{N_d^2} = \frac{g_t}{g_d^2}[1 + \text{Non-Gaussian terms}]$$

terms like $\langle r^4\rangle_c, \langle p^4\rangle_c, \langle r^2 p\rangle_c \cdots$

$$R \equiv \left(\frac{\langle N_B\rangle}{N_p}\right)^{A-1}\left(\frac{N_p}{\langle N_A\rangle}\right)^{B-1} - \frac{g_B^{A-1}}{g_A^{B-1}}$$

Thanks !