Low-energy Phenomenology of Family Gauge Symmetry

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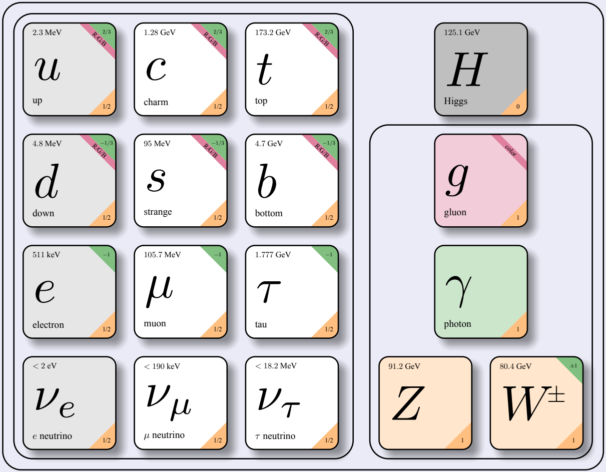
Introduction

$SU(3) \times SU(2)_L \times U(1)$



粒子物理标准模型

Introduction

$\sin^2(\theta_{12}) = 0.307 \pm 0.013, \sin^2(\theta_{23}) = 0.539 \pm 0.02,$
$\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ PDG(2020)

$$V_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \quad (1)$$

: Tri-Bimaximal mixing: H. P. S. (2002)

$$V_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}}c_\nu & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}}s_\nu \\ -\frac{1}{\sqrt{6}}c_\nu - \frac{1}{\sqrt{2}}s_\nu & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}c_\nu - \frac{1}{\sqrt{6}}s_\nu \\ -\frac{1}{\sqrt{6}}c_\nu + \frac{1}{\sqrt{2}}s_\nu & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}c_\nu - \frac{1}{\sqrt{6}}s_\nu \end{pmatrix} \quad (2)$$

$$\equiv V_{TB} \begin{pmatrix} c_\nu & 0 & s_\nu \\ 0 & 1 & 0 \\ -s_\nu & 0 & c_\nu \end{pmatrix} \quad (3)$$

$SO(3)$ YLWu(2008)

Family Gauge Symmetry: SO(3)

Representations

$$\Psi^U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \qquad \Psi^D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \tag{4}$$

$$T^a/2: \quad (T^a)_{bc} = -i\epsilon_{abc}. \quad \text{and} \quad A^a_\mu, \quad (a,b,c=1,2,3) \tag{5}$$

$$\mathcal{L} \supset -i\frac{g_f}{2}\epsilon_{abc}\bar{\Psi}^U_a\gamma^\mu\Psi^U_b A^c_\mu + (U \to D) \tag{6}$$

$$= \frac{g_f}{2}\bar{\Psi}^U V^c_U \gamma^\mu A^c_\mu \Psi^U + (U \to D) \tag{7}$$

$$V_{\mathrm{CKM}} = V_U^\dagger V_D \tag{8}$$

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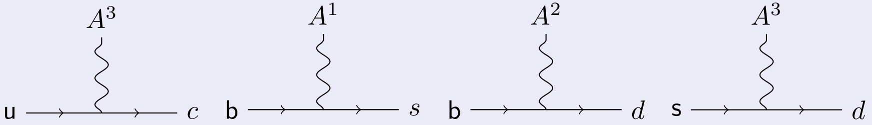
$$V_{\mathrm{CKM}} = V_U^\dagger V_D \tag{8}$$

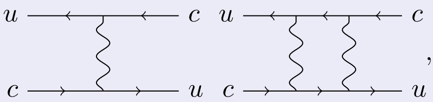
In SM, V_u and V_d are not detectable. Only V_{CKM} is physical.

Family Gauge Symmetry: SO(3)

FCNC Coupling



$\Delta F = 2$ Process: $P - \bar{P}$ Mixing



$$, \quad \begin{array}{l}(c \to b, u \to d/s),\\ (c \to s, u \to d)\end{array} \tag{9}$$

$\Delta F = 1$ processes are suppressed by mixing.

Family Gauge Symmetry : SO(3)

The effective Hamiltonian for the $\Delta F = 2$ processes,

$$\mathcal{H}_{\rm eff}(\Delta F = 2) \;\; = \;\; \mathcal{H}^{\rm SM}_{\rm eff}(\Delta F = 2) + \mathcal{H}^{\rm NP}_{\rm eff}(\Delta F = 2). \qquad (10)$$

The $\mathcal{H}^{\rm SM}_{\rm eff}$ is from the box diagrams in SM, while the $\mathcal{H}^{\rm NP}_{\rm eff}$ is from the new gauge interactions,

$$\begin{aligned}\mathcal{H}^{\rm NP}_{\rm eff}(\Delta F = 2) \;\; = \;\; & C_1^{D-\bar{D}}(\bar{u}c)_V(\bar{u}c)_V + C_2^{D-\bar{D}}(\bar{u}_\alpha c_\beta)_V(\bar{u}_\beta c_\alpha)_V \\ & + C_1^{K-\bar{K}}(\bar{d}s)_V(\bar{d}s)_V + C_2^{K-\bar{K}}(\bar{d}_\alpha s_\beta)_V(\bar{d}_\beta s_\alpha)_V \\ & + C_1^{B-\bar{B}}(\bar{d}b)_V(\bar{d}b)_V + C_2^{B-\bar{B}}(\bar{d}_\alpha b_\beta)_V(\bar{d}_\beta b_\alpha)_V \\ & + C_1^{B_s-\bar{B}_s}(\bar{s}b)_V(\bar{s}b)_V + C_2^{B_s-\bar{B}_s}(\bar{s}_\alpha b_\beta)_V(\bar{s}_\beta b_\alpha)_V. \qquad (11)\end{aligned}$$

At tree level, the Wilson coefficients are

$$\begin{aligned} C_1^{D-\bar{D}} &= g^2\left(\sum_{i=1}^{3}\frac{{U^i_{12}}^2}{4M_i^2}\right), \qquad C_1^{K-\bar{K}} = g^2\left(\sum_{i=1}^{3}\frac{{V^i_{12}}^2}{4M_i^2}\right), \\ C_1^{B-\bar{B}} &= g^2\left(\sum_{i=1}^{3}\frac{{V^i_{13}}^2}{4M_i^2}\right), \qquad C_1^{B_s-\bar{B}_s} = g^2\left(\sum_{i=1}^{3}\frac{{V^i_{23}}^2}{4M_i^2}\right). \qquad (12)\end{aligned}$$

$C_2^{K-\bar{K}} = C_2^{B-\bar{B}} = C_2^{B_s-\bar{B}_s} = C_2^{D-\bar{D}} = 0$ at tree level.

Master Formulas

$P - \bar{P}$ Mixing

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_W \tag{13}$$

$$|\psi\rangle = a(t)|P\rangle + b(t)|\bar{P}\rangle + \sum_i c_i(t)|n_i\rangle \tag{14}$$

Weisskopf-Wigner Approximation:

$$i\frac{d}{dt}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \tag{15}$$

$$H = M - i\Gamma/2 \tag{16}$$

$$H_{ij} = m_0\delta_{ij} + \langle i|\mathcal{H}_W|j\rangle + \sum_n P\left\{\frac{\langle i|\mathcal{H}_W|n\rangle\langle n|\mathcal{H}_W|j\rangle}{m_0 - E_n}\right\} \tag{17}$$

$$- i\pi\sum\delta(m_0 - E_n)\langle i|\mathcal{H}_W|n\rangle\langle n|\mathcal{H}_W|j\rangle \tag{18}$$

$$\Delta M = M_H - M_L = 2Re\sqrt{H_{12}H_{21}} \tag{19}$$

Master Formulas

$$H_{12} = \langle P|\mathcal{H}_{\rm eff}|\bar{P}\rangle \tag{21}$$

The factorizations of the hadron matrix elements

$$\langle\bar{P}|(V-A)(V-A)|P\rangle = \frac{4}{3}m_P f_P^2 B_1^P(\mu), \tag{22}$$

$$\langle\bar{P}|(V-A)(V+A)|P\rangle = -\frac{2}{3}R(\mu)m_P f_P^2 B_2^P(\mu), \tag{23}$$

where the factor $R = M^2/(m_q + m_q')^2$ and the M is the average mass of the P and $\bar{P}$. m_q and $m_{q'}$ are the mass of the quarks which are the components of the meson. f_P is the decay constant and B_is are the bag parameters which are unit in naive factorization.

Input Parameters

	K	D	B	B_s
M_P	0.498GeV	1.86 GeV	5.28GeV	5.37GeV
f_P	156 ± 0.8MeV	191 ± 23MeV	220 ± 40MeV	205 ± 10MeV
B_1	0.571 ± 0.048	0.87 ± 0.03	0.87 ± 0.04	0.86 ± 0.02
B_2	0.562 ± 0.039	1.46 ± 0.09	1.91 ± 0.04	1.94 ± 0.03

SM with above input (Refs. can be found in the paper)

$$\Delta M_K = (3.483 \pm 0.006) \times 10^{-15}\text{GeV}.$$
$$\Delta M_B = (3.337 \pm 0.033) \times 10^{-13}\text{GeV}$$
$$\Delta M_{B_s} = (1.170 \pm 0.008) \times 10^{-11}\text{GeV}$$
$$\Delta M_D = (1.4 \pm 0.5) \times 10^{-14}\text{GeV}.$$

$$\Delta M_K = 2.312^{+0.024+0.466}_{-0.024-0.462} \times 10^{-15}\mathrm{GeV}$$
$$\Delta M_{B_d} = 3.483^{+0.991+0.161}_{-0.789-0.159} \times 10^{-13}\mathrm{GeV}$$
$$\Delta M_{B_s} = 1.20^{+0.47+0.03}_{-0.77-0.03} \times 10^{-11}\mathrm{GeV}$$
$$\Delta M_D = 3.84^{+0.38+0.12}_{-0.36-0.14} \times 10^{-18}\mathrm{GeV}$$

Results

Result I

$$\frac{g_f^2}{M_1^2} \leq 1.4 \times 10^{-10}\mathrm{GeV}^{-2} \qquad M_1 \geq 27TeV \tag{24}$$

$$\frac{g_f^2}{M_2^2} \leq 4.2 \times 10^{-12}\mathrm{GeV}^{-2} \qquad M_2 \geq 160TeV \tag{25}$$

$$\frac{g_f^2}{M_3^2} \leq 1.0 \times 10^{-13}\mathrm{GeV}^{-2} \qquad M_3 \geq 10^3TeV \tag{26}$$

Result II

$$\Delta M_D = 0.85^{+0.08+0.04+0.12}_{-0.08-0.03-0.11} \times 10^{-14}\mathrm{GeV} \tag{27}$$

Li,Bao,Si (2012)

Family Gauge Symmetry: SU(3)

$$F_\mu = F_\mu^a \frac{\lambda^a}{2} = \begin{pmatrix} \frac{1}{2}\left(F_3 + \frac{F_8}{\sqrt{3}}\right) & \frac{1}{2}\left(F_1 - iF_2\right) & \frac{1}{2}\left(F_4 - iF_5\right) \\ \frac{1}{2}\left(F_1 + iF_2\right) & \frac{1}{2}\left(\frac{F_8}{\sqrt{3}} - F_3\right) & \frac{1}{2}\left(F_6 - iF_7\right) \\ \frac{1}{2}\left(F_4 + iF_5\right) & \frac{1}{2}\left(F_6 + iF_7\right) & -\frac{F_8}{\sqrt{3}} \end{pmatrix}_\mu . \tag{28}$$

$$\mathcal{L} \supset g_F^2 \,\mathrm{tr}\left(F^\mu \Phi_\nu {F_\mu}^* \Phi_\nu^* + F^\mu \Phi_\nu \Phi_\nu^* {F_\mu}^\dagger + \Phi_\nu {F_\mu}^T F^{\mu *} \Phi_\nu^* + \Phi_\nu {F_\mu}^T \Phi_\nu^* F^{\mu \dagger}\right) . \tag{29}$$

$$\langle \Phi_\nu \rangle = V_0 + \begin{pmatrix} V_1 & V_2 & V_2 \\ V_2 & V_2 & V_1 \\ V_2 & V_1 & V_2 \end{pmatrix} \tag{30}$$

Bao, Liu, Wu(2016)

Family Gauge Symmetry: SU(3)

The masses of the five heavy gauge bosons are

$$\begin{aligned} M_1 &= 2g_F V_0, \quad M_2 = 2g_F(V_0^2 + 2V_1V_0 + V_2V_0)^{1/2}, \\ M_3 &= 2g_F(V_0^2 + 3V_2V_0)^{1/2}, \\ M_4 &= \frac{2\sqrt{3}}{3} g_F V_0^{1/2} \left(2V_0 + 2V_1 + 4V_2 + 2\sqrt{4V_1^2 - 2V_1V_2 + 7V_2^2}\right)^{1/2}, \\ M_5 &= \frac{2\sqrt{3}}{3} g_F V_0^{1/2} \left(2V_0 + 2V_1 + 4V_2 - 2\sqrt{4V_1^2 - 2V_1V_2 + 7V_2^2}\right)^{1/2}. \end{aligned} \quad (31)$$

And the masses of the three light gauge bosons, which are related to the $SO(3)_F$ symmetry, are

$$M_6 = 2g_F|V_2 - V_1|, \quad M_7 = 3g_F|V_2|, \quad M_8 = g_F|2V_1 + V_2|. \quad (32)$$

$$V_0 > V_2 > V_1 : \quad V_1, r_1 \equiv \frac{V_1}{V_0} \simeq \lambda, r_2 \equiv \frac{V_2 - V_1}{V_0} \simeq 2\lambda \quad (33)$$

Family Gauge Symmetry: SU(3)

The Interactions

$$\begin{aligned}\mathcal{L}_{int} \supset \quad & g_F\left[\overline{u}_L\gamma^\mu(U_L^{u\dagger}F_\mu U_L^u)u_L + \overline{u}_R\gamma^\mu(U_R^{u\dagger}F_\mu U_R^u)u_R\right] \\ + \quad & g_F\left[\overline{d}_L\gamma^\mu(U_L^{d\dagger}F_\mu U_L^d)d_L + \overline{d}_R\gamma^\mu(U_R^{d\dagger}F_\mu U_R^d)d_R\right] \\ + \quad & g_F\left[\overline{e}_L\gamma^\mu(U_L^{e\dagger}F_\mu U_L^e)e_L + \overline{e}_R\gamma^\mu(U_R^{e\dagger}F_\mu U_R^e)e_R\right] \\ + \quad & g_F\overline{\nu}_L\gamma^\mu(U_L^{\nu\dagger}F_\mu U_L^\nu)\nu_L, \end{aligned} \tag{34}$$

Mixing Matrix Parameterization

$$\begin{aligned} & U_L^u = U_R^u = U^u, \quad U_L^d = U_R^d = U^d, \\ & U_L^e = U_R^e = U_e, \quad U_L^\nu = U_\nu = U_{TB}, \end{aligned} \tag{35}$$

$$U_{CKM} = U^{u\dagger}U^d, \quad U_{PMNS} = U_e^\dagger U_{TB}, \tag{36}$$

Family Gauge Symmetry: SU(3)

Mixing Matrix Parameterization

We can also assume that U^u, U^d and U_e have the same hierarchy structures as U_{CKM} and can be parameterized via Wolfenstein method.

$$U_{CKM} \sim \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\rho e^{-i\delta} \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (37)$$

For the mixing matrix in up(down) quark sectors, we have mixing matrix $U^u(U^d)$ with the parameters A, λ, ρ, δ replaced by $A_u, \lambda_u, \rho_u, \delta_u$ $(A_d, \lambda_d, \rho_d, \delta_d)$.

$\Delta F = 2$: $K - \bar{K}$ mixing

$$\mathcal{H}_K^{New} = \mathcal{C}_K (\bar{s}\gamma_\mu d) \otimes (\bar{s}\gamma^\mu d) + H.c.. \quad (38)$$

Here we treat λ_d as a small parameter and get the coefficient in Eq.(38) to the order of λ_d^2. At higher order the heavy family gauge bosons' effects should be take into consideration. The coefficient $\mathcal{C}_K$ is

$$\mathcal{C}_K = \frac{1}{16}[F_K(V_1, V_2) + G_K(V_1, V_2) A_d \lambda_d^2] + \mathcal{O}(\lambda_d^3), \quad (39)$$

where

$$\begin{aligned} F_K(V_1, V_2) &= \frac{1}{6\,(V_2 - V_1)^2} + \frac{1}{3\,(2V_1 + V_2)^2} + \frac{1}{9V_2^2}. \\ G_K(V_1, V_2) &= \frac{1}{3\,(V_2 - V_1)^2} + \frac{2}{3\,(2V_1 + V_2)^2} - \frac{2}{9V_2^2}. \end{aligned} \quad (40)$$

Family Gauge Symmetry: SU(3)

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\rm NP} | K^0 \rangle = \frac{F_K(V_1, V_2) f_K^2 M_K}{96} \left[1 + 2R(\mu)\right]. \quad (41)$$

$$\frac{1}{F_K(V_1, V_2)} = \frac{f_K^2 M_K}{48 \Delta m^{New}} \left[1 + 2R(\mu)\right] \geq \frac{f_K^2 M_K}{48 \Delta m_K}. \quad (42)$$

Results I:

$P^0 - \bar{P}^0$	$[\Delta m_{meson}]^{PDG}$	M_{meson}	f_{meson}	$V_1 \geq$
$K - \bar{K}$	$(3.483 \pm 0.006) \times 10^{-12}$	497.6	156 ± 1.2	7.0×10^7
$D - \bar{D}$	$(1.57^{+0.39}_{-0.41}) \times 10^{-11}$	1864.86 ± 0.13	206 ± 11	8.4×10^7
$B_d - \bar{B}_d$	$(3.337 \pm 0.033) \times 10^{-10}$	5279.58 ± 0.17	195 ± 11	2.9×10^7
$B_s - \bar{B}_s$	$(116.4 \pm 0.5) \times 10^{-10}$	5366.77 ± 0.24	243 ± 11	0.7×10^7

$$V_1 \geq 69.8 TeV, \quad V_2 \geq 209 TeV, \quad V_0 \geq 317 TeV. \quad (43)$$

Family Gauge Symmetry: SU(3)

Result II. $\Delta F = 1$

$$Br(K^+ \to \pi^+ \nu\bar{\nu})|_{NP} \approx Br(K_L \to \pi^0 \nu\bar{\nu})|_{NP} \leq 4.6 \times 10^{-16}. \tag{44}$$

$$Br(K^+ \to \pi^+ \nu\bar{\nu})|_{SM} = (1.5^{+3.4}_{-1.2}) \times 10^{-10}, \tag{45}$$

$$Br(K_L \to \pi^0 \nu\bar{\nu})|_{SM} = (2.6 \pm 1.2) \times 10^{-11}. \tag{46}$$

$$Br(\mu \to e^- e^+ e^-) \leq 4.1 \times 10^{-16}. \tag{47}$$

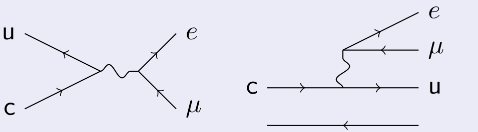
$$Br(\mu \to 3e)|_{SM} \sim 10^{-56}. \tag{48}$$

$$Br(\mu^- \to 3e)|_{Exp} < 1.0 \times 10^{-12}. \tag{49}$$

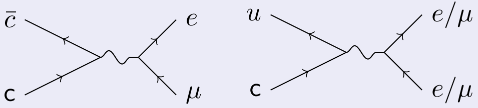
Refs. can be found in the paper.

Family Gauge Symmetry

Something missed: Lepton flavor violation process



Favored (50)



Not Favored (51)

Summary

- Introduce a new gauge symmetry or interaction
- $\Delta F = 2$ at tree-level
- SO(3):
  - New Physics scale $\Lambda_{\texttt{Family}} \sim 100\text{TeV}$ in addition to $\Lambda_{\texttt{QCD}} \sim 100\text{MeV}$ and $\Lambda_{EW} \sim 100\text{GeV}$.

$$M_1 \geq 27\text{TeV}, \quad M_2 \geq 160\text{TeV}, \quad M_3 \geq 10^3\text{TeV}. \tag{52}$$

  - Consistent result for ΔM_D

$$\Delta M_D = 3.84^{+0.38+0.12}_{-0.36-0.14} \times 10^{-18}\text{GeV} \tag{53}$$

- SU(3):
  - $\Delta F = 2$ gives constrains to the vev

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  - $\Delta F = 1$
- Lepton number violating channels to be considered.

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Thanks for your attention

Buckups

The particles in the model

	Fields	Representation
SM fermions	$\begin{pmatrix} u, c, t \\ d, s, b \end{pmatrix}_L$	$(3_F, 3_C, 2_L, (1/6)_Y)$
	$(u, c, t)_R$	$(3_F, 3_C, 1_L, (2/3)_Y)$
	$(d, s, b)_R$	$(3_F, 3_C, 1_L, (-1/3)_Y)$
	$\begin{pmatrix} e, \mu, \tau \\ \nu_e, \nu_\mu, \nu_\tau \end{pmatrix}_L$	$(3_F, 1_C, 2_L, (-1/2)_Y)$
	$(e, \mu, \tau)_R$	$(3_F, 1_C, 1_L, (-1)_Y)$
SM Higgs	H	$(1_F, 1_C, 2_L, (1/2)_Y)$
New fermions	U	$(3_F, 3_C, 1_L, (2/3)_Y)$
	D	$(3_F, 3_C, 1_L, (-1/3)_Y)$
	E	$(3_F, 1_C, 1_L, (-1)_Y)$
	N_R	$(3_F, 1_C, 1_L, 0_Y)$
New Higgs	Φ_1, Φ_2	$(8_F, 1_C, 1_L, 0_Y)$
	Φ_ν	$(6_F, 1_C, 1_L, 0_Y)$
	ϕ_s	$(1_F, 1_C, 1_L, 0_Y)$

The Lagrangian of the model

The field strengths of all gauge fields, including the $SU(3)$ family symmetry, are defined as

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_{F,\nu} - \partial_\nu A^a_{F,\mu} + g_F f^{abc} A^b_{F,\mu} A^c_{F,\nu}, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_w \epsilon^{abc} W^b_\mu W^c_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \tag{55}$$

We define the covariant derivative as

$$\begin{aligned} D_\mu &= \partial_\mu - i g_F A^a_{F,\mu} T^a - i g_s G_\mu - g_w W_\mu + i g'_w Y B_\mu \\ &= D^{SM}_\mu - i g_F A^a_{F,\mu} T^a. \end{aligned} \tag{56}$$

The full Lagrangian is

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_k + \mathcal{L}_H + \mathcal{L}_Y + \mathcal{L}_N, \tag{57}$$

The Lagrangian

$$
\begin{aligned}
\mathcal{L}_G &= -\frac{1}{4}\left(F^a_{\mu\nu}F^{a\mu\nu} + G^b_{\mu\nu}G^{b\mu\nu} + W^c_{\mu\nu}W^{c\mu\nu} + B_{\mu\nu}B^{\mu\nu}\right) \\
\mathcal{L}_k &= \overline{u}_{L,R} i\gamma^\mu D_\mu u_{L,R} + \overline{d}_{L,R} i\gamma^\mu D_\mu d_{L,R} + \overline{e}_{L,R} i\gamma^\mu D_\mu e_{L,R} + \overline{\nu}_L i\gamma^\mu D_\mu \nu_L, \\
\mathcal{L}_H &= \mathcal{L}_{DH} - V[H, \Phi_1, \Phi_2, \Phi_\nu, \phi_s] \\
&= \left(D^{SM}_\mu H\right)^\dagger \left(D^{\mu,SM} H\right) + \mathrm{Tr}\left(D_\mu \Phi_1 (D^\mu \Phi_1)^\dagger\right) + \mathrm{Tr}\left(D_\mu \Phi_2 (D^\mu \Phi_2)^\dagger\right) \\
&\quad + \mathrm{Tr}\left(D_\mu \Phi_\nu (D^\mu \Phi_\nu)^*\right) + \partial_\mu \phi_s \partial^\mu \phi_s - V\left(H, \Phi_1, \Phi_2, \Phi_\nu, \phi_s\right). \\
\mathcal{L}_Y &= y^u_L \bar{l} H U + y^u_R \overline{u}_R \phi_s U + \frac{1}{2}\overline{U}(\Delta^U_1 \Phi_1 + \Delta^U_2 \Phi_2) U \\
&\quad + y^d_L \bar{l}\tilde{H} D + y^d_R \overline{d}_R \phi_s D + \frac{1}{2}\overline{D}(\Delta^D_1 \Phi_1 + \Delta^D_2 \Phi_2) D \\
&\quad + y^e_L \bar{l} H E + y^e_R \overline{e}_R \phi_s E + \frac{1}{2}\overline{E}(\Delta^E_1 \Phi_1 + \Delta^E_2 \Phi_2) E \\
&\quad + y^\nu_L \bar{l}\tilde{H} N_R + \frac{1}{2}\xi^\nu \overline{N}_R \Phi_\nu N^c_R + H.C.. \\
\mathcal{L}_N &= i\overline{U}\gamma^\mu(\partial_\mu - i g_s G_\mu - i g_F A^a_{F,\mu} T^a) U + i\overline{D}\gamma^\mu(\partial_\mu - i g_s G_\mu - i g_F A^a_{F,\mu} T^a) D \\
&\quad + i\overline{E}\gamma^\mu(\partial_\mu - i g_F A^a_{F,\mu} T^a) E + i\overline{N_R}\gamma^\mu(\partial_\mu - i g_F A^a_{F,\mu} T^a) N_R.
\end{aligned}
$$