Spin transport & relaxation

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Outline

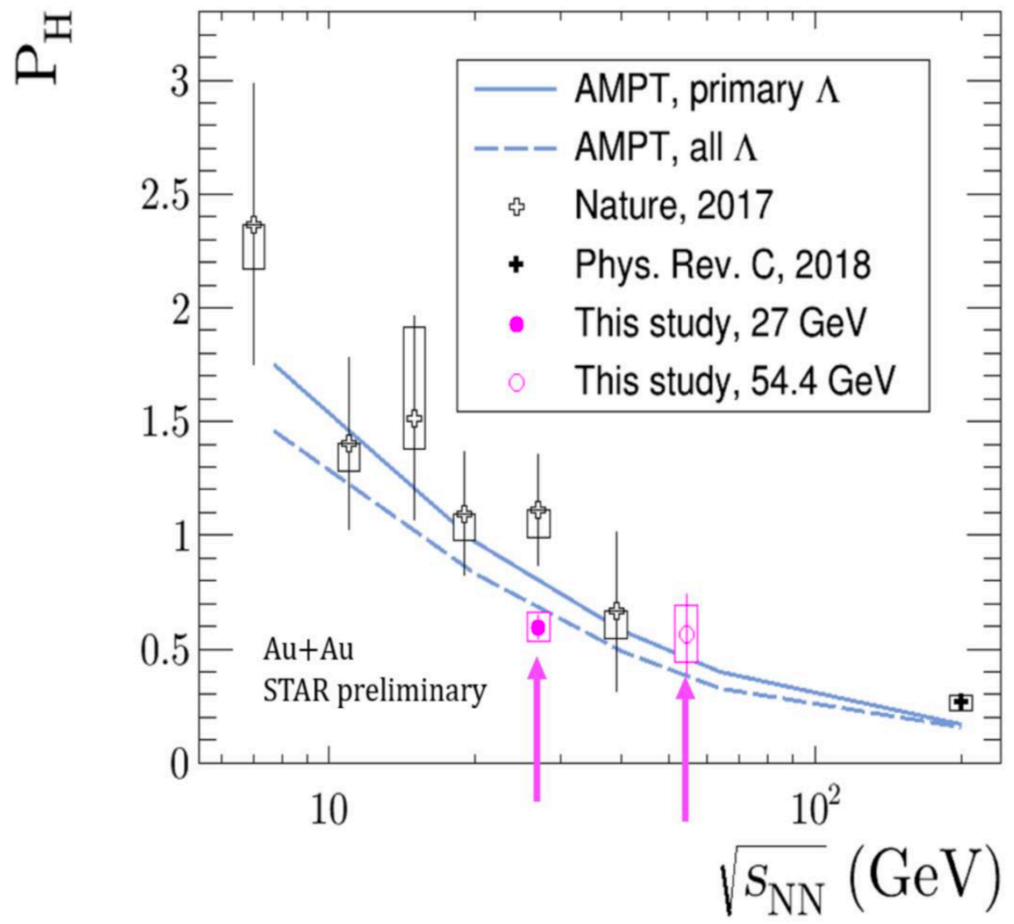
- Background
- Spin Kinetic Equation
  - Contact interaction
  - QED
- Decomposition and static solution in local equilibrium
- Summary

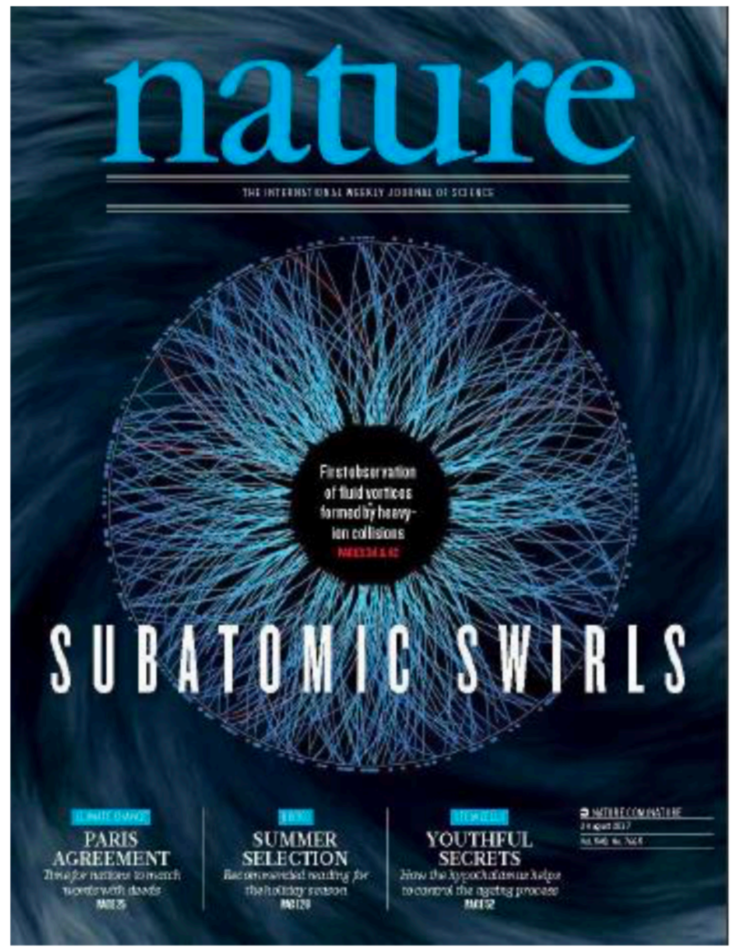
Why kinetic theory of spin necessary?

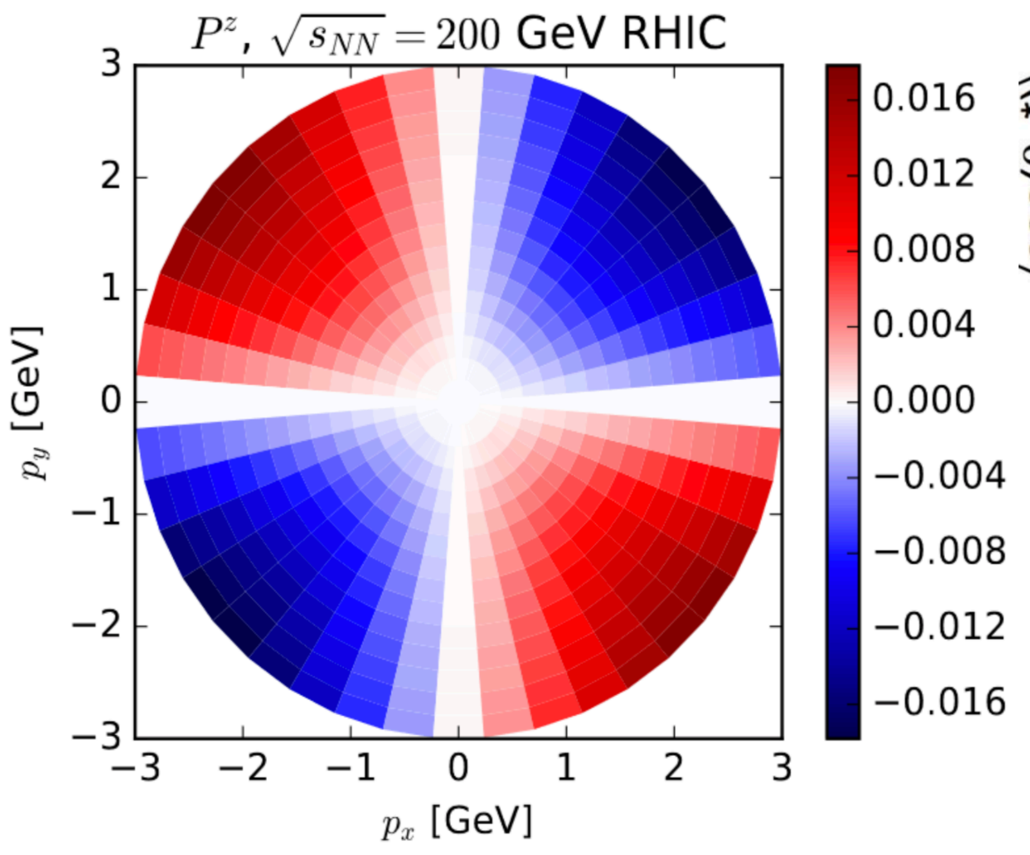
"With **thermodynamics**, one can calculate almost everything crudely;
with **kinetic theory**, one can calculate fewer things, but more accurately;
and with **statistical mechanics**, one can calculate almost nothing exactly."

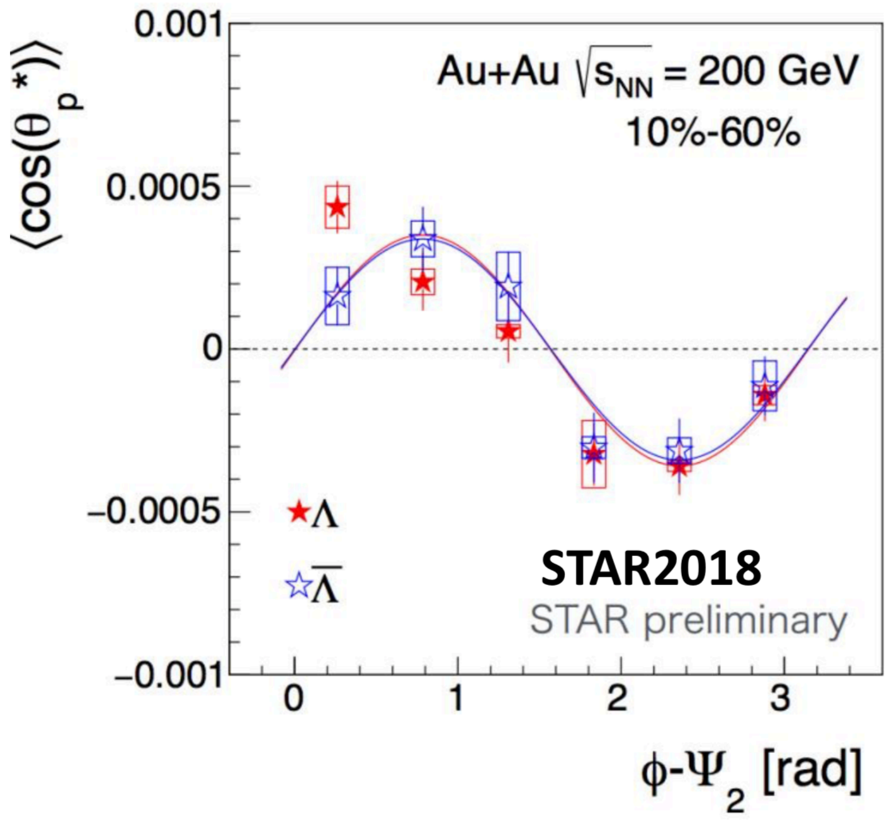
— Eugene Wigner

- **Accurate microscopic description of non-equilibrium evolution of spin 1/2 particles.**
- **Large OAM in non-central heavy ion collisions, spin polarization, collective property of strong interaction matter**
- **Robust global polarization v.s. "spin sign problem"**
- **Shear tensor tends to solve the problem, indicating that at least global equilibrium not enough.**



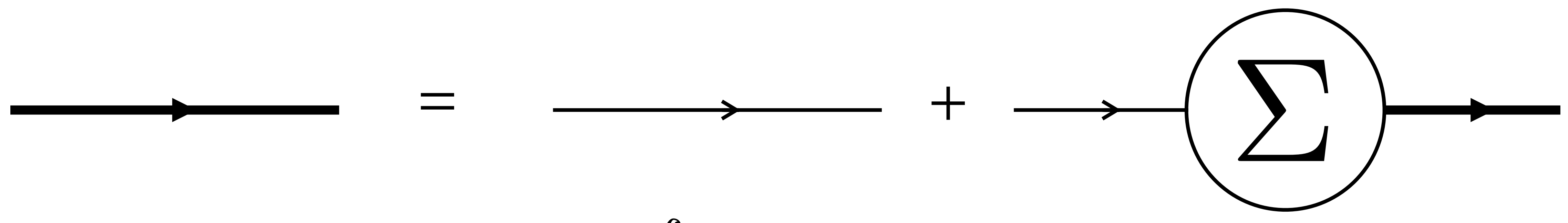






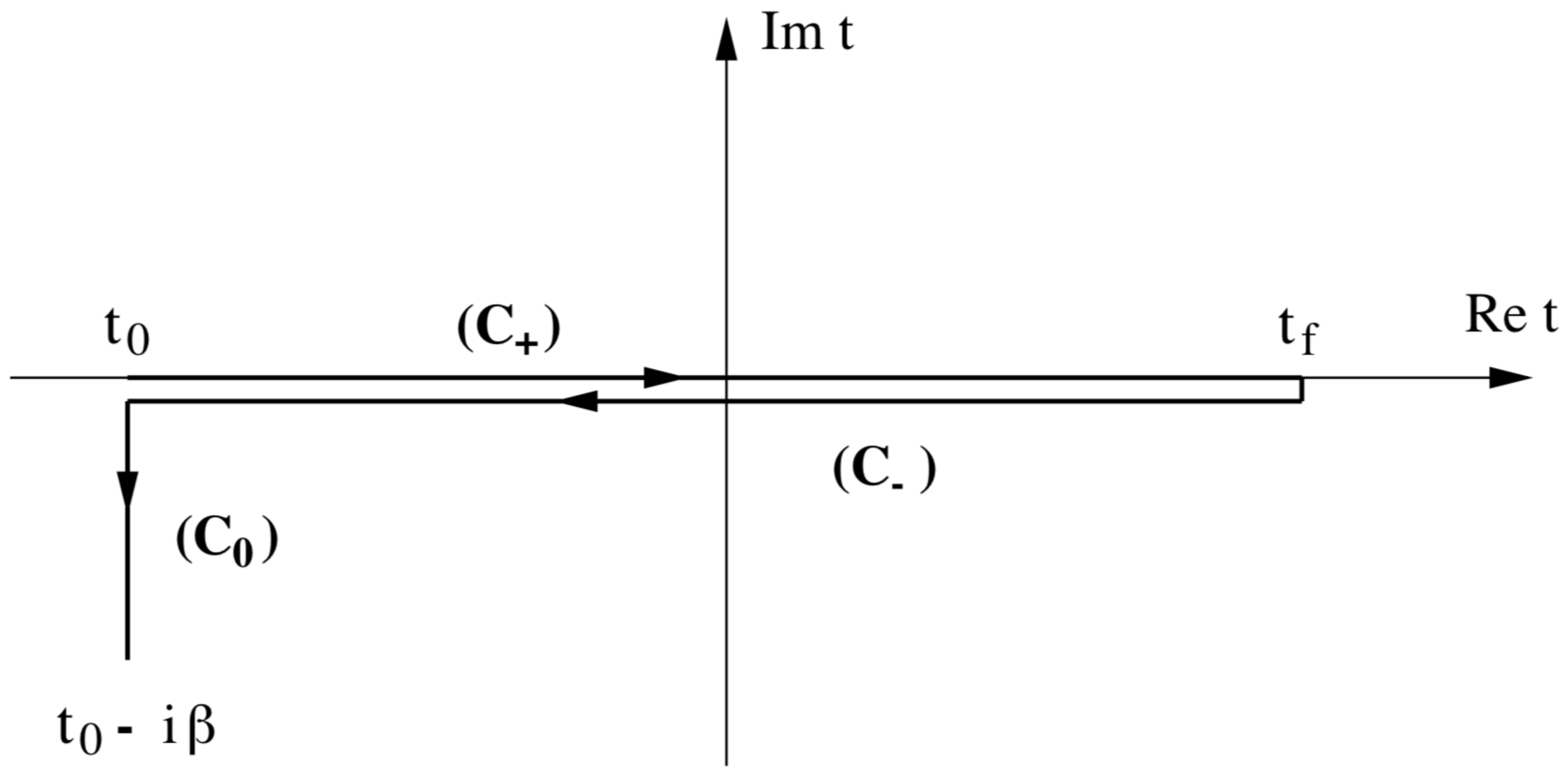
Theoretical methodology

Dyson Schwinger equation



$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Keldysh contour



Wigner transformation

$$S(X,p) = \int d^4u e^{ip\cdot u/\hbar} S\left(X + \frac{u}{2}, X - \frac{u}{2}\right)$$

Kadanoff-Baym equation

$$\left(\gamma^\mu p_\mu - M(X)\right)S^< + \frac{i\hbar}{2}\gamma^\mu \nabla_\mu S^< + \frac{i\hbar}{2}(\nabla_\mu M)(\partial^p_\mu S^<) \;=\; -\hbar\Sigma^<\hat{\Lambda}\mathrm{Re}S_R + \frac{i\hbar}{2}\left(\Sigma^<\hat{\Lambda}S^> - \Sigma^>\hat{\Lambda}S^<\right)$$

Eugene Wigner

Leo Kadanoff

Gordon Baym

"There are two kinds of people in the world: Johnny Von Neumann and the rest of us."

In 1962 he and Leo Kadanoff collaborated on *Quantum Statistical Mechanics: Green's Function Methods in Equilibrium and Nonequilibrium Problems*. In 1969 he published *Lectures on Quantum Mechanics*, a widely used graduate textbook that, unconventionally, begins with photon polarization.

Current Development

Jing-Yuan Chen, Dam T. Son, Mikhail A. Stephanov, Ho-Ung Yee, Yi Yin, *PRL* 113, 182302 [1404.5963]

Jing-Yuan Chen, Dam T. Son, Mikhail A. Stephanov, *PRL* 115, 021601 [1502.06966]

Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, *PRD* 95, 091901 [1612.04630]

Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, *PRD* 97, 016004 [1710.00278]

Nora Weickgenannt, Xin-Li Sheng, Enrico Speranza, Qun Wang, Dirk H. Rischke*PRD*100, 056018 [1902.06513]

ZyW, Xingyu Guo, Shuzhe Shi, Pengfei Zhuang, *PRD* 100, 014015 [1903.03461]

Koichi Hattori, Di-Lun Yang, Yoshimasa Hidaka, *PRD*100, 096011 [1903.01653]

Nora Weickgenannt, Xin-Li Sheng, Enrico Speranza, Qun Wang, Dirk H. Rischke*PRL*127, 052301 [2005.01506]

Xin-Li Sheng, Nora Weickgenannt, Enrico Speranza, Dirk H. Rischke, Qun Wang, *PRD*104, 016029 [2103.10636]

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Xin-Li Sheng, Qun Wang, Dirk H. Rischke [2202.10160]

Di-Lun Yang, Koichi Hattori, Yoshimasa Hidaka, *JHEP* 07, 070 [2002.02612]

ZyW, Xingyu Guo, Pengfei Zhuang, *EPJC* 81, 799 [2009.10930]

Defu Hou, Shu Lin, *PLD* 818, 136386 [2008.03862]

Shu Lin, *PRD* 105, 076017 [2109.00184]

Shuo Fang, Shi Pu, Di-Lun Yang, *PRD* 106, 016002 [2204.11519]

ZyW [2205.09334]

Shu Lin, ZyW [2206.12573]

…….

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Spin Kinetic Equation — 1. General

- **General formalism**

Kadanoff-Baym equation (quasi-particle assumption)

$$\frac{i}{2}\gamma^\mu\partial_\mu S^< + (\gamma^\mu P_\mu - m)S^< = \frac{i}{2}\left(\Sigma^<\hat{\Lambda}S^> - \Sigma^>\hat{\Lambda}S^<\right)$$

Spin decomposition

$$S^< = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{S}_{\mu\nu}\sigma^{\mu\nu},$$

$$\Sigma^< = \Sigma_S + i\Sigma_P\gamma^5 + \Sigma_{V\mu}\gamma^\mu + \Sigma_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\Sigma_{T\mu\nu}\sigma^{\mu\nu}$$

- **Gradient counting**

$$\mathcal{V}_\mu = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)P_\mu f_V, \quad \sim \mathcal{O}(\partial^0)$$
$$\mathcal{A}_\mu = 2\pi\epsilon(P\cdot u)\delta(P^2-m^2)n_\mu(P), \quad \sim \mathcal{O}(\partial^1)$$

$$p_\mu\mathcal{V}^\mu - m\mathcal{S} = \frac{i}{4}C_S \qquad \partial_\mu\mathcal{V}^\mu = \frac{1}{2}D_S$$
$$2m\mathcal{P} + \partial_\mu\mathcal{A}^\mu = -\frac{i}{2}C_P \qquad 2p_\mu\mathcal{A}^\mu = \frac{1}{2}D_P$$
$$2p_\mu\mathcal{S} - 2m\mathcal{V}_\mu - \partial^\nu\mathcal{S}_{\nu\mu} = \frac{i}{2}C_{V\mu} \qquad 2p^\nu\mathcal{S}_{\nu\mu} + \partial_\mu\mathcal{S} = \frac{1}{2}D_{V\mu}$$
$$\partial_\mu\mathcal{P} - \epsilon_{\mu\nu\rho\sigma}p^\sigma\mathcal{S}^{\nu\rho} - 2m\mathcal{A}^\mu = \frac{i}{2}C_{A\mu} \qquad 2p_\mu\mathcal{P} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma\mathcal{S}^{\nu\rho} = -\frac{1}{2}D_{A\mu}$$
$$\partial_{[\mu}\mathcal{V}_{\nu]} - 2\epsilon_{\rho\sigma\mu\nu}p^\rho\mathcal{A}^\sigma - 2m\mathcal{S}_{\mu\nu} = \frac{i}{2}C_{T\mu\nu} \qquad 2p_{[\mu}\mathcal{V}_{\nu]} + \epsilon_{\mu\nu\rho\sigma}\partial^\rho\mathcal{A}^\sigma = -\frac{1}{2}D_{T\mu\nu}$$

- **General Kinetic of spin up to $\mathcal{O}(\partial)$**

Massless

$$\partial\cdot\mathcal{A}^< = -\widehat{\Sigma_V^\nu\mathcal{A}_\nu} + \widehat{\Sigma_A^\nu\mathcal{V}_\nu}$$

Massive

$$p\cdot\partial\mathcal{A}_\mu = -p_\mu\widehat{\Sigma_{V\nu}\mathcal{A}^\nu} + p_\mu\widehat{\Sigma_{A\nu}\mathcal{V}^\nu} - m\widehat{\Sigma_S\mathcal{A}_\mu} - p_\nu\widehat{\Sigma_{A\mu}\mathcal{V}^\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma(\widehat{\Sigma_V^\nu\mathcal{V}^\rho}) - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_V^\rho\mathcal{S}^{\sigma\lambda}} - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_T^{\rho\sigma}\mathcal{V}^\lambda}$$

Spin Kinetic Equation — 2. Collision

Massive

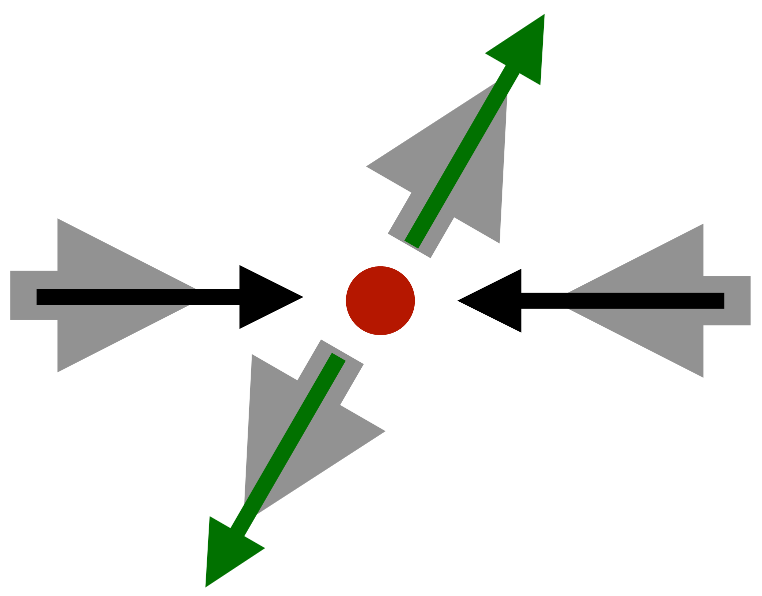
$$p\cdot\partial\mathcal{A}_\mu = -p_\mu\widehat{\Sigma_{V\nu}\mathcal{A}^\nu} + p_\mu\widehat{\Sigma_{A\nu}\mathcal{V}^\nu} - m\widehat{\Sigma_S\mathcal{A}_\mu} - p_\nu\widehat{\Sigma_{A\mu}\mathcal{V}^\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma(\widehat{\Sigma_V^\nu\mathcal{V}^\rho}) - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_V^\rho\mathcal{S}^{\sigma\lambda}} - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_T^{\rho\sigma}\mathcal{V}^\lambda}$$

- **Specify system & interaction**

e.g. 2 by 2 scattering of righthand chiral fermion

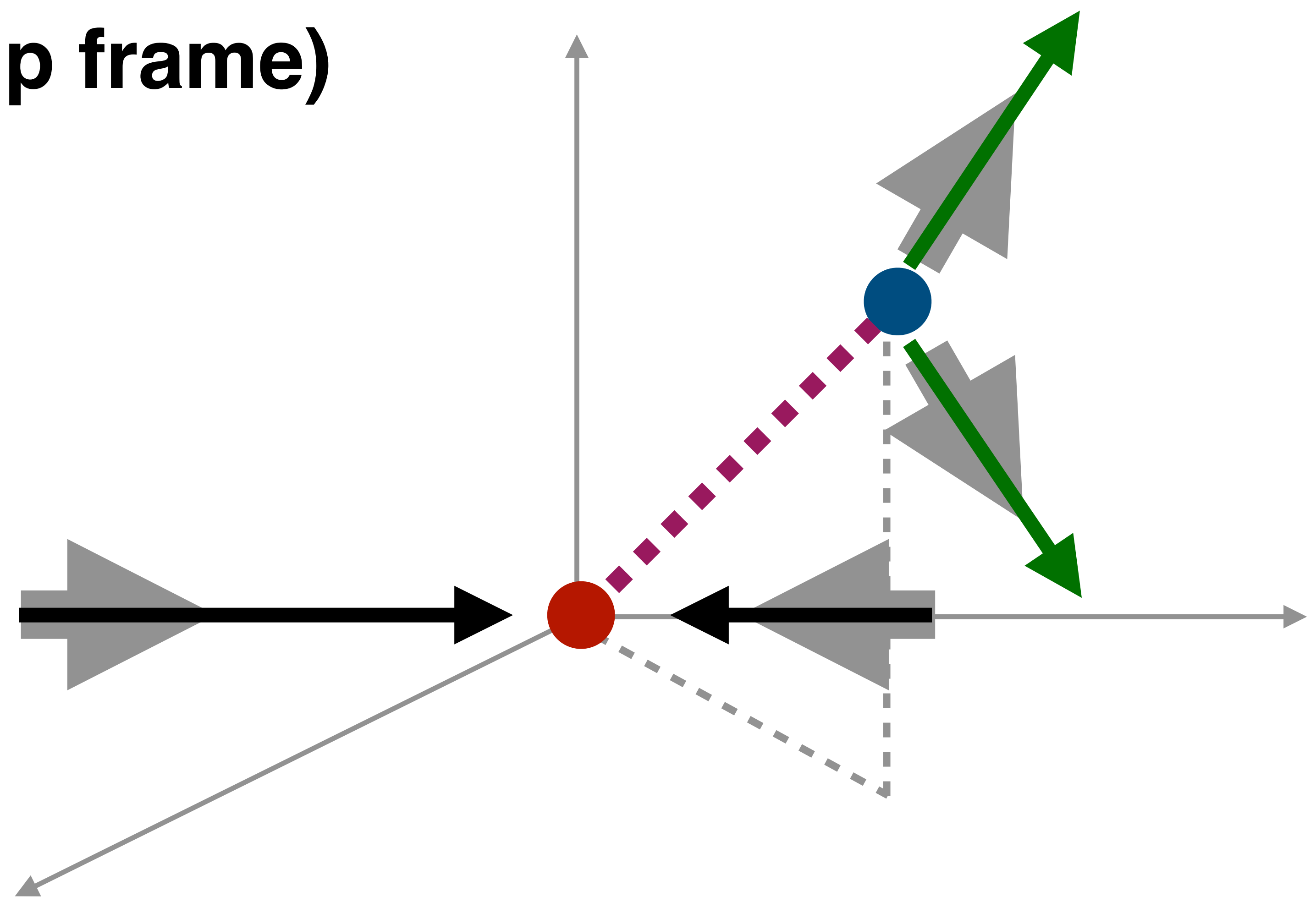
tunneling movement: side-jump

Center of mass frame
(no jump frame)



No OAM, spin conserved
— angular momentum conserved

Lab frame
(jump frame)



relative OAM, spin not conserved
— angular momentum conserved

Lorenze covariance & angular momentum conservation

Spin Kinetic Equation — 2. Collision

Massive

$$p\cdot\partial\mathcal{A}_\mu = -p_\mu\widehat{\Sigma_{V\nu}\mathcal{A}^\nu} + p_\mu\widehat{\Sigma_{A\nu}\mathcal{V}^\nu} - m\widehat{\Sigma_S\mathcal{A}_\mu} - p_\nu\widehat{\Sigma_{A\mu}\mathcal{V}^\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma(\widehat{\Sigma_V^\nu\mathcal{V}^\rho}) - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_V^\rho\mathcal{S}^{\sigma\lambda}} - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_T^{\rho\sigma}\mathcal{V}^\lambda}$$

- **Specify system & interaction**

Contact

$$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - m)\psi + G(\bar\psi\psi)^2$$

simple
does not allow for side-jump, not real

Yukawa

$$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - g\phi - m)\psi + \frac{1}{2}(\partial_\mu\phi)^2$$

simple, no gauge issue
Meson spin zero, not real

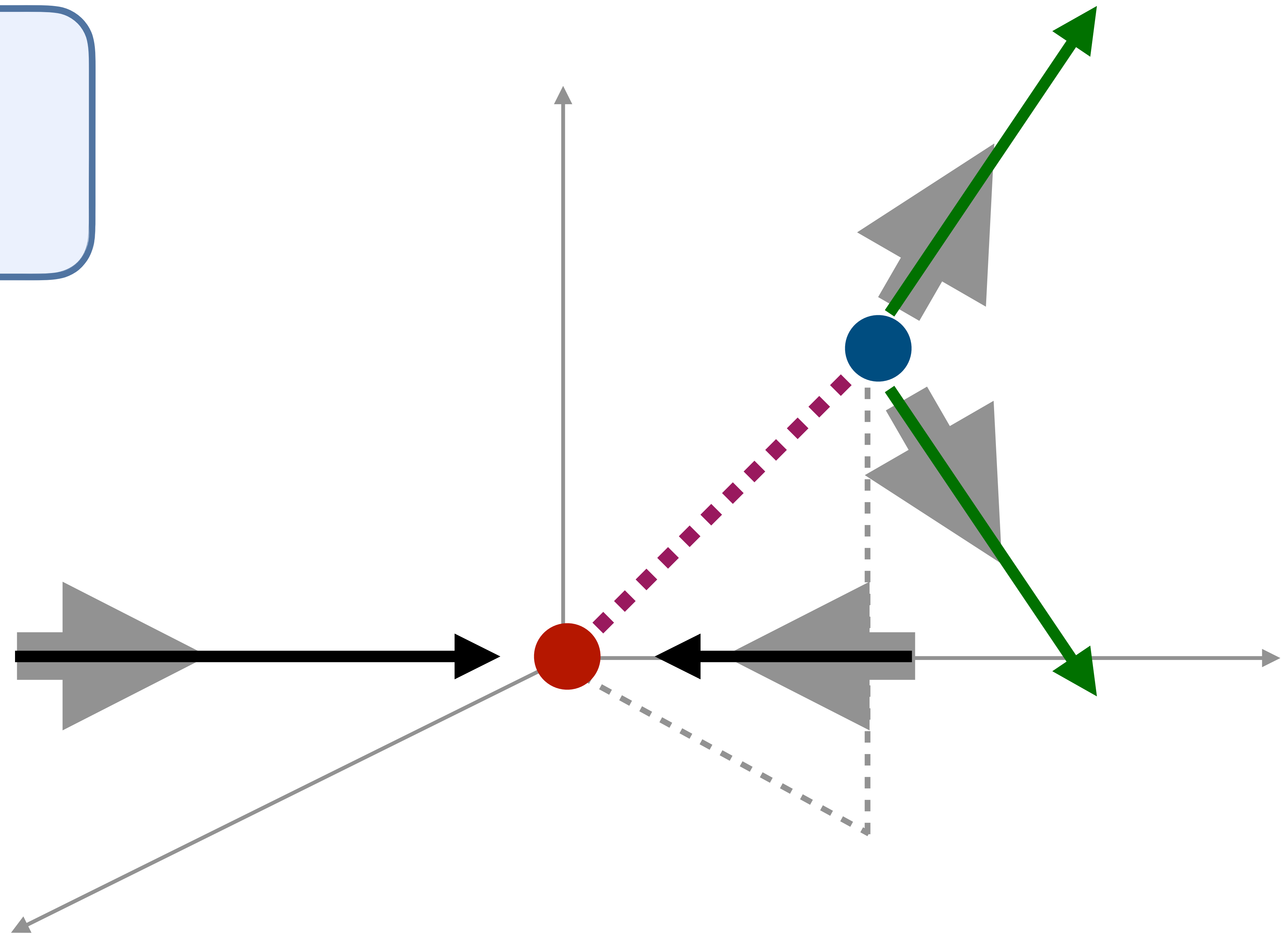
QED

$$\mathcal{L} = \bar\psi(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Comparable simple
Gauge issue

QCD

$$\mathcal{L} = \bar\psi_i(i\gamma^\mu(\partial_\mu\delta_{ij} - igA_\mu^a t_{a,ij}) - m\delta_{ij})\psi_j - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu}$$



Spin Kinetic Equation — 2. Collision

Massive

$$p\cdot\partial\mathcal{A}_\mu = -p_\mu\widehat{\Sigma_{V\nu}\mathcal{A}^\nu} + p_\mu\widehat{\Sigma_{A\nu}\mathcal{V}^\nu} - m\widehat{\Sigma_S\mathcal{A}_\mu} - p_\nu\widehat{\Sigma_{A\mu}\mathcal{V}^\nu} + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\sigma(\widehat{\Sigma_V^\nu\mathcal{V}^\rho}) - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_V^\rho\mathcal{S}^{\sigma\lambda}} - \frac{m}{2}\epsilon_{\rho\sigma\lambda\mu}\widehat{\Sigma_T^{\rho\sigma}\mathcal{V}^\lambda}$$

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Gauge issue

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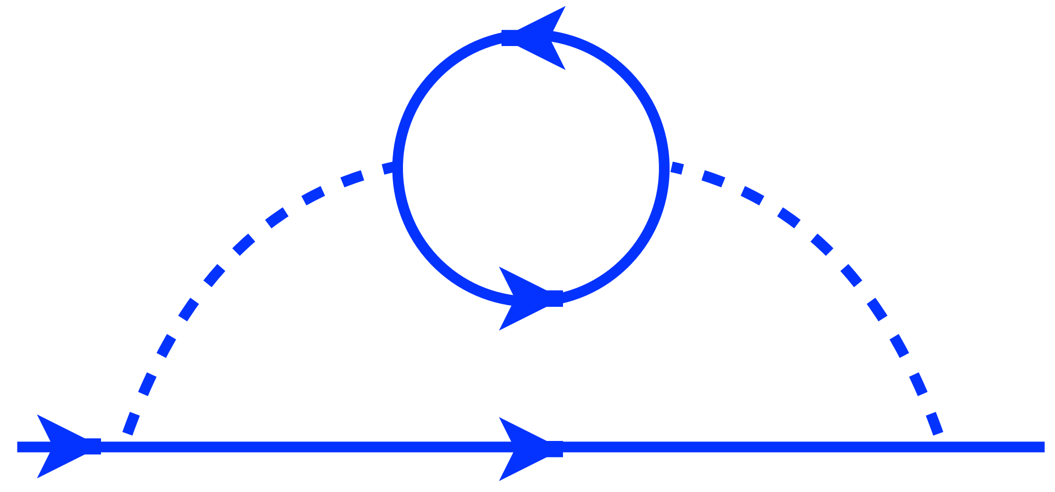
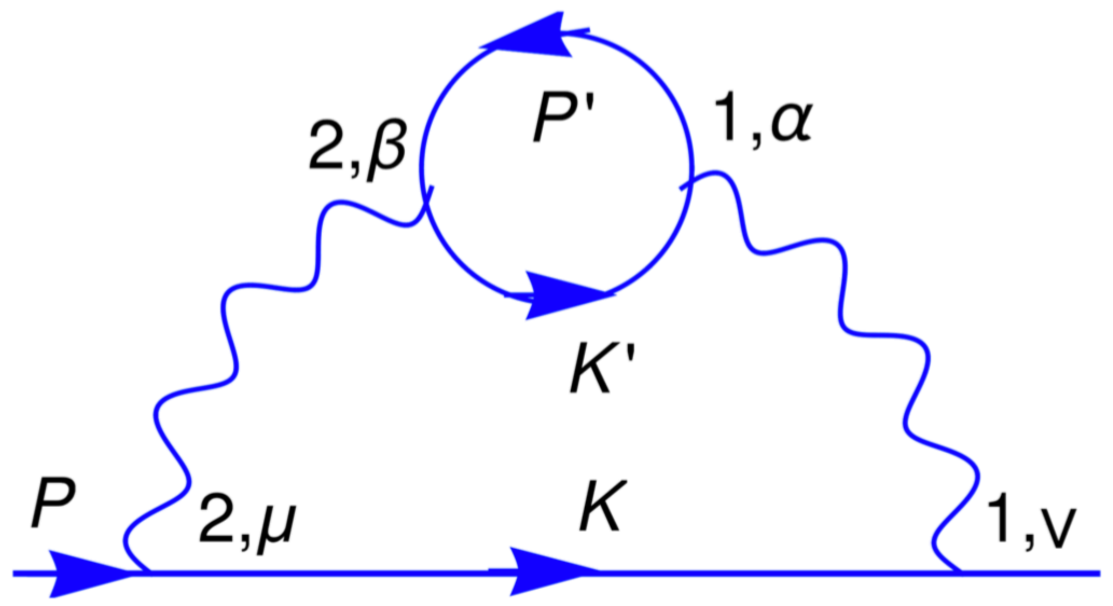
$\Sigma^{\lessgtr} =$ 

Detailed balance

Relaxation near global equilibrium

$\Sigma^{\lessgtr} =$ 

Collision term to leading logarithm order

global equilibrium, relaxation

Spin Kinetic Equation — 3.Contact Interaction

Contact interaction

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2$$

ZyW and P.Zhuang, [arXiv:2105.00915 [hep-th]].
ZyW, X.Guo and P.Zhuang, Eur.Phys.J.C 81 (2021) 9, 799.

Qualitative analysis

Act like a source

$$p\cdot\nabla\mathcal{A}^{(1)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p_\nu\widehat{\Sigma^{(0)\nu}_V\mathcal{A}^{(1)}_\mu} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} + p_\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)\nu}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}} + \frac{m}{2}\epsilon_{\rho\nu\lambda\mu}\widehat{\Sigma^{(1)\rho\nu}_T\mathcal{V}^{(0)\lambda}}$$

- **Find equilibrium $\mathcal{A}_\mu$ from detailed balance**

$$\mathcal{A}^{(1)global}_\mu(p) = -\frac{1}{(2\pi)^3 4E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f'_V(p)$$

$$\nabla^\mu\beta^\nu + \nabla^\nu\beta^\mu = 0$$

Killing condition -- global equilibrium

- **Relaxation rate near global equilibrium**

$$p\cdot\partial f(x,p) = -\frac{f(x,p) - f_{eq}(x,p)}{\tau(p)}$$

$f_\pm = (f_V \pm f_A)/2$ Direction of spin a_μ

$$p\cdot\partial\delta f_+ = -\tau^{-1}_{++}\delta f_+ - \tau^{-1}_D(\delta f_+ - \delta f_-),$$
$$p\cdot\partial\delta f_- = -\tau^{-1}_{--}\delta f_- - \tau^{-1}_D(\delta f_- - \delta f_+),$$
$$p\cdot\partial\,\delta\vec{a} = -\tau^{-1}_+\delta\vec{a} + \vec{\tau}_a^{\,-1}(\delta f_+ + \delta f_-)/(f^{\rm eq}_+ - f^{\rm eq}_-)$$

$$\tau^{-1}_{++} = \tau^{-1}_+ - 2\tau^{-1}_D + \tau^{-1}_-,$$
$$\tau^{-1}_{--} = \tau^{-1}_+ - 2\tau^{-1}_D - \tau^{-1}_-,$$

Separation of time scale

$\tau^{-1}_+ \sim \mathcal{O}(\partial^0)$, **damping rate**

$\tau^{-1}_- \sim \mathcal{O}(\partial^1)$, **difference of damping rates**

$\tau^{-1}_D \sim \mathcal{O}(\partial^2)$, **spin flipping rate**

Spin Kinetic Equation — 4.QED

ZyW [2205.09334]

Aim: the spin dynamics of s-quark in the quark gluon plasma

Toy model: hard massive fermion (m » eT) probing into a hot massless QED plasma at local equilibrium

Simplification: Compton scattering is suppressed in case m » eT, dominated by Coulomb scattering.

Competing processes: diffusion process — scattering with medium — drives the fluctuation of spin back to equilibrium,

polarization process— collective motion of the medium, hydrodynamic gradients

Diffusion process

$$P\cdot\partial n^\mu(P) = -\kappa_{LL}\frac{T}{mv}\Big\{C^{(1)}n^\mu(P)+C^{(2)}u^\mu+C^{(3)}\hat{P}_\perp^\mu+C^{(4)}\hat{P}_\perp^\nu\partial_{P_{\perp\mu}}n_\nu(P)$$
$$+C^{(5)}\hat{P}_\perp^\nu\partial_{P_\perp^\nu}n^\mu(P)+C^{(6)}g^{\nu\rho}\partial_{P_\perp^\nu}\partial_{P_\perp^\rho}n^\mu(P)+C^{(7)}\hat{P}_\perp^\nu\hat{P}_\perp^\rho\partial_{P_\perp^\nu}\partial_{P_\perp^\rho}n^\mu(P)$$
$$+C^{(8)}(\omega^\mu+\hat{P}_\perp^\mu\hat{P}_\perp^\nu\omega_\nu)+C^{(9)}\frac{1}{2}\Big(\epsilon^{\mu\nu\rho\alpha}u_\nu\hat{P}_{\perp\rho}\hat{P}_\perp^\beta+\epsilon^{\mu\nu\rho\beta}u_\nu\hat{P}_{\perp\rho}\hat{P}_\perp^\alpha\Big)\sigma_{\langle\alpha\beta\rangle}$$
$$+C^{(10)}\epsilon^{\mu\nu\alpha\beta}u_\alpha\hat{P}_{\perp\beta}Du_\nu+C^{(11)}\epsilon^{\mu\nu\alpha\beta}u_\alpha\hat{P}_{\perp\beta}\partial_\nu\ln T\Big\},$$

Polarization process

$$\kappa_{LL}=e^4\ln\frac{1}{e}\frac{T^2}{8\pi}$$

$C^{(i)}$ Functions of momentum and temperature

Spin Kinetic Equation — 4.QED

$P \cdot \partial n^\mu(P) =$ **Diffusion process** + **Polarization process**

Both processes balance with each other in equilibrium — detailed balance

Global equilibrium : Killing condition, purely rotating fluid — detailed balance satisfied

$$n_\mu^{\rm geq}(P) = \left(\frac{P\cdot\omega\, u_\mu}{2} - \frac{P\cdot u\omega_\mu}{2}\right) f'_P.$$

Local equilibrium : **Chiral fermion** — satisfied

Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, *PRD* 97, 016004 [1710.00278]
Shuo Fang, Shi Pu, Di-Lun Yang, *PRD* 106, 016002 [2204.11519]

$$S^\mu_{leq} = 2\pi\delta(P^2)\left(P^\mu + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha u_\beta}{2P\cdot u}\partial_\nu\right) f_P$$

There are more first order terms, self-energy and gauge.
what is the complete local equilibrium solution ?

Massive fermion — under question

Decomposition of $\mathcal{A}_\mu$

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Decomposition and static solution

Lorenze covariance and Decomposition

Chiral fermion

$$\mathcal{A}^{<\mu} = 2\pi\delta(P^2)\Big(P^\mu f_A + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha n_\beta}{2P\cdot n}(\partial_\nu f + \Sigma^>_\nu f - \Sigma^<_\nu(1-f))\Big)$$

Jing-Yuan Chen, Dam T. Son, Mikhail A. Stephanov, *PRL* 115, 021601 [1502.06966]
Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, *PRD* 95, 091901 [1612.04630]
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Shu Lin, ZyW, [2206.12573]

The same structure $T_{\nu,\alpha\beta}\sigma^{\alpha\beta} = \Sigma^>_\nu f - \Sigma^<_\nu(1-f)$

when projecting to P^ν, gives the collision terms of $P\cdot\partial f$

$$P^\nu(\partial_\nu f + \Sigma^>_\nu f - \Sigma^<_\nu(1-f)) = 0$$

consider only shear

$$\mathcal{A}^{<\mu} = 2\pi\delta(P^2)\Big(P^\mu f_A + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha n_\beta}{2P\cdot n}(\partial_\nu f + \Sigma^>_\nu f - \Sigma^<_\nu(1-f))\Big)$$
$$= 2\pi\delta(P^2)\Big(P^\mu f^<_A + (\frac{3}{2} + \frac{T}{2p(1-2f)})\frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha n_\beta}{2P\cdot n}\partial_\nu f\Big)$$

1. shear contribution enhanced
2. Detail balance still satisfied

Massive fermion

$$\mathcal{A}^{<\mu} = 2\pi\delta(P^2-m^2)\Big(a^\mu f_A + \frac{\epsilon^{\mu\nu\alpha\beta}P_\alpha u_\beta}{2(P\cdot u+m)}(\partial_\nu f + \Sigma^>_\nu f + \Sigma^<_\nu(1-f))\Big)$$

Di-Lun Yang, Koichi Hattori, *Yoshimasa Hidaka*, *JHEP* 07, 070 [2002.02612]

Shu Lin, ZyW, [2206.12573]

- Self-energy part is necessary for massive kinetic equation to return to chiral limit.
- Static solution $a^\mu f_A$ need to be solved from collision term.
- $\mathcal{A}^{<\mu}$ is frame independent, while each part are frame dependent.

Full static solution… under progressing

Summary

- Qualitative analysis using contact interaction
- Global equilibrium — detailed balance
- Different relaxation rate for different degrees of freedom
- Spin kinetic equation in QED plasma
- Full expression to first order of gradient