Quantum kinetic theory and its applications to spin polarization

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Recent invited reviews:

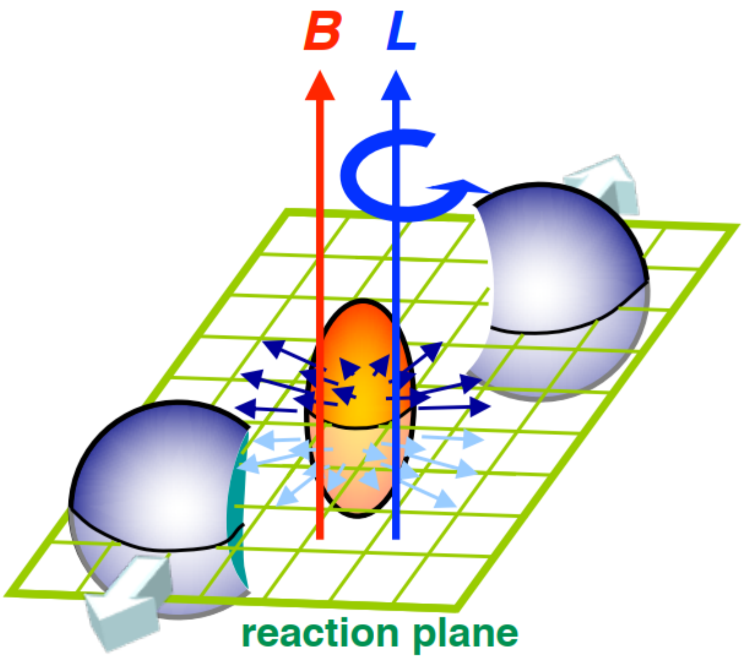
- Y. Hidaka, SP, Q.Wang, D.L. Yang, *Foundations and Applications of Quantum Kinetic Theory*, **arXiv:2201.07644** (Invited by Progress in Particle and Nuclear Physics)
- J.H. Gao, G.L. Ma, SP, Q. Wang, *Recent developments in chiral and spin polarization effects in heavy-ion collisions*, **Nucl.Sci.Tech.** 31 (2020) 9, 90

Outline

- Spin polarization in relativistic heavy ion collisions
- Quantum kinetic theory and its recent development
- Applications to spin polarization
- Summary

Spin polarization in relativistic heavy ion collisions

Huge angular momentum

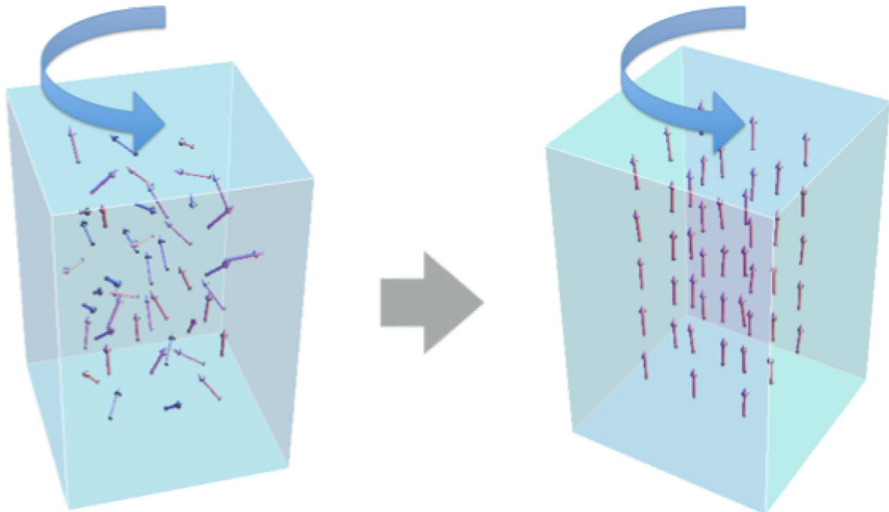


- Huge global orbital angular momenta are produced

$$L \sim 10^5 \hbar$$

- How do orbital angular momenta be transferred to the matter created?

Barnet effects and Einstein-de Hass effects



Barnett effect:

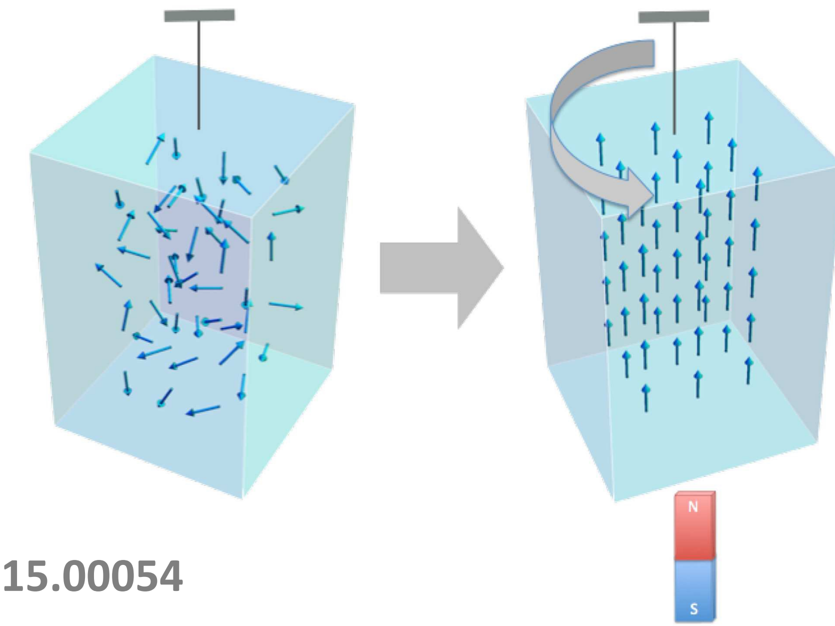
Rotation $\Longrightarrow$ Magnetization

Barnett, Magnetization by rotation, Phys Rev. (1915) 6:239–70.

Einstein-de Haas effect:

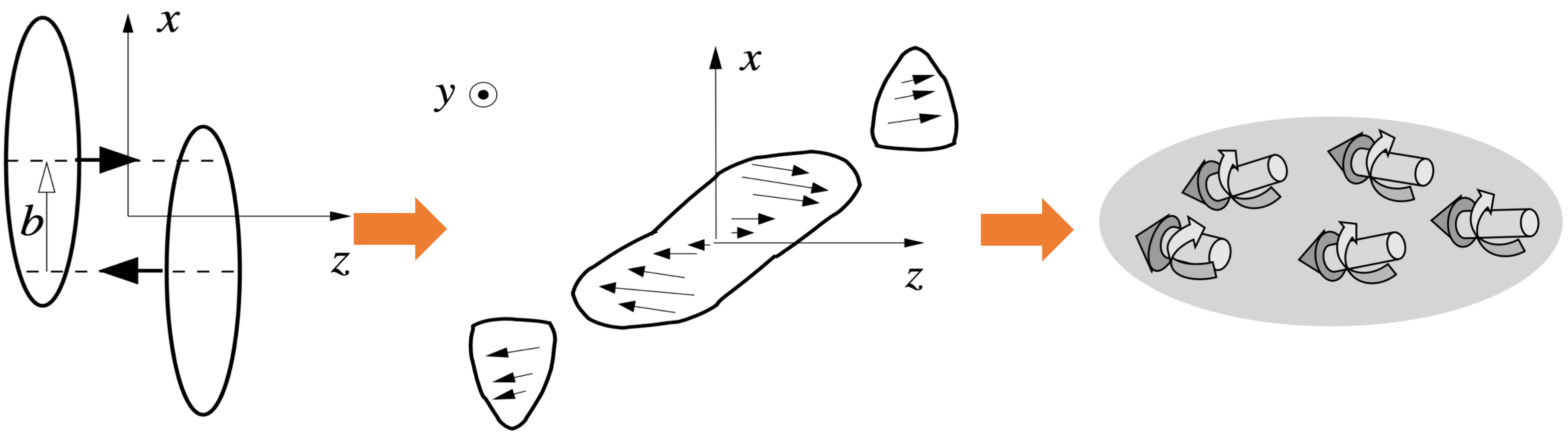
Magnetization $\Longrightarrow$ Rotation

Einstein, de Haas, Experimental proof of the existence of Ampere's molecular currents. Verh Dtsch Phys Ges. (1915) 17:152.



Figures: copy from paper doi: 10.3389/fphy.2015.00054

Global orbital angular momentum in HIC

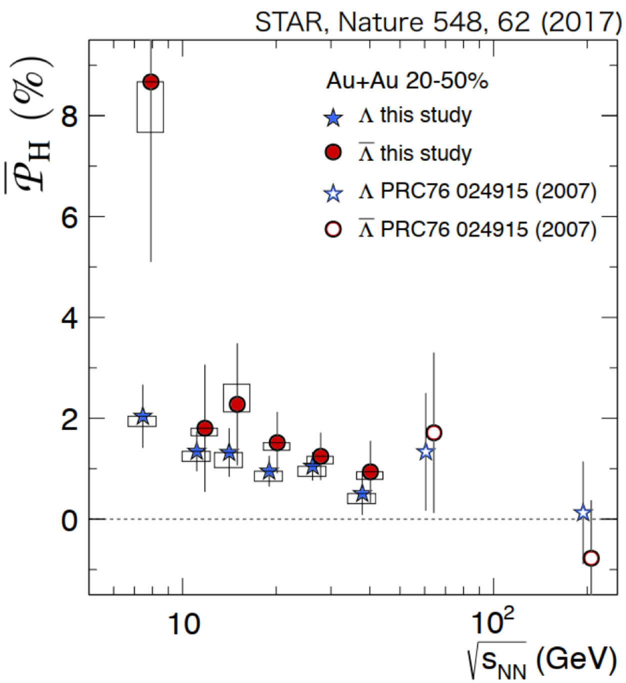


- Global orbital angular momentum leads to the polarizations of Λ hyperons and vector mesons through spin-orbital coupling.

Liang and Wang, PRL 94,102301(2005); PLB 629, 20(2005)

Gao, Chen, Deng, Liang, Wang, Wang, PRC (2008)

Global Polarization of Λ and $\overline{\Lambda}$

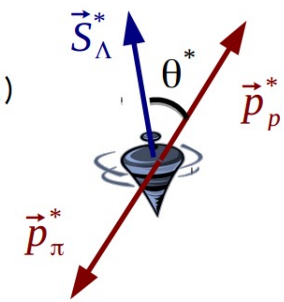


parity-violating decay of hyperons

In case of Λ's decay, daughter proton preferentially decays in the direction of Λ's spin (opposite for anti-Λ)

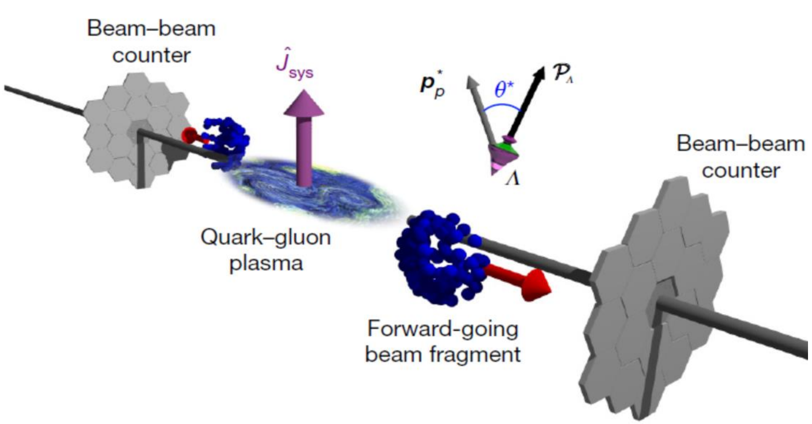
$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha \mathbf{P_\Lambda} \cdot \mathbf{p_p^*})$$

α: Λ decay parameter (=0.642±0.013)
P_Λ: Λ polarization
p_p^*: proton momentum in Λ rest frame



$$\Lambda \rightarrow p + \pi^+$$

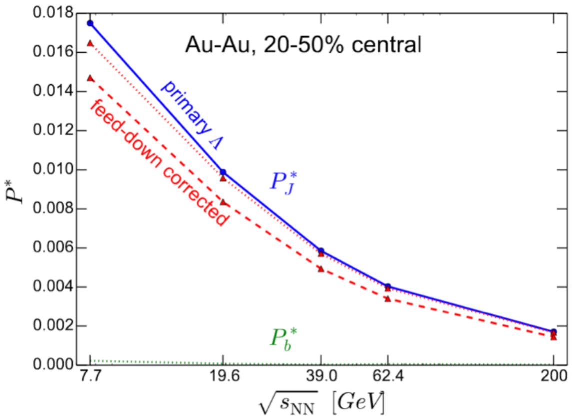
(BR: 63.9%, cτ~7.9 cm)



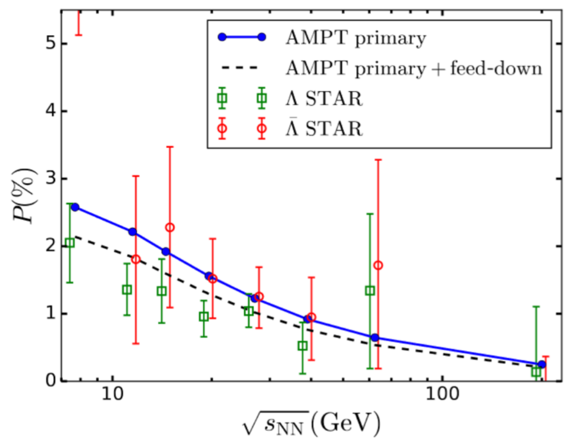
- **The lower energy, the stronger polarization effects.**
- **ω = (9 ± 1)x10²¹/s, greater than previously observed in any system.**

Liang, Wang, PRL (2005)
Betz, Gyulassy, Torrieri, PRC (2007)
Becattini, Piccinini, Rizzo, PRC (2008)
Becattini, Karpenko, Lisa, Upsal, Voloshin, PRC (2017)
Fang, Pang, Q. Wang, X. Wang, PRC (2016)

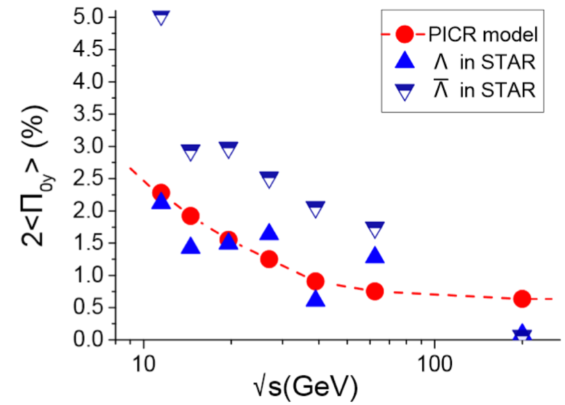
Global Polarization from different models



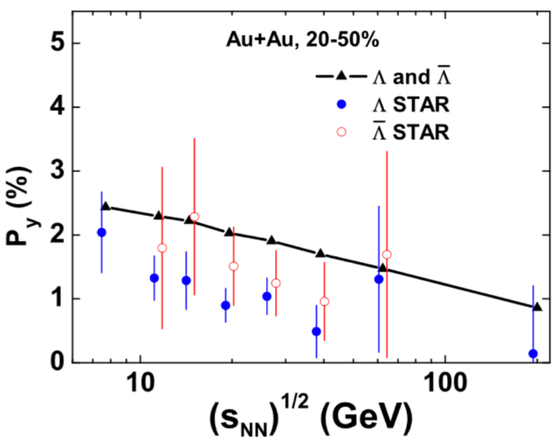
Karpenko, Becattini, EPJC(2017)



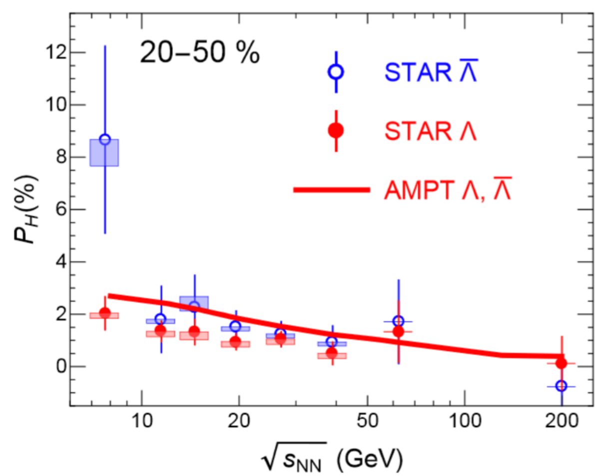
Li, Pang, Wang, Xia PRC(2017)



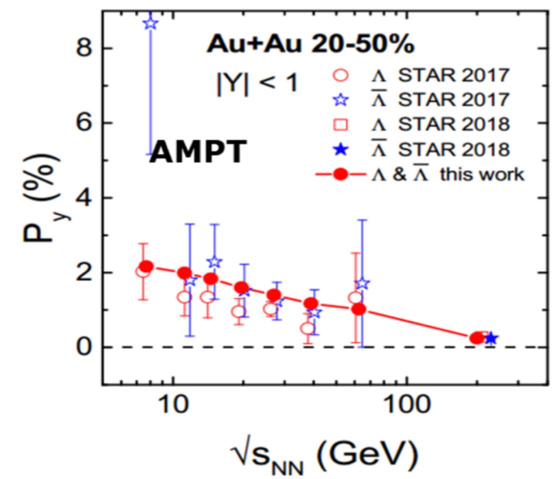
Xie, Wang, Csernai, PRC(2017)



Sun, Ko, PRC(2017)

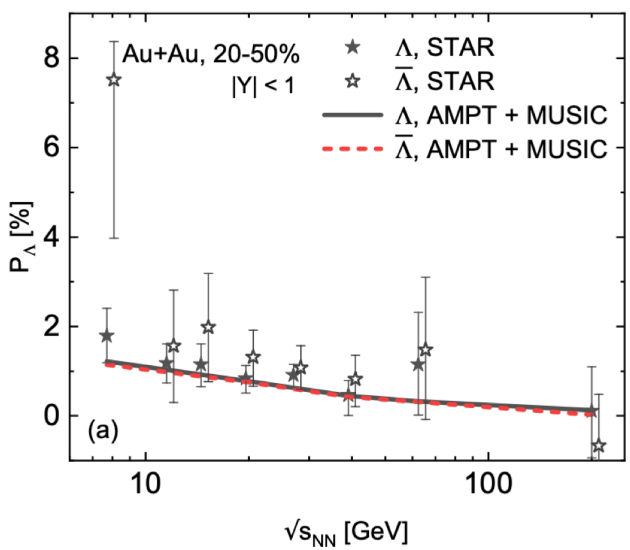


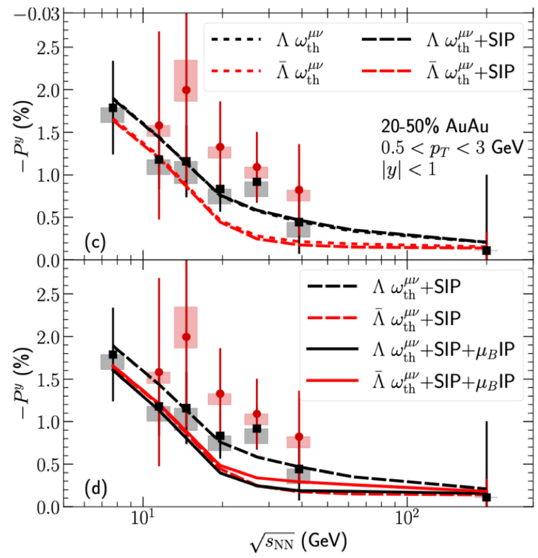
Shi, Li, Liao, PLB(2018)

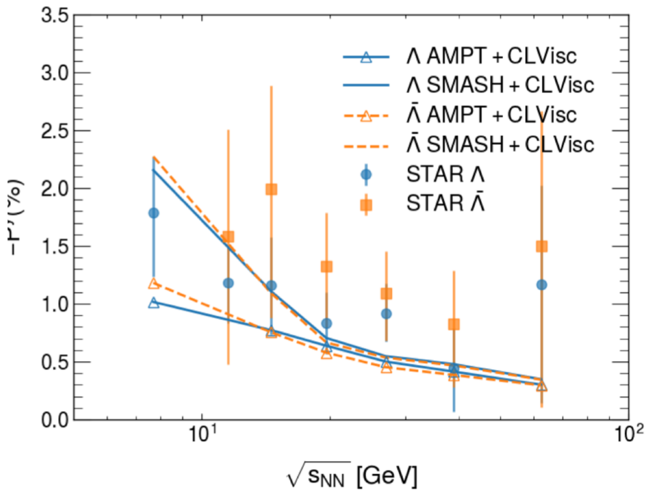


Wei, Deng, Huang, PRC(2019)

Global Polarization from different models





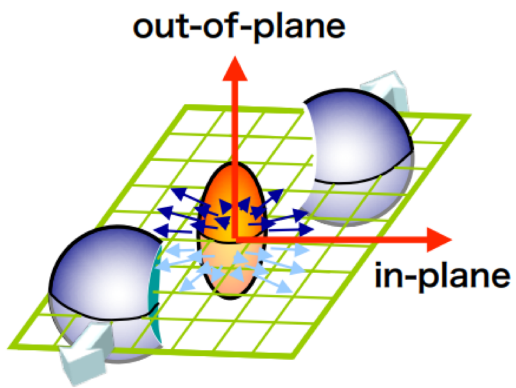


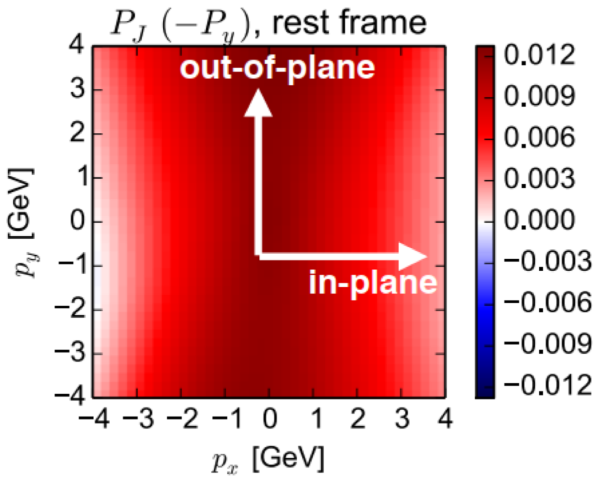
B.C. Fu, K. Xu, X.G. Huang, H.C. Song, Phys. Rev. C 103, 024903 (2021)

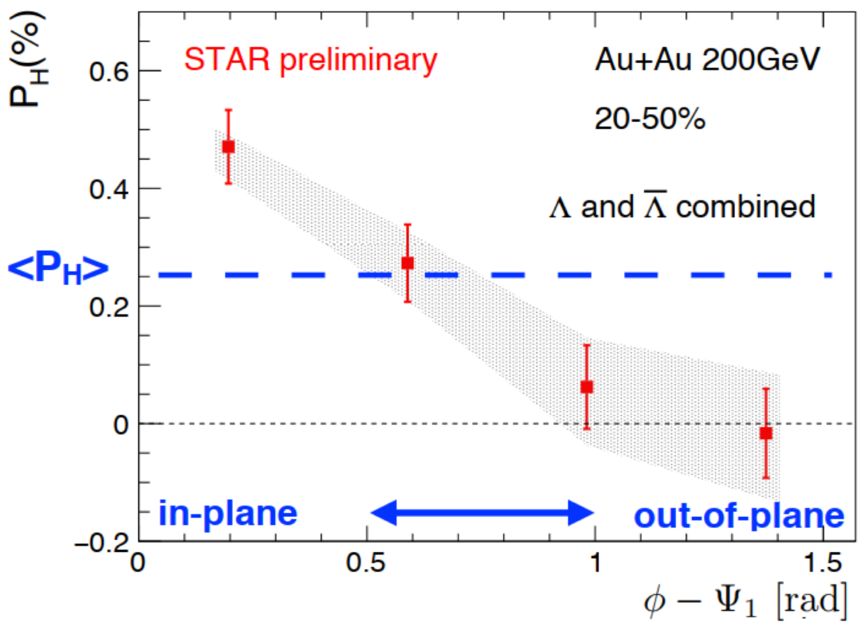
S. Ryu, V. Jupic, C. Shen, arXiv:2106.08125

Y.X. Wu, C. Yi, G.Y. Qin, SP arXiv:2204.02218

Local Polarization







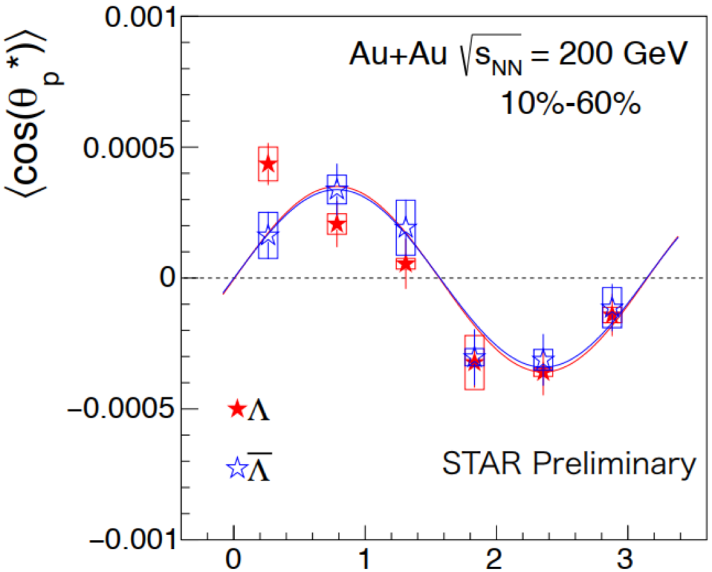
- **Exp data:**

  P_H **in-plane >** P_H **out-of-plane**

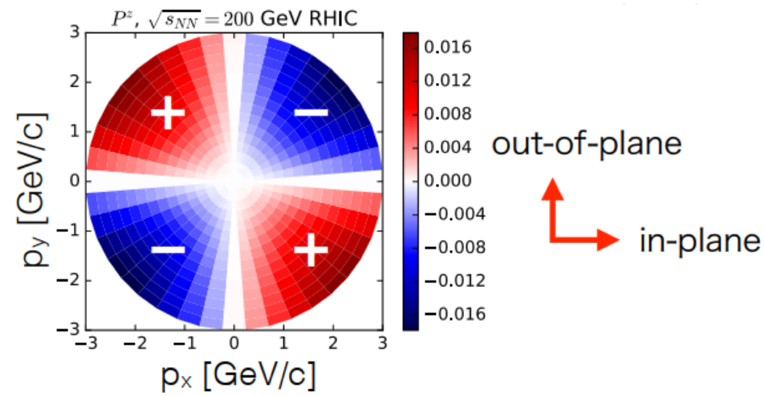
- **Simulations:**

  P_H **out-of-plane >** P_H **in-plane**

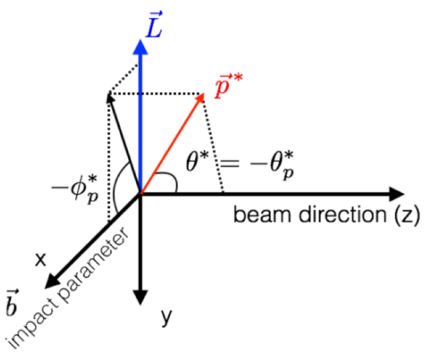
Local Polarization alone beam direction



Sign problem in polarization.



UrQMD : ***Becattini, Karpenko, PRL (2018)***



$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_H \mathbf{P_H} \cdot \mathbf{p}_p^*)$$

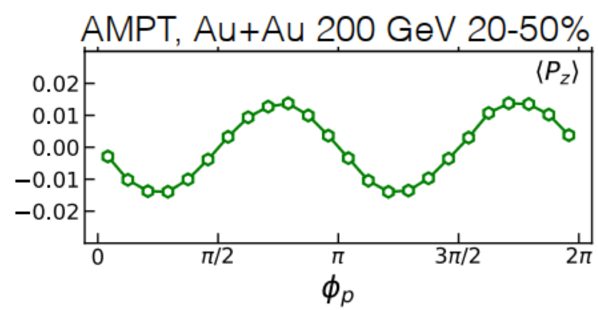
$$\langle \cos\theta_p^* \rangle = \int \frac{dN}{d\Omega^*} \cos\theta_p^* d\Omega^*$$

$$= \alpha_H P_z \langle (\cos\theta_p^*)^2 \rangle$$

$$\therefore \ P_z = \frac{\langle \cos\theta_p^* \rangle}{\alpha_H \langle (\cos\theta_p^*)^2 \rangle}$$

$$= \frac{3\langle \cos\theta_p^* \rangle}{\alpha_H} \quad \text{(if perfect detector)}$$

α_H: hyperon decay parameter
θ_p^*: θ of daughter proton in Λ rest frame

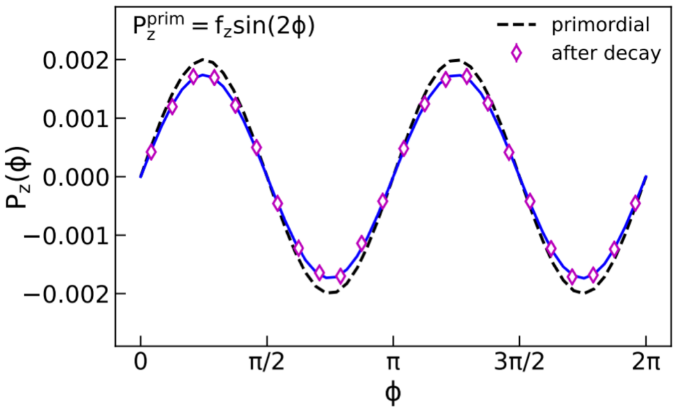


AMPT: ***Xia, Li, Tang, Wang, PRC (2018)***

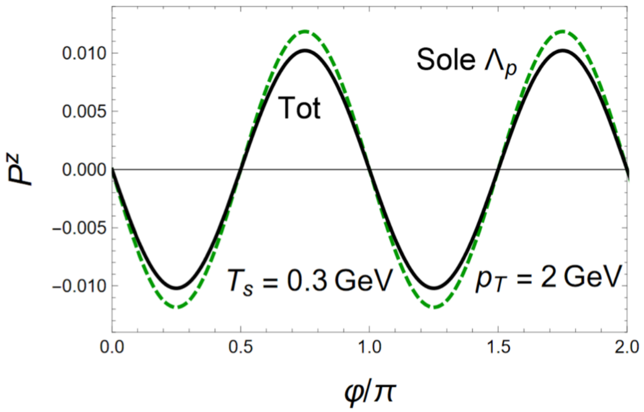
Feed-down effects: NO!

- **Feed-down effects**

  Lambda may come from decays of heavier particles



Xia, Li, Huang, Huang PRC(2019)



Becattini, Cao, Speranza, EPJC(2019)

Different approaches

- **Spin hydrodynamics**

  Florkowski, Friman, Jaiswal, Ryblewski, Speranza (2017-2018);
  Montenegro, Tinti, Torrieri (2017-2019);
  Hattori, Hongo, Huang, Matsuo, Taya PLB(2019) ; arXiv: 2201.12390; arXiv: 2205.08051
  Fukushima, SP, Lecture Note (2020); PLB(2021); Wang, Fang, SP, PRD(2021); Wang, Xie, Fang, SP, PRD (2022)
  S.Y. Li, M.A Stephanov, H.U Yee, arXiv:2011.12318
  D. She, A. Huang, D.F. Hou, J.F Liao, arXiv: 2105.04060

- **Quantum kinetic theory for massive fermions and collisions**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD 100, 056018 (2019)
  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.
  Liu, Mameda, Huang, arXiv:2002.03753.
  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019) ； Z.Y. Wang, arXiv:2205.09334;
  Li ,Yee, PRD100, 056022 (2019)
  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184; Lin, Wang, arXiv:2206.12573
  Fang, SP, Yang, PRD (2022)

- **Other approaches:**
  - Side-jump effect Liu, Sun, Ko PRL(2020)
  - Mesonic mean-field Csernai, Kapusta, Welle, PRC(2019)
  - Using different vorticity Wu, Pang, Huang, Wang, PRR (2019)

Quantum kinetic theory and its recent development

Kinetic theory

- **Assumptions:**

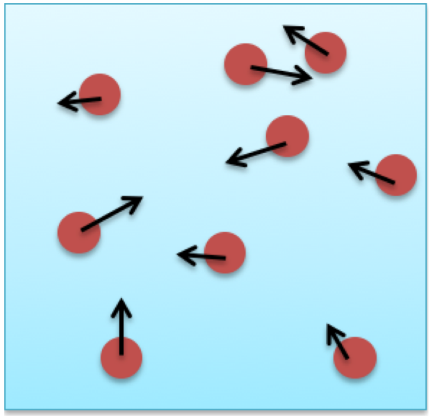
Mean free path >> collision length scaling

- **"distribution function" f(x,p,t)**

how many particles in a small

volume of phase space (x+dx, p+dp)

e.g. Fermi-Dirac distribution function



- **Ordinary kinetic theory: Boltzmann equation**

Dynamical evolution equation for f(x,p,t)

Ordinary Boltzmann equation

Particle's velocity:

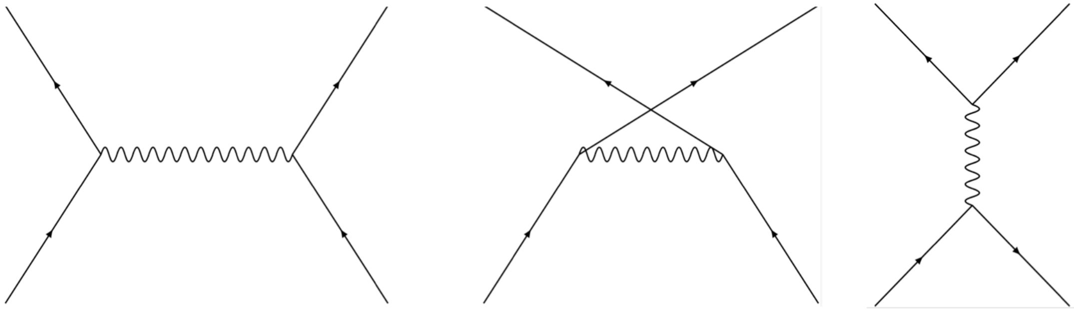
$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}},$$

ε : Particle's energy

Lorentz force:

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B},$$

$$\partial_t f + \dot{\mathbf{x}} \cdot \nabla_x f + \dot{\mathbf{p}} \cdot \nabla_p f = C[f],$$



Collison term:

Chiral kinetic theory (massless fermions)

- Hamiltonian formulism, effective theory

  Son, Yamamoto, PRL, (2012); PRD (2013)

- Path integration

  Stephanov, Yin, PRL (2012);

  Chen, Son, Stephanov, Yee, Yin, PRL, (2014);

  J.W. Chen, J.Y. Pang, SP, Q. Wang, PRD (2014)

- Wigner function (Quantum field theory)
  - hydrodynamics, equilibrium

    Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012) ; J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);
  - out-of-equilibrium, quantum field theory

    Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)
  - Other studies

    A.P. Huang, S.Z. Su, Y. Jiang, J.F. Liao, P.F. Zhuang, arXiv:1801.03640

- World-line formulism

  N. Muller, R, Venugopalan PRD 2017

  Also see recent review:

  Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

  Hidaka, SP, D.L. Yang, Q. Wang, arXiv:2201.07644

Wigner function (I)

- **Wigner operator**

$$\hat{W}_{\alpha\beta} = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta(x_+) U(x_+, x_-) \psi_\alpha(x_-), \quad x = x_+ + x_- \quad y = x_+ - x_-$$

Gauge link $U(x_+, x_-) \equiv e^{-iQ\int_{x_-}^{x_+} dz^\mu A_\mu(z)},$

- **Wigner function:**

$$W(x,p) = \left\langle : \hat{W}(x,p) : \right\rangle$$

W operator in thermal ensemble average and normal ordering of the operators

- **Physical meaning: "density matrix" in quantum field theory**

Vasak, Gyulassy, Elze, Ann. Phys. (N.Y.) 173, 462 (1987);

Elze, Heinz,Phys.Rep.183,81(1989).

Wigner function (II)

- By using the Dirac equation, one can get the master equations for Wigner function

$$\gamma_\mu\left(p^\mu + \frac{i}{2}\nabla^\mu\right)W(x,p) = 0, \qquad \nabla^\mu \equiv \partial_x^\mu - QF^\mu{}_\nu\partial_p^\nu$$

- Matrix decomposition

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right],$$

Charge current

$$\mathscr{V}^\mu = \int\frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <: \bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right) :>$$

Chiral current

$$\mathscr{A}_\mu = \int\frac{d^4y}{(2\pi)^4}e^{-ip\cdot y} <: \bar{\psi}_\beta\left(x+\frac{1}{2}y\right)\gamma^\mu\gamma^5 U\left(x+\frac{1}{2}y, x-\frac{1}{2}y\right)\psi_\alpha\left(x-\frac{1}{2}y\right) :>$$

Solving Wigner function in hbar expansion

- **We consider the hbar (gradient) expansion,**

$$\mathscr{J}^s_\mu(x,p) = \frac{1}{2}[\mathscr{V}_\mu(x,p) + s\mathscr{A}_\mu(x,p)], \qquad s = \pm$$

$$\mathscr{J}^s_\mu(x,p) = \mathscr{J}^s_{\mu,(0)}(x,p) + \mathscr{J}^s_{\mu,(1)}(x,p) + \cdots ,$$

- **Leading order is the classical currents. We introduce the initial distribution function f(x,p) as input.**

$$\mathscr{J}^\rho_{(0)s}(x,p) = p^\rho f_s \delta(p^2),$$

- **We can then solve the next-to-leading order.**

$$\mathscr{J}^\rho_{(1)s}(x,p) = -\frac{s}{2}\tilde{\Omega}^{\rho\lambda} p_\lambda \frac{df_s}{dp_0}\delta(p^2) - \frac{s}{p^2} e\tilde{F}^{\rho\lambda} p_\lambda f_s \delta(p^2).$$

$$\Omega_{\nu\sigma} = \tfrac{1}{2}(\partial_\nu u_\sigma - \partial_\sigma u_\nu), \text{ and } \Omega^{\mu\nu} = -\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}\tilde{\Omega}_{\rho\sigma}.$$

Connection to Currents and chiral anomaly

- **Integral over momentum p, we get CME and other quantum transport effects**

$$j_s^\mu = \int d^4p\, \mathscr{J}_s^\mu = n_s u^\mu + \xi_{B,s} B^\mu + \xi_s \omega^\mu,$$

Charge current

$$j^\mu = \sum_{s=\pm} j_s^\mu = n u^\mu + \xi_B B^\mu + \xi \omega^\mu,$$

Chiral current

$$j_5^\mu = \sum_{s=\pm} s j_s^\mu = n_5 u^\mu + \xi_{B5} B^\mu + \xi_5 \omega^\mu,$$

$$\xi_B = \frac{e}{2\pi^2}\mu_5, \quad \xi = \frac{1}{\pi^2}\mu\mu_5, \quad \xi_{B5} = \frac{e}{2\pi^2}\mu, \quad \xi_5 = \frac{1}{6}T^2 + \frac{1}{2\pi^2}\left(\mu^2 + \mu_5^2\right)$$

- **We also reproduce the chiral anomaly from the kinetic theory.**

$$\partial_\mu j^\mu = 0, \qquad \partial_\mu j_5^\mu = -\frac{e^2}{2\pi^2} E \cdot B.$$

Gao, Liang, Pu, Q. Wang, X.N. Wang, PRL 109, 232301 (2012)

Derivation of chiral kinetic theory

- There is a constrain equation.

$$\nabla^{\mu} \mathscr{J}^{s}_{\mu}(x,p) \;=\; 0,$$

- We can insert our results into this equation and get the constraint equation for distribution function.
- Then, we need to integral over p0 to get 3-dim form.

J.W. Chen, SP, Q. Wang, X.N. Wang, PRL (2013);

Y. Hidaka, SP, D.L. Yang, PRD(RC) (2017)

Review: Gao, Liang, Wang, Int.J.Mod.Phys A 36 (2021), 2130001

Hidaka, SP, D.L. Yang, Q. Wang, 2201.07644

Chiral kinetic equation

- Chiral kinetic theory is a useful tool to study CME.

$$\sqrt{G}\partial_t f + \sqrt{G}\dot{\mathbf{x}} \cdot \nabla_x f + \sqrt{G}\dot{\mathbf{p}} \cdot \nabla_p f = C[f].$$

- Particle's effective velocity:

$$\sqrt{G}\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \left(\frac{\partial \varepsilon}{\partial \mathbf{p}} \cdot \mathbf{\Omega} \right) \mathbf{B} + \hbar \mathbf{E} \times \mathbf{\Omega},$$

- Effective force:

$$\sqrt{G}\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar (\mathbf{E} \cdot \mathbf{B}) \mathbf{\Omega},$$

- Berry curvature

$$\mathbf{\Omega} = \frac{\mathbf{p}}{2|\mathbf{p}|^3}, \qquad \sqrt{G} = 1 + \hbar \mathbf{B} \cdot \mathbf{\Omega},$$

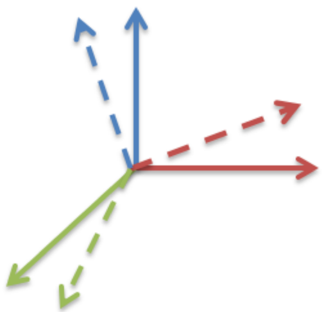
Side-jump effects and Lorentz symmetry

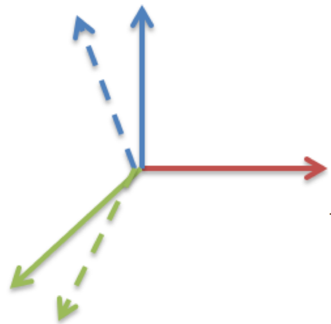
- The subgroup for Lorentz group for massless fermions and massive fermions are different.

Massive particles:
(Rest frame)

$$p^\mu = (m, 0, 0, 0)$$

Subgroup: SO(3)





Massless particles:
(No rest frame)

$$p^\mu = (|p_z|, 0, 0, p_z)$$

Subgroup: ISO(2)

- From quantum field theory, the distribution function is no longer a scalar.

$$f'(x', p', t') = f(x', p', t') + \hbar N^\mu(\partial_\mu^x + F_{\nu\mu}\partial_p^\nu) f,$$

Infinitesimal Lorentz Transform

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$

$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

Chen, Son, Stephanov, PRL, (2015);
Y. Hidaka, SP, D.L. Yang, PRD (2016)

Quantum kinetic theory (massive fermions)

- **Collision term with quantum corrections**

  Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD (2019); PRL (2021)

  Hattori, Hidaka, Yang, PRD100, 096011 (2019); Yang, Hattori, Hidaka, arXiv: 2002.02612.

  Liu, Mameda, Huang, arXiv:2002.03753.

  Wang, Guo, Shi, Zhuang, PRD100, 014015 (2019); Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

  Li ,Yee, PRD100, 056022 (2019)

  Hou, Lin, arXiv: 2008.03862; Lin, arXiv: 2109.00184

  Fang, SP, Yang, PRD (2022)

  Z.Y. Wang, arXiv:2205.09334; Lin, Wang, arXiv:2206.12573

Recent reviews:

Gao, Ma, SP, Wang, NST 31 (2020) 9, 90

Gao, Liang, Wang, IJMPA 36 (2021), 2130001

Hidaka, SP, Yang, Wang, arXiv:2201.07644

Collisional kernel

- **An example for collision kernel of NJL type interactions:**

Eq. for Particle distribution function

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[f^{(1)}(x,p)\right] = -\frac{1}{\pi\hbar}\int_0^\infty dp_0\, \mathrm{Im}\mathrm{Tr}\left(I^{(2)}_{\mathrm{coll}}\right) - \frac{1}{2\pi\hbar m}\mathrm{Re}\,\mathrm{Tr}\left(\gamma\cdot\partial_x I^{(1)}_{\mathrm{coll}}\right)$$

$$\equiv \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right) + \mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)\ ,$$

Eq. for Spin distribution function

$$\frac{1}{E_p} p\cdot\partial_x \mathrm{tr}\left[n_j^{(+)\mu}\tau_j^T f^{(1)}(x,p)\right] = \frac{1}{2\pi\hbar m}\int_0^\infty dp_0\left[\epsilon^{\mu\nu\alpha\beta}p_\nu \mathrm{Im}\mathrm{Tr}\left(\sigma_{\alpha\beta}I^{(2)}_{\mathrm{coll}}\right) + \mathrm{Re}\,\mathrm{Tr}\left(\gamma^5\partial_x^\mu I^{(1)}_{\mathrm{coll}}\right)\right]$$

$$\equiv \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right) + \mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)\ .$$

$\mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right)$ $\quad$ $\mathscr{C}_{\mathrm{scalar}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right)$ $\quad$ $\mathscr{C}_{\mathrm{scalar}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)$ $\quad$ $\mathscr{C}_{\mathrm{scalar}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$

$\mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll,\,qc}}\right)$ $\quad$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(\Delta I^{(1)}_{\mathrm{coll},\,\nabla}\right)$ $\quad$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(\partial_x I^{(1)}_{\mathrm{coll}}\right)$ $\quad$ $\mathscr{C}^\mu_{\mathrm{pol}}\left(I^{(0)}_{\mathrm{coll,PB}}\right)$

Perturbative Correction to Ordinary terms

Non-local terms related to the space derivatives may be the key to describe the spin-orbital transformation.

Sheng, Weickgenannt, Speranza, Rischke, Wang PRD (2021)

Challenge the collisional kernel

- **Theory:**
  - Collison kernel needs to be further simplified.
- **Simulations: it is changeling to simulate the QKT:**
  - Collison kernel is high dimensional integrals.
  - One needs to consider the non-local terms.
- **Usually, to solve kinetic theory, one can use the cross section + MC sampling instead of integrating the collision kernel. But, it would fail in quantum kinetic theory with collisions.**
- **We may need to face the high dimensional integrations in collision kernel.**

Theory: Collisions for gauge fields and spin polarization (I)

- **We have derived collision kernel for QED in HTL approximation.**

Eq. for Particle distribution function

$$(p\cdot\partial)f_V^<(x,p)=\mathcal{C}_V^{\mathrm{HTL}}[f_V]+\mathcal{O}(\hbar^2),$$

Eq. for Spin distribution function

$$(p\cdot\partial)f_A^<(x,p)+\hbar\partial_\mu S_{(u)}^{\mu\nu}\partial_\nu f_V^<(x,p)=\mathcal{C}_A^{\mathrm{HTL}}[f_V,f_A]+\mathcal{O}(\hbar^2),$$

- **For the first time, the real QED type collision kernel for axial part:**

$$\begin{aligned}\mathcal{C}_A[f_V,f_A]=&-\frac{e^4\delta(p^2)}{8\pi^2|\boldsymbol{p}|}\ln\frac{T}{m_D}\left\{\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|F(p)f_A^<(p)+\frac{\pi^2}{3\beta^2}|\boldsymbol{p}|^2F(p)[(\hat{p}_\perp\cdot\partial_{p_\perp})-\frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})]f_A^<(p)\right.\\&-\frac{2\pi^2}{3\beta^2}|\boldsymbol{p}|^2f_A^<(p)(\hat{p}_\perp\cdot\partial_{p_\perp})f_V^<(p)+\hbar F(p)|\boldsymbol{p}|H_{3,\alpha}\partial^\alpha_{p_\perp}f_V^<(p)\\&-\hbar\frac{\pi^2}{12\beta^2}F(p)|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\partial_{p_\perp,\rho}\partial_\alpha f_V^<(p)+\hbar\frac{\pi^2}{6\beta^3}\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\rho}u_\beta\partial_{p_\perp,\nu}\partial_\alpha f_V^<(p)\\&+\hbar\frac{\pi^2}{6\beta^2}\epsilon^{\mu\xi\lambda\kappa}p_\lambda u_\kappa\partial_\xi f_V^<(p)\partial_{p_\perp,\mu}f_V^<(p)\\&\left.-\hbar\frac{\pi^2}{12\beta^3}|\boldsymbol{p}|\epsilon^{\rho\alpha\nu\beta}\hat{p}_{\perp,\nu}u_\beta\hat{p}_{\perp,(\gamma}g_{\lambda)\rho}\hat{p}_{\perp,\lambda}\partial^\lambda_{p_\perp}\partial^\gamma_{p_\perp}\partial_\alpha f_V^<(p)\right\}+\mathcal{O}(\hbar^2).\end{aligned}\quad(65)$$

S. Fang, SP, D.L. Yang, PRD (2022), arXiv: 2204.11519

Theory: Collisions for gauge fields and spin polarization (II)

- **We have proved that dynamical spin polarization for a probe is much slower than its thermalization.**

$$\frac{\text{Spin polarization time}}{\text{Thermalization time}} \approx \frac{\Gamma_A(p)}{\Gamma_V(p)} \approx \frac{\hbar H_{3,\alpha}}{T^2|\boldsymbol{p}|} \sim \mathcal{O}\left(\frac{\partial}{|\boldsymbol{p}|}\right),$$

Also see Wang, Guo, Zhuang, EPJC (2021);Wang, Zhuang, arXiv:2105.00915

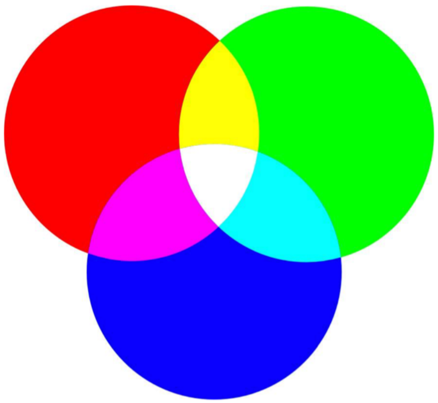
- **We also derive the Boltzmann equation for spin evolution:**

$$(p\cdot\partial)f_A^{<}(x,p)+\hbar\partial_\mu S^{\mu\nu}_{(u)}\partial_\nu f^{<}_{V,leq}(x,p)=\mathcal{C}_A^{\mathrm{HTL}}[f_{V,leq},f_A]+\mathcal{O}(\hbar^2),$$

$$\begin{aligned}C_A^{\mathrm{HTL}}[f_{V,leq},f_A] = &-\frac{e^4}{16\pi^3}\frac{\pi^2}{3\beta^2}\ln\frac{T}{m_D}\Big\{2\left(f^{>}_{V,leq}(p)-f^{<}_{V,leq}(p)\right)+2|\boldsymbol{p}|\beta f^{<}_{V,leq}(p)f^{>}_{V,leq}(p)\\ &+|\boldsymbol{p}|\left[\left(f^{>}_{V,leq}(p)-f^{<}_{V,leq}(p)\right)\hat{p}_\perp\cdot\partial_{p_\perp}-\frac{1}{\beta}(\partial_{p_\perp}\cdot\partial_{p_\perp})\right]\Big\}f_A^{<}(p)\\ &+\hbar\frac{e^4}{16\pi^3|\boldsymbol{p}|}\frac{\pi^2}{3\beta^3}\ln\frac{T}{m_D}S^{\alpha\nu}_{(u)}\Omega_{\alpha\nu}f^{<}_{V,leq}(p)f^{>}_{V,leq}(p)+\mathcal{O}(\hbar^2),\end{aligned}$$

S. Fang, SP, D.L. Yang, PRD (2022), arXiv: 2204.11519

Simulations: Relativistic Boltzmann equations on GPU



Relativistic Boltzmann equations on GPU

RBG

Basic, but nontrivial.

- **We introduce a new numerical framework to derive full solutions of a relativistic BE on GPUs.**
  - **Full 2->2 collisional term:**

    high dimensional integrals.
  - **High performance:**

    space 10x10x10,

    momentum 30x30x30,

    Time steps: 10^4-10^6,

    on one Nvidia Tesla V100 card costs a few days!
  - **Particle number is strictly conserved.**
  - **Dynamical Debye mass**

J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

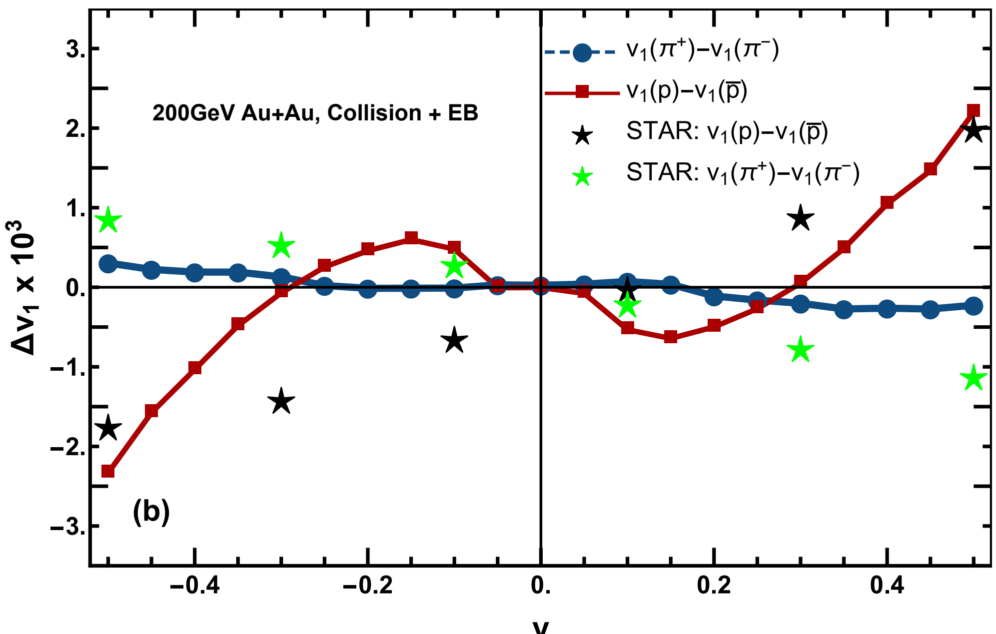
Simulations: RBG-Maxwell equations and v1, dv1 problem

- We solve the Boltzmann equation coupled to Maxwell equations.

$$[p^\mu \partial_\mu + Q_a p_\mu F^{\mu\nu} \partial_{p^\nu}] f_a(t, \mathbf{x}, \mathbf{p}) = \mathcal{C}[f_a],$$
$$\partial_\mu F^{\mu\nu} = j^\nu_{\text{ext}} + j^\nu_{\text{med}},$$ (Spectators + Participant)

QCD 2->2 scattering full collision term

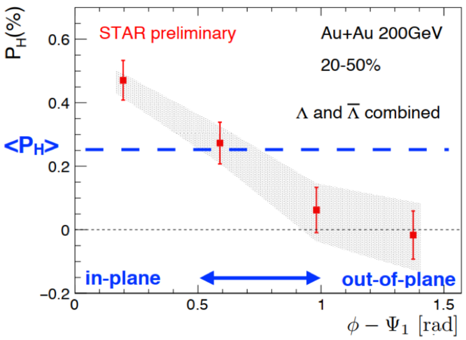
- A good example to show the effects from EB fields. Qualitative consistent with exp. Could help us to understand the dv1 difference for pions and protons.



J.J. Zhang, X.L. Sheng, SP, et. al, arXiv:2201.06171, accepted by PRR

Applications to spin polarization

Modified Cooper-Frye formula



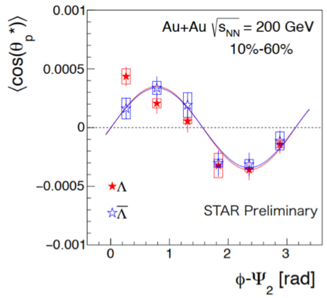
Thermal vorticity

$$\omega^{\rm th}_{\rho\sigma} = \frac{1}{2}\left[\partial_\rho\left(\frac{u_\sigma}{T}\right) - \partial_\sigma\left(\frac{u_\rho}{T}\right)\right]$$

Distribution function: f_0

$$\mathcal{S}^\mu(p) = \frac{1}{8m_\Lambda}\epsilon^{\mu\nu\rho\sigma}p_\nu\frac{\int d\Sigma_\lambda p^\lambda f_0(1-f_0)\omega^{\rm th}_{\rho\sigma}}{\int d\Sigma_\lambda p^\lambda f_0}$$

Freezeout surface



Karpenko, F. Becattini, Eur. Phys. J. C 77 (2017) 213
R.-H. Fang, L.-G. Pang, Q. Wang, X.-N. Wang, Phys. Rev. C94, 024904 (2016)

Polarization and axial current

- **Recalling the original equations**

$$\mathcal{S}^{\mu}(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p,X)}{2m_{\Lambda}\int d\Sigma \cdot \mathcal{N}(p,X)},$$

- **For massless fermions, the left and right handed currents read**

$$\mathcal{J}_{\lambda}^{\mu}(p,X) \;=\; 2\pi \mathrm{sign}(u\cdot p)\left\{p^{\mu} + \lambda\frac{\hbar}{2}\delta(p^2)\left[u^{\mu}(p\cdot\omega) - \omega^{\mu}(u\cdot p)\right.\right.$$
$$\left.\left. -2S_{(u)}^{\mu\nu}\tilde{E}_{\nu}\right]\partial_{u\cdot p} + \lambda\frac{\hbar}{4}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\partial_{\nu}^{p}\delta(p^2)\right\} f_{\lambda}^{(0)},$$

$$S_{(u)}^{\mu\nu} \;=\; \epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}/(2u\cdot p),$$

$\lambda = \pm$

+: right

-: left

$$\tilde{E}_{\nu} \;=\; E_{\nu} + T\partial_{\nu}\frac{\mu_{\lambda}}{T} + \frac{(u\cdot p)}{T}\partial_{\nu}T - p^{\sigma}[\partial_{<\sigma}u_{\nu>} + \frac{1}{3}\Delta_{\sigma\nu}(\partial\cdot u) + u_{\nu}Du_{\sigma}].$$

$$f_{\lambda}^{(0)} = 1/(e^{(u\cdot p - \mu_{\lambda})/T} + 1),$$

Y. Hidaka, SP, and D.L. Yang, Phys. Rev. D97, 016004 (2018)

Polarization induced by different sources

- **Axial currents can be decomposed as**

$$\mathcal{J}_5^\mu = \mathcal{J}^\mu_{\rm thermal} + \mathcal{J}^\mu_{\rm shear} + \mathcal{J}^\mu_{\rm accT} + \mathcal{J}^\mu_{\rm chemical} + \mathcal{J}^\mu_{\rm EB},$$

where they are related to:

Thermal vorticity

$$\mathcal{J}^\mu_{\rm thermal} = a\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$

Shear viscous tensor

$$\mathcal{J}^\mu_{\rm shear} = -a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta p^\sigma\partial_{<\sigma}u_{\nu>}$$

Fluid acceleration

$$\mathcal{J}^\mu_{\rm accT} = -a\frac{1}{2T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha\left(Du_\beta - \frac{1}{T}\partial_\beta T\right).$$

Gradient of chemical potential

$$\mathcal{J}^\mu_{\rm chemical} = a\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta\partial_\nu\frac{\mu}{T},$$

Electromagnetic fields

$$\mathcal{J}^\mu_{\rm EB} = a\frac{1}{(u\cdot p)T}\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta E_\nu + a\frac{B^\mu}{T},$$

Y. Hidaka, SP, D.L. Yang, PRD97, 016004 (2018); C. Yi, SP, D.L. Yang, PRC 2021

Out-of-equilibrium corrections

- **Polarization vector**

$$\mathcal{P}^z(p) = \int_{-1}^{+1} dY \mathcal{S}^z(p),$$
$$\mathcal{P}^y(p) = \int_{-1}^{+1} dY \mathcal{S}^y(p),$$

- **Polarization induced by thermal vorticity, shear viscous tensor and residual part of fluid acceleration**

$$\mathcal{S}^\mu_{\rm thermal}(\mathbf{p}) = \frac{\hbar}{8m_\Lambda N}\int d\Sigma^\sigma p_\sigma f_V^{(0)}(1-f_V^{(0)})\epsilon^{\mu\nu\alpha\beta}p_\nu\partial_\alpha\frac{u_\beta}{T},$$
$$\mathcal{S}^\mu_{\rm shear}(\mathbf{p}) = -\frac{\hbar}{4m_\Lambda N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{\epsilon^{\mu\nu\alpha\beta}p_\alpha u_\beta}{(u\cdot p)T}\frac{1}{2}\{p^\sigma(\partial_\sigma u_\nu+\partial_\nu u_\sigma)-Du_\nu\}$$
$$\mathcal{S}^\mu_{\rm accT}(\mathbf{p}) = -\frac{\hbar}{8m_\Lambda N}\int d\Sigma\cdot p f_V^{(0)}(1-f_V^{(0)})\frac{1}{T}\epsilon^{\mu\nu\alpha\beta}p_\nu u_\alpha(Du_\beta-\frac{1}{T}\partial_\beta T),$$

C. Yi, SP, D.L. Yang, PRC 2021

Shear induced polarization

- Shear induced polarization draws some attentions.
- Shear induced Polarization from massless fermions (Theory):

  Y. Hidaka, SP, and D.L. Yang, PRD97, 016004 (2018);

- Shear induced Polarization from massive fermions:
  - Theory:

    S. Y. F. Liu, Y. Yin, 2103.09200

    F. Becattini, M. Buzzegoli, A. Palermo, 2103.10917

  - Hydrodynamic simulations:

    B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, 2103.10403

    F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, 2103.14621

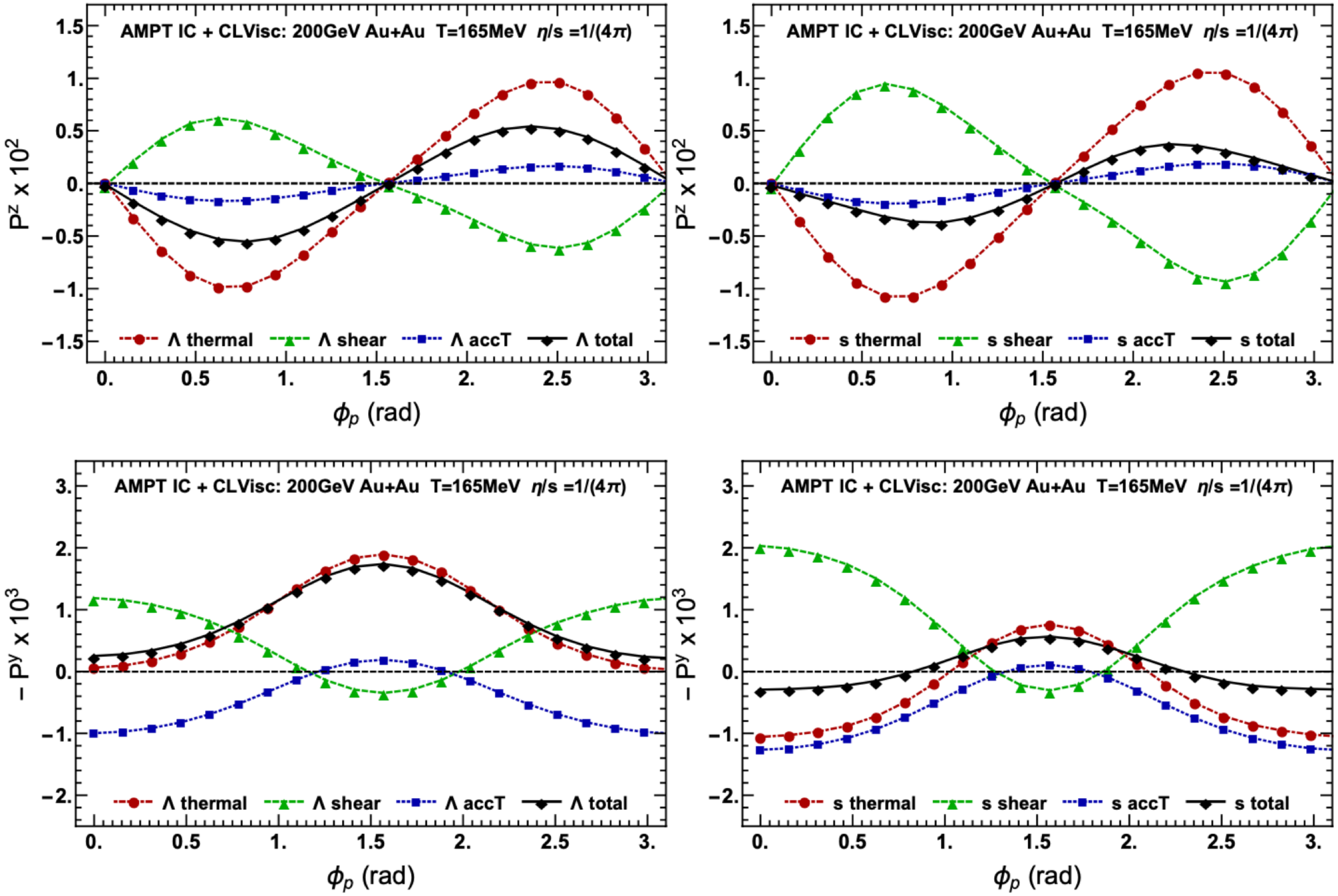
    C. Yi, SP, D.L. Yang, PRC 2021

- Global polarization induced by shear and gradient of chemical potential

  S. Ryu, V. Jupic, C. Shen, arXiv:2106.08125

Main result for shear induced polarization

- **Shear induced polarization always give a "correct" sign.**
- **Total local polarization is sensitive to mass of s quark, EoS, freeze out temperature and eta / s.**



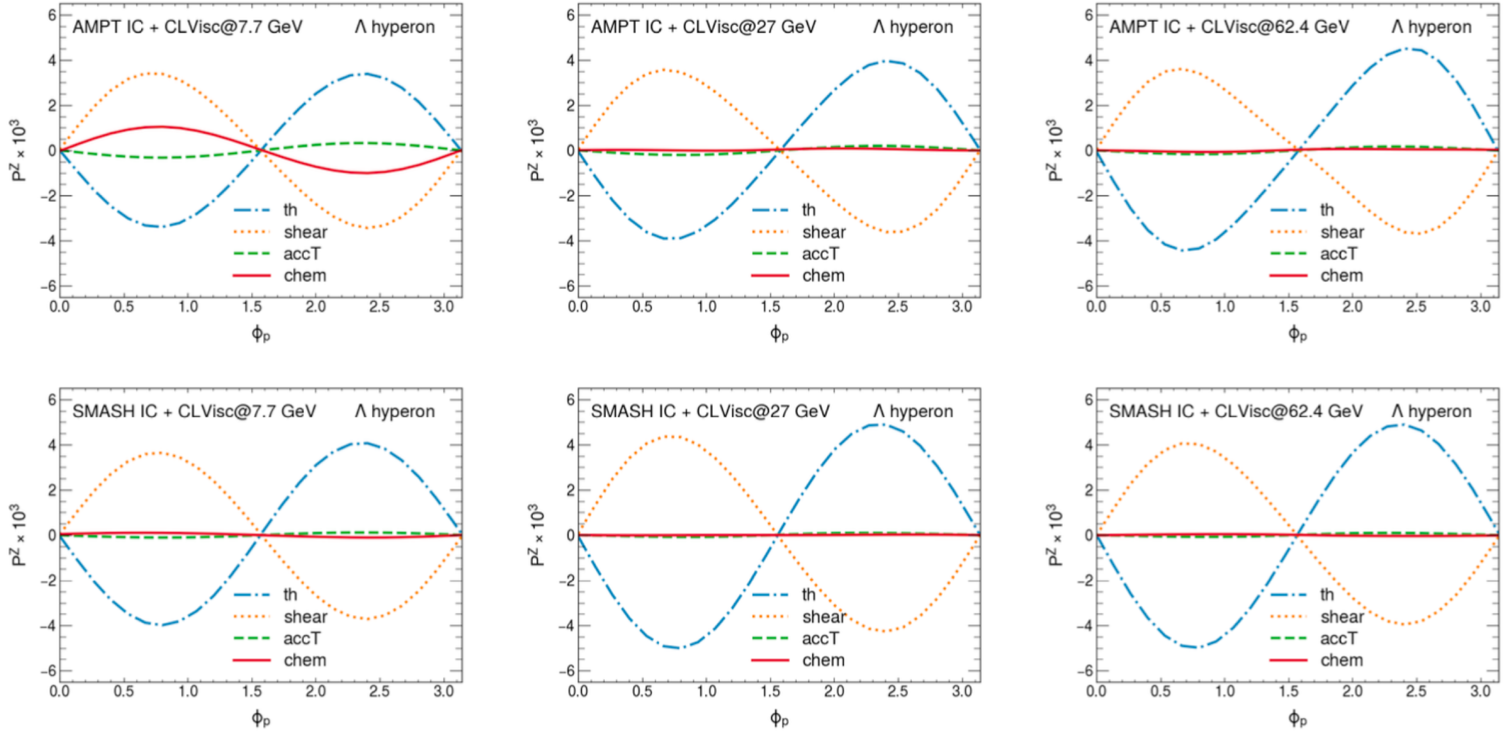
C. Yi, SP, D.L. Yang, PRC 2021

Simulations for spin Hall effects

- "Spin Hall effect": polarization induced by the gradient of chemical potential

$$\mathcal{S}^{\mu}_{\text{chemical}}(\mathbf{p}) = 2\int d\Sigma^{\sigma}F_{\sigma}\frac{1}{(u\cdot p)}\epsilon^{\mu\nu\alpha\beta}p_{\alpha}u_{\beta}\partial_{\nu}\frac{\mu}{T},$$

- We study the polarization at RHIC beam energy scan energies via the (3+1)-dimensional CLVisc hydrodynamics model with AMPT and SMASH initial conditions. The results depend on initial condition and baryon diffusion.



X.Y. Wu, C. Yi, G.Y. Qin, SP, arXiv:2204.02218, accepted by PRC

Other possible corrections

- Electromagnetic fields make the differences between Λ and Λ_bar
- Collisional effects to modified Cooper-Frye formula
  - Fang, SP, Yang, PRD (2022)
  - Z.Y. Wang, arXiv:2205.09334
  - Lin, Wang, arXiv:2206.12573
- Corrections from spin potential to modified Cooper-Frye formula
  - Liu, Huang, arXiv: 2109.15301
- Hadronization
- Hadronic interaction after chemical freezeout

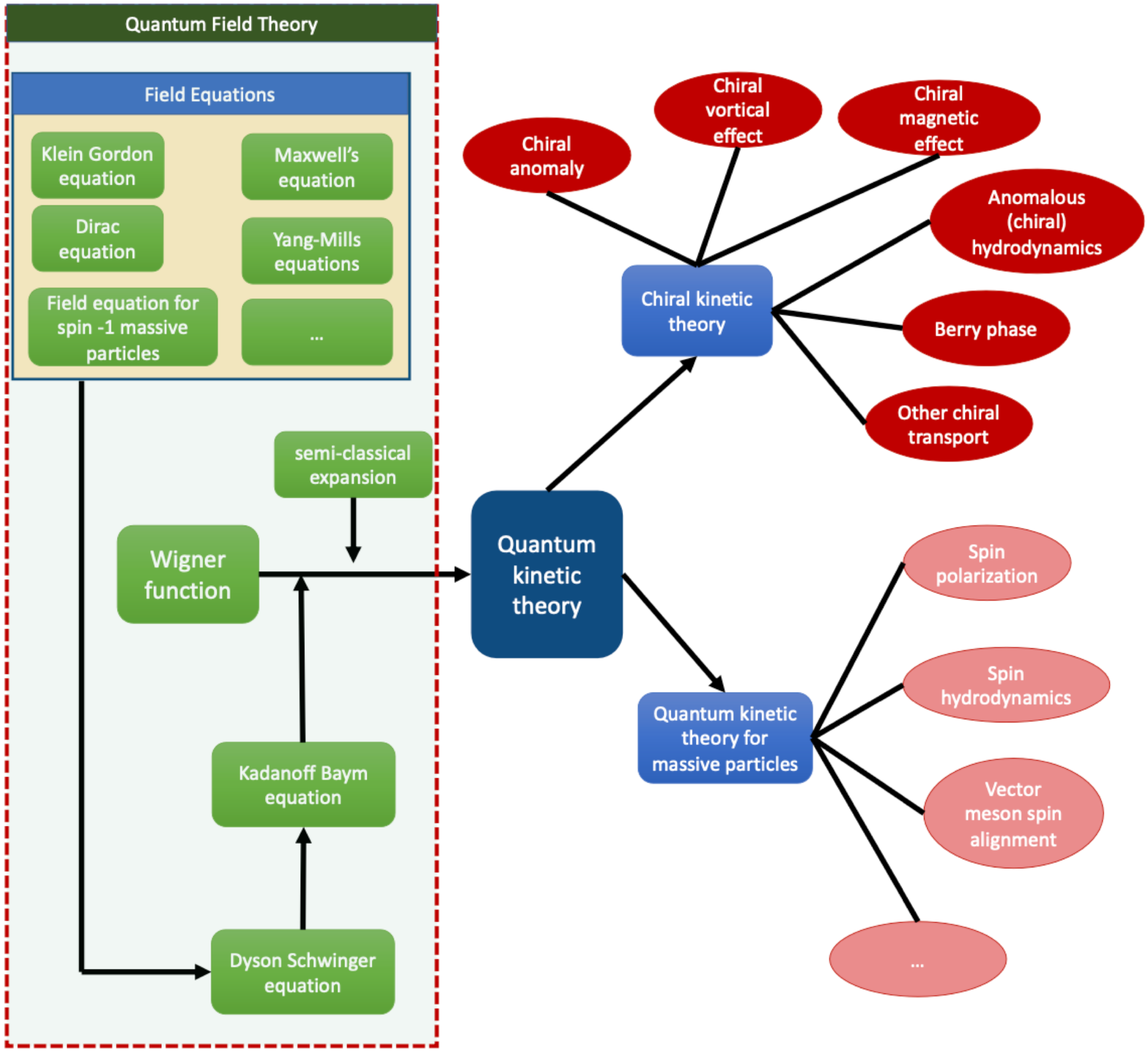
What we get for spin polarization?

- Shear induced polarization always give a "correct" sign.

- Total local polarization is very sensitive to EoS, freeze out temperature and eta / s.

- The local spin polarization is still an open question. We still need to consider the out-of-equilibrium effects carefully through the spin hydrodynamics and the quantum kinetic theory with collisions.

Summary



Quantum Field Theory
Field Equations
Klein Gordon equation
Maxwell's equation
Dirac equation
Yang-Mills equations
Field equation for spin -1 massive particles
...
semi-classical expansion
Wigner function
Kadanoff Baym equation
Dyson Schwinger equation
Quantum kinetic theory
Chiral kinetic theory
Chiral anomaly
Chiral vortical effect
Chiral magnetic effect
Anomalous (chiral) hydrodynamics
Berry phase
Other chiral transport
Quantum kinetic theory for massive particles
Spin polarization
Spin hydrodynamics
Vector meson spin alignment
...

Summary

- Relativistic heavy ion collisions provide us a nice platform to study the relativistic quantum matter under extreme conditions. The strongest EM fields and fastest vortical fluid have been found in these collisions.

- Quantum kinetic theory is a framework to study the quantum effects far away equilibrium. It successfully describes the chiral magnetic effects and is used for the studies of spin polarization.

Thank you for your time!

欢迎批评指正！

Backup

Wigner function (III)

- **Left and right handed currents**

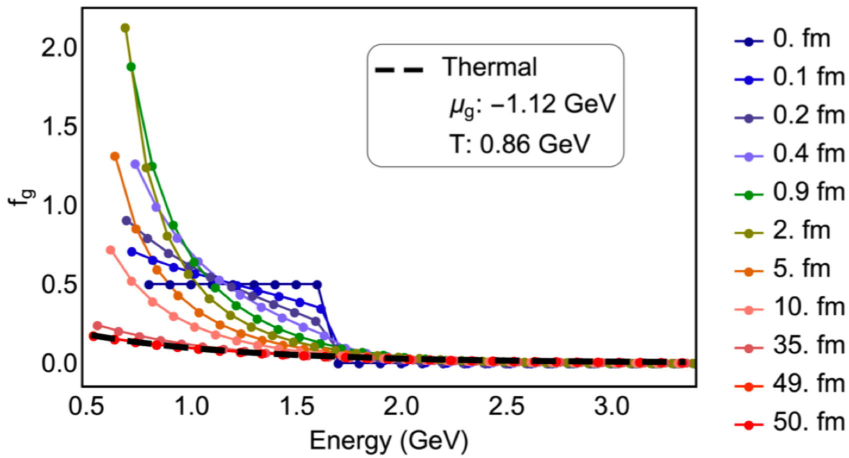
$$\mathscr{J}^s_\mu(x,p) = \frac{1}{2}[\mathscr{V}_\mu(x,p) + s\mathscr{A}_\mu(x,p)], \qquad s = \pm$$

- **In massless limit,**

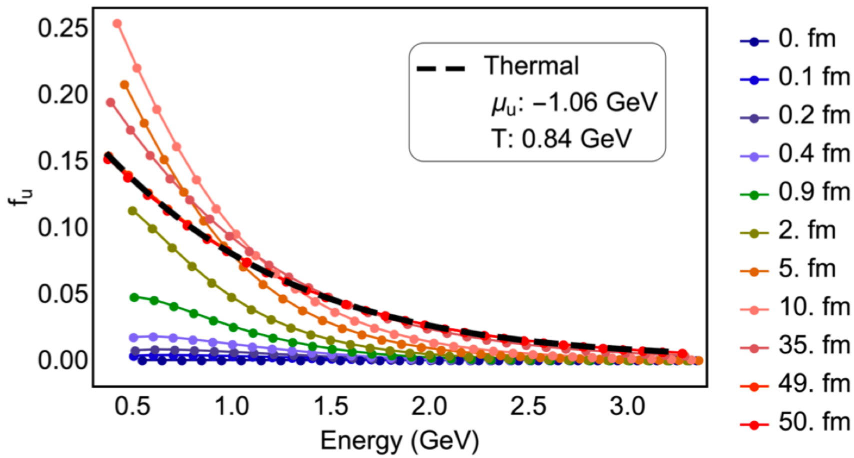
$$\begin{aligned} p^\mu \mathscr{J}^s_\mu(x,p) &= 0, \\ \nabla^\mu \mathscr{J}^s_\mu(x,p) &= 0, \\ 2s(p^\lambda \mathscr{J}^\rho_s - p^\rho \mathscr{J}^\lambda_s) &= -\epsilon^{\mu\nu\lambda\rho}\nabla_\mu \mathscr{J}^s_\nu. \end{aligned}$$

Tests for time evolution of quarks and gluons

Gluons



Quarks



Grids: 1 grid (space) ; momentum: 30x30x30=27,000

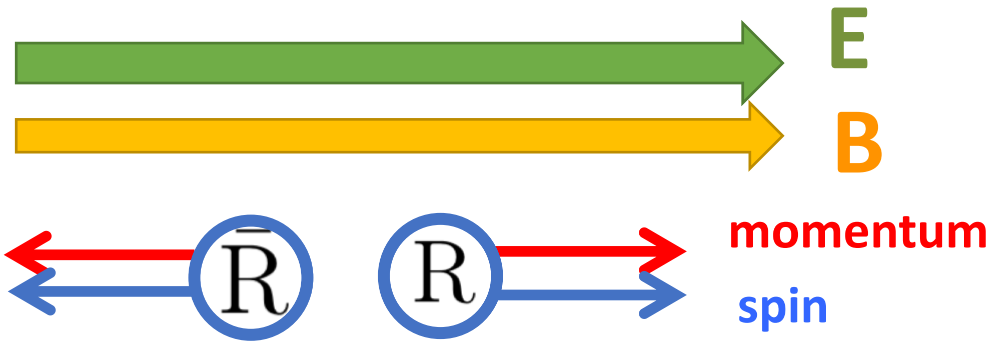
Phase space size: $[-3fm,3fm]^3$ x $[-2GeV,2GeV]^3$

Time step: dt=0.0005fm ； 100,000 steps

Time cost: around 50 hours on a Nvidia Tesla V100 card

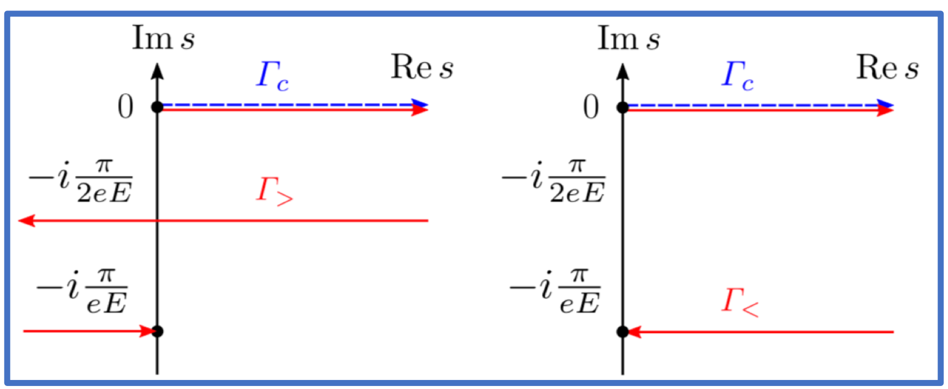
J.J Zhang, H.Z. Wu, SP, G.Y. Qin, Q. Wang, PRD (2019)

Connection to the Schwinger pair production



$$\frac{1}{2}\partial_t n_5 = \text{Schwinger Pair Production rate}$$

Fukushima, Kharzeev, Warringa, PRL(2010)



- **We rediscover a non-perturbative method to compute dynamical quantities in strong EB fields.**
  - ➢**Axial Ward identity, correct mass correction!**

$$\partial_\mu j_5^\mu = \frac{e^2 EB}{2\pi^2}\exp\left(-\frac{\pi m^2}{eE}\right)$$

  - ➢**Mass correction to CME**

$$j^3 = \frac{e^2 EB}{2\pi^2}\coth\left(\frac{B}{E}\pi\right)\exp\left(-\frac{\pi m^2}{eE}\right)t$$

  - ➢**Dynamical chiral condensate**

Copinger, Fukushima, SP, PRL(2018)

Also see recent review:
Copinger, SP, IJMPA (2020)

What is Chiral kinetic theory?

Chiral Kinetic Theory = **Boltzmann equation** + **Massless fermions** + **Quantum Corrections**

Boltzmann equation ⟷

$$\partial_t f + \dot{\mathbf{x}} \cdot \nabla_x f + \dot{\mathbf{p}} \cdot \nabla_p f = C[f]$$

+

Quantum Corrections ⟷

$$\varepsilon = |\mathbf{p}| + \hbar \ldots$$

$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \hbar \ldots$$

$$\dot{\mathbf{p}} = \mathbf{E} + \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{B} + \hbar \ldots$$

Hydrodynamic setup

- (3+1) dimensional viscous hydrodynamic CLVisc

  L.G. Pang, H. Petersen, and X.N. Wang, Phys. Rev. C 97, 064918 (2018)

- AMPT initial conditions

  Z.W. Lin, C. M. Ko, B.A. Li, B. Zhang, and S. Pal, Phys. Rev. C 72, 064901

- EoS “sp95-pce”

  P. Huovinen and P. Petreczky, Nucl. Phys. A 837, 26 (2010)

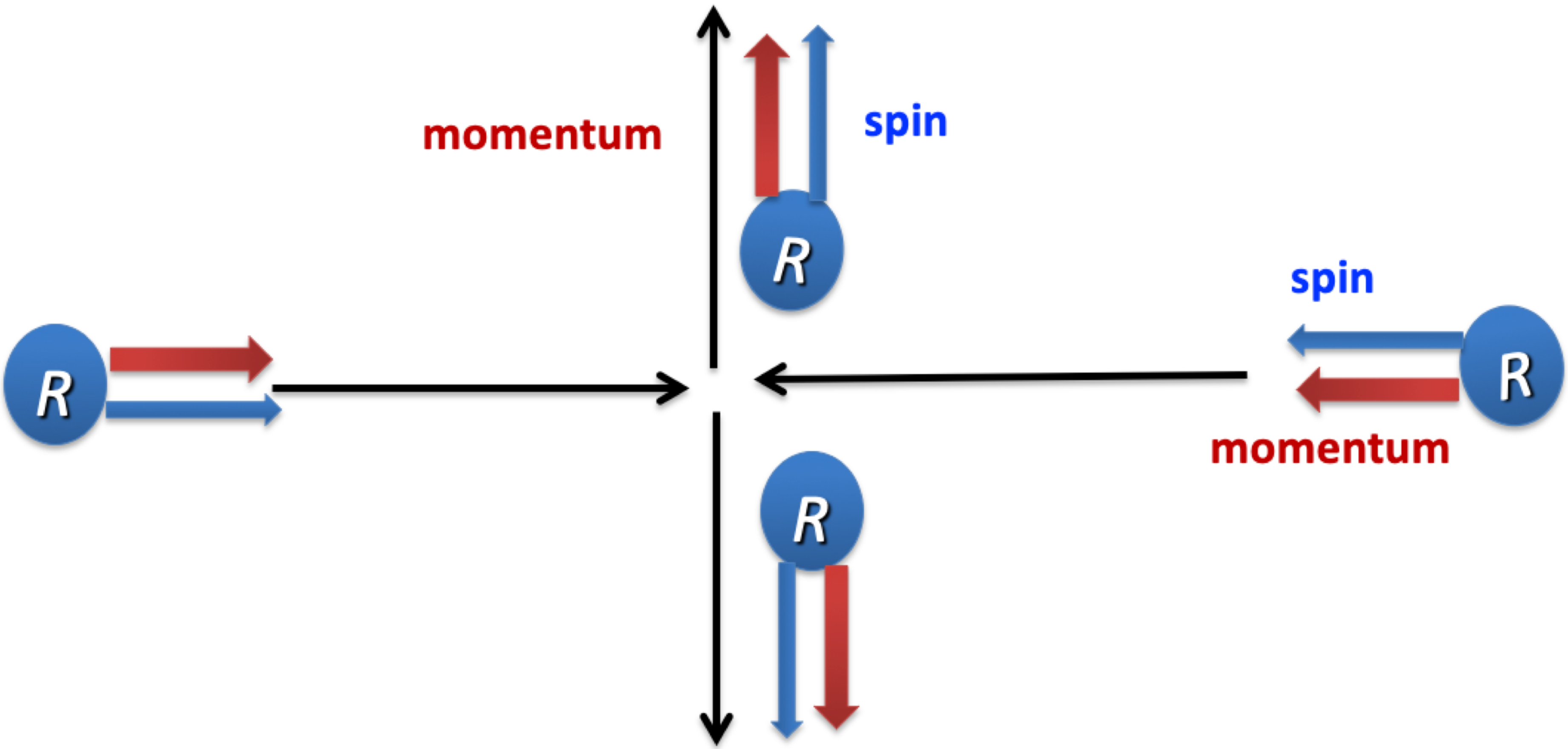
- Two scenarios

  B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, (2021), 2103.10403

  - Lambda equilibrium scenario
  - s quark equilibrium scenario

Side-jump (I)

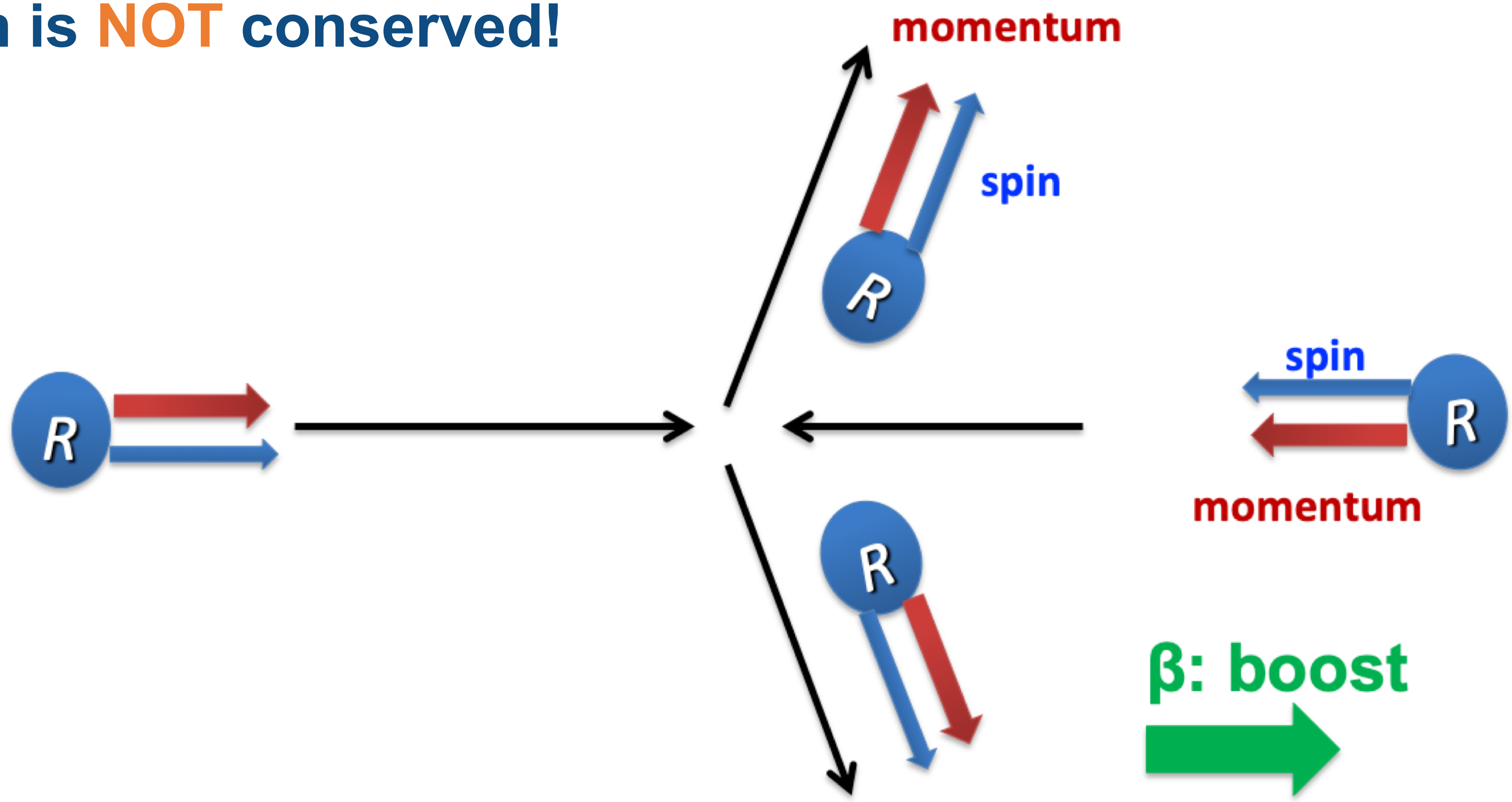
Orbital angular momentum and spin are conserved separately



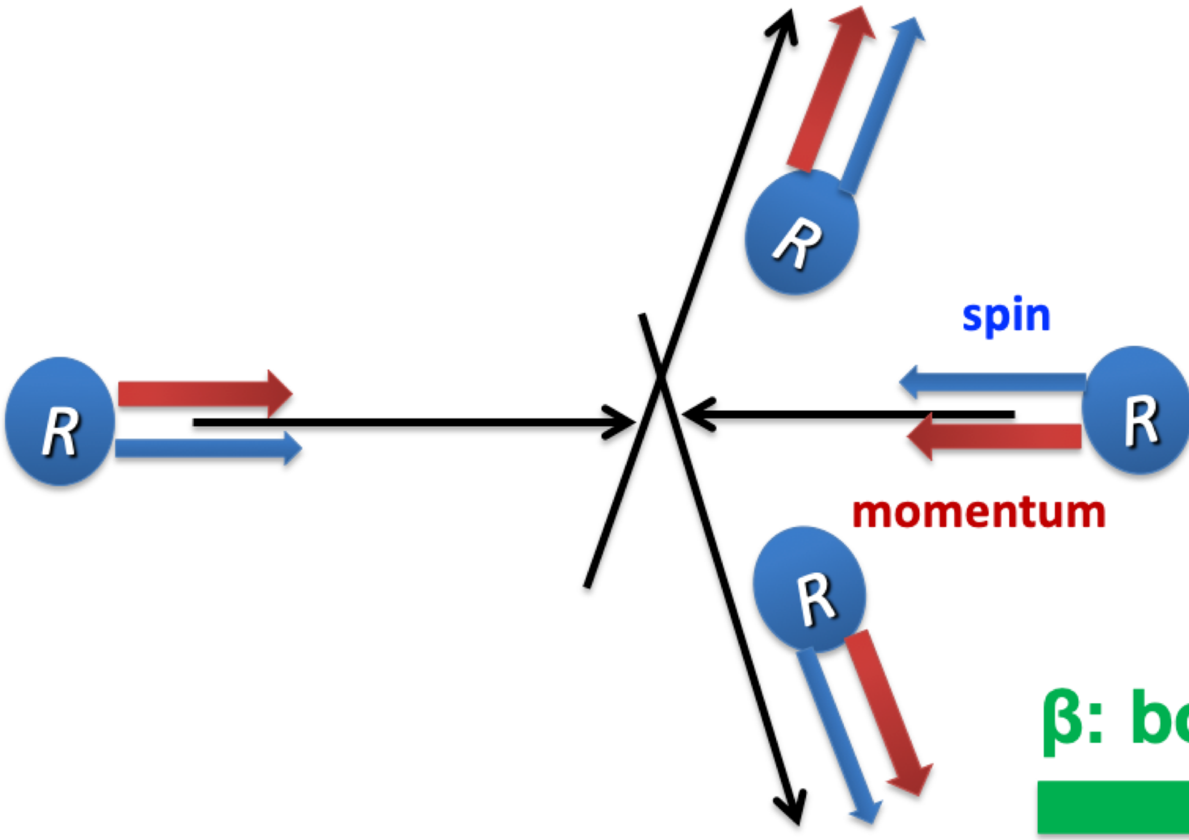
Chen, Son, Stephanov, Yee, Yin, PRL, (2014)

Side-jump (II)

Orbital angular momentum ?
Spin is NOT conserved!



Side-jump (III)



x has a shift!!!

"Side-jump":

$$\mathbf{x}' = \mathbf{x} + \boldsymbol{\beta} t + \delta \mathbf{x},$$
$$\mathbf{p}' = \mathbf{p} + \boldsymbol{\beta} \varepsilon + \delta \mathbf{p},$$

$$\delta \mathbf{x} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|},$$
$$\delta \mathbf{p} = \hbar \frac{\boldsymbol{\beta} \times \hat{\mathbf{p}}}{2|\mathbf{p}|} \times \mathbf{B}$$

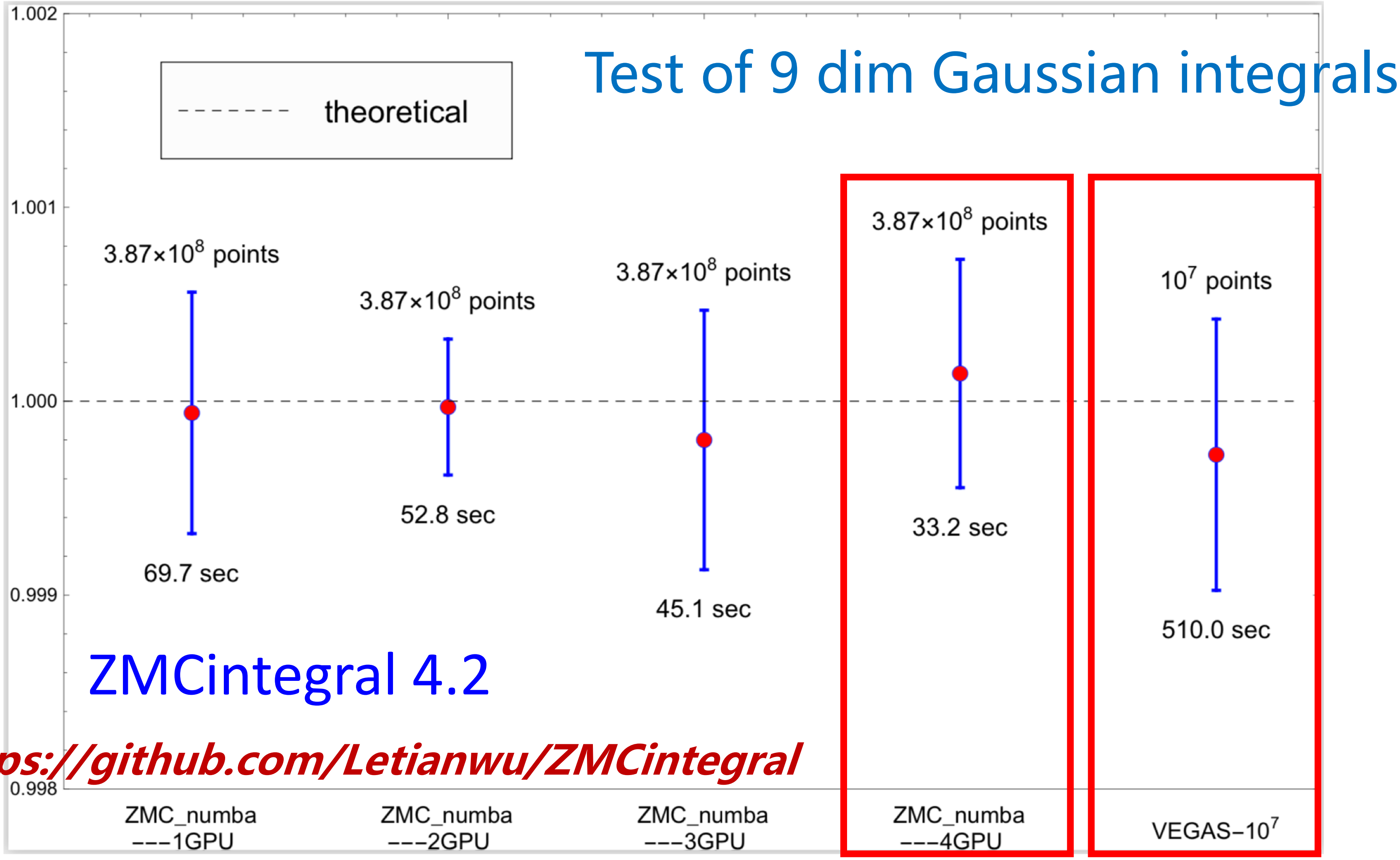
β: boost

Chen, Son, Stephanov, PRL, (2015);
Y. Hidaka, SP, D.L. Yang, PRD (2016)

Collisional term via ZMCintegral

- **5 dimensional integral on each phase space grid:**

Wu, Zhang, Pang, Q. Wang, Computer Physics Communications (2019)



https://github.com/Letianwu/ZMCintegral