Fluctuation dynamics and QCD critical point

1. Evolving critical fluctuations.

2. Freezing out critical fluctuations.

3. Outlook.

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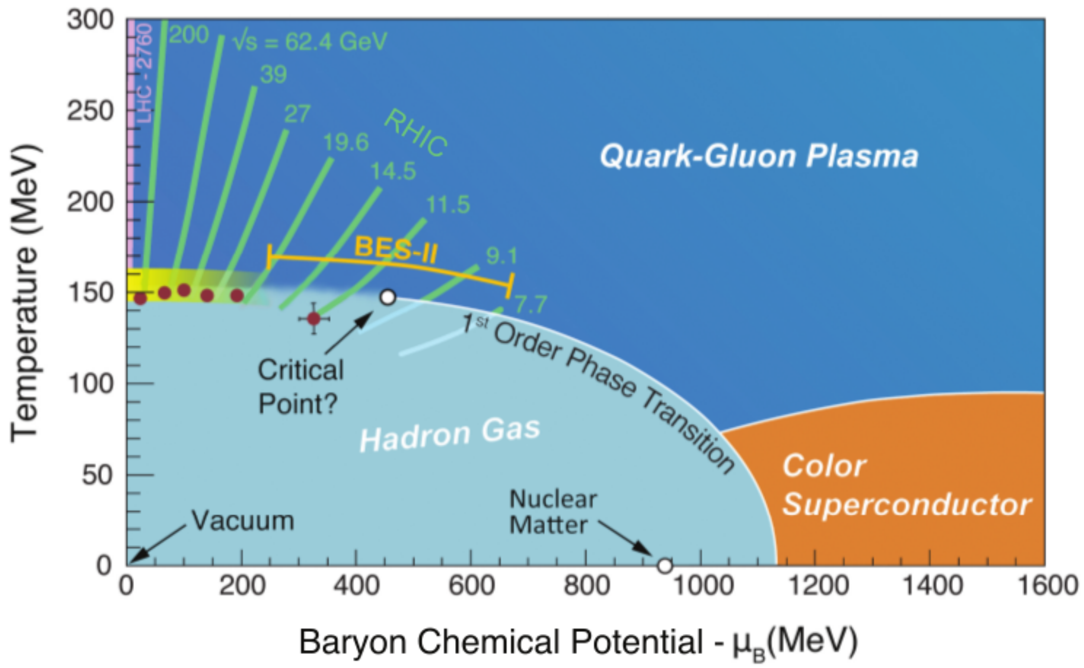
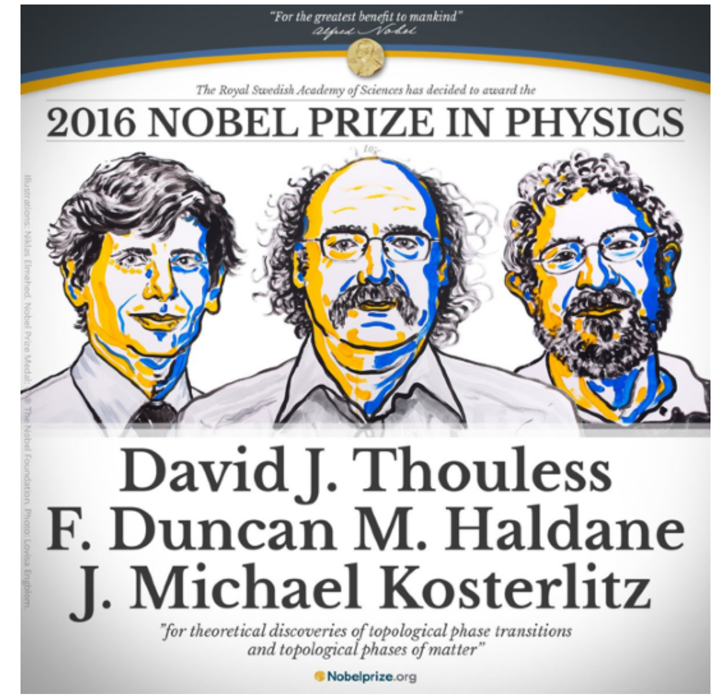
重大项目研讨会,July.29，2022

Introduction

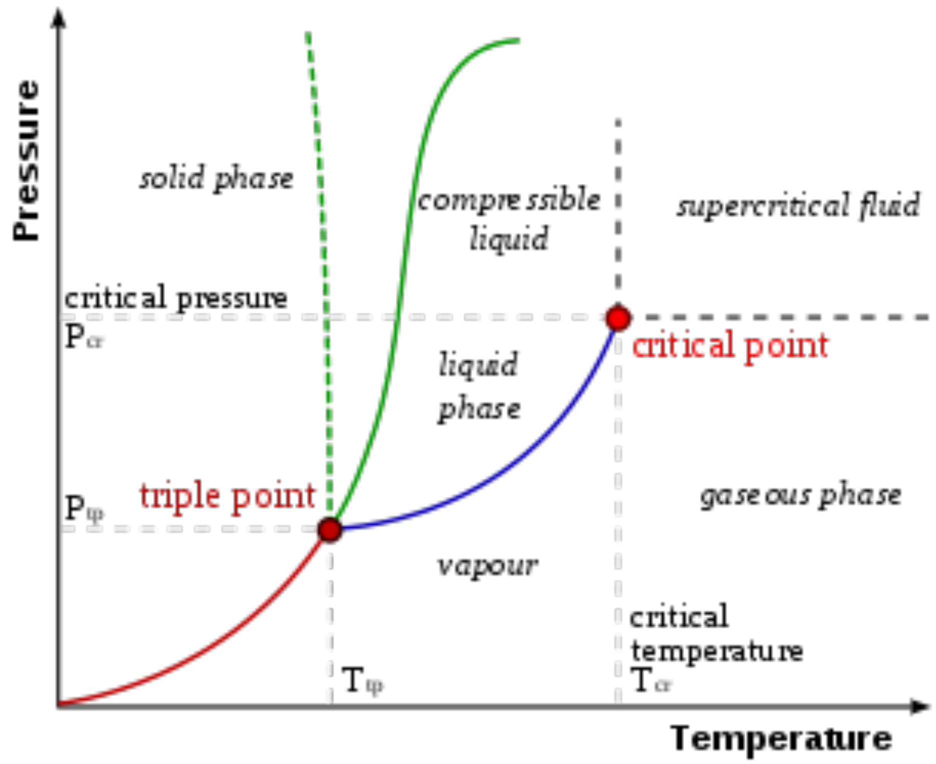
QCD phase diagram



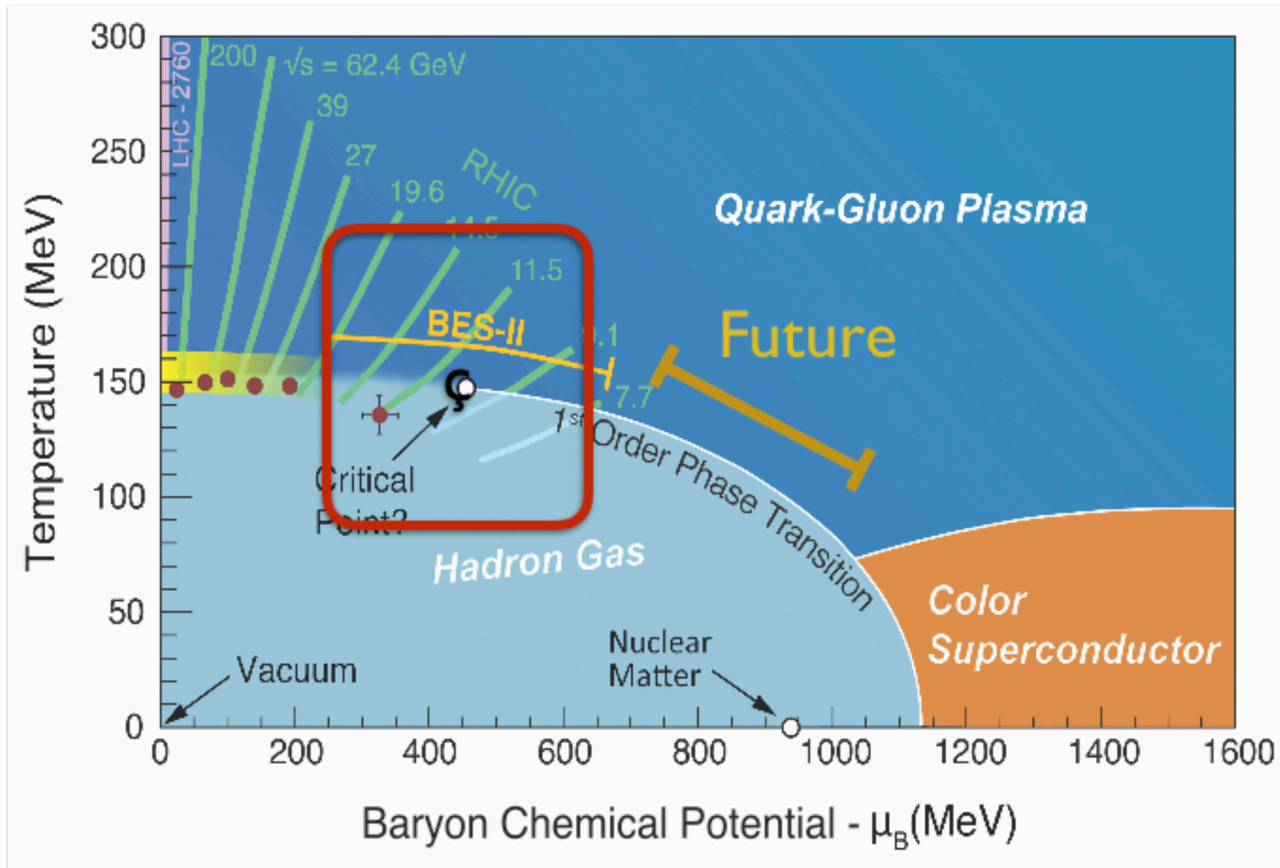


- QCD: simply equation, rich phases.
- Traditional paradigm (Landau-Ginzburg): symmetry breaking pattern + thermodynamics.
- Modern view: quantum effects are important in characterizing and understanding phases. (not covered in this talk: spin and QCD phases)

Critical point



Phase diagram of water (from wiki)

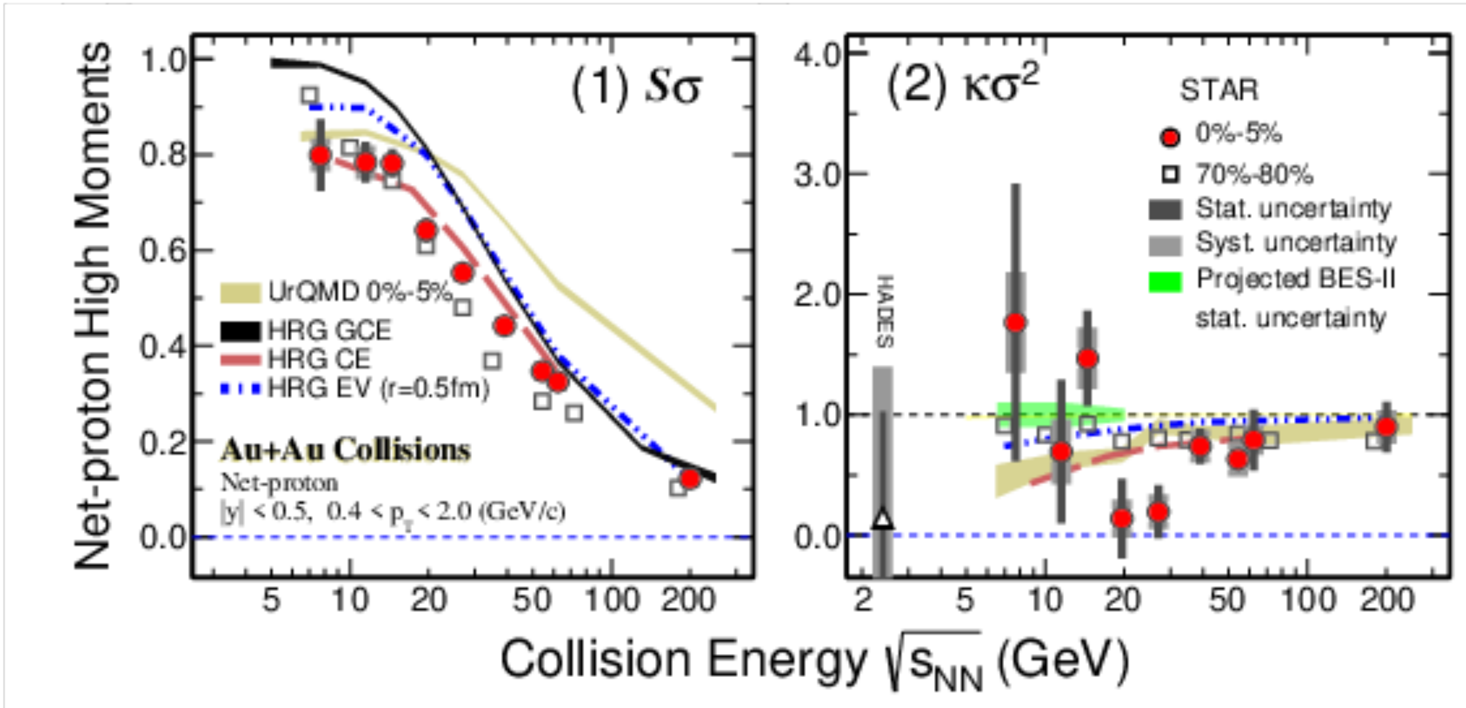


ubiquitous phenomenon.

the landmark point on the QCD phase diagram (if exists).

The discovery of the QCD critical point and first order transition would break new frontier in the field.

Discovery potential of BESII



STAR collaboration, 2001.02852

Intriguing hints in fluctuation observables from BESI

Theory: comprehensive dynamical framework for criticality.

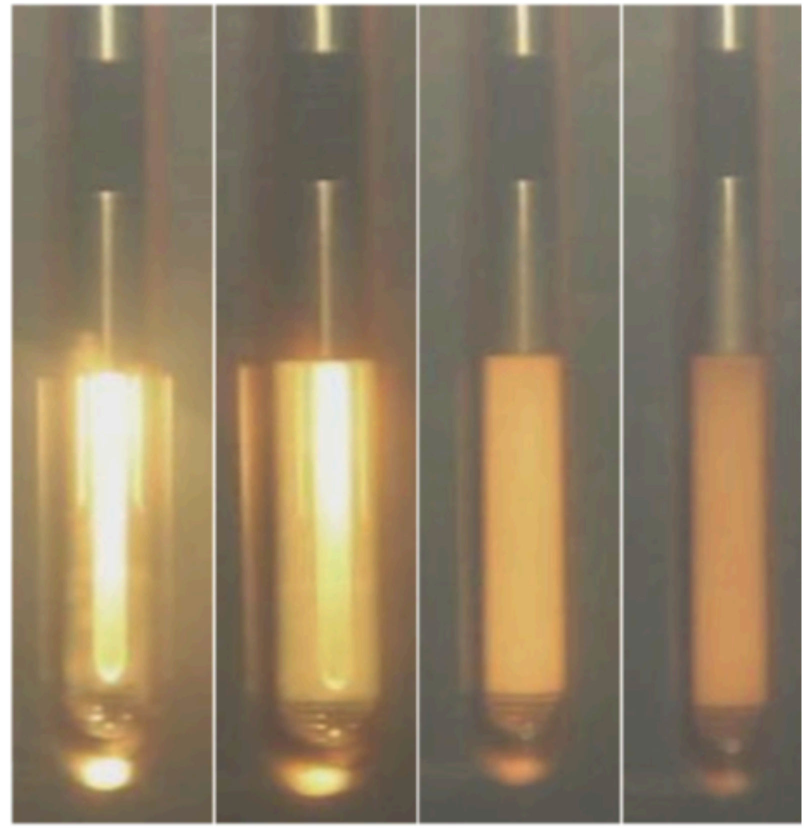
Status: the most important ingredients as well as the connections among them have already been clarified.

This talk: critical fluctuations.

The BEST framework for the search for the QCD critical point and the chiral magnetic effect

Xin An,[1] Marcus Bluhm,[2] Lipei Du,[3] Gerald V. Dunne,[4] Charles Gale,[5] Joaquin Grefa,[6] Ulrich Heinz,[3] Anping Huang,[7] Jamie M. Karthein,[6] Dmitri E. Kharzeev,[8,9] Volker Koch,[10] Jinfeng Liao,[7] Shiyong Li,[11] Mauricio Martinez,[12] Michael McNelis,[3] Debora Mroczek,[13] Swagato Mukherjee,[9] Marlene Nahrgang,[2] Angel Nava Acuna,[6] Jacquelyn Noronha-Hostler,[13] Dmytro Oliinychenko,[14] Paolo Parotto,[15] Israel Portillo,[6] Maneesha Sushama Pradeep,[11] Scott Pratt,[16] Krishna Rajagopal,[17] Claudia Ratti,[6] Gregory Ridgway,[17] Thomas Schäfer,[12] Björn Schenke,[9] Chun Shen,[18,19] Shuzhe Shi,[5] Mayank Singh,[5,20] Vladimir Skokov,[12,19] Dam T. Son,[21] Agnieszka Sorensen,[10,22] Mikhail Stephanov,[11] Raju Venugopalan,[9] Volodymyr Vovchenko,[10] Ryan Weller,[17] Ho-Ung Yee,[11] and Yi Yin[23,24]

See Sec IIIA and Sec IVD for more details; 2108.13867



Hou et al, Journal of Chemistry 16'.

"As the density fluctuations become of a size comparable to the wavelength of light, the light is scattered and causes the normally transparent liquid to appear cloudy." -- wiki

Fluctuations, correlations and criticality

The correlation function of the order parameter field δM (Fourier momentum Q ~ inverse of the size of the fluctuation)

$$\phi_{\rm eq}(Q) \sim \langle \delta M \delta M \rangle \sim \frac{1}{\xi^{-2} + Q^2}$$

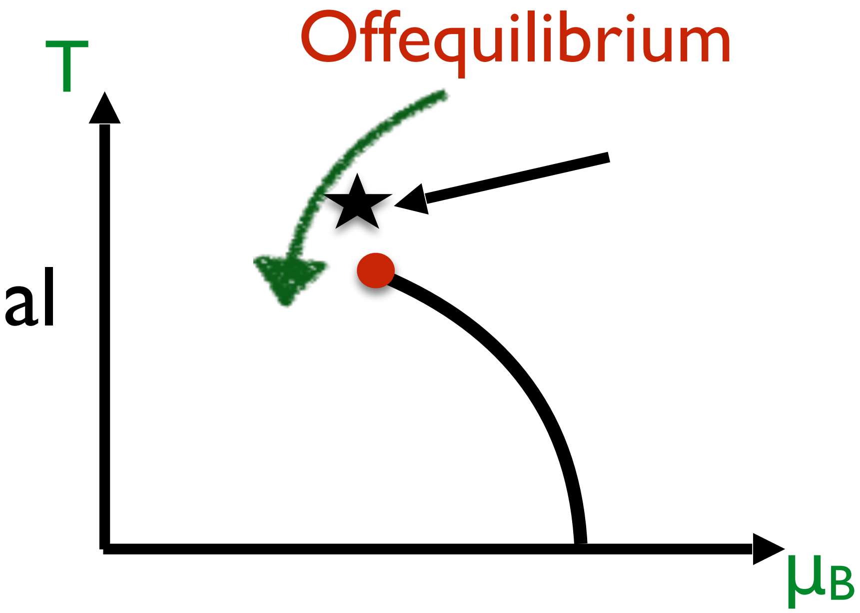
Enhanced $\phi_{eq}(Q)$ near the critical point for Q~1/ ξ $\Rightarrow$ *phenomenon of the critical opalescence*.

QCD critical point: $\phi_{eq}(Q)$ $\Rightarrow$ i) the critical scaling of E.o.S. ii) the growth of non-Gaussian fluctuations of proton numbers

Two keys to understand observables: non-equilibrium evolution and freezeout prescription.

Evolving critical fluctuations

Real-time critical fluctuations



Inescapably fall out of equilibrium near the critical point. ("Critical slowing down")

Could be different from the equilibrium expectation both quantitatively and qualitatively !

e.g. S. Mukherjee, R. Venugopalan and YY, PRC15' ; see YY, 1811.06519 for a concise review on related development.

Back-reacts on bulk evolution.

$\boxed{\text{gradient of p (?)} \simeq \text{acceleration of flow}}$

How to describe the interplay among fluctuations and bulk evolutions?

Hydro+

Key idea: couples hydro. modes with critical fluctuations using deterministic equations. *Stephanov-YY, PRD '18;*

Further development: e..g. additional slow modes, non-Gaussian fluctuations. Xin An-Basar-Stephanov -H.-U. Yee, PRC19'&20'& 2009.10742.

Broad application: e.g. hydro with chiral anomaly.

A generalized hydro+ model to describing jet-medium interaction in non-hydro. non-perturbative regime of QGP. *Weiyao Ke YY, to appear*

(Stochastic hydro: i. Implementation for expanding system are challenging. ii Results depend on cut-off (lattice space). Significant progress recently.)

The construction of "hydro+"

Stephanov-YY, PRD '18;

"+": (Wigner transform of) the two point function of δM (For QCD critical point, M ~ s/n):

$$\phi(t,x;Q) = \int d\Delta x e^{-i\Delta x Q}\left\langle \delta M(t, x+\Delta x/2)\delta M(t, x-\Delta x/2)\right\rangle$$

The evolution of "+" is modelled by relaxation rate equation.

$$u^\mu \partial_\mu \phi = \Gamma_\phi(Q)\left(\phi(Q) - \phi_{\rm eq}(e,n;Q)\right)$$

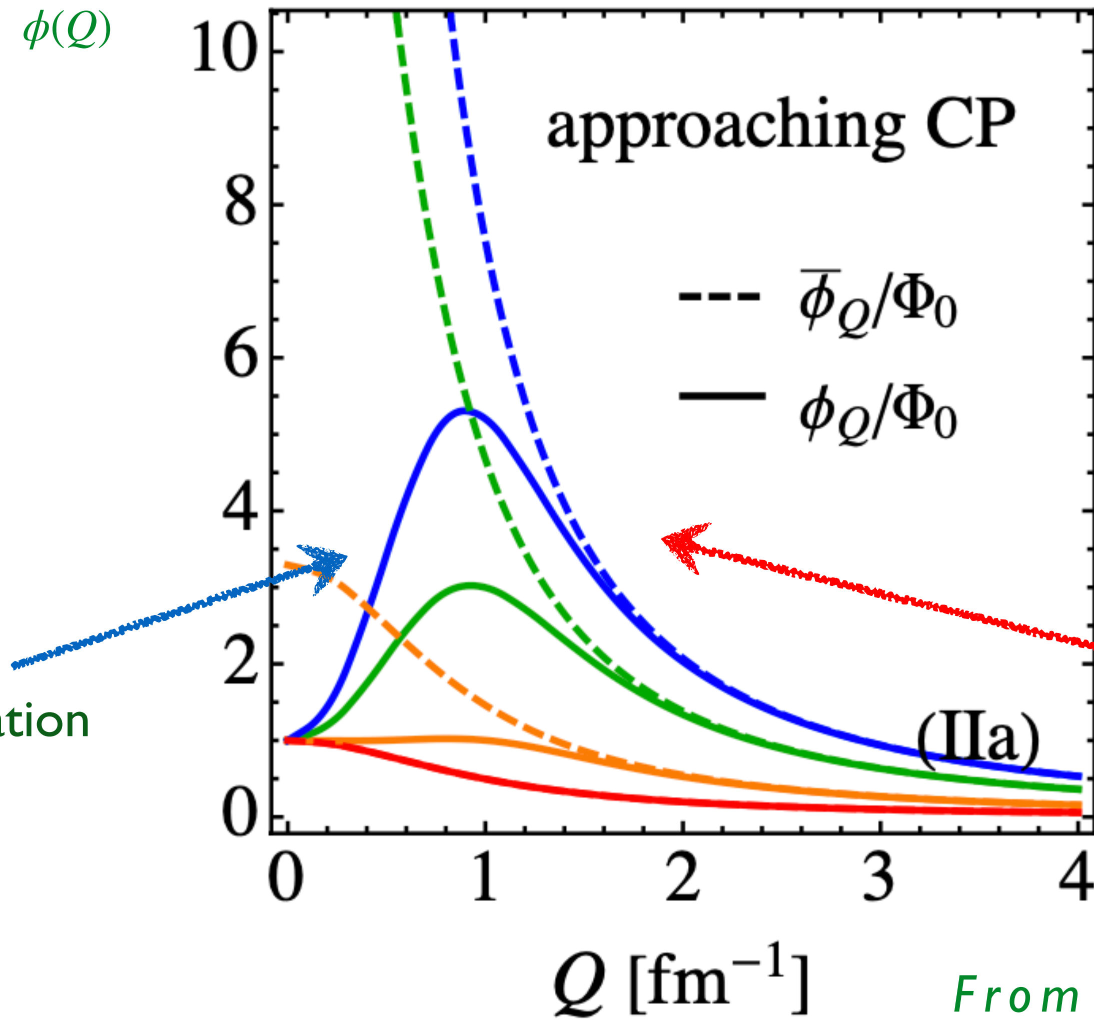
Feedback to hydro. $\partial_\mu T^{\mu\nu} = 0; \partial_\mu J^\mu = 0$:

$$T^{\mu\nu} = e\, u^\mu\, u^\nu + p_{(+)}\left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial) \qquad p(e,n) \to p_{(+)}(e,n,\phi)$$

Similar for transport coefficients.

New lesson learned: feeback effect is suppressed in practice.

A representative evolution of off-equilibrium fluctuation.



Conservation

Q^{-2} tail

$\sqrt{D\Delta\tau}$

size of correlation

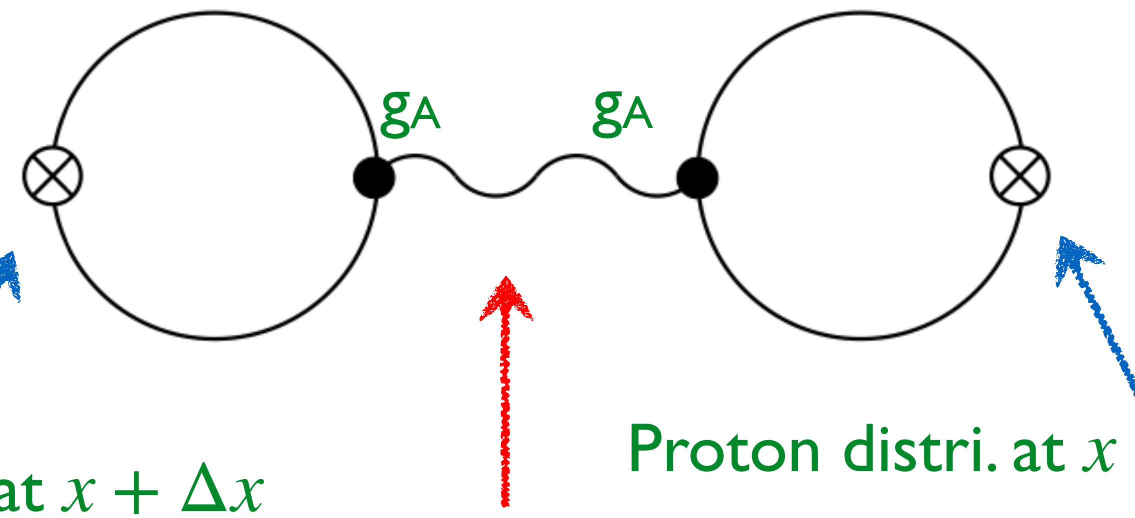
From Lipei Du-Heinz-Rajagopal-YY, 2004.02719;see also Akamatsu-Teaney-Yan-Yin, PRC 19'

Diffusive equilibration leads to emergent length scale (off-equilibrium correlation length).

Next: converting density fluctuations into observable correlations/ fluctuations.

Freezeout of Gaussian fluct.

Pradeep-Rajagopal-Stephanov-YY, 2204.00639



Pradeep, graduate@UIC

Proton distri. at $x + \Delta x$

Proton distri. at $x - \Delta x$

$\phi(x, Q) \to \tilde{\phi}(x, \Delta x)$

$$\boxed{\langle \delta N_A^2 \rangle_\sigma = g_A^2 Z \int dS_\mu J_A^\mu(x_+) \int dS'_\nu J_A^\nu(x_-) \tilde{\phi}(x, \Delta \tilde{x})}$$

(Expression for non-Gaussian cumulants is similar)

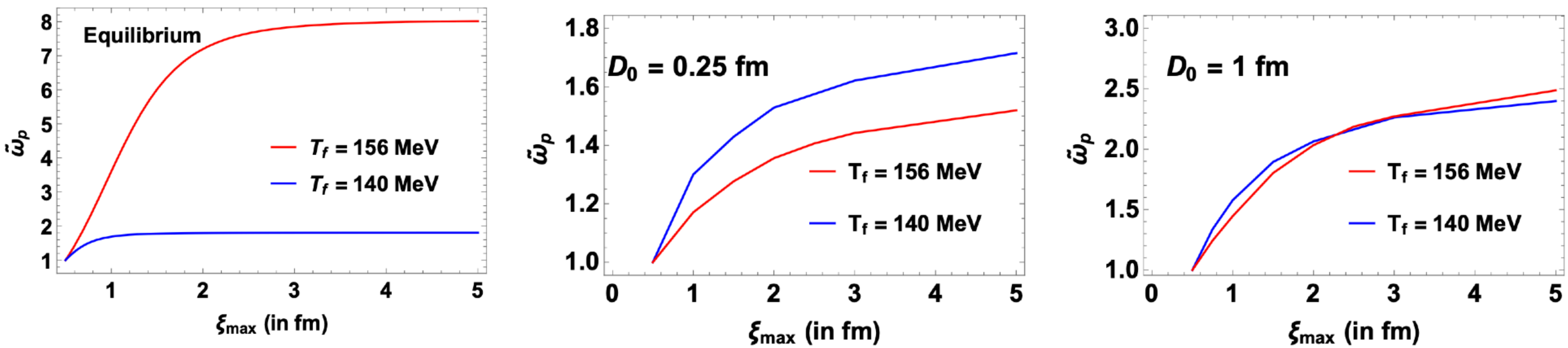
Key idea: the fluctuation of the order parameters induce the fluctuation of hadron mass, including proton mass. Stephanov-Rajagopal-Shuryak 1999'

Freezing out non-critical fluctuations is also important.

Oliinychenko-Koch PRL 19'; Oliinychenko-Shuzhe Shi-Koch PRC 20'

Rescaled proton number fluctuations

Pradeep-Rajagopal-Stephanov-YY, 2204.00639



- Effects of charge conservation are built-in; an alternative to putting in sub-volume scale by hand.

- The manifestation of non-equilibrium effect in Gaussian fluctuation observables. The quantitative description of Gaussian fluctuation reaches its maturity!

Non-Gaussian fluctuations

- Stochastic hydrodynamic simulation (numerically formidable for realistic situations.)

- Solving evolution equations for n-point functions.

S. Mukherjee, R.Venugopalan and YY, PRC15' ;An Xin et al, PRL 21

- New developments based on non-equilibrium EFT (constructed via action principle).

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …

Newly-developed EFT approaches

- A systematic treatment for fluctuations; a number of novel phenomena have been uncovered.

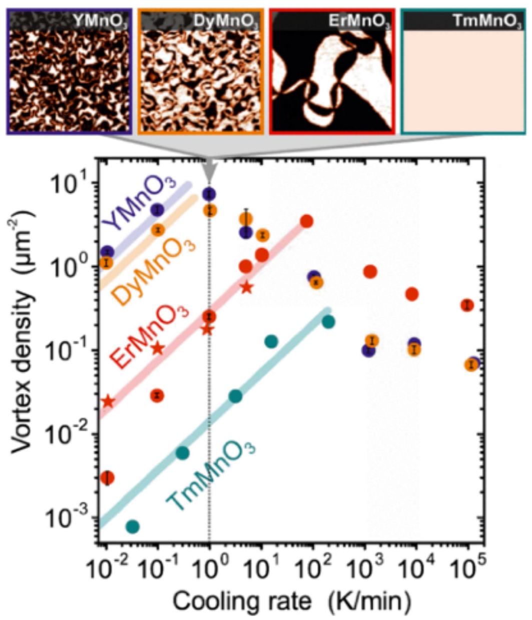
Xinyi Chen Li-Delacrétaz-Hartnoll PRL 18'; Delacrétaz-Glorioso PRL 20'; Jain-Kovtun PRL 22'

- A new framework for the future (personal view):
  - suitable for implementation on quantum computer.
  - the formalism might be extended in a way which does not depend on any long wavelength expansion. (a basis for studying QGP in non-hydrodynamic yet non-perturbative regime)

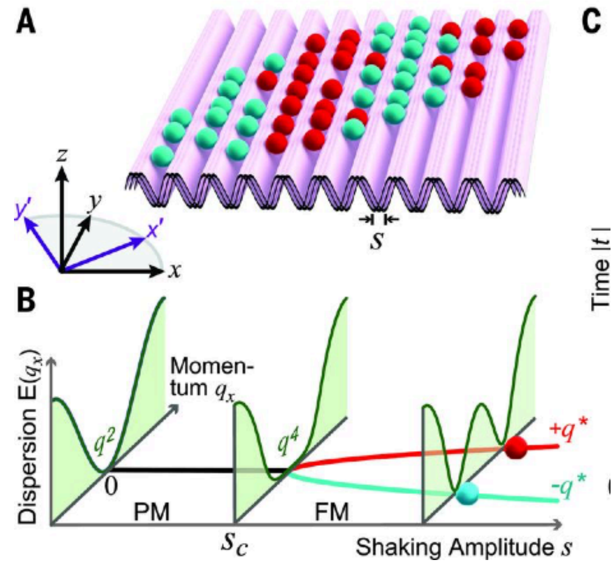
- Based on non-equilibrium EFT, we derive Dyson-Schwinger like equations for non-Gaussian fluctuations; numerically simulation is needed!

Sogabe-YY, JHEP 22'

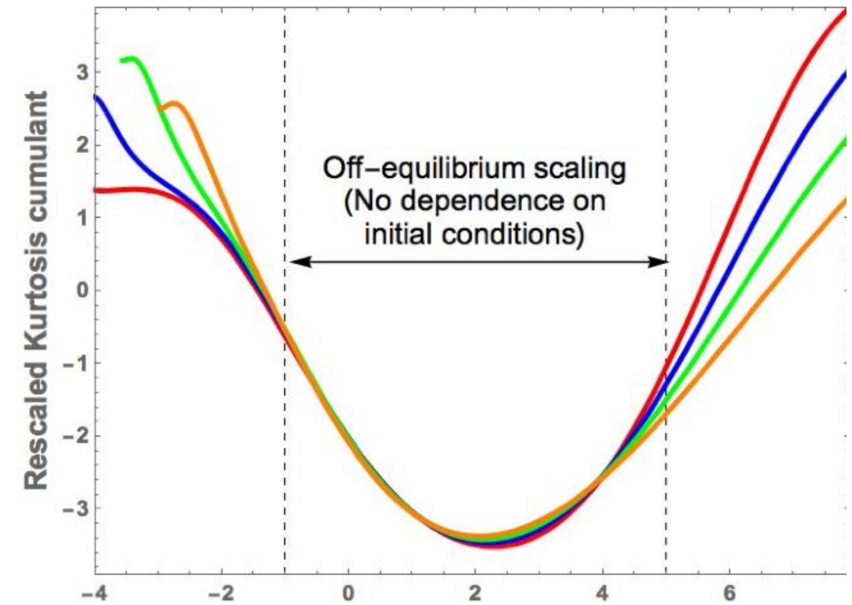
Other ideas: Kibble-Zurek scaling



(Bosons in a shaken optical lattice, W. Clark et al, Science' 16)



Hexagonal Manganites, M. Griffin et al, Phys.Rev.X ' 12



(Conjecture for the QCD point, Mukherjee, Venugopalan and YY, PRL, 16.)

Kibble-Zurek scaling: critical fluctuations and correlations scale with off-equilibrium correlation length.

Once observed, a beautiful demonstration of the unity of physics.

Conclusion

Summary and outlook

The evolution and freeze out of critical fluctuations are ready for implementation in realistic simulation.

Future challenges: non-Gaussian fluctuations? first-order transitions?

Broad view: we are embracing the quantitative era of BES dynamics.

Spin observables: e.g. "Baryonic spin Hall effect" probes the dense QCD matter.

Baochi Fu, Longgang Pang, Huichao Song, YY, 2201.12970.

Important yet unanswered question: what is the criterion for the discovery QCD critical point.

Back-up

Approaches to fluctuating hydro.

Stochastic approach: adding noise to hydro. eqns

$$\partial_t \vec{u} = -(\vec{u}\cdot\vec{\partial})\vec{u} - \nu\nabla^2\vec{u} + \vec{F} \qquad \langle F(t,\vec{x})F(t',\vec{x}')\rangle \sim 2T\eta\delta(t-t')\delta(\vec{x}-\vec{x}')$$

Landau-Lifshitz; Kapusta-Mueller-Stephanov; ..

Deterministic approach: formulating and solving a set of deterministic equations, which couple fluctuations with hydro modes.

$$\partial_\mu\left[T^{\mu\nu}_{\rm ave}(\epsilon, n, u^\mu) + \Delta T^{\mu\nu}(\text{2pt,3pt}, \ldots)\right] = 0$$

E.o.Ms for 2pt, 3pt,…

Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …
Akamatsu-Mazeliauskas-Teaney, PRC 16' & 18'; Stephanov-YY PRD 18'; Mauricio-Schaefer PRC 19; Xin An-Basar-Stephanov -H.-U. Yee, PRC19'&20'& 2009.10742;

EFT approaches: based on action principle.

See Teaney's talk

$$Z = \int D\psi_{\rm hydro}\, e^{iI_{hydro}[\psi_{\rm hydro}]}$$

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …