



# QCD phase structure and inhomogeneous instabilities at finite temperature and densities

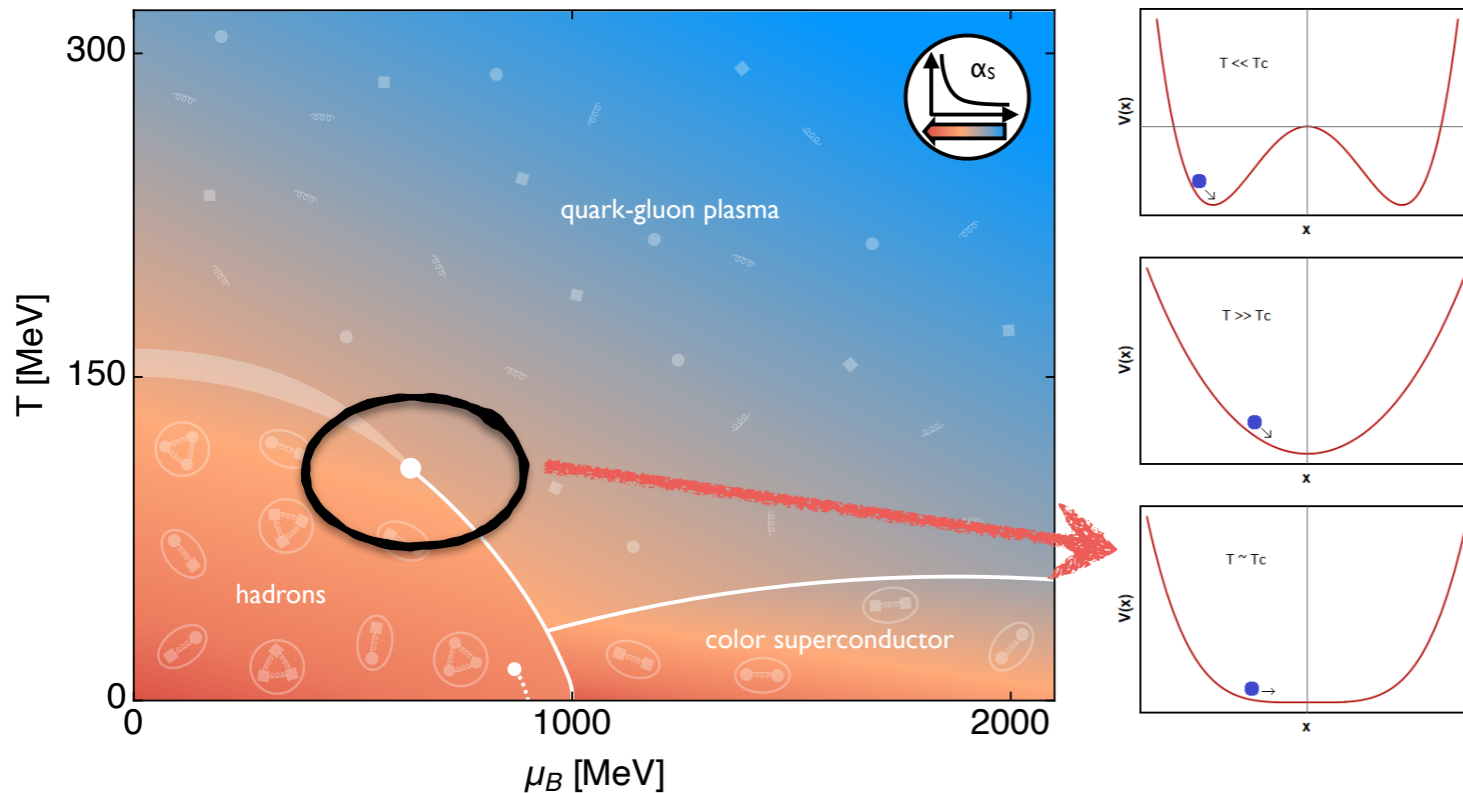
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QCD物理研讨会暨基金委重大项目学术交流会  
青岛，2022年7月29-31日

# Critical end point and fluctuations

## QCD phase diagram



## Scaling analysis of fluctuations

Baryon number fluctuations of the  $n$ -th order:

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{p}{T^4}$$

pressure

$$p \sim \xi^{-d}$$

when  $\mu_B \rightarrow \mu_{B_c}$

$$\mu_B \sim \xi^{-1/\nu}$$

which leads us to

$$\chi_n^B \sim \xi^{\frac{n}{\nu}-d} (\mu_B \neq 0)$$

For  $d = 3$  and the  $O(4)$  symmetry

$$\nu = 0.75$$

one has

$$\chi_2^B \sim \xi^{-0.33}, \quad \chi_3^B \sim \xi^{1.00}, \quad \chi_4^B \sim \xi^{2.33}$$

$$\chi_5^B \sim \xi^{3.67}, \quad \chi_6^B \sim \xi^{5.00}$$

For the  $Z(2)$  symmetry

$$\nu = 0.63$$

then one has

$$\chi_2^B \sim \xi^{0.17}, \quad \chi_3^B \sim \xi^{1.76}, \quad \chi_4^B \sim \xi^{3.35}$$

$$\chi_5^B \sim \xi^{4.93}, \quad \chi_6^B \sim \xi^{6.52}$$

However, former analysis showed

$$\chi_n^B = \xi^{\frac{n}{2}(5-\eta)-3}$$

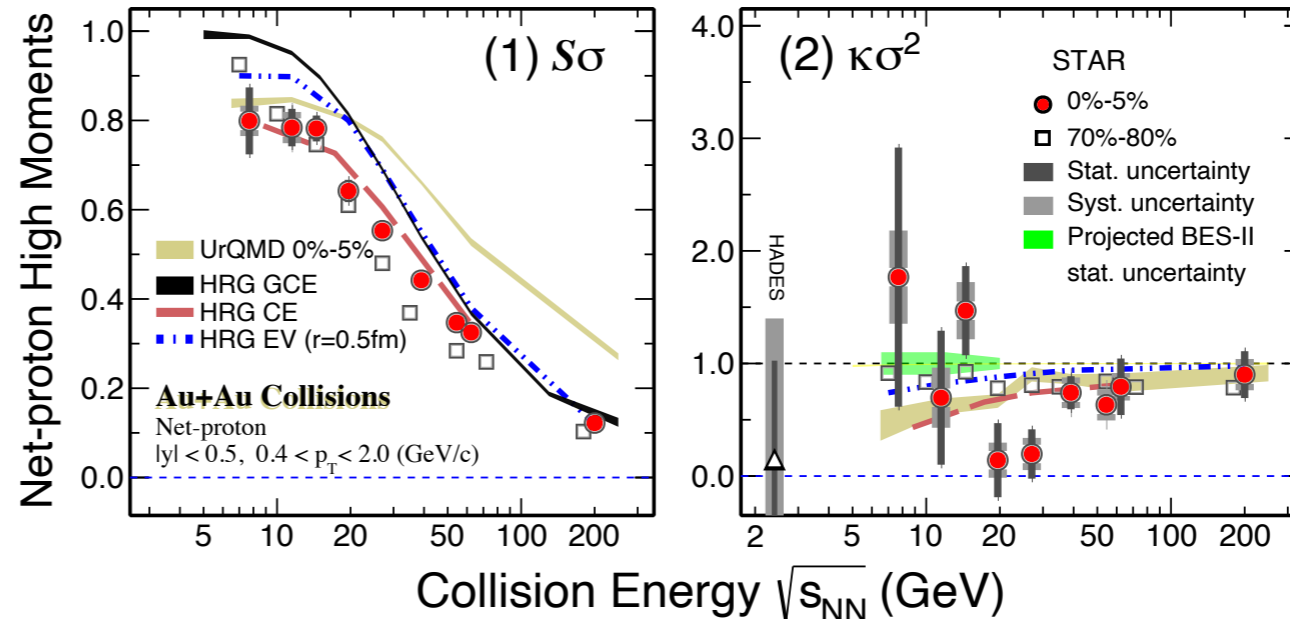
$\eta \ll 1$

$$\chi_2^B \sim \xi^2, \quad \chi_3^B \sim \xi^{4.5}, \quad \chi_4^B \sim \xi^7$$

$$\chi_5^B \sim \xi^{9.5}, \quad \chi_6^B \sim \xi^{12}$$

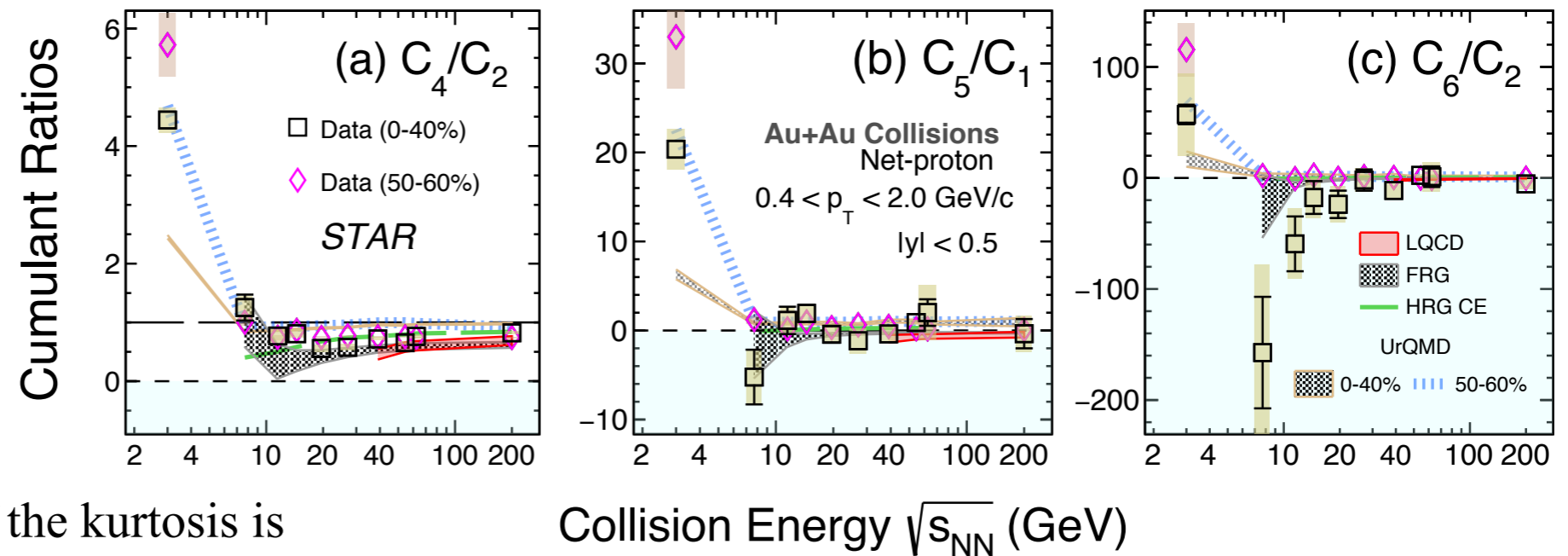
# Fluctuation measurements at RHIC

Skewness  
and kurtosis  
of net-proton  
distributions



J. Adam *et al.* (STAR), *PRL* 126 (2021) 092301

Hyper-order  
net-proton  
fluctuations  
at RHIC



- The non-monotonicity of the kurtosis is observed with  $3.1\sigma$  significance.
- Hyper-order fluctuations are increasingly negative.
- Is CEP not far away?

STAR: The STAR Collaboration, arXiv:2207.09837

LQCD: A. Bazavov *et al.*, *PRD* 101 (2020) 074502

fRG: WF, Luo, Pawłowski, Rennecke, Wen, Yin, *PRD* 104 (2021) 094047

HRG CE: P. Braun-Munzinger *et al.*, *NPA* 1008 (2021) 122141

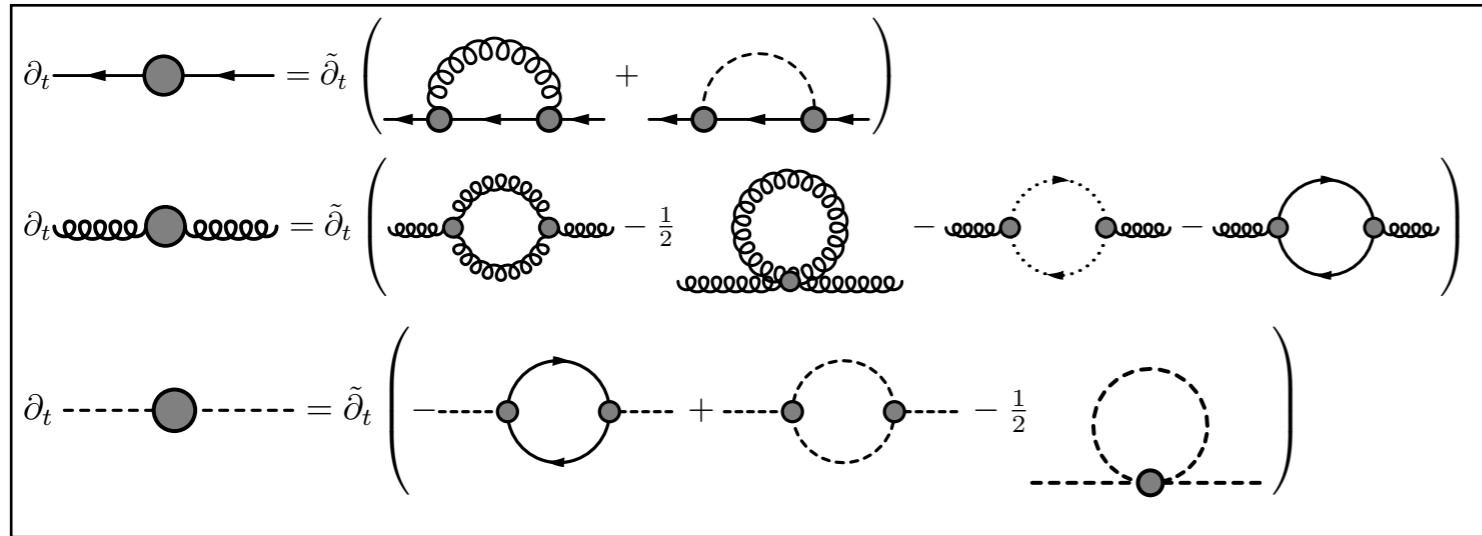
# Outline

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- \* **Introduction**
- \* **fRG approach to QCD**
- \* **QCD phase structure from fRG**
- \* **Inhomogeneous instabilities**
- \* **Summary**

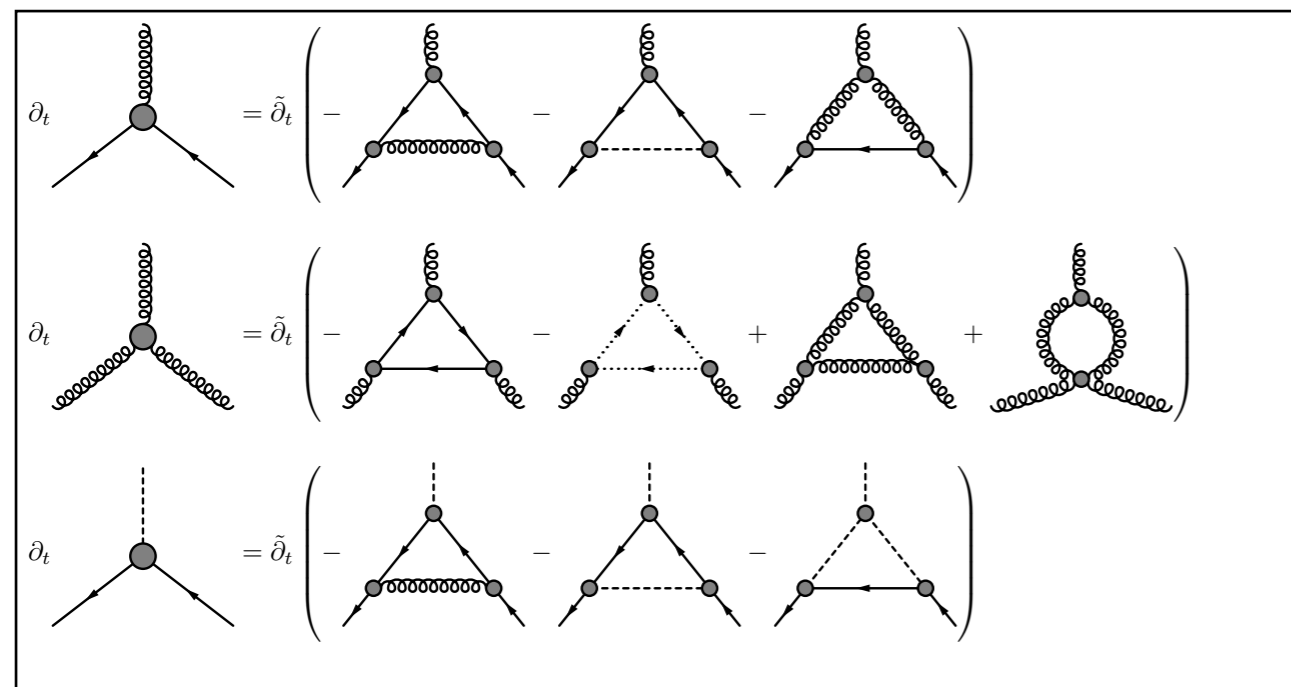
# Functional renormalization group to QCD

## Propagators:

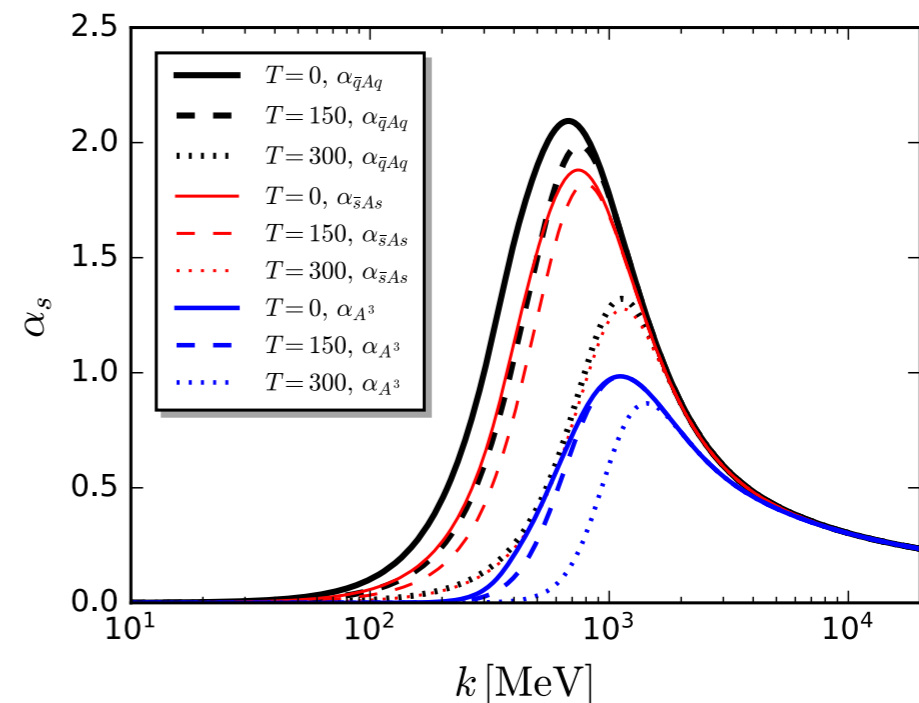


- Vertex expansion: apparent convergence.
- Expansion around  $N_f = 2$  gluon propagator in vacuum QCD.
- Effective mesonic potential of  $N_f = 2$ , improved to the full potential of  $N_f = 2+1$  recently.

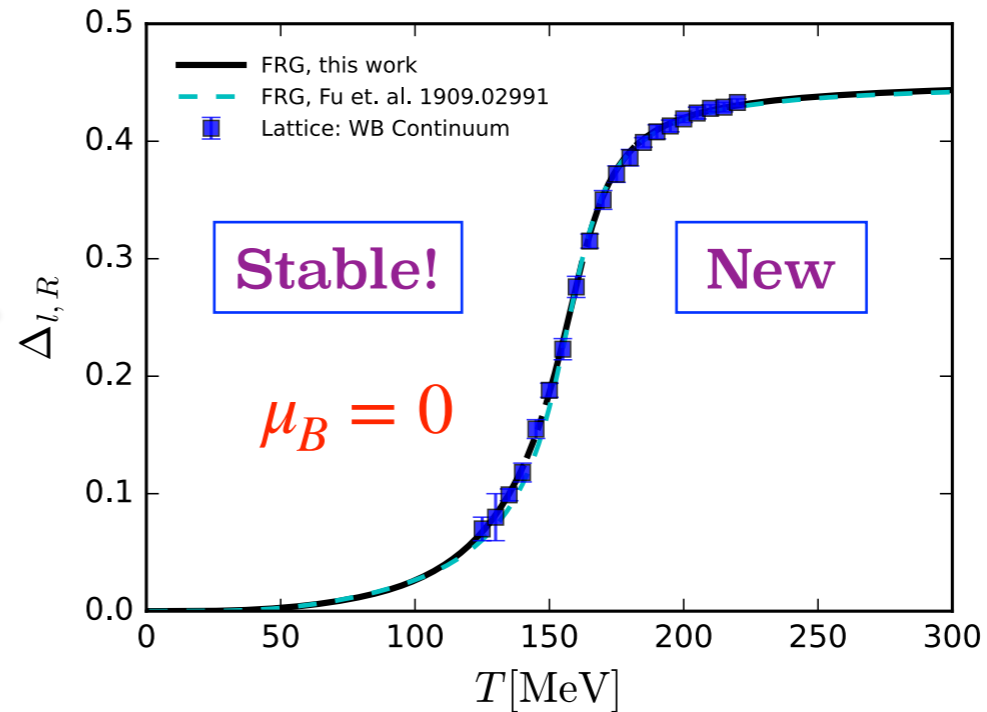
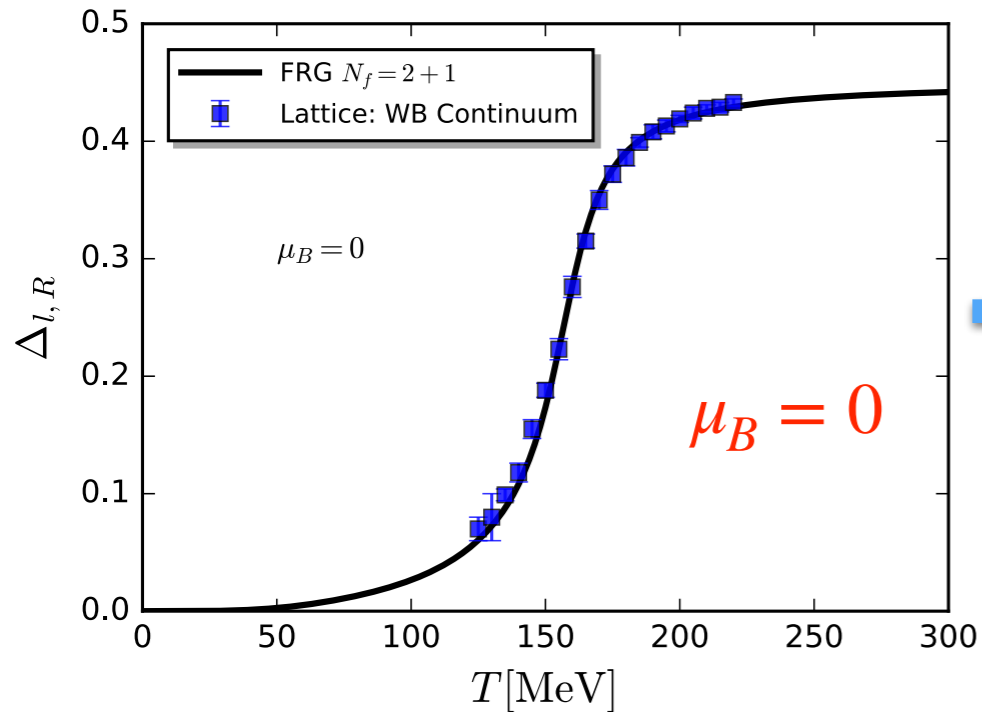
## Three-point functions:



## Strong couplings:



# Renormalized light quark condensate

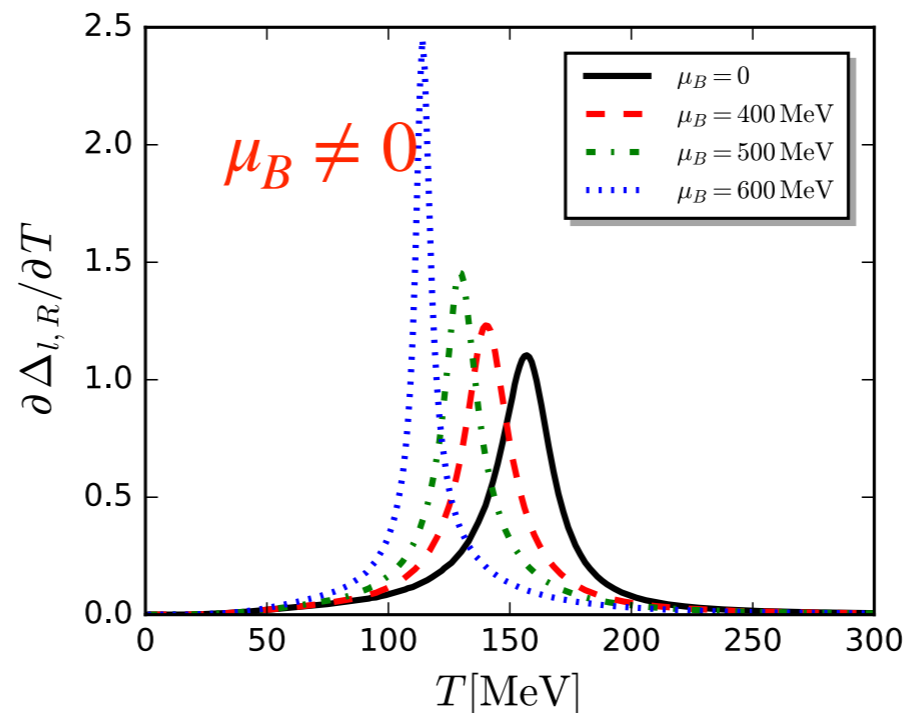
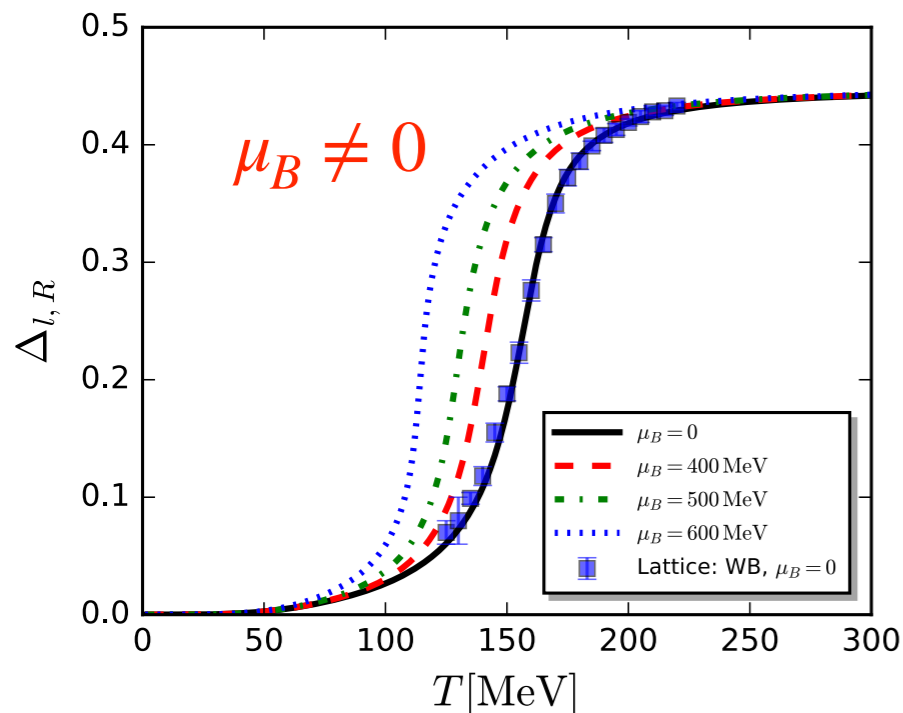


improved truncations for the sector of  $s$  quark and the full mesonic potential of  $N_f = 2+1$ .

Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

fRG: WF, Pawłowski, Rennecke, *PRD* 101 (2020) 054032

fRG: WF, Pawłowski, Rennecke, Wen, Yin, (2022) in preparation

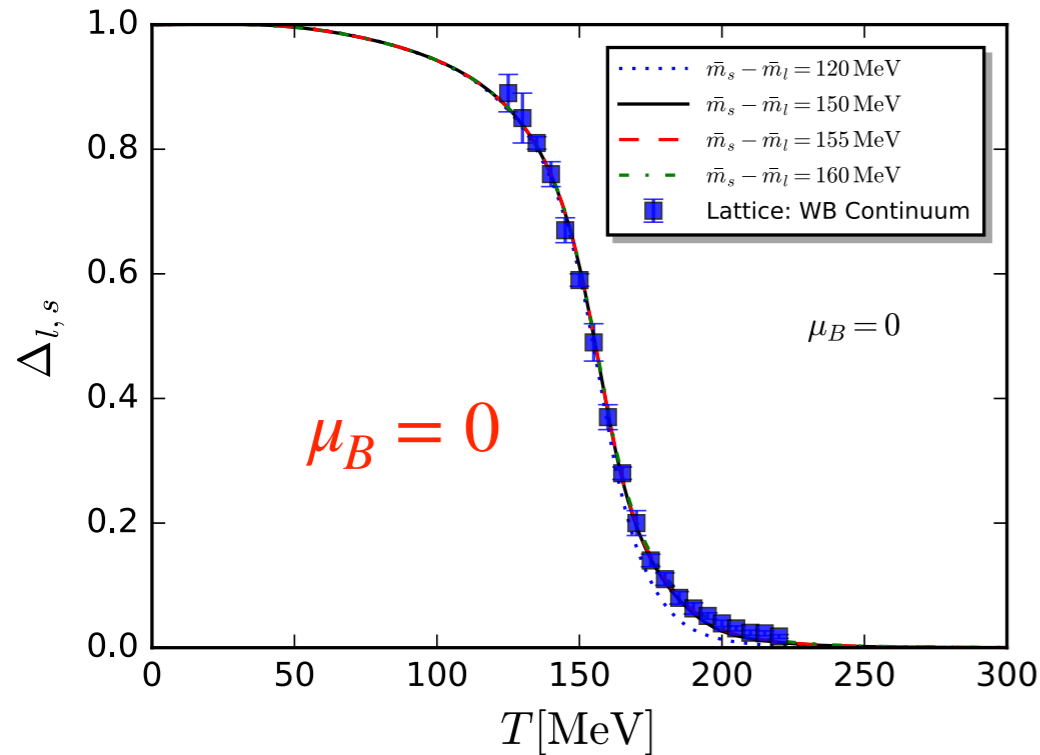


quark condensate:

$$\Delta_{q_i} \simeq -m_{q_i}^0 T \sum_{n \in \mathbb{Z}} \int \frac{d^3 q}{(2\pi)^3} \text{tr} G_{q_i \bar{q}_i}(q),$$

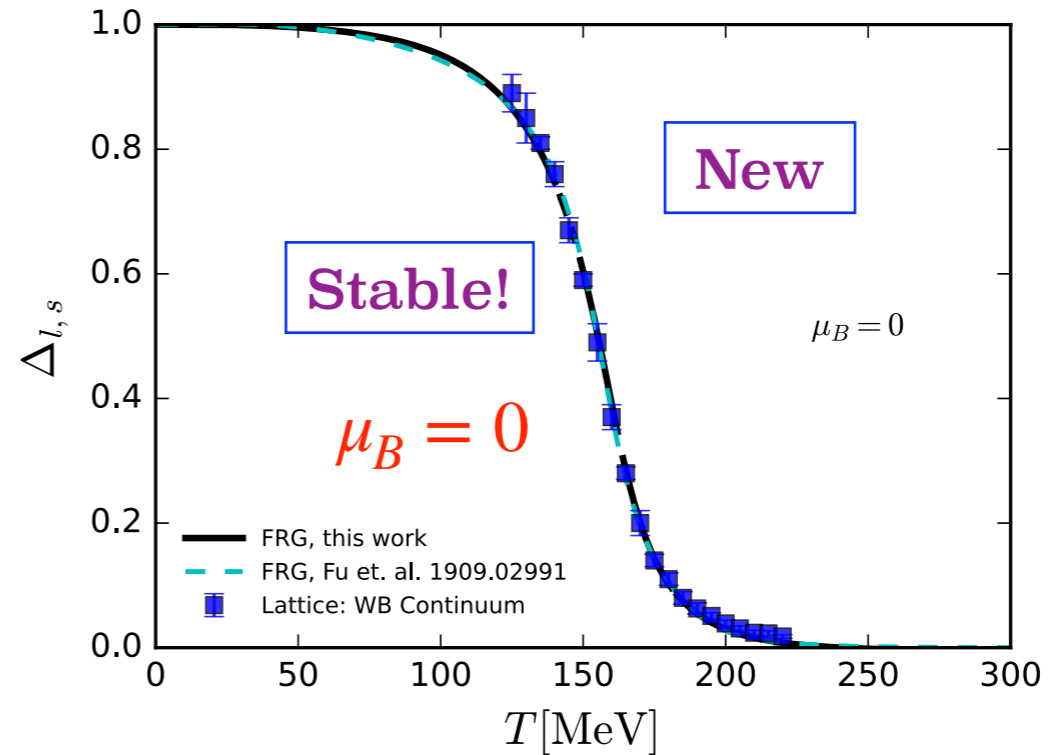
$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R} \left[ \Delta_{q_i}(T, \mu_q) - \Delta_{q_i}(0,0) \right].$$

# Other fermionic observables



Lattice: Borsanyi *et al.* (WB), *JHEP* 09 (2010) 073

fRG: WF, Pawłowski, Rennecke, *PRD* 101 (2020) 054032



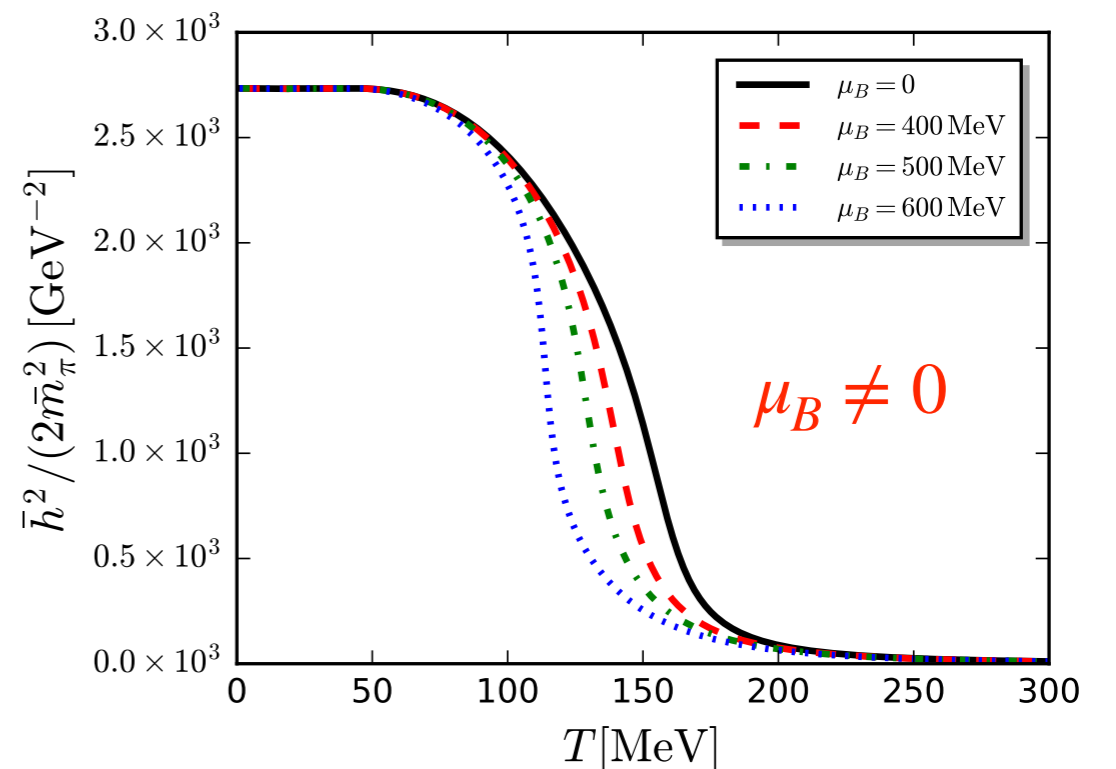
fRG: WF, Pawłowski, Rennecke, Wen, Yin, (2022) in preparation

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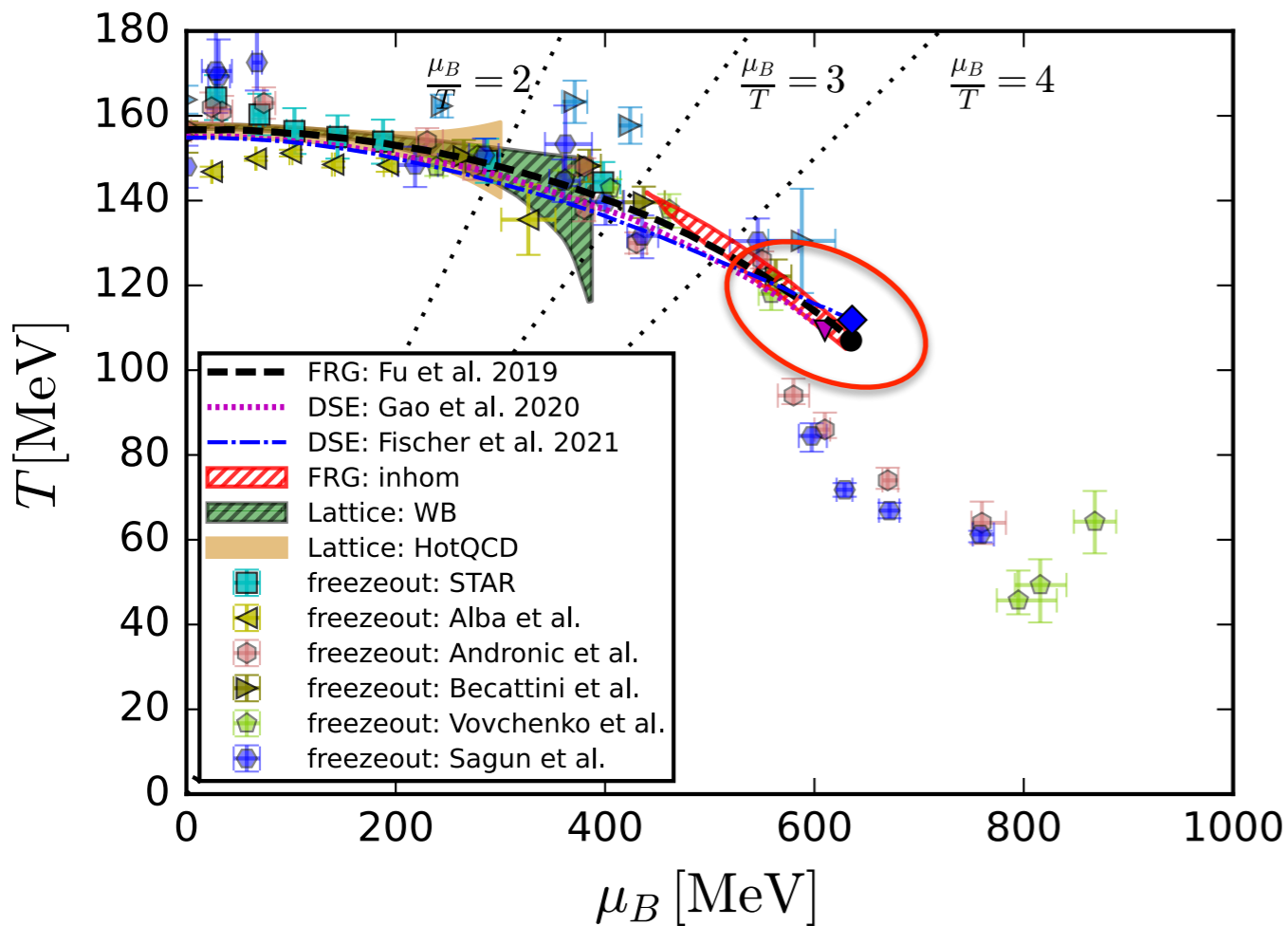
Reduced condensate:

$$\Delta_{l,s}(T, \mu_q) = \frac{\Delta_l(T, \mu_q) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T, \mu_q)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)}$$

Effective four-quark coupling:



# CEP from functional QCD



Prediction of location of CEP from functional QCD in literature

fRG:

●  $(T, \mu_B)_{\text{CEP}} = (107, 635)\text{MeV}$

fRG: WF, Pawłowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

▼  $(T, \mu_B)_{\text{CEP}} = (109, 610)\text{MeV}$

DSE (fRG): Gao, Pawłowski, *PLB* 820 (2021) 136584

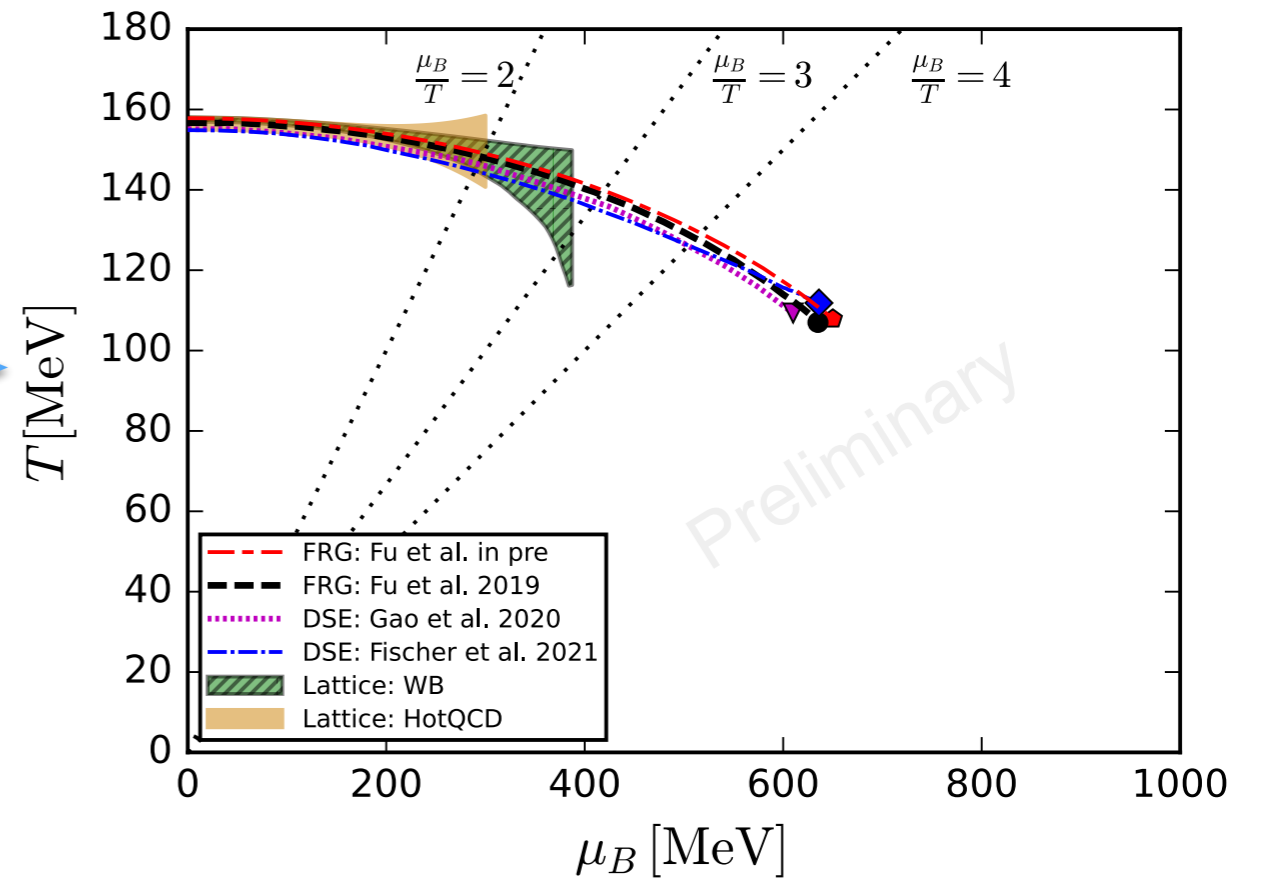
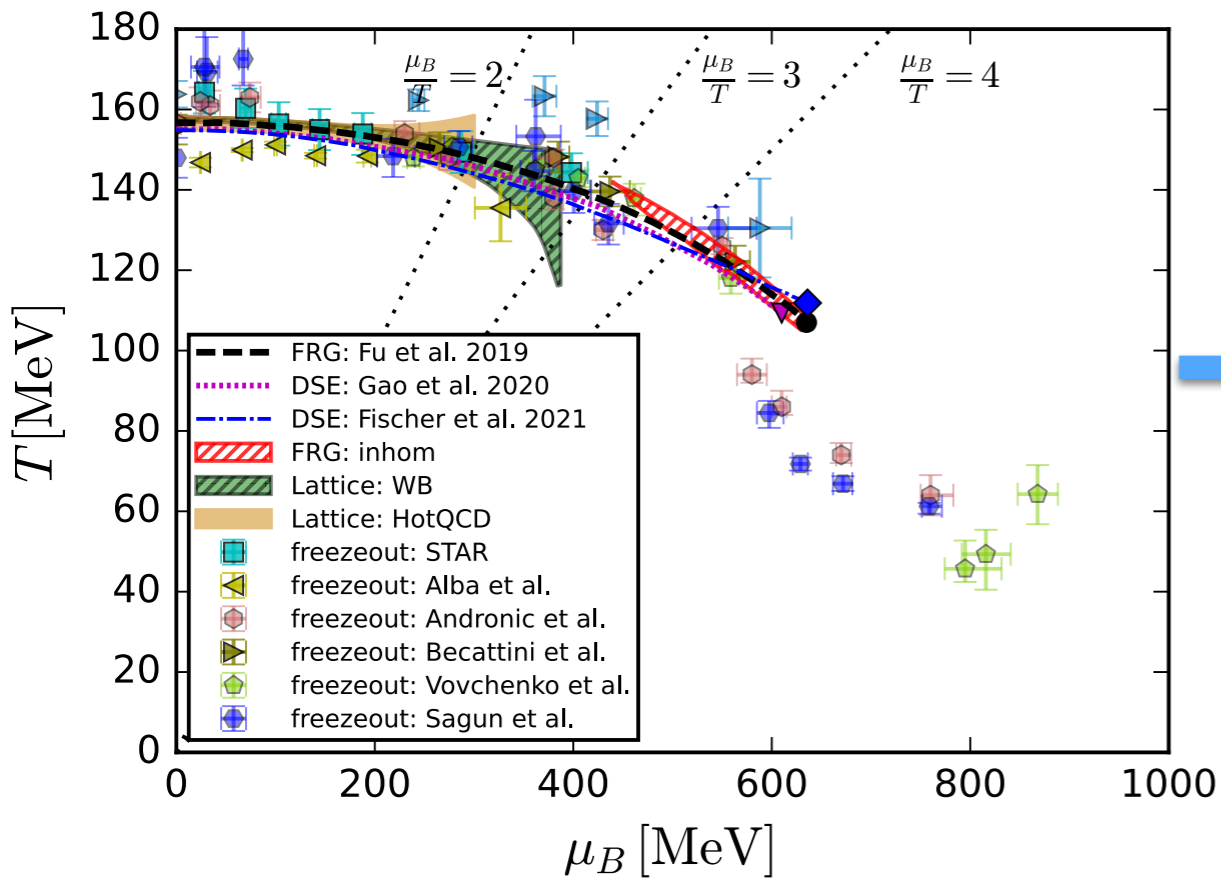
◆  $(T, \mu_B)_{\text{CEP}} = (112, 636)\text{MeV}$

DSE: Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- No CEP observed in  $\mu_B/T \lesssim 2 \sim 3$  from lattice QCD. Karsch, *PoS CORFU2018* (2019)163
- Recent studies of QCD phase structure from both fRG and DSE have shown convergent estimate for the location of CEP.
- Considering relatively larger errors when  $\mu_B/T \gtrsim 4$ , one arrives at a reasonable estimation :  $450 \text{ MeV} \lesssim \mu_{B\text{CEP}} \lesssim 650 \text{ MeV}$ .



# Update: CEP from fRG



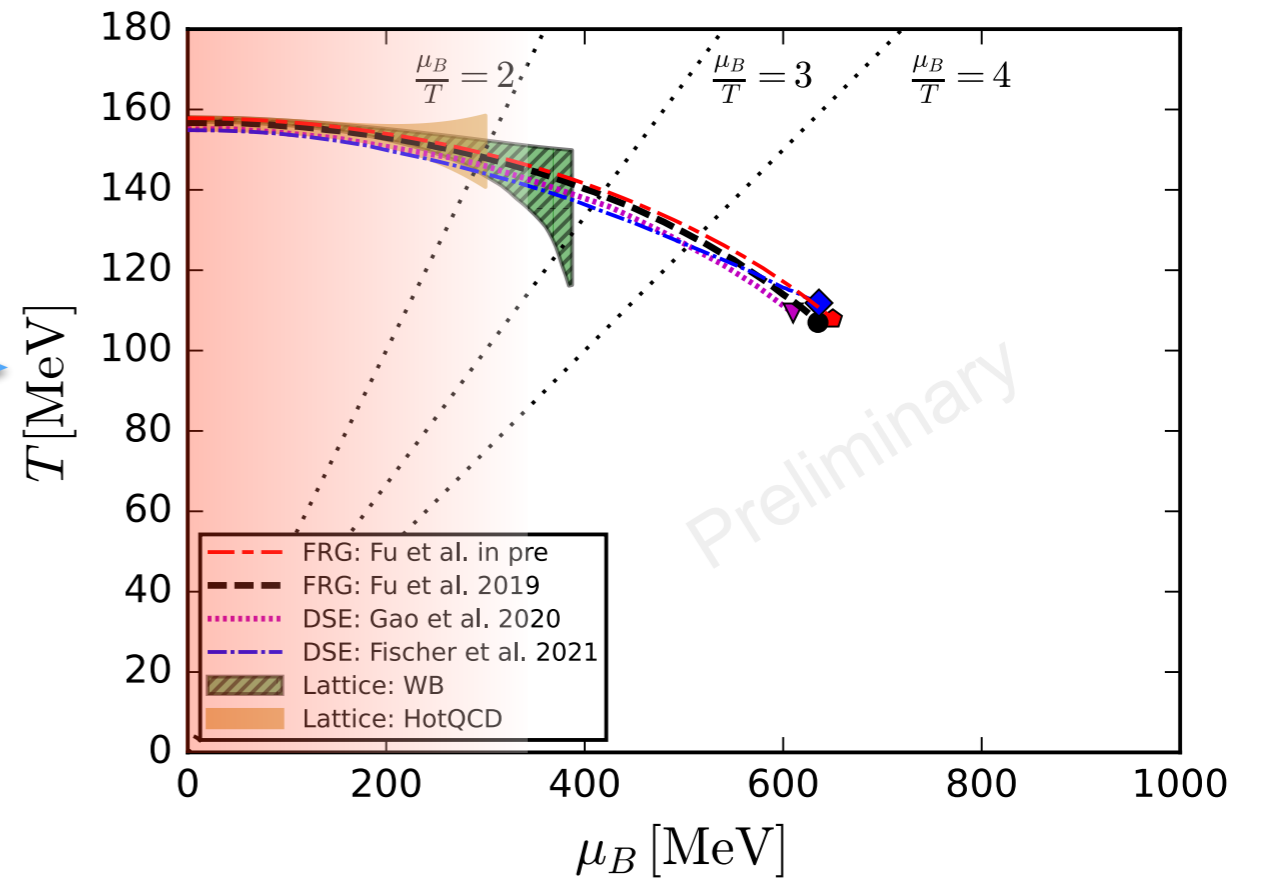
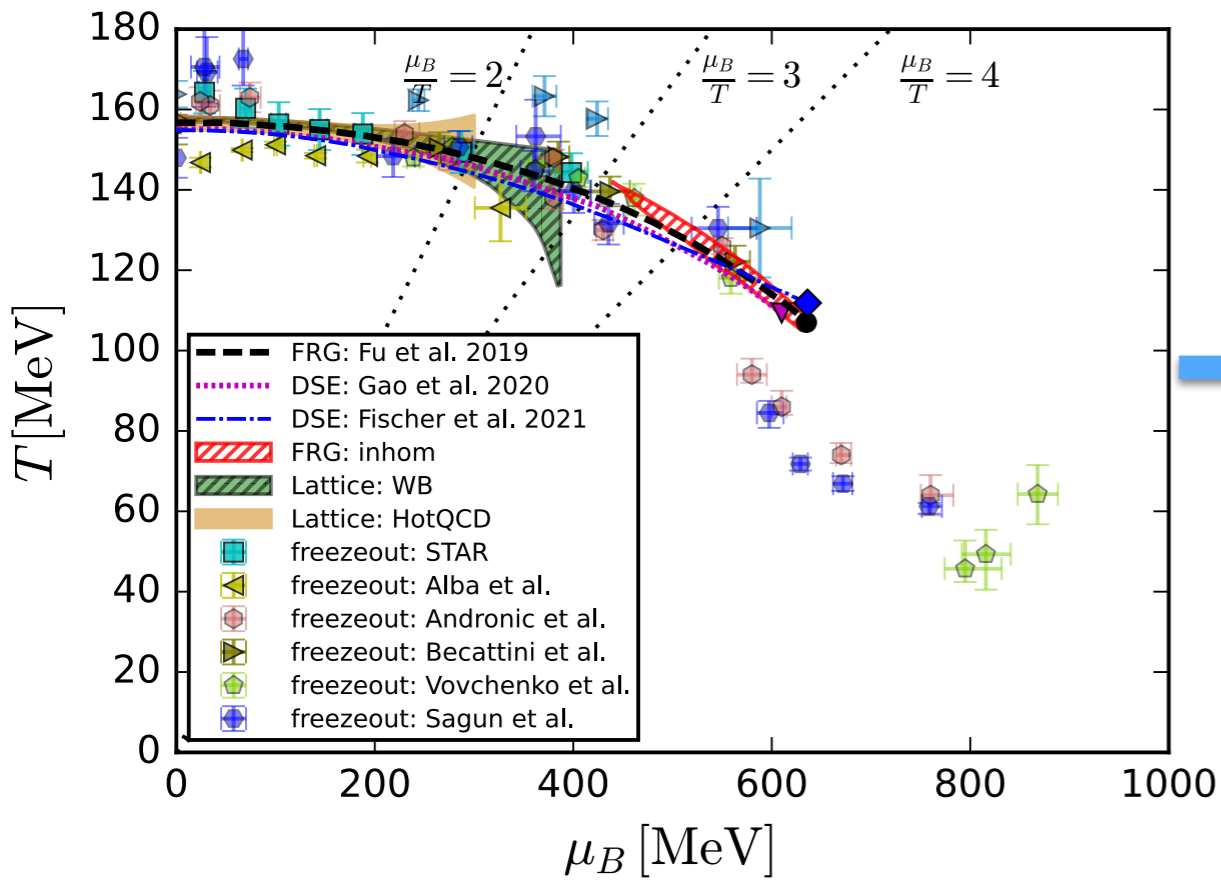
fRG:

$\blacklozenge (T, \mu_B)_{\text{CEP}} = (108, 650)\text{MeV}$

improved truncations for the sector of  $s$  quark and the full mesonic potential of  $N_f = 2+1$ .

fRG: WF, Pawłowski, Rennecke, Wen, Yin, (2022) in preparation

# Update: CEP from fRG



fRG:

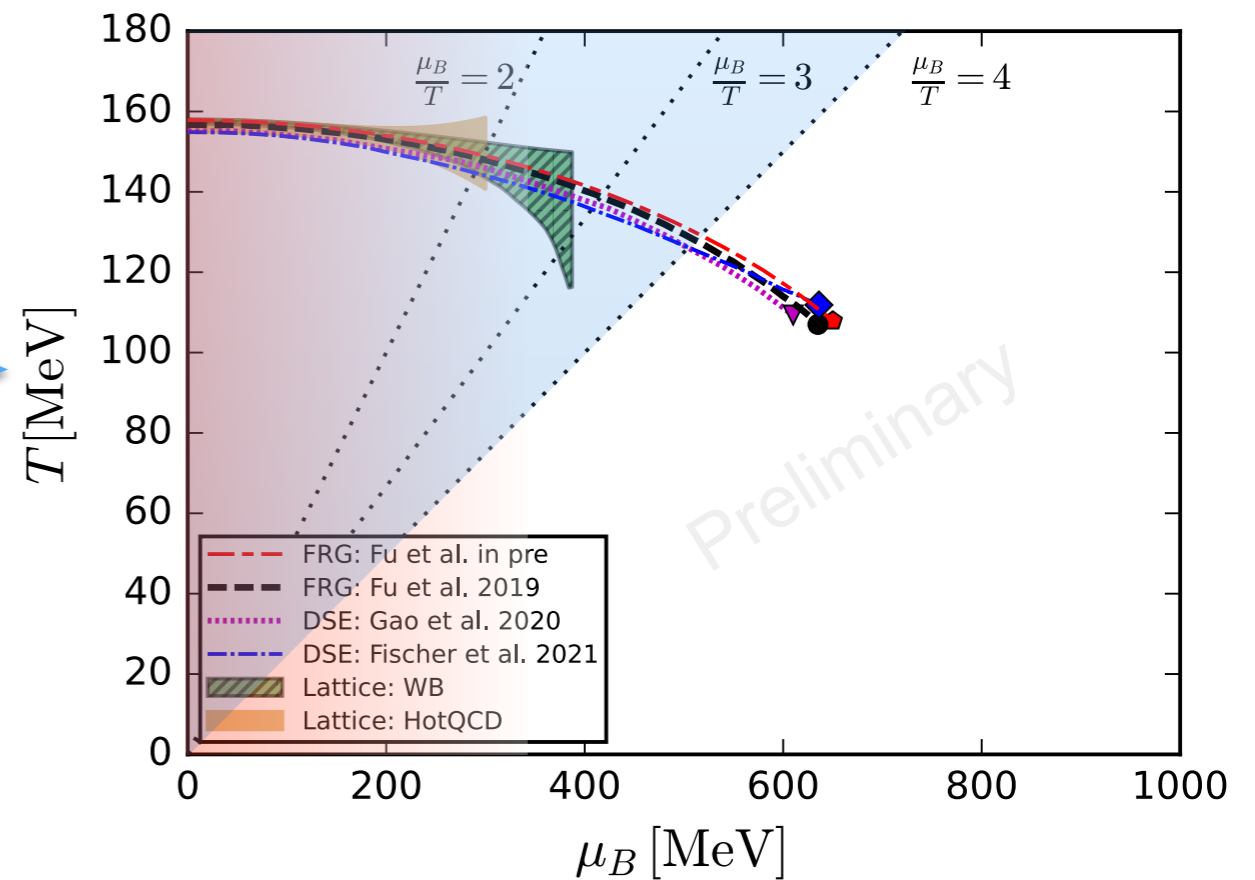
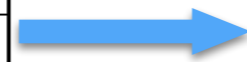
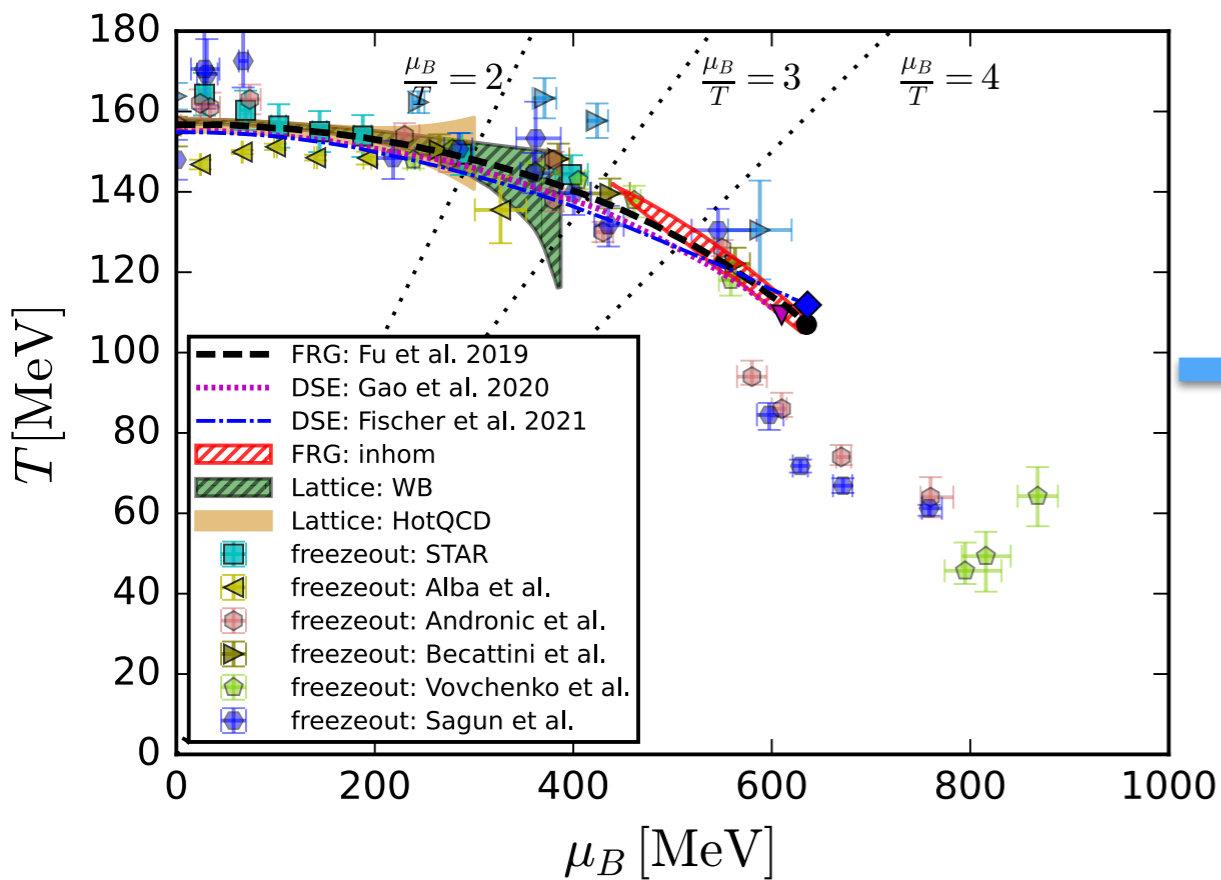
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Passing lattice benchmark tests at vanishing  $\mu_B$ .

# Update: CEP from fRG



fRG:

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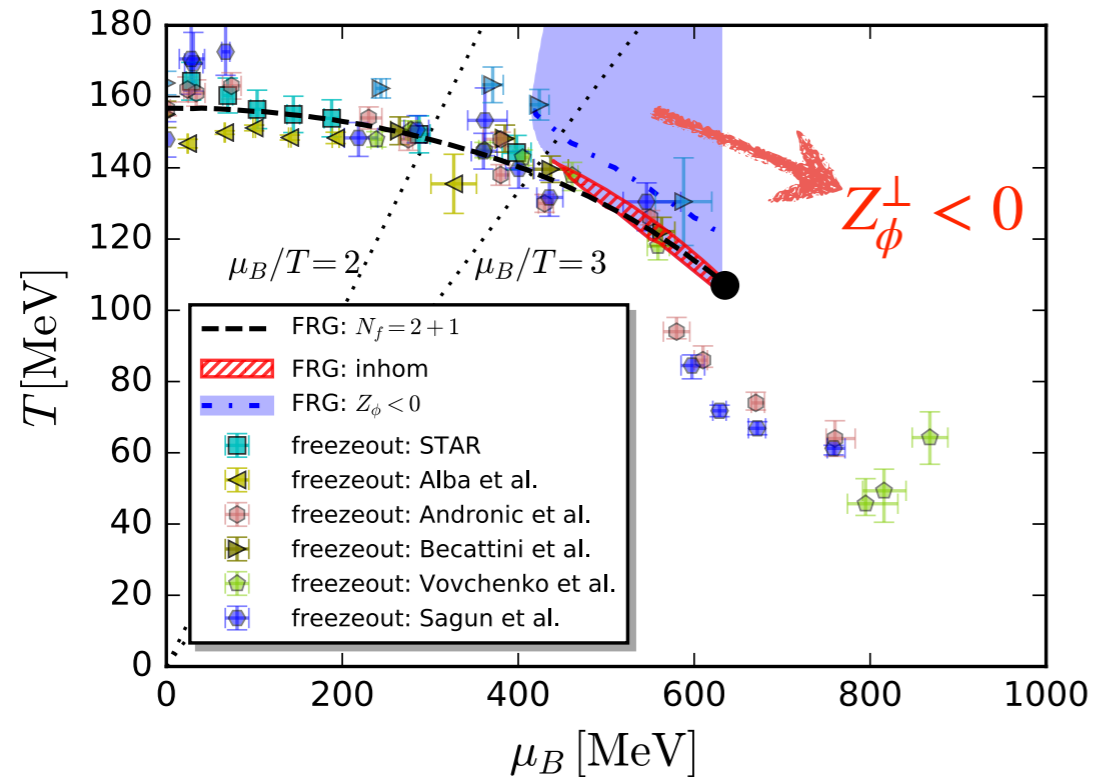
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Passing lattice benchmark tests at vanishing  $\mu_B$ .

Regime of reliability of current best truncation.

# Inhomogeneous instabilities in QCD phase diagram



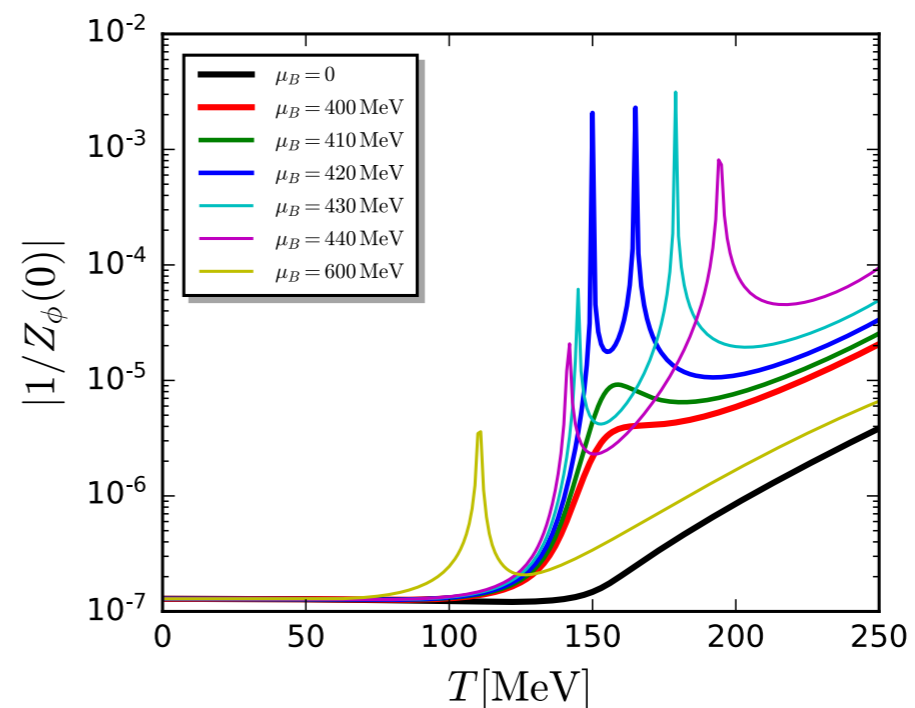
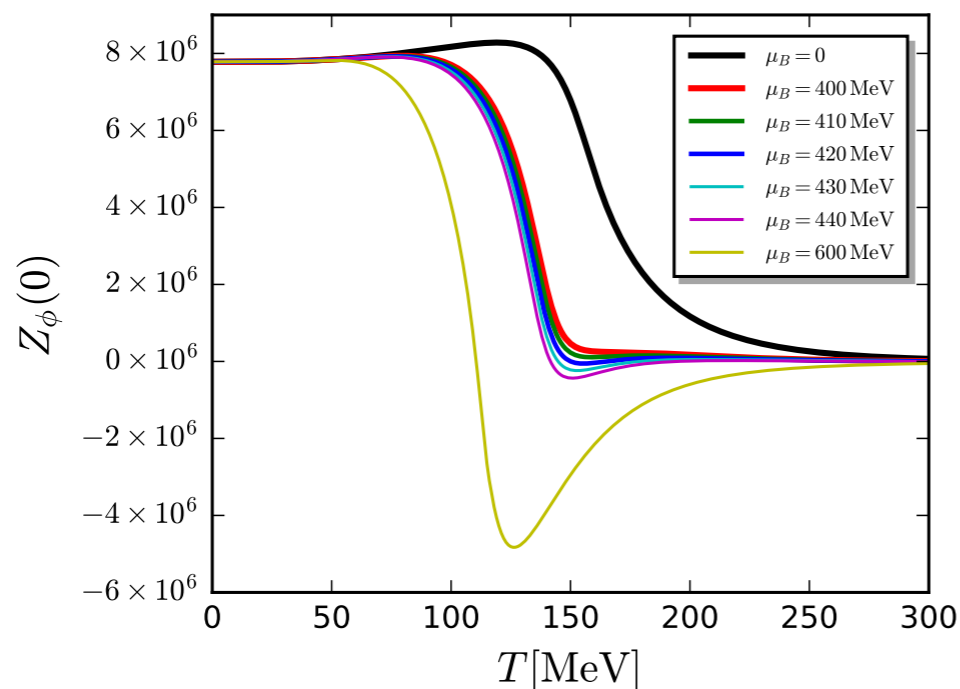
Mesonic two-point correlation function:

$$\Gamma_{\phi\phi}^{(2)}(p) = [Z_\phi^\parallel(p_0, \mathbf{p}) p_0^2 + Z_\phi^\perp(p_0, \mathbf{p}) \mathbf{p}^2] + m_\phi^2$$

with

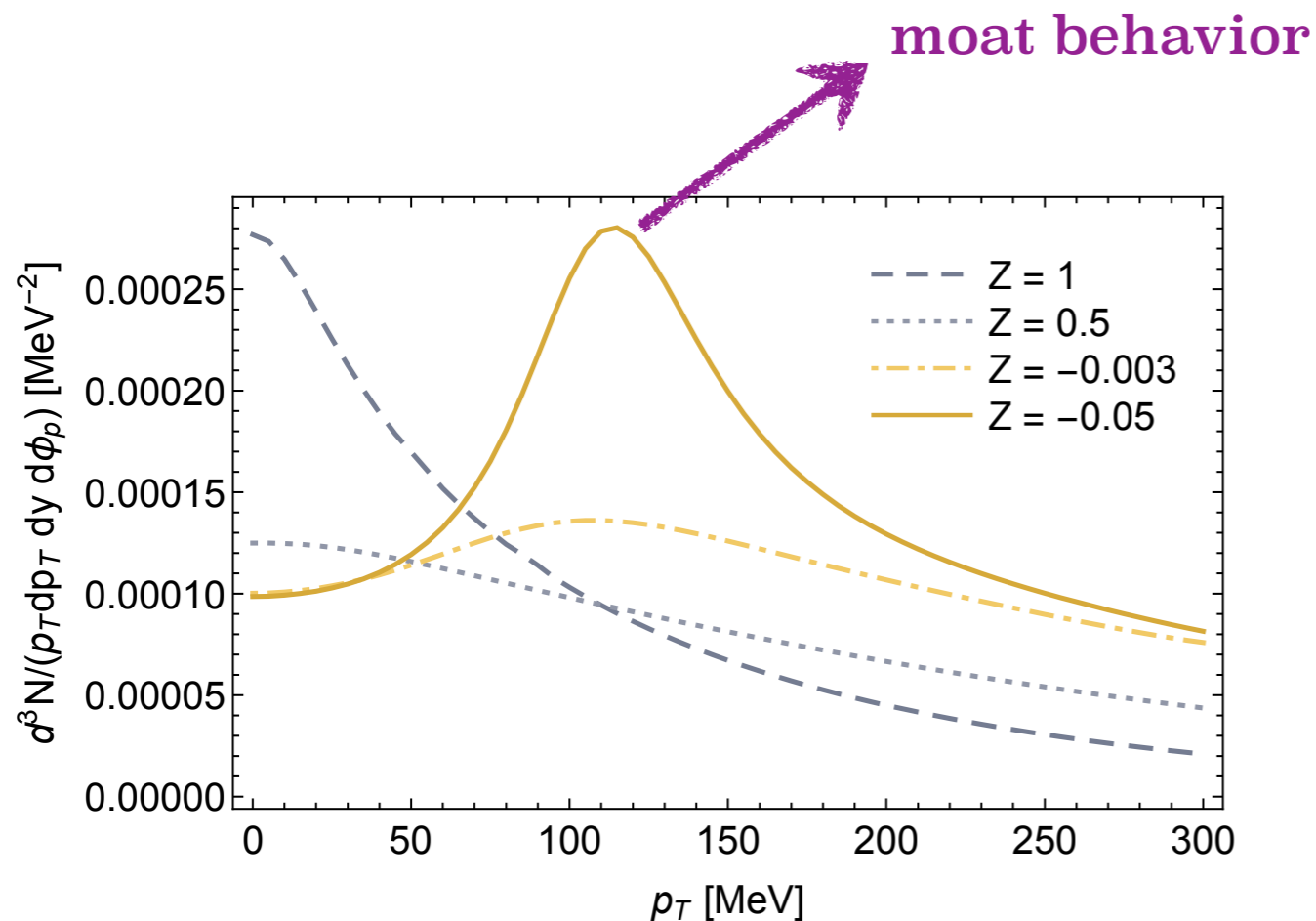
$$\Gamma_{\phi\phi,k}^{(2)} = \left. \frac{\delta^2 \Gamma_k[\Phi]}{\delta\phi\delta\phi} \right|_{\Phi=\Phi_{\text{EoM}}}$$

WF, Pawłowski, Rennecke, *PRD* 101 (2020) 054032



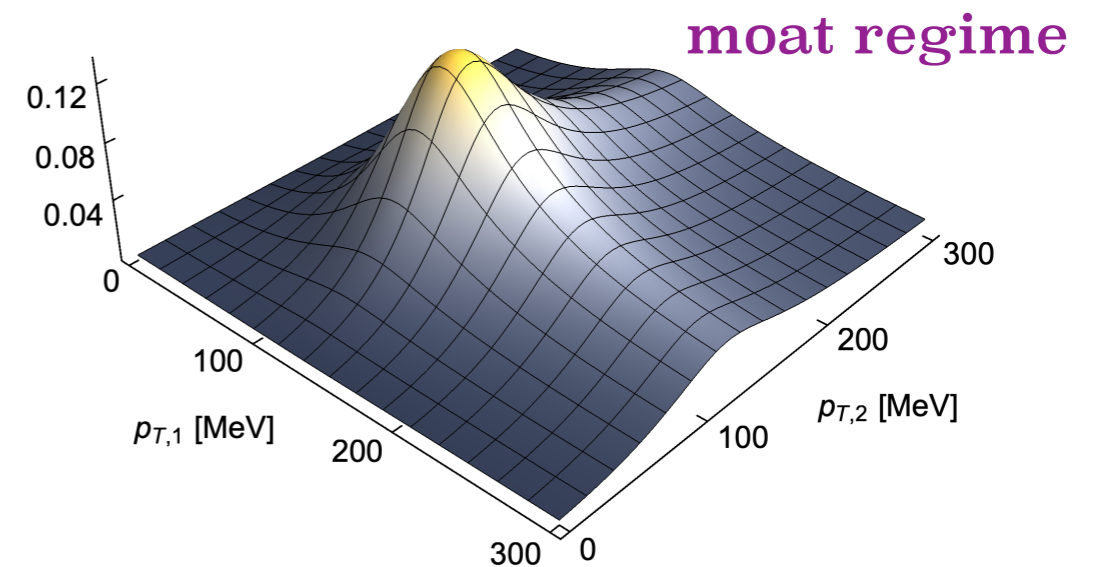
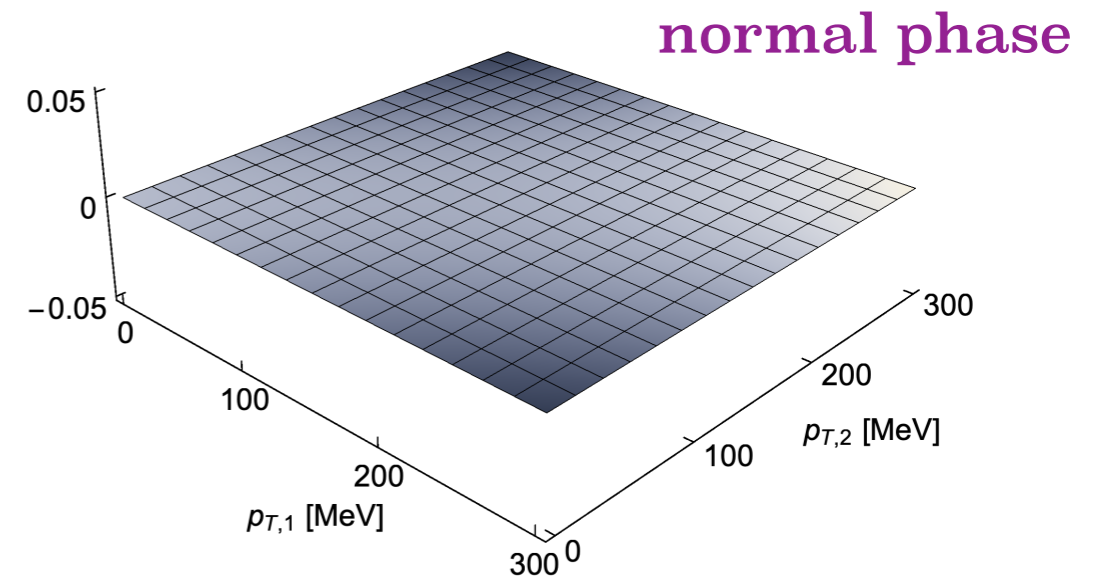
# Signature of inhomogeneous instability in heavy-ion collisions—“moat” spectrum

- transverse momentum spectrum of one particle:



Pisarski, Rennecke, *PRL* 127 (2021) 152302;  
Rennecke, Pisarski, arXiv:2110.02625

- two-particle correlation:

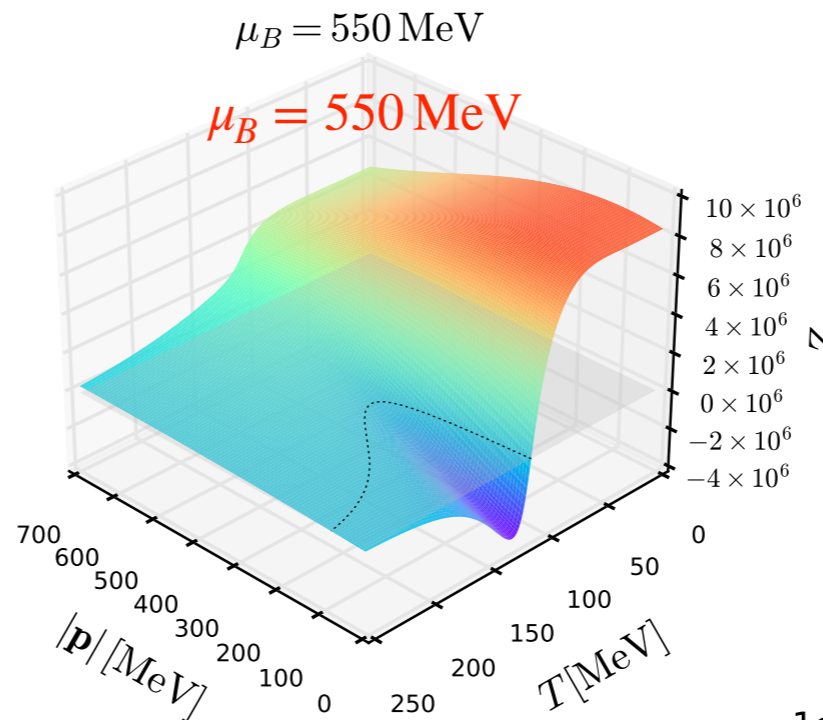
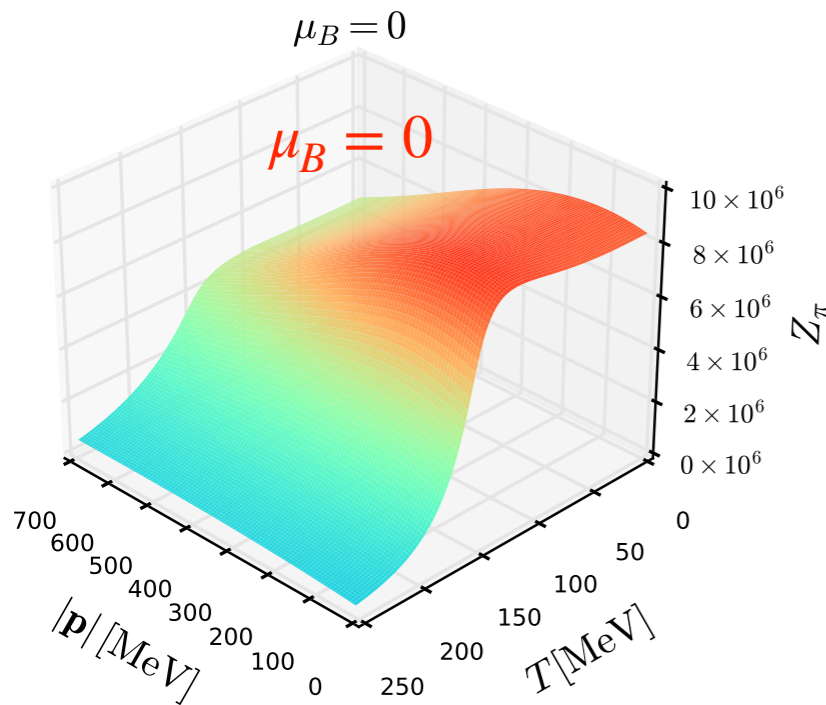


$$\Delta n_{12} = \left\langle \left( \frac{d^3 N}{d\mathbf{p}^3} \right)^2 \right\rangle_c / \left\langle \frac{d^3 N}{d\mathbf{p}^3} \right\rangle^2$$

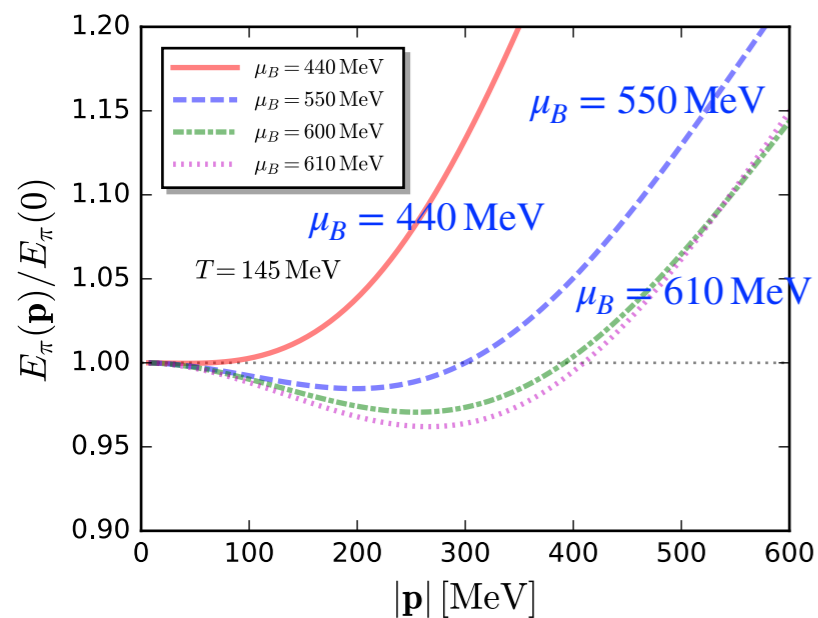
# Momentum-dependent mesonic wave function

Flow equation for mesonic two-point functions:

$$\partial_t \text{---} \bullet \text{---} = \tilde{\partial}_t \left( \text{---} \bullet \text{---} \text{---} \bullet \text{---} + \frac{1}{2} \text{---} \bullet \text{---} \text{---} \bullet \text{---} + \frac{1}{2} \text{---} \bullet \text{---} \text{---} \bullet \text{---} \right)$$



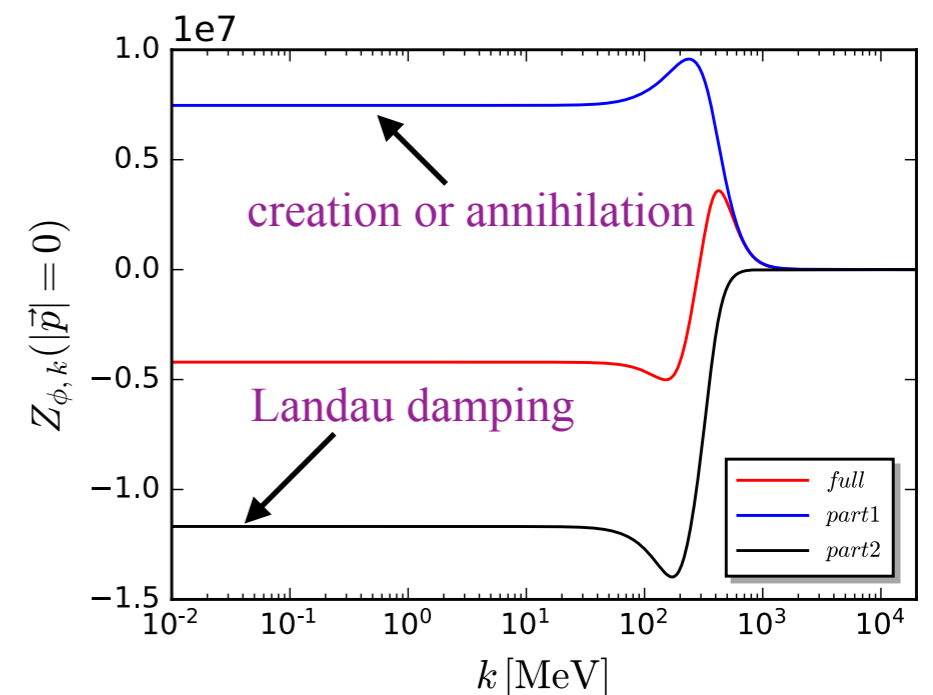
- Inhomogeneous instability is resulted from **Landau damping** of two quarks in thermal bath in the regime of large baryon chemical potential.



Dispersion relation:

$$E_\phi(\mathbf{p}) = \left[ Z_\phi^\perp(\mathbf{p}) \mathbf{p}^2 + m_\phi^2 \right]^{1/2}$$

WF, Pawłowski, Pisarski, Rennecke, Wen, Yin, in preparation.

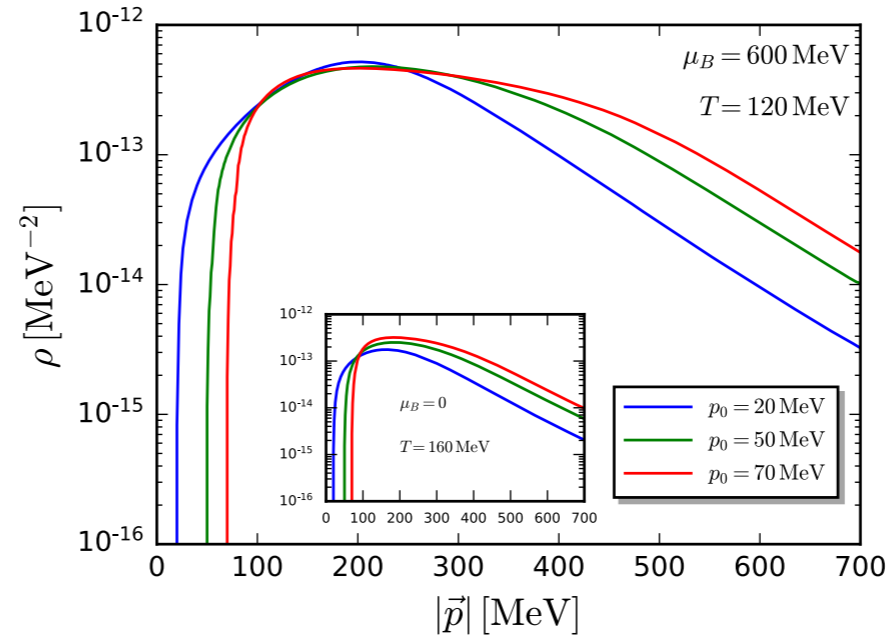


# Real-time mesonic two-point functions

Analytic continuation on the flow equation:

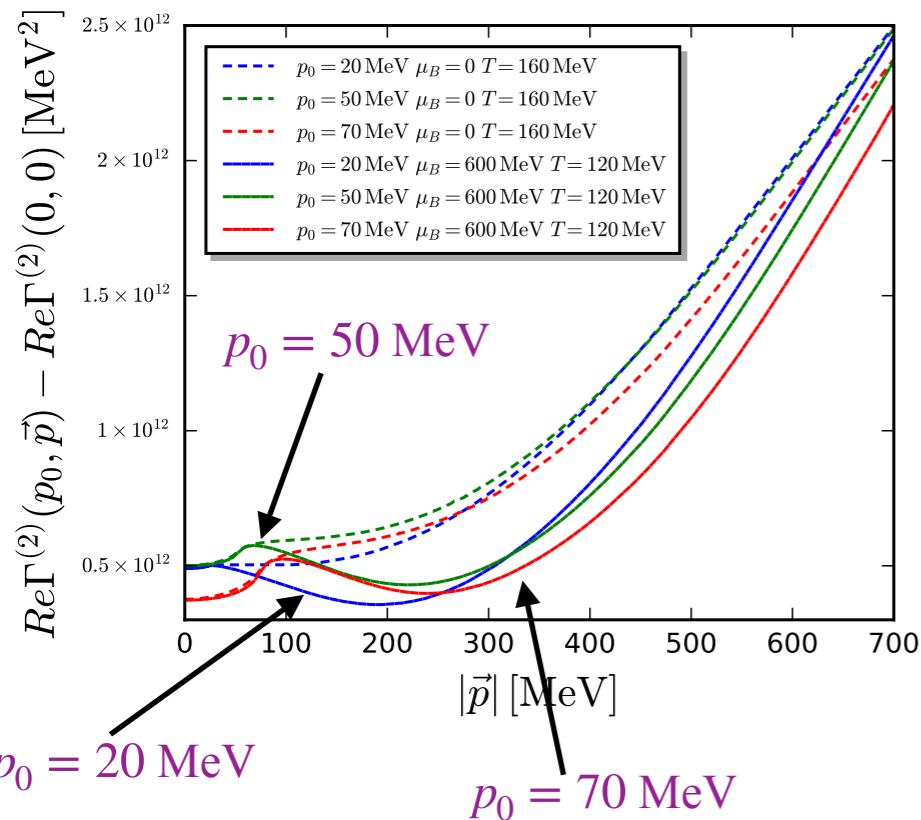
$$\Gamma_{\phi\phi,R}^{(2)}(\omega, \mathbf{p}) = \lim_{\epsilon \rightarrow 0^+} \Gamma_{\phi\phi}^{(2)}(-i(\omega + i\epsilon), \mathbf{p})$$

**Note: not on data!**

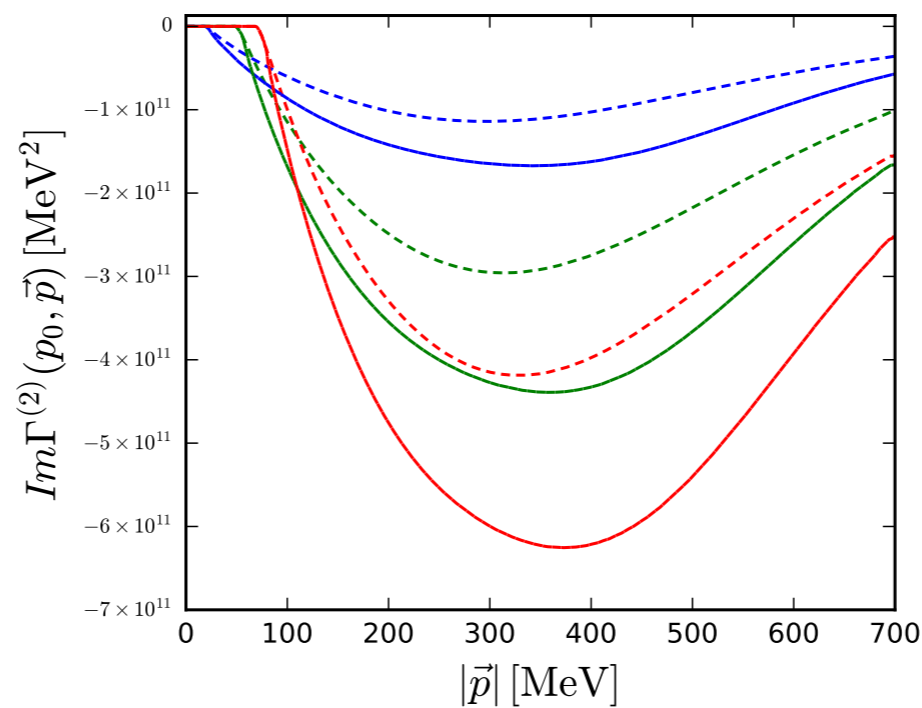


Spectral function

Real part of  $\Gamma_{\phi\phi,R}^{(2)}(p_0, \mathbf{p})$ :



Imaginary part of  $\Gamma_{\phi\phi,R}^{(2)}(p_0, \mathbf{p})$ :



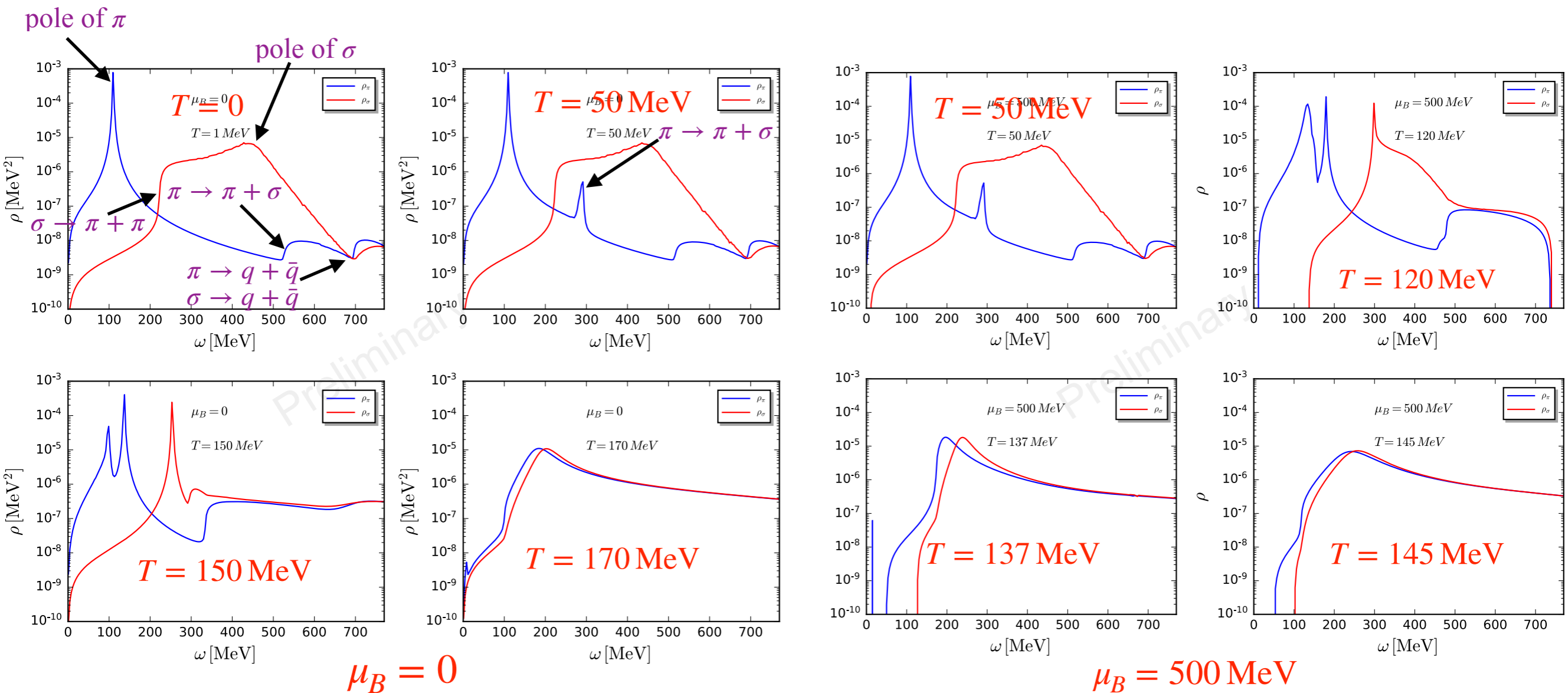
- Imaginary part of the mesonic two-point functions and spectral function are enhanced by the Landau damping effect

WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

# Spectral functions for mesons

- spectral function:

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \frac{\text{Im}\Gamma^{(2),R}(\omega, \vec{p})}{(\text{Re}\Gamma^{(2),R}(\omega, \vec{p}))^2 + (\text{Im}\Gamma^{(2),R}(\omega, \vec{p}))^2}$$





# Summary

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- ★ Estimates for the location of the CEP or the onset of new physics from fRG and Dyson-Schwinger Equations converge in a rather small region at baryon chemical potentials of about 600 MeV.
- ★ It is found that inhomogeneous instabilities in the regime of large baryon chemical potential arise from the Landau damping.

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- ★ Estimates for the location of the CEP or the onset of new physics from fRG and Dyson-Schwinger Equations converge in a rather small region at baryon chemical potentials of about 600 MeV.
- ★ It is found that inhomogeneous instabilities in the regime of large baryon chemical potential arise from the Landau damping.

**Thank you very much for your attentions!**