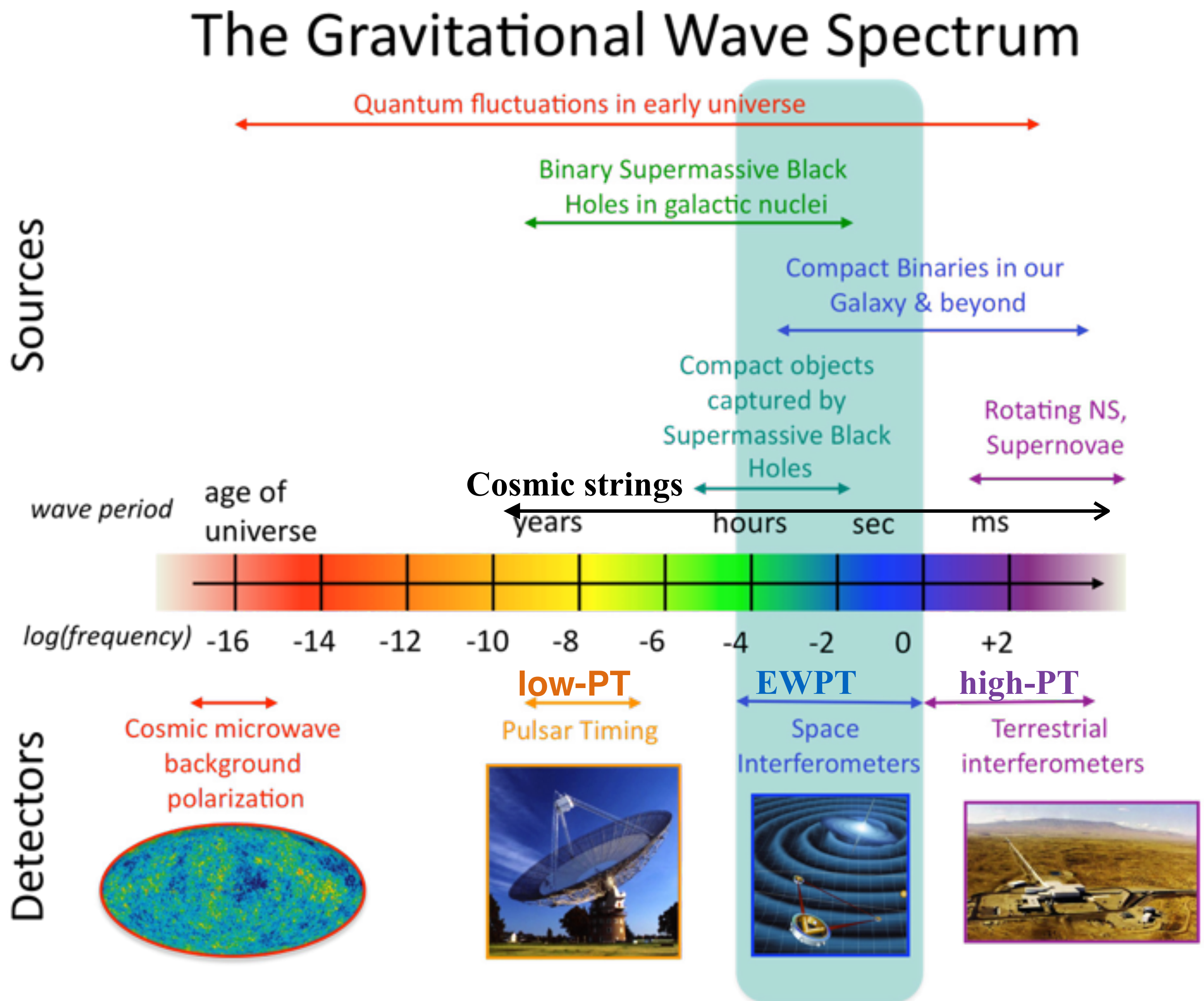
FOPT&Symmetry breaking in the early Universe

Ligong Bian(边立功)
Chongqing University（重庆大学）
lgbycl@cqu.edu.cn

CLHCP2021

Outline

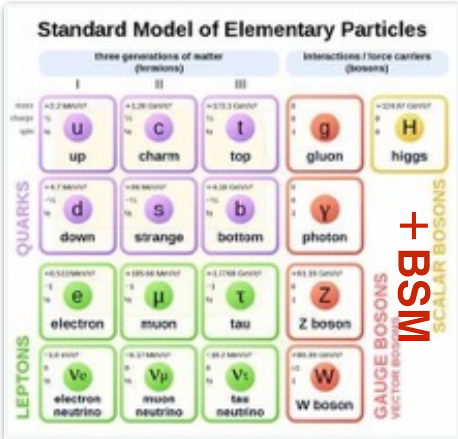
- Motivation
- First-order phase transition
- Cosmic strings and symmetry breaking
- Summary



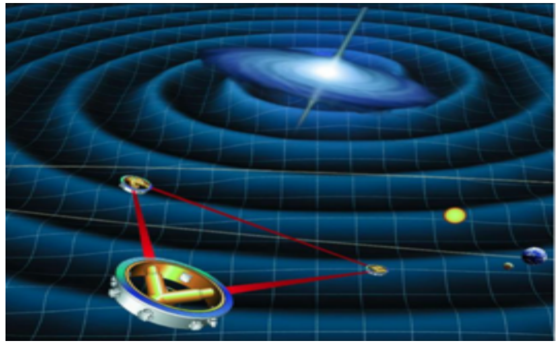
LISA,TianQin,Taiji,...

BSM for FOPT GW

Particle physics model



PTA,LIGO,LISA,TianQin,Taiji,...



PT parameters

Effective action $\rightarrow \beta, H_*$

Energy budget $\rightarrow \alpha, \kappa(\alpha, v_w)$

Bubble wall dynamics $\rightarrow v_w$

GW power spectrum

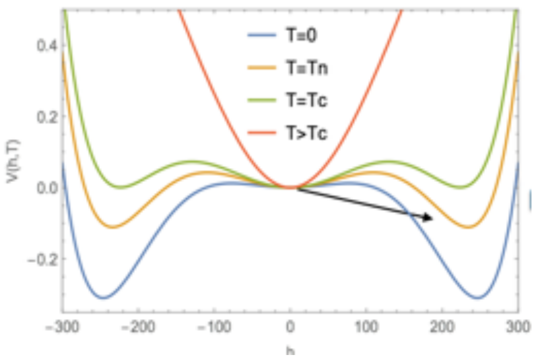
Numerical simulations $\rightarrow$ $h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

LISA sensitivity

Configuration + noise level $\rightarrow$ $h^2\Omega_{sens}(f)$

Thermal field theory

Lattice simulation



Signal-to-noise ratio

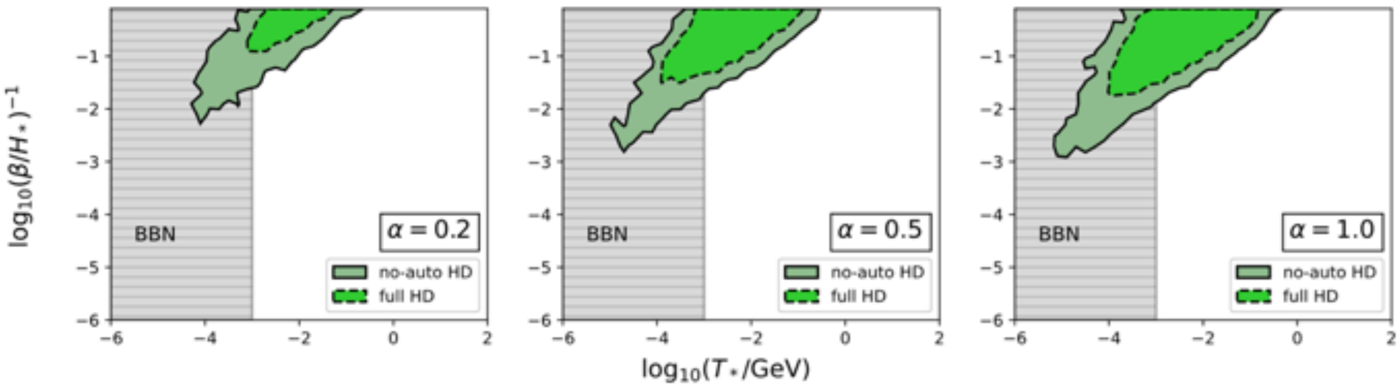
Chiara Caprini *et al* JCAP03(2020)024

PTGW experimental constraints

Low-scale QCD&Dark PT

Xue, **Bian***, Shu*, Yuan*, Zhu*, etal, 2110.03096, PRL in press, editor suggestion

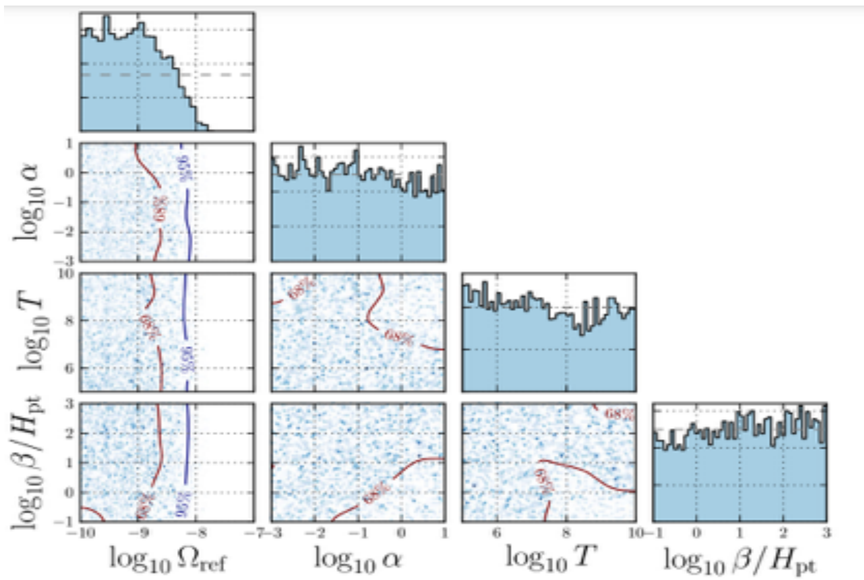
PPTA



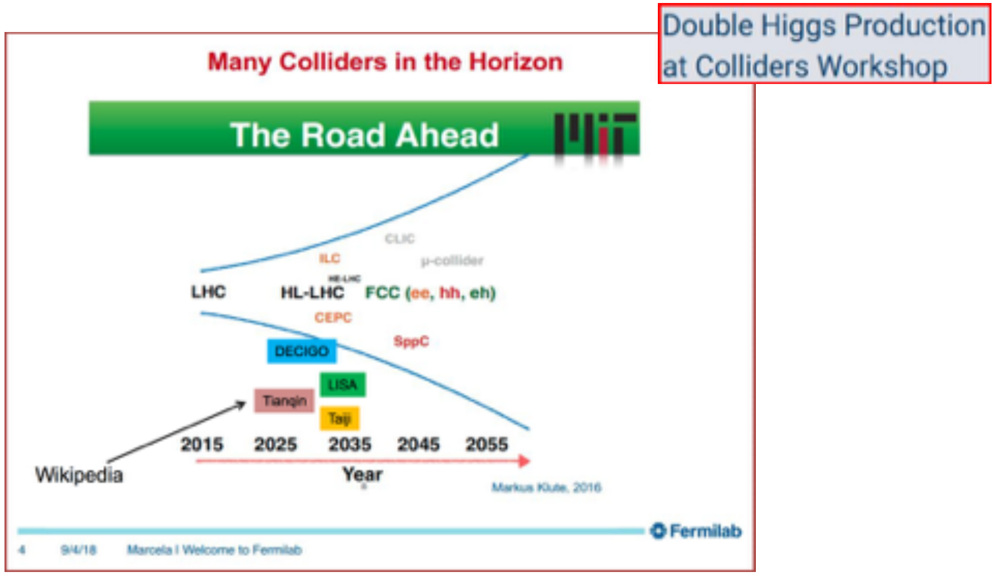
High-scale PT

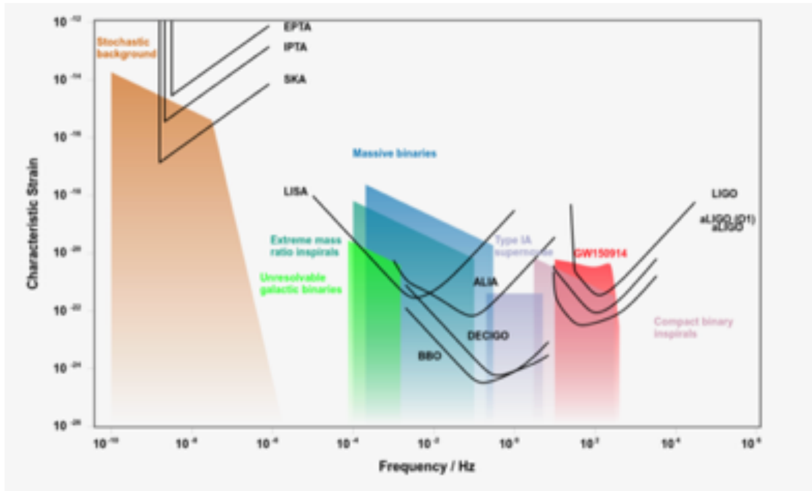
Romero, Martinovic,Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3

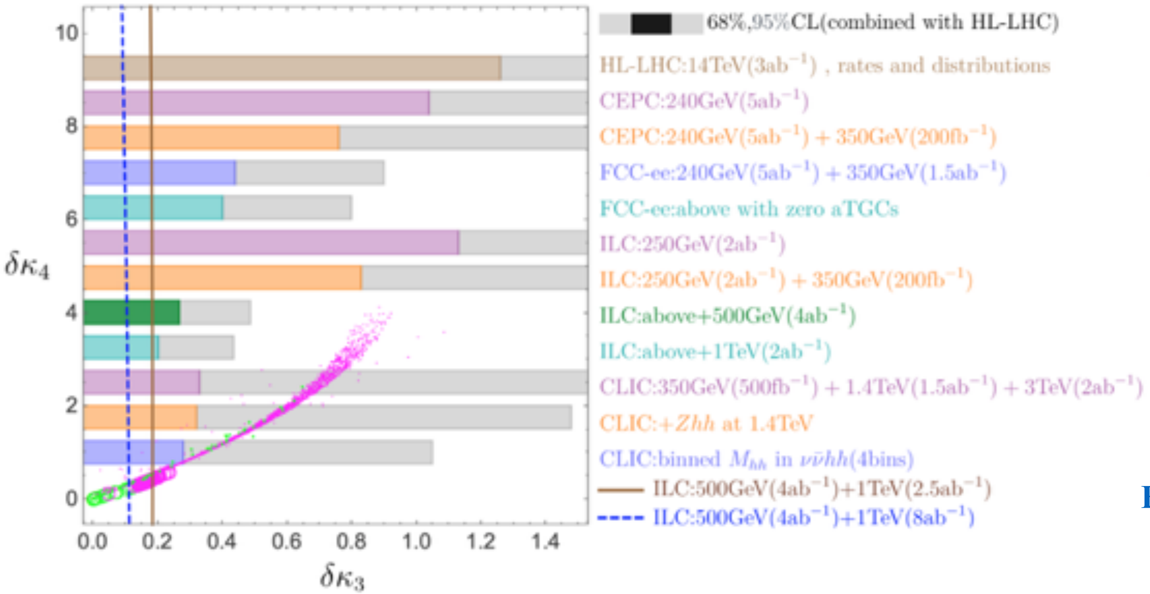


PTGW and collider search





C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.

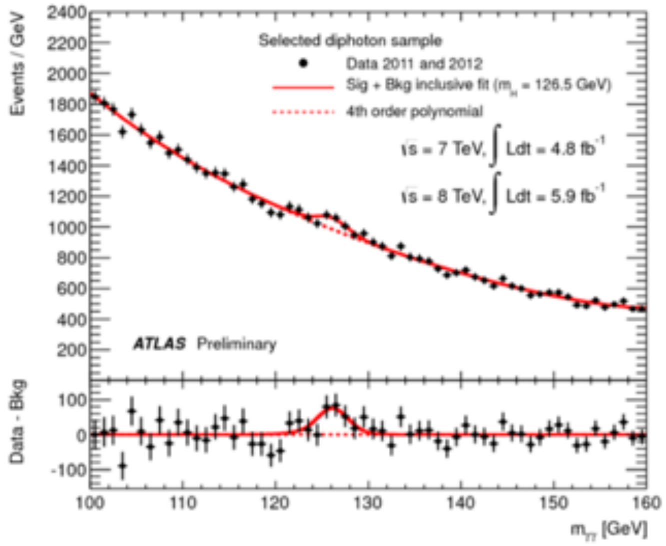


$$\Delta\mathscr{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$

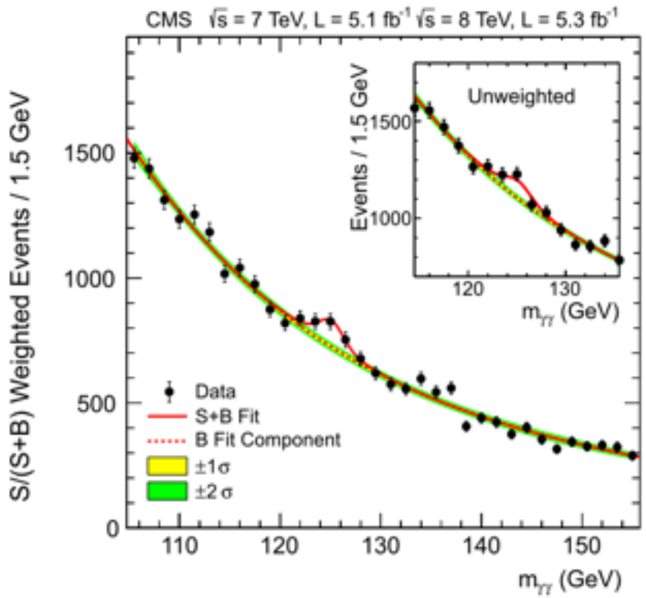
SNR > 10 for two-step and one-step SFOEWPT

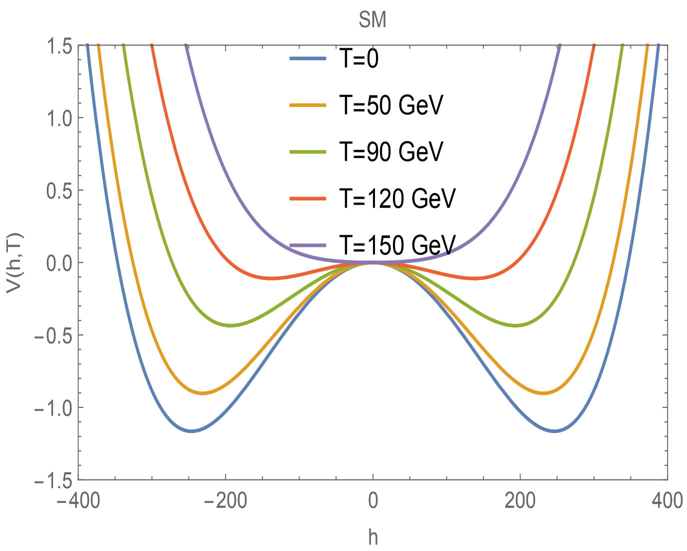
Bian, Guo, Wu, Zhou, Phys.Rev.D 101 (2020) 3, 035011

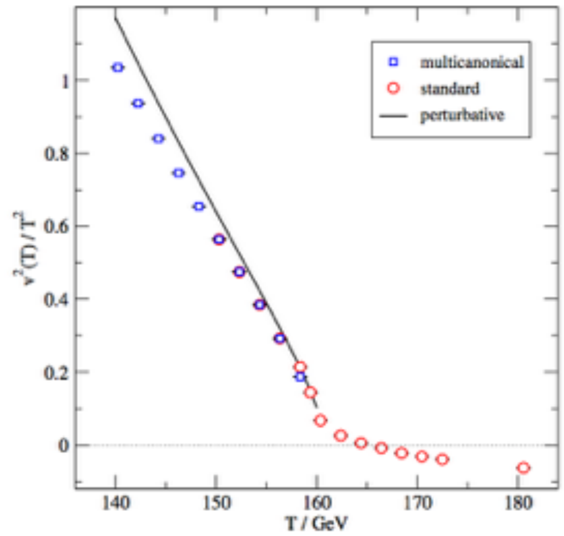
FOPT 125 Higgs exclude FOPT in the SM



Higgs announcement seminar on 4 July 2012

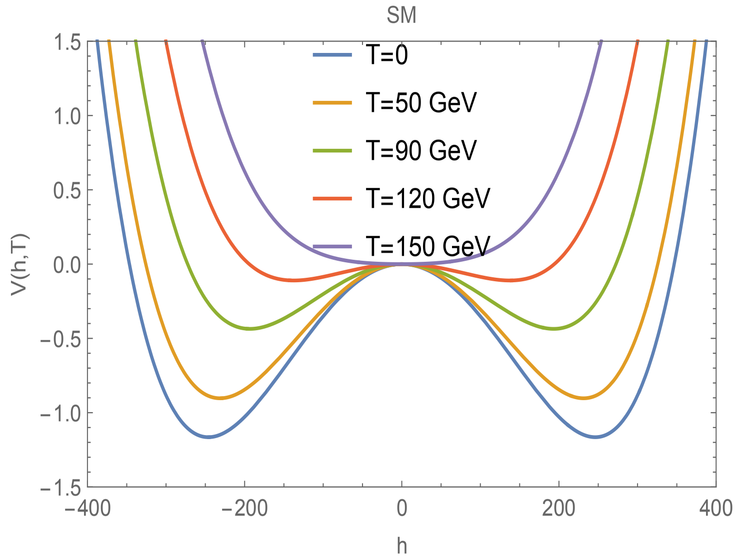


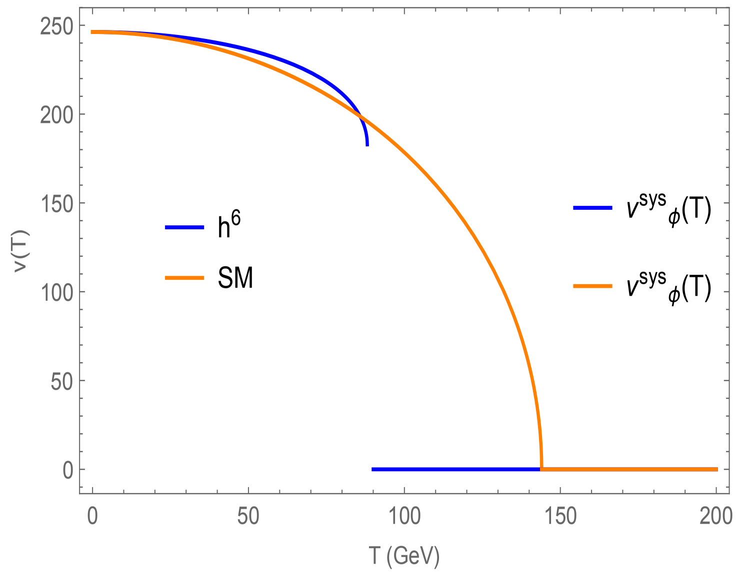


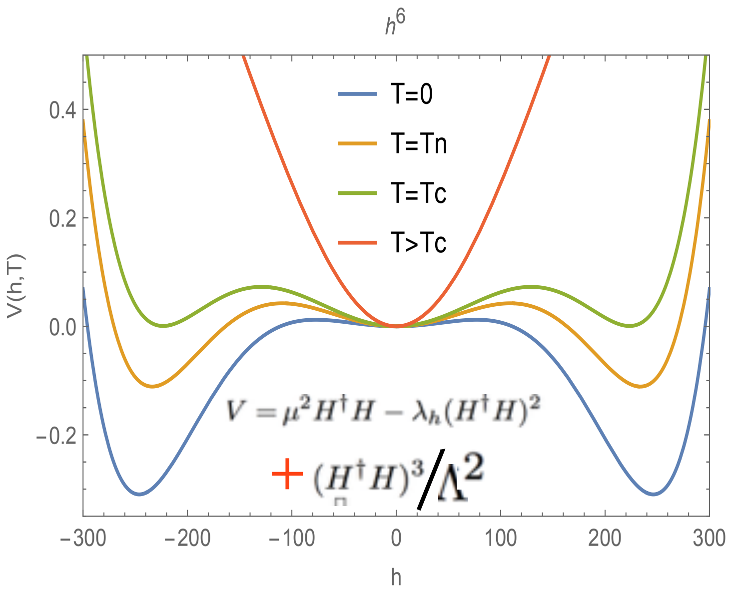


PRL 113, 141602 (2014)

Higgs potential & FOPT



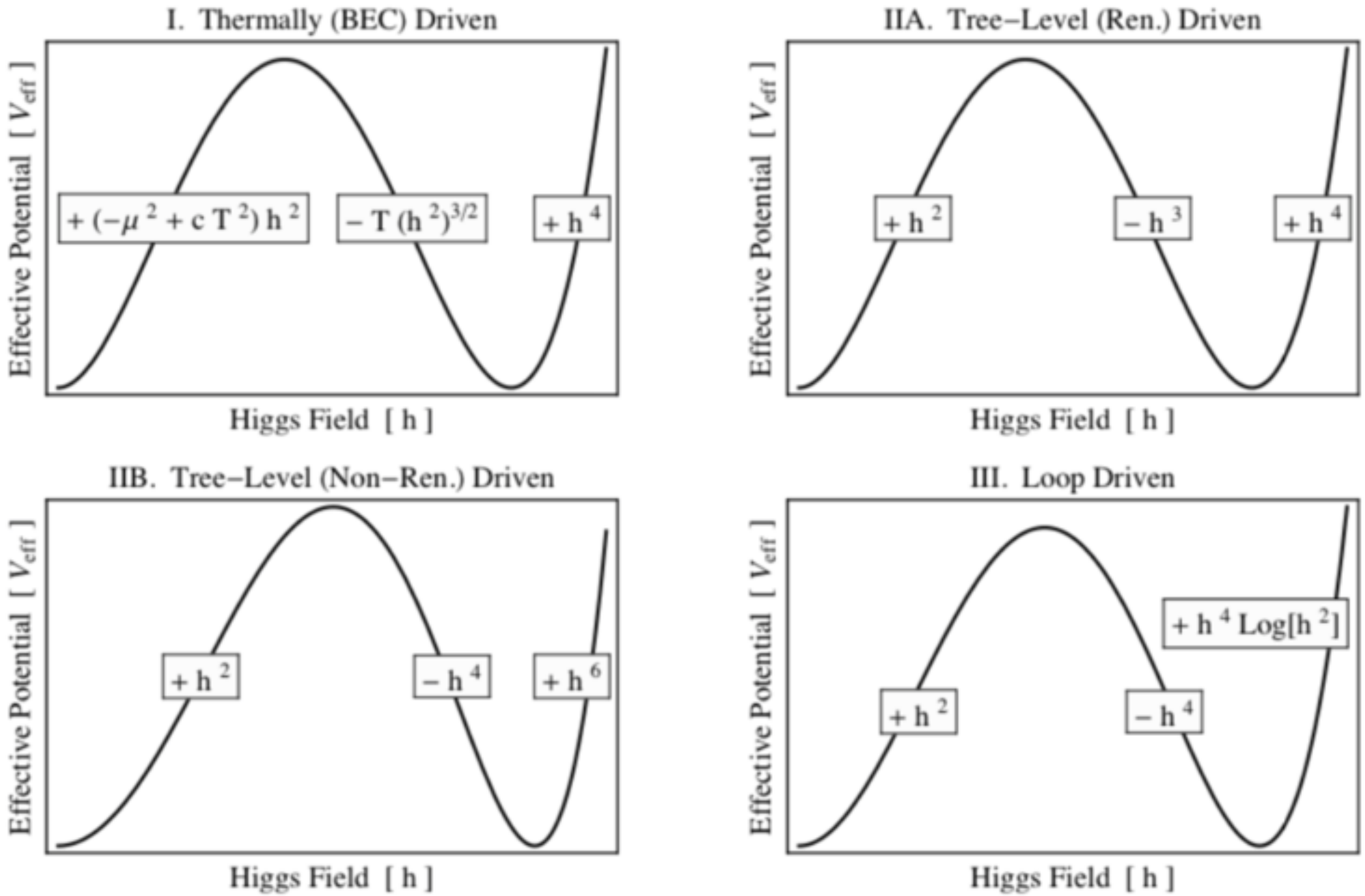




Grojean, Servant, Wells 05, Huang, Jokelar, Li, Wagner 15, Cao, Huang, Xie, & Zhang 17, Zhou, **Bian**, Guo 19

LHC say the quantum fluctuation (quadratic oscillation) around h=v with mh=126 GeV, not sensitive to the specifically potential shape

Model classes for FOPT



PRD87, 023509 (2013)

BSM & EWPT

- **SM+Scalar Singlet**

Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, Jiang, **Bian,** Huang, Shu 15, Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17,Cheng, **Bian** 17, Bian, Tang 18,Chen, Li, Wu, **Bian**,19…

- **SM+Scalar Doublet**

Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, Bernon, **Bian,** Jiang 17, **Bian**, Liu 18,…

- **SM + Scalar Triplet**

Profumo, Ramsey-Musolf 12, Chiang 14, Zhou, Cheng, Deng, **Bian**, Wu 18,Zhou, **Bian**, Guo,Wu 19,…

- **NMSSM**

Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, Bi, **Bian**, Huang, Shu, Yin 15, **Bian,** Guo, Shu 17,…

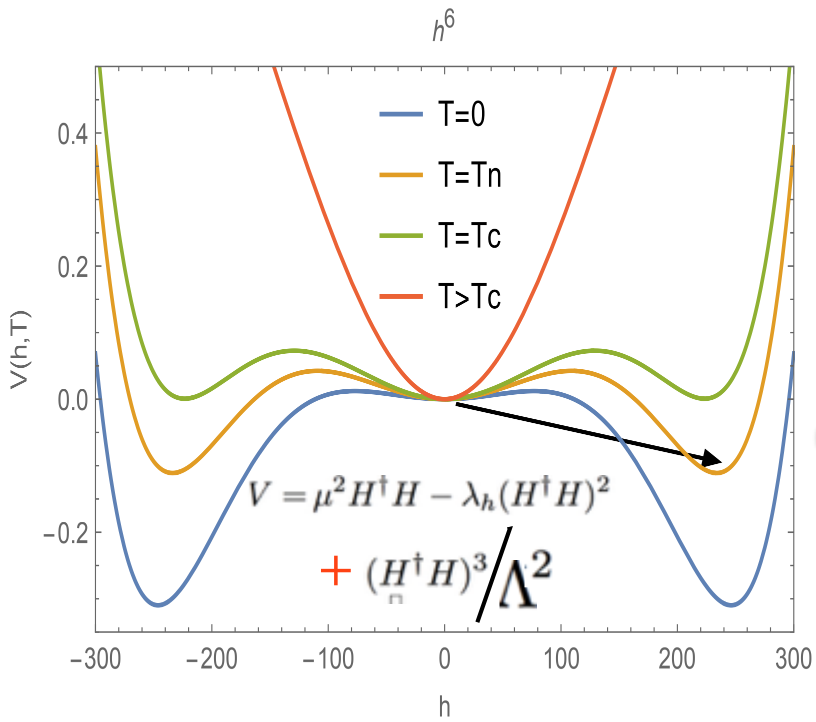
- **Composite Higgs**

Espinosa, Gripaios, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, **Bian**,Wu,Xie 19, De Curtis, Delle Rose, Panico 19, **Bian**,Wu,Xie 20,…

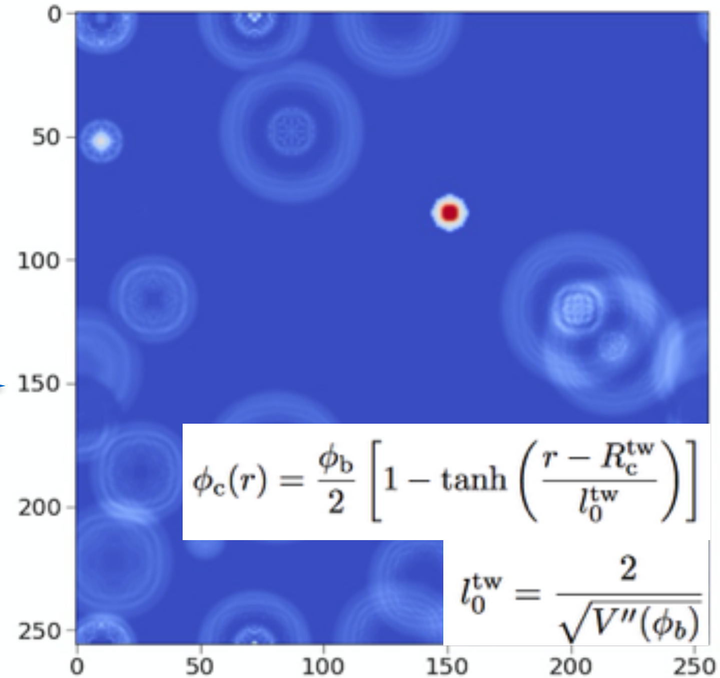
- **EFT**

Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15,Cai,Sasaki ,Wang17,Zhou, **Bian**, Guo 19, …

PT process & thermal potential



$$V = \mu^2 H^\dagger H - \lambda_h (H^\dagger H)^2 + (H^\dagger H)^3 / \Lambda^2$$



$$\phi_c(r) = \frac{\phi_b}{2}\left[1 - \tanh\left(\frac{r - R_c^{tw}}{l_0^{tw}}\right)\right]$$

$$l_0^{tw} = \frac{2}{\sqrt{V''(\phi_b)}}$$

$$\alpha = \frac{\Delta\rho}{\rho_R} = \frac{1}{\rho_R}\left[-V(\vec{\phi}_b, T) + T\frac{\partial V(\vec{\phi}_b, T)}{\partial T}\right]\Bigg|_{T=T_n}$$

$$S(t_N) \sim 4\ln(m_P/T_N)$$

$$\frac{\beta}{H_*} \simeq \frac{2S(t_N)}{(1 - T_N/T_c)}$$

$$p(t_N) = H^4(t_N)$$

$$p(t) \simeq \Gamma_0 e^{-S(t_N)+\beta(t-t_N)} \quad \Gamma 0 \sim \alpha^5_{W} T_N^4$$

$$\beta = -\,d\ln p(t)/dt|_{t_f}$$

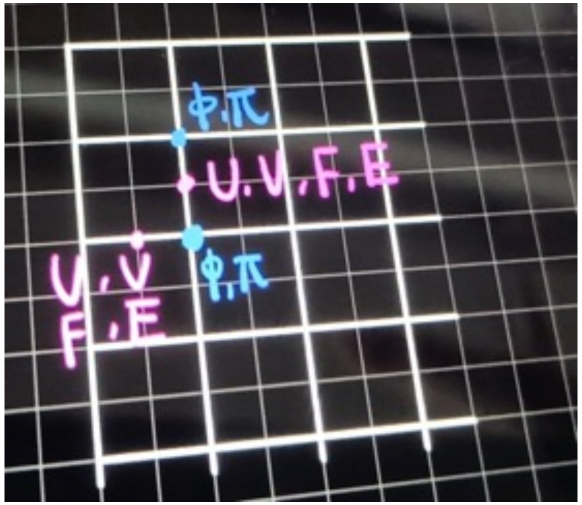
Lattice EW field foundation

$\Phi(t, x)$: Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps $t + \Delta t, t + 2\Delta t, \ldots$;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps $t + \Delta t/2, t + 3\Delta t/2$.

$$U_i(t,x) = \exp\left(-\frac{i}{2} g\Delta x \sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2} g\Delta t \sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2} g\Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2} g\Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$
$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t\Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2}g'\Delta x\Delta t E_i(t+\Delta t/2,x)V_i(t,x)$$
$$U_i(t+\Delta t,x) = g\Delta x\Delta t F_i(t+\Delta t/2,x)U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

Di, Wang, Zhou, Bian*, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

PT process simulation

Field basis

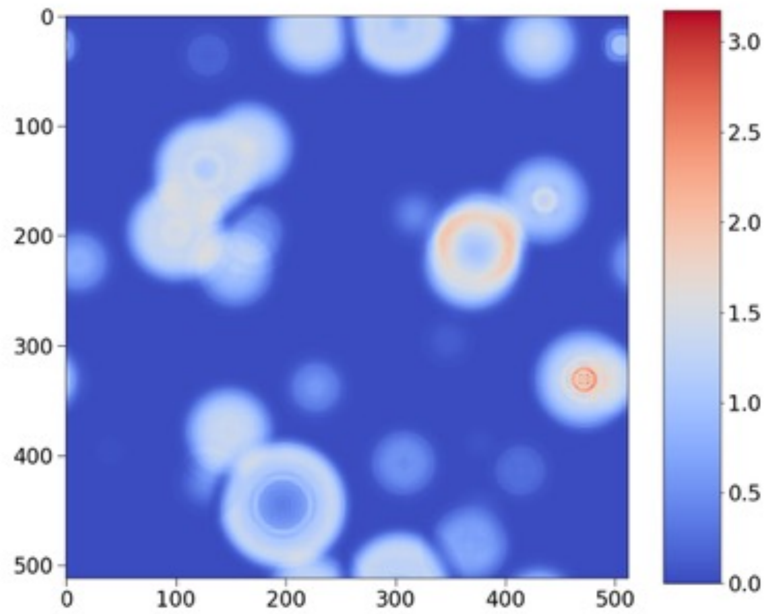
$$\partial_0^2\Phi = D_i D_i\Phi - 2\lambda(|\Phi|^2-\eta^2)\Phi - 3(\Phi^\dagger\Phi)^2\Phi/\Lambda^2,$$
$$\partial_0^2 B_i = -\partial_j B_{ij} + g'\,\mathrm{Im}[\Phi^\dagger D_i\Phi],$$
$$\partial_0^2 W_i^a = -\partial_k W_{ik}^a - g\,\epsilon^{abc}W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger\sigma^a D_i\Phi].$$
$$\partial_0\partial_j B_j - g'\,\mathrm{Im}[\Phi^\dagger\partial_0\Phi] = 0,$$
$$\partial_0\partial_j W_j^a + g\,\epsilon^{abc}W_j^b\partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger\sigma^a\partial_0\Phi] = 0.$$

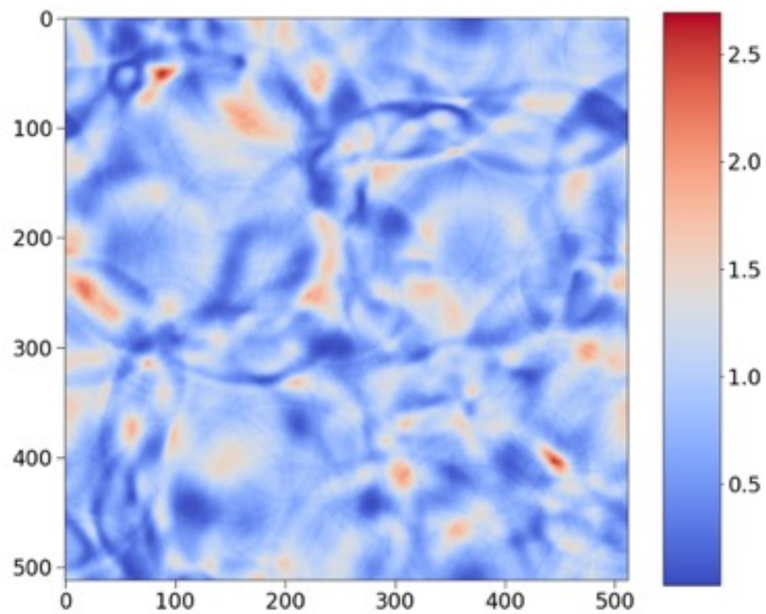
Lattice implementation

$$\Pi(t+\Delta t/2,x) = \Pi(t-\Delta t/2,x) + \Delta t\Big\{\frac{1}{\Delta x^2}\sum_i [U_i(t,x)V_i(t,x)\Phi(t,x+i) - 2\Phi(t,x) + U_i^\dagger(t,x-i)V_i^\dagger(t,x-i)\Phi(t,x-i)] - \frac{\partial U}{\partial\Phi^\dagger}\Big\}$$

$$\mathrm{Im}[E_k(t+\Delta t/2,x)] = \mathrm{Im}[E_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g'}{\Delta x}\mathrm{Im}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)\Phi(t,x)] - \frac{2}{g'\Delta x^3}\sum_i \mathrm{Im}[V_k(t,x)V_i(t,x+k)V_k^\dagger(t,x+i)V_i^\dagger(t,x) + V_i(t,x-i)V_k(t,x)V_i^\dagger(t,x+k-i)V_k^\dagger(t,x-i)]\Big\}$$

$$\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2,x)] = \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2,x)] + \Delta t\Big\{\frac{g}{\Delta x}\mathrm{Re}[\Phi^\dagger(t,x+k)U_k^\dagger(t,x)V_k^\dagger(t,x)i\sigma^m\Phi(t,x)] - \frac{1}{g\Delta x^3}\sum_i \mathrm{Tr}[i\sigma^m U_k(t,x)U_i(t,x+k)U_k^\dagger(t,x+i)U_i^\dagger(t,x) + i\sigma^m U_k(t,x)U_i^\dagger(t,x+k-i)U_k^\dagger(t,x-i)U_i(t,x-i)]\Big\},$$





Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

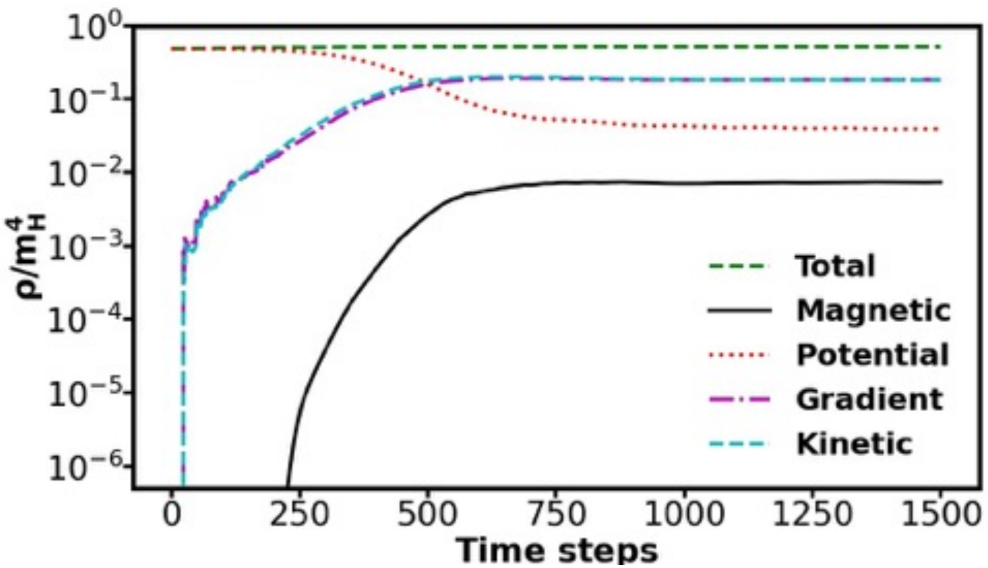
GW from Bubble collisions

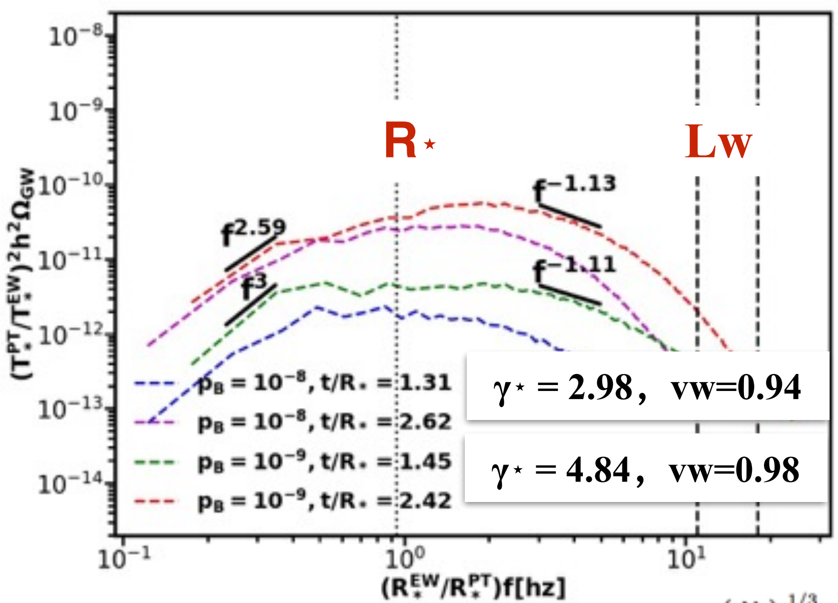
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \mathrm{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t) \dot{h}_{ij}^{TT}(\mathbf{k}',t) \rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3 \delta(\mathbf{k}+\mathbf{k}')$$

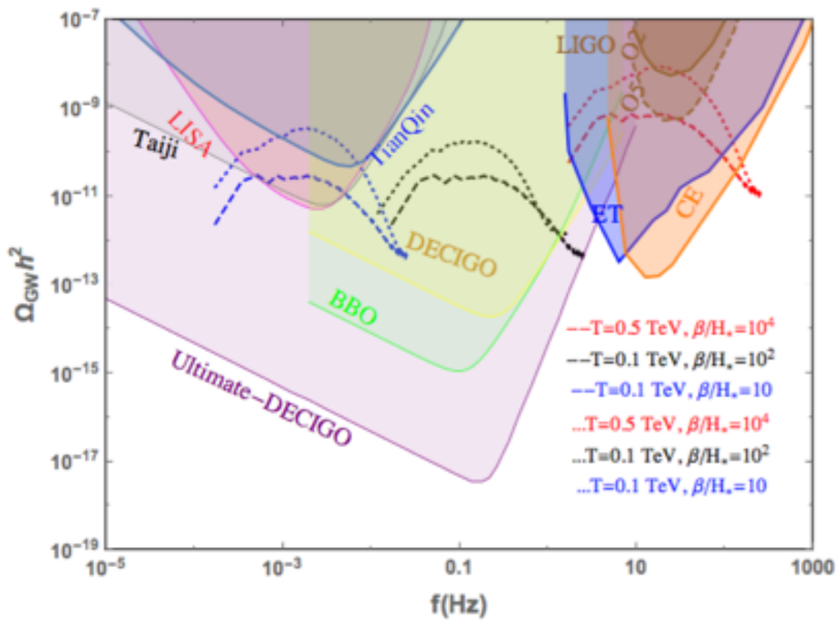
$$\frac{d\Omega_{gw}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_{\dot{h}}(\mathbf{k},t)$$





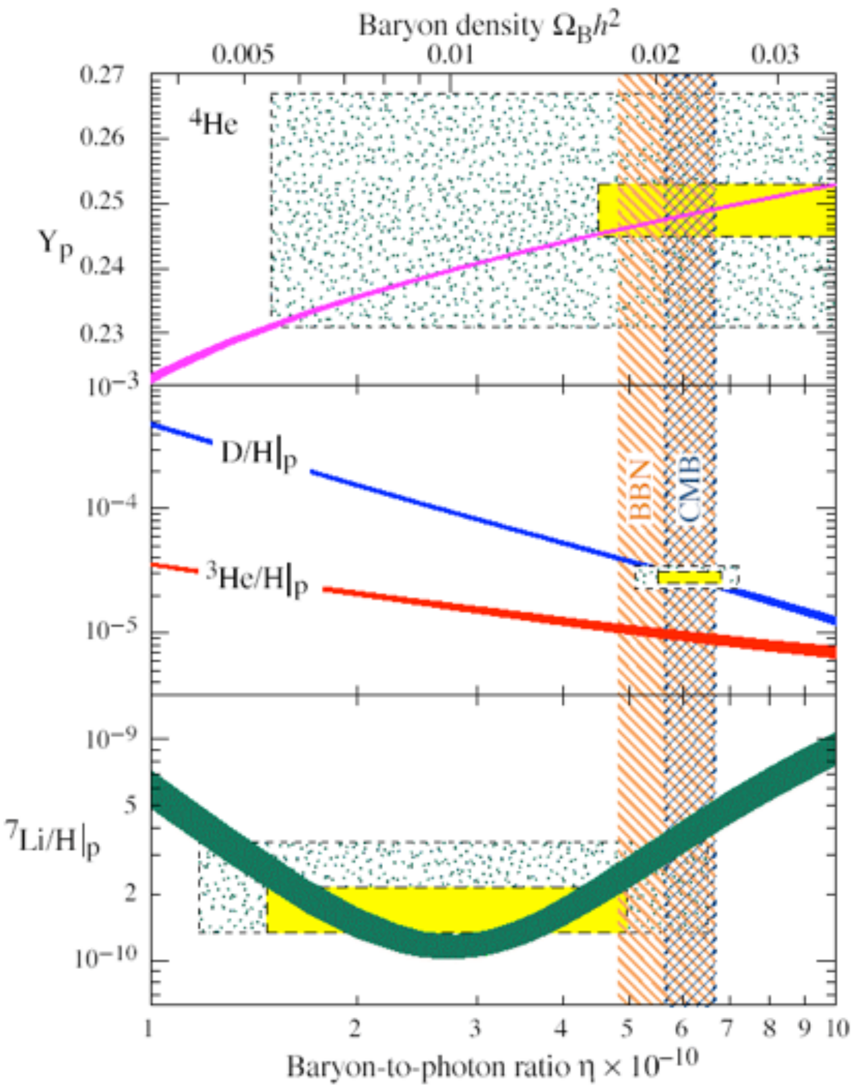
$\gamma_\star = R_\star/(2Rc)$

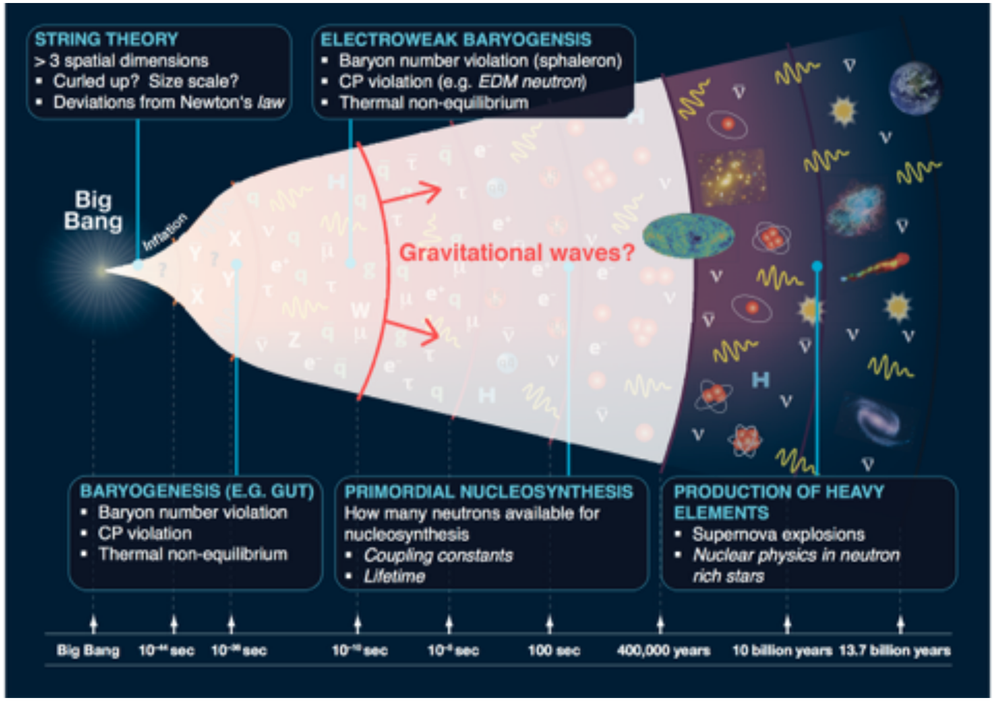
$R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$

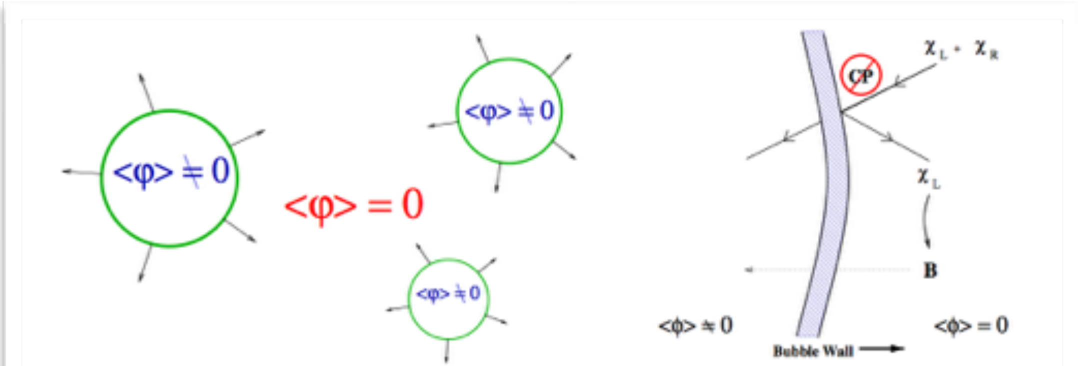


Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

BAU& CPV

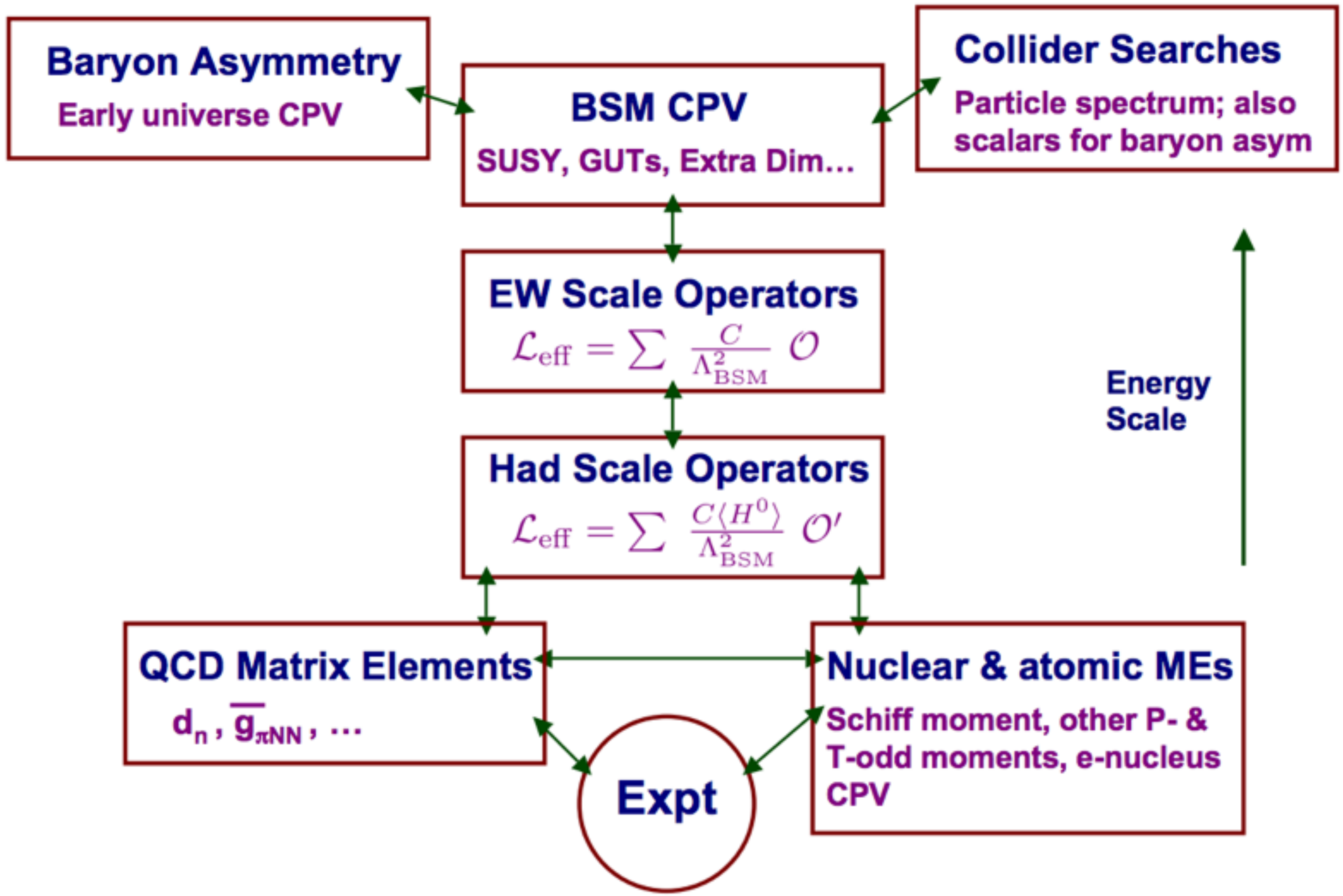






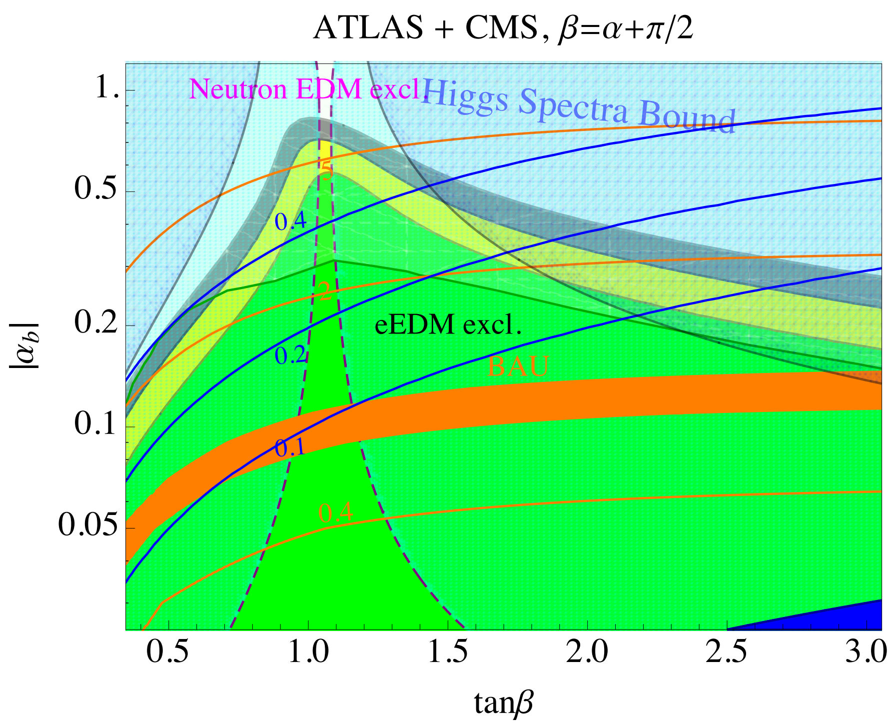
Chup etal, Rev Mod Phys.91.015001

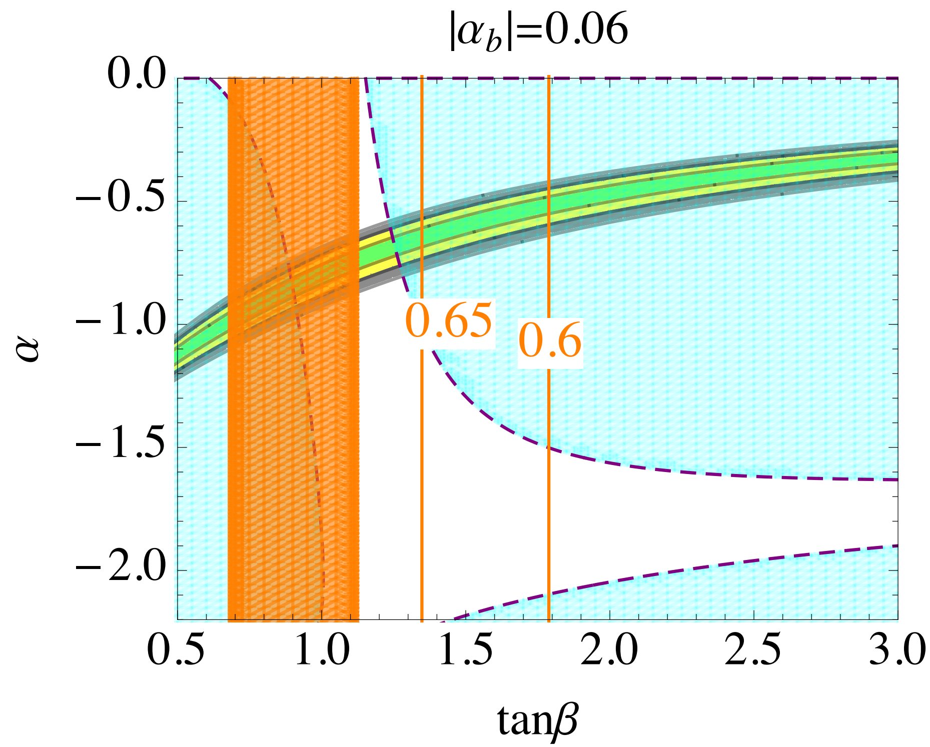
BAU& CPV



Engel etal, Prog.Part.Nucl.Phys. 71 (2013) 21-74

BAU& CPV





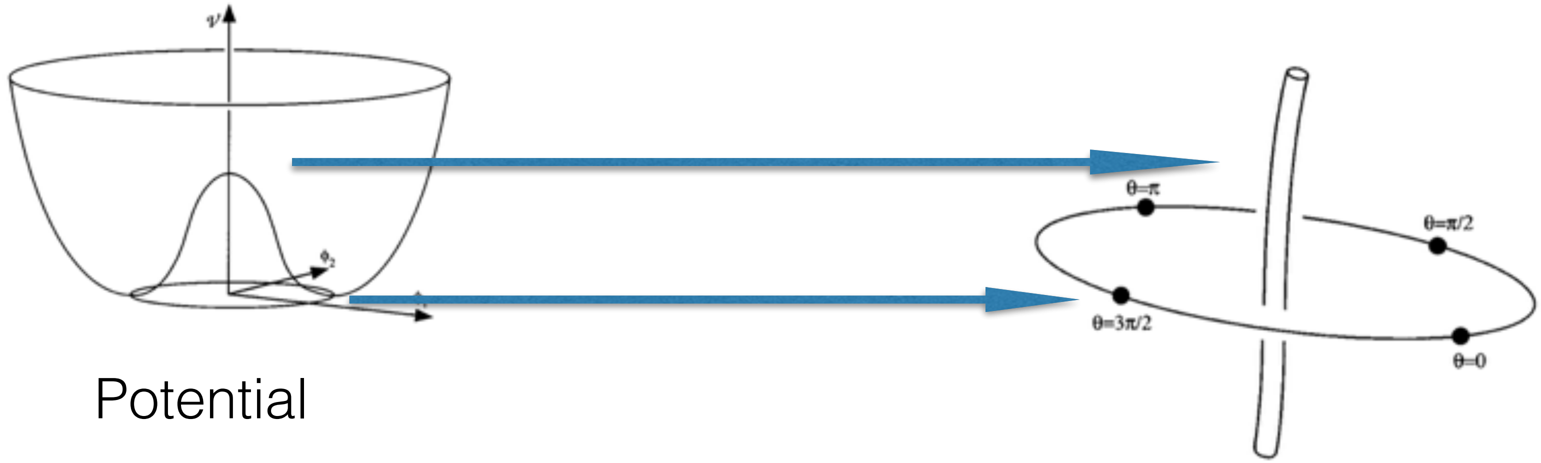
α, αb are mixing angles between two CP-even Higgs, the light CP-even and the CP-odd Higgs,

Bian, Liu, Shu, Phys.Rev.Lett. 115 (2015) 021801

CS formation at FOPT-Theory

CS: SSB of U(1) symmetry

The one-dimension topological defects: cosmic string



Potential

T. W. B. Kibble

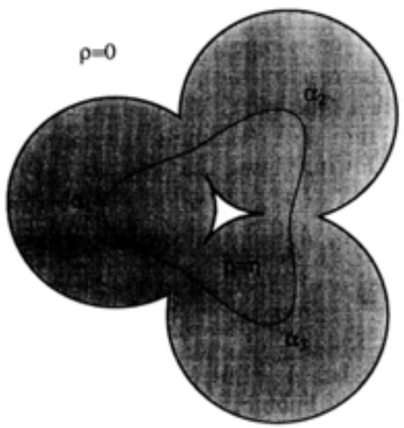
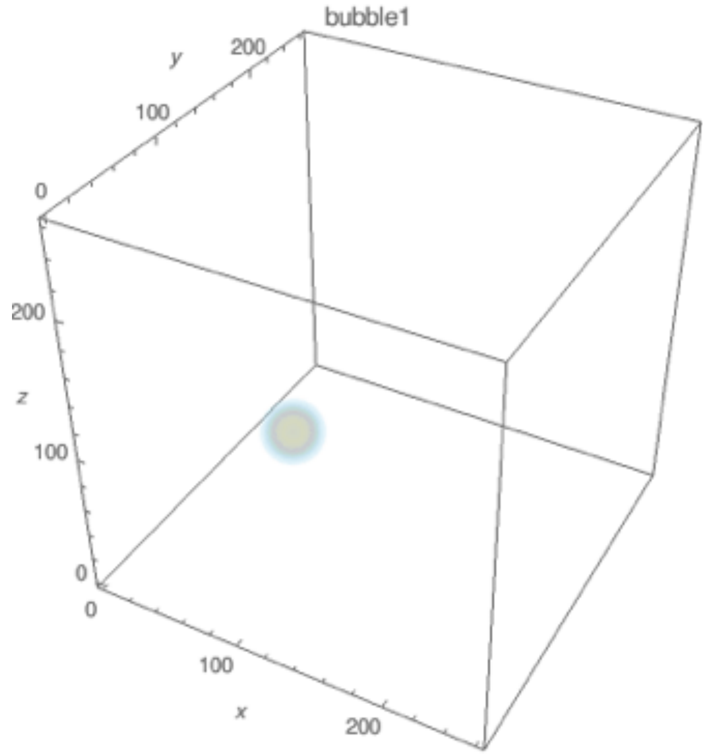


FIG. 1. Three bubbles of the broken symmetry phase ($\rho=\eta$) colliding. If the phase change of the scalar field around the loop γ is $\pm 2\pi$, a string (or antistring) is formed. If the phases α_i are ordered, then the requirement for a string is $\alpha_1+\pi<\alpha_3<\alpha_2+\pi$.

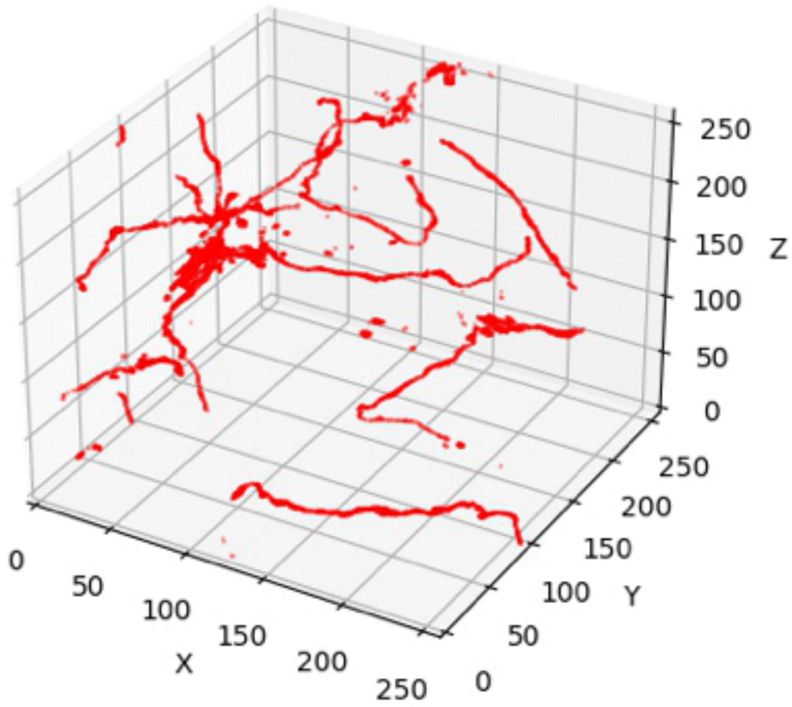
ξ_{str} of the string network is essentially the typical bubble diameter for SFOPT.

Aust. J. Phys., 1997, 50, 697–722
Phys.Rev.D 49 (1994) 1944-1950

CS formation at FOPT-Simulation



Bubbles evolution

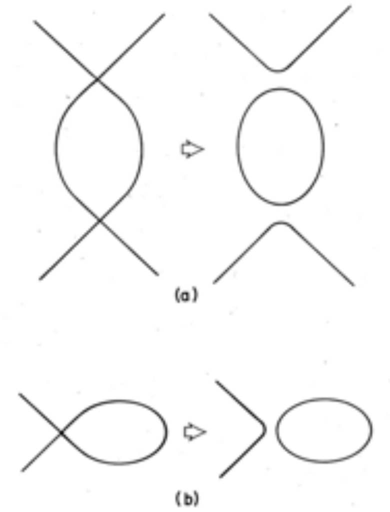


CS evolution

Bian,Yang,Zhao,to appear

CS loops and GWs

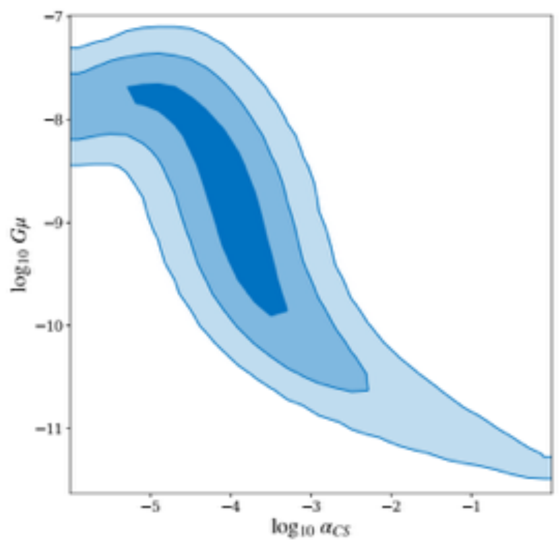
Loop formation



Closed-loop formation by intercommuting strings.

Cosmic string loop formation. A loop forms (a) when two strings interacts in 2 separate points or (b) when a string crosses itself.

Phys.Rev.D 30 (1984) 2036



GW emission

Cusps: a pointed and highly Lorentz-boosted region which appear few times per oscillation period

Nambu-Goto string tension and the breaking scale $\mu \approx 2\pi v_S{}^2$

$$\Omega_{\rm GW}^{cs}(f) = \sum_k \Omega_{\rm GW}^{(k)}(f)$$

$$\Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c}\frac{2k}{f}\frac{\mathcal{F}_\alpha \Gamma^{(k)} G\mu^2}{\alpha_{CS}(\alpha_{CS}+\Gamma G\mu)}\int_{t_F}^{t_0} d\tilde{t}\,\frac{C_{\rm eff}(t_i^{(k)})}{t_i^{(k)4}} \times \left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 \left[\frac{a(t_i^{(k)})}{a(\tilde{t})}\right]^3 \Theta(t_i^{(k)} - t_F).$$

Bian, Cai, Liu, Zhou, Phys.Rev.D 103 (2021) 8, L081301

Summary and future

- **Probing the EWPT with GW production and Colliders complementarily Higgs Potential shape**

1) **The future collider prospect, with dihiggs, Zh and/or Zhh production**
2) **Thin wall or thick wall tell by gravitational wave, wall profile and GW spectrum**

- **CPV @LHC&LHCb & EDMs**

- **Cosmic strings can form during the FOPT, with high scale symmetry breaking induce SSB of local or global U(1)**

Thanks

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0\,, \qquad \frac{d\phi_b}{dr}\Big|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

Latent heat

$$\alpha = \frac{1}{\rho_R}\left[-(V_{\rm EW} - V_f) + T\left(\frac{dV_{\rm EW}}{dT} - \frac{dV_f}{dT}\right)\right]\Big|_{T=T_*}$$

Phase transition inverse duration

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}\Big|_{T=T_n}$$

GW spectrum from FOPT

- **Bubble collisions**

$$\Omega_{\rm col}h^2 = 1.67\times10^{-5}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}\left(\frac{0.11v_b^3}{0.42+v_b^2}\right)\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}$$

peak frequency: $f_{\rm env} = 16.5\times10^{-6}\left(\frac{f_*}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}{\rm Hz}$

- **Sound Wave**

$$\Omega h^2_{\rm sw}(f) = 2.65\times10^{-6}(H_*\tau_{sw})\left(\frac{\beta}{H}\right)^{-1} v_b\left(\frac{\kappa_\nu\alpha}{1+\alpha}\right)^2\left(\frac{g_*}{100}\right)^{-\frac{1}{3}}\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

phase transition duration: $\tau_{sw} = min\left[\frac{1}{H_*},\frac{R_*}{\bar{U}_f}\right],\ H_*R_* = v_b(8\pi)^{1/3}(\beta/H)^{-1}$

Root-mean-square four-velocity of the plasma: $\bar{U}_f^2 \approx \frac{3}{4}\frac{\kappa_\nu\alpha}{1+\alpha}$

peak frequency: $f_{\rm sw} = 1.9\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$

- **MHD turbulence**

$$\Omega h^2_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{\beta}{H}\right)^{-1}\left(\frac{\epsilon\kappa_\nu\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{g_*}{100}\right)^{-\frac{1}{3}} v_b\frac{(f/f_{\rm turb})^3(1+f/f_{\rm turb})^{-\frac{11}{3}}}{[1+8\pi f a_0/(a_*H_*)]}$$

peak frequency: $f_{\rm turb} = 2.7\times10^{-5}\frac{\beta}{H}\frac{1}{v_b}\frac{T_*}{100}\left(\frac{g_*}{100}\right)^{\frac{1}{6}}{\rm Hz}$