

New probes of the proton internal structures

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HETH-Forum @ IHEP, 2021



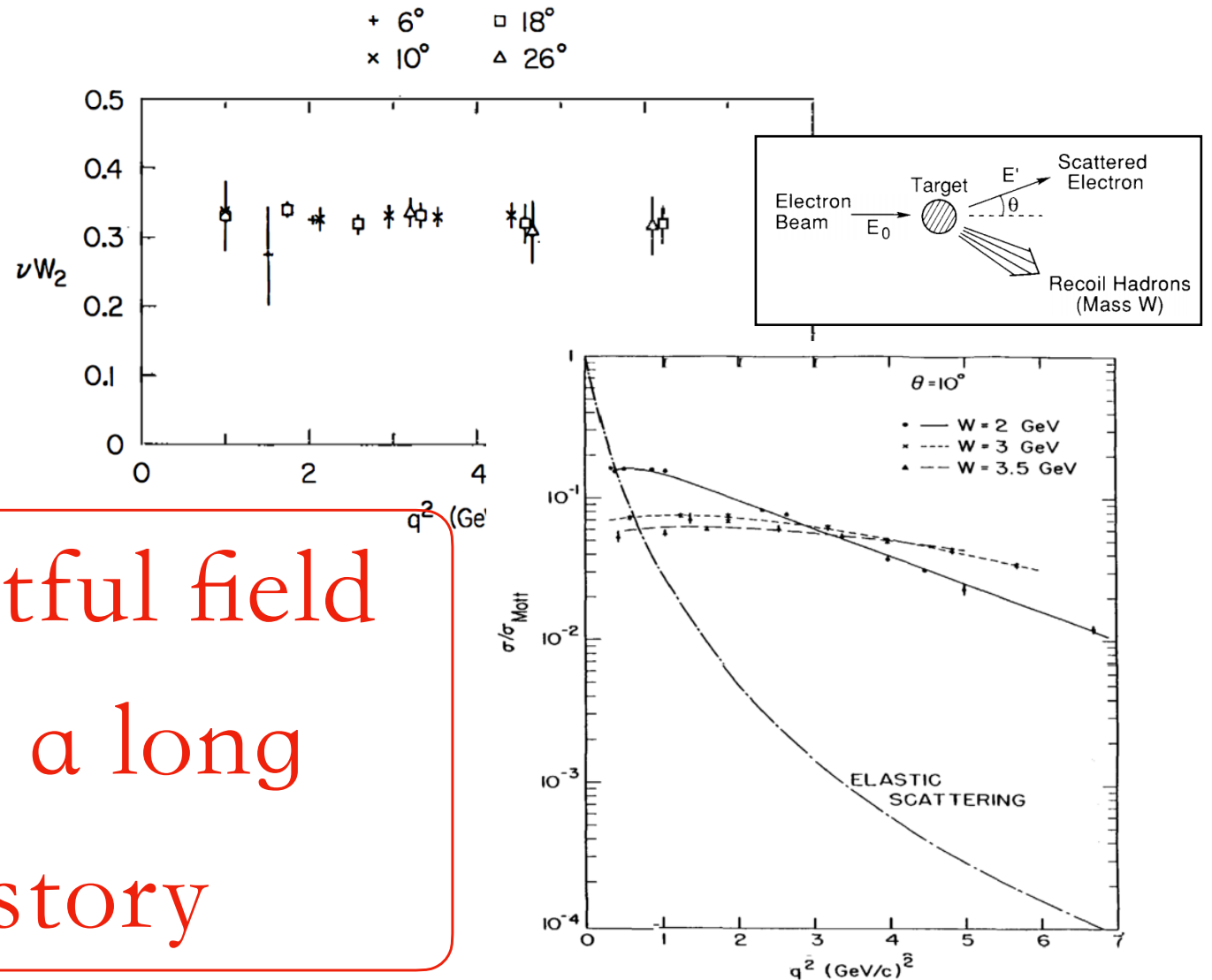
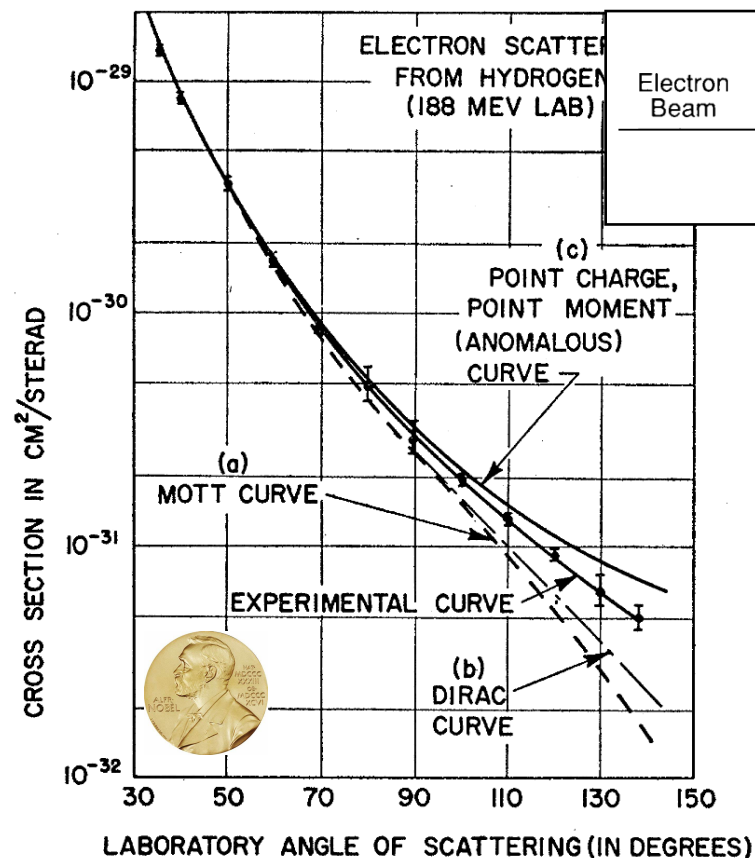
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Outline

- Proton structure studies (basic ideas)
 - By jet probes
 - By quantum simulation (toy model)
- Conclusion

Proton Structure Studies

McAllister, Hofstadter, 1956



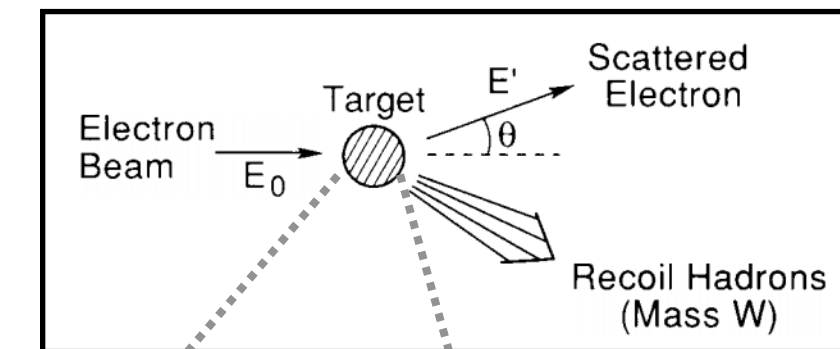
A fruitful field
with a long
history

Proton is not
elementary

- Feynman Parton Model
- Confirm the existence of quarks
- Asymptotic free
- Discovery of QCD

Proton Structure Studies

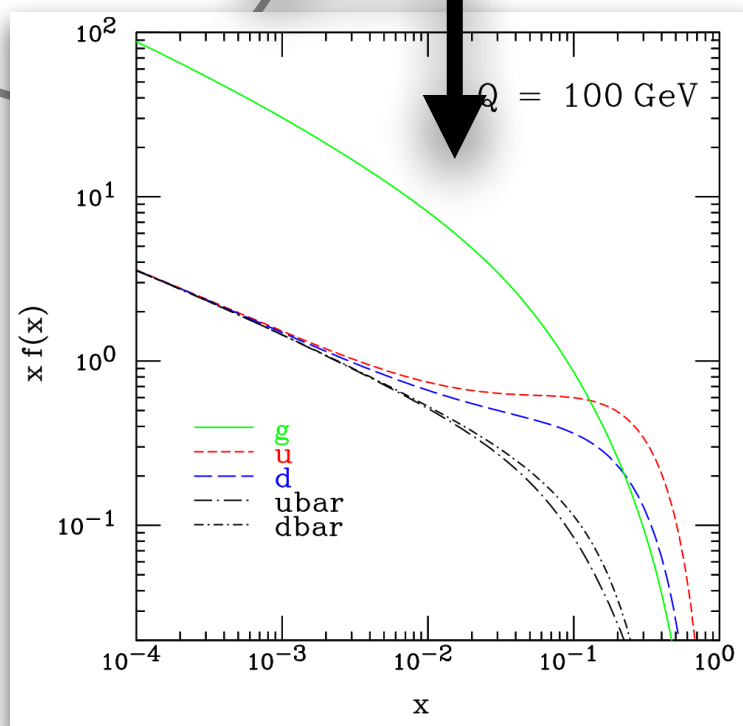
- Experimentally: Hard probes



$$\sigma = \hat{\sigma}_i f_{i/P}(x), \quad f_{i/P}(x) = \hat{\sigma}_i / \sigma$$

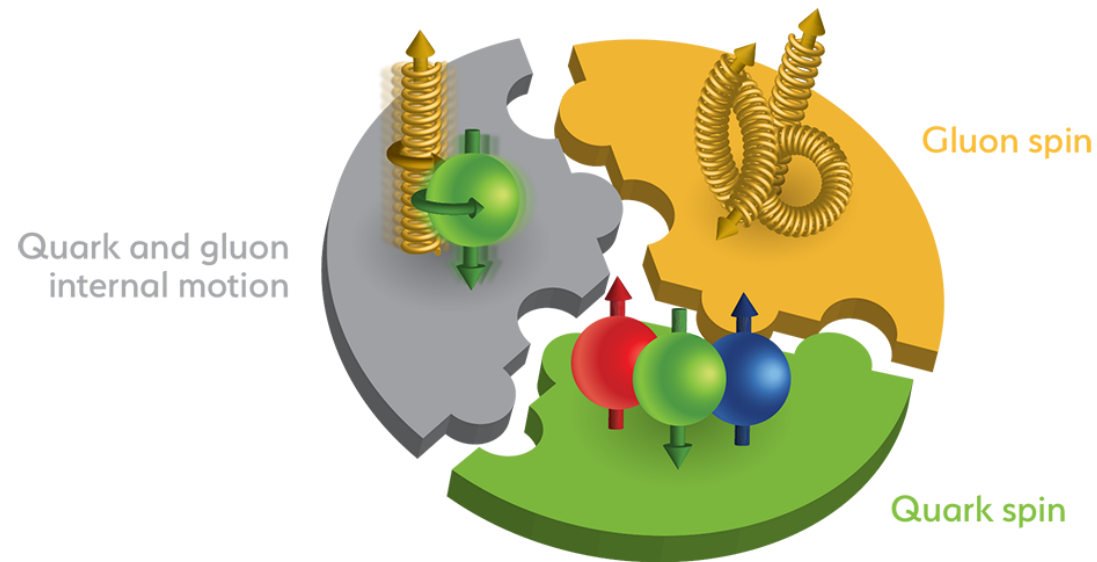
- Very good understanding of the proton **longitudinal** component
- Crucial for the LHC physics

e.g., $\sigma_{ggH} = 48.58 \text{ pb}^{+4.56\%}_{-6.72\%}(\text{theory}) \pm 3.2\%(\text{PDF} + \alpha_s)$

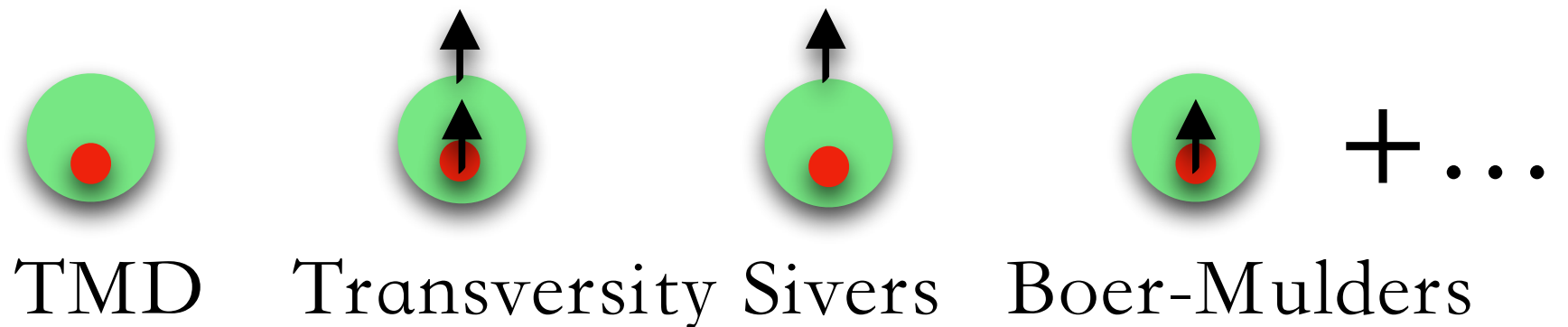


Anastasou, et.al., PRL 2016

Proton Structure Studies



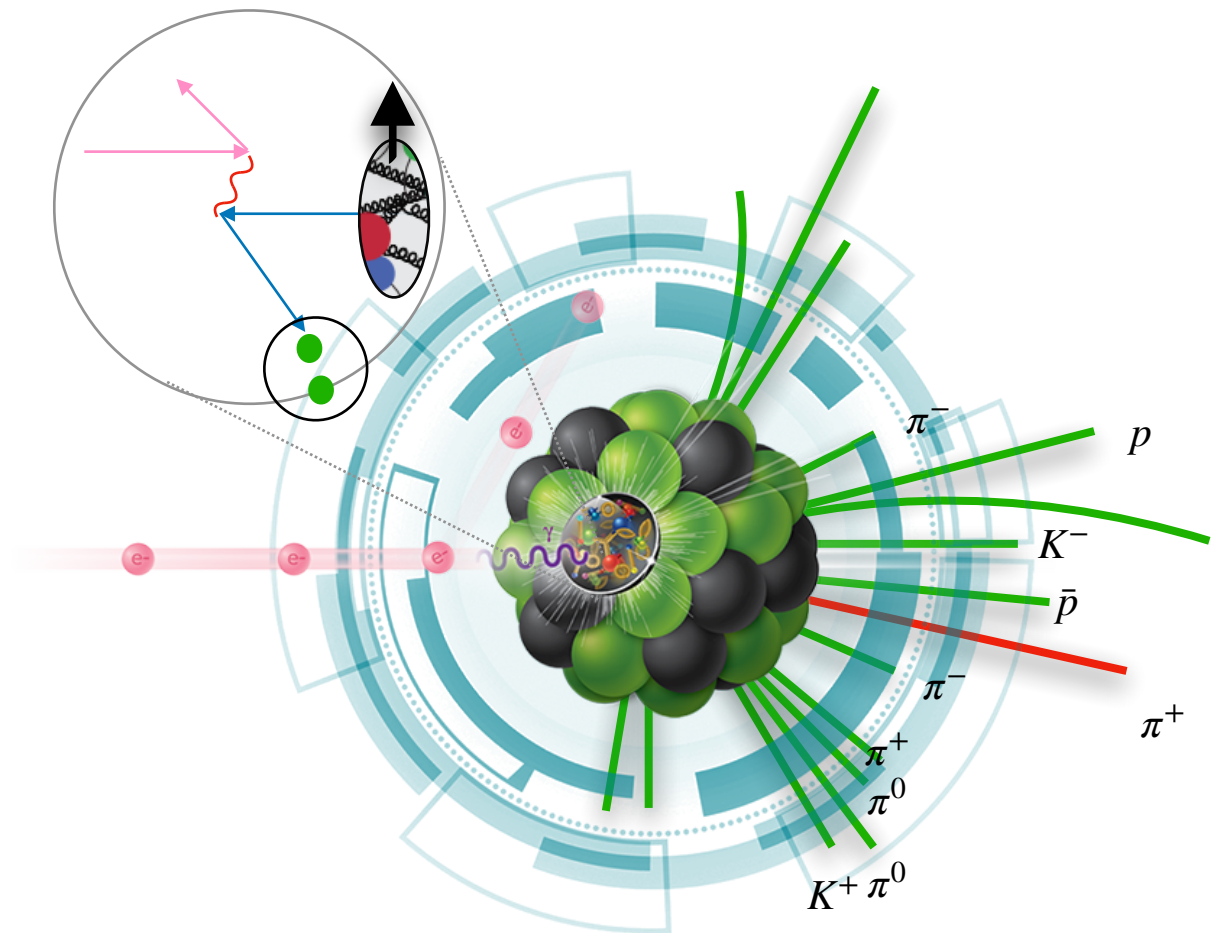
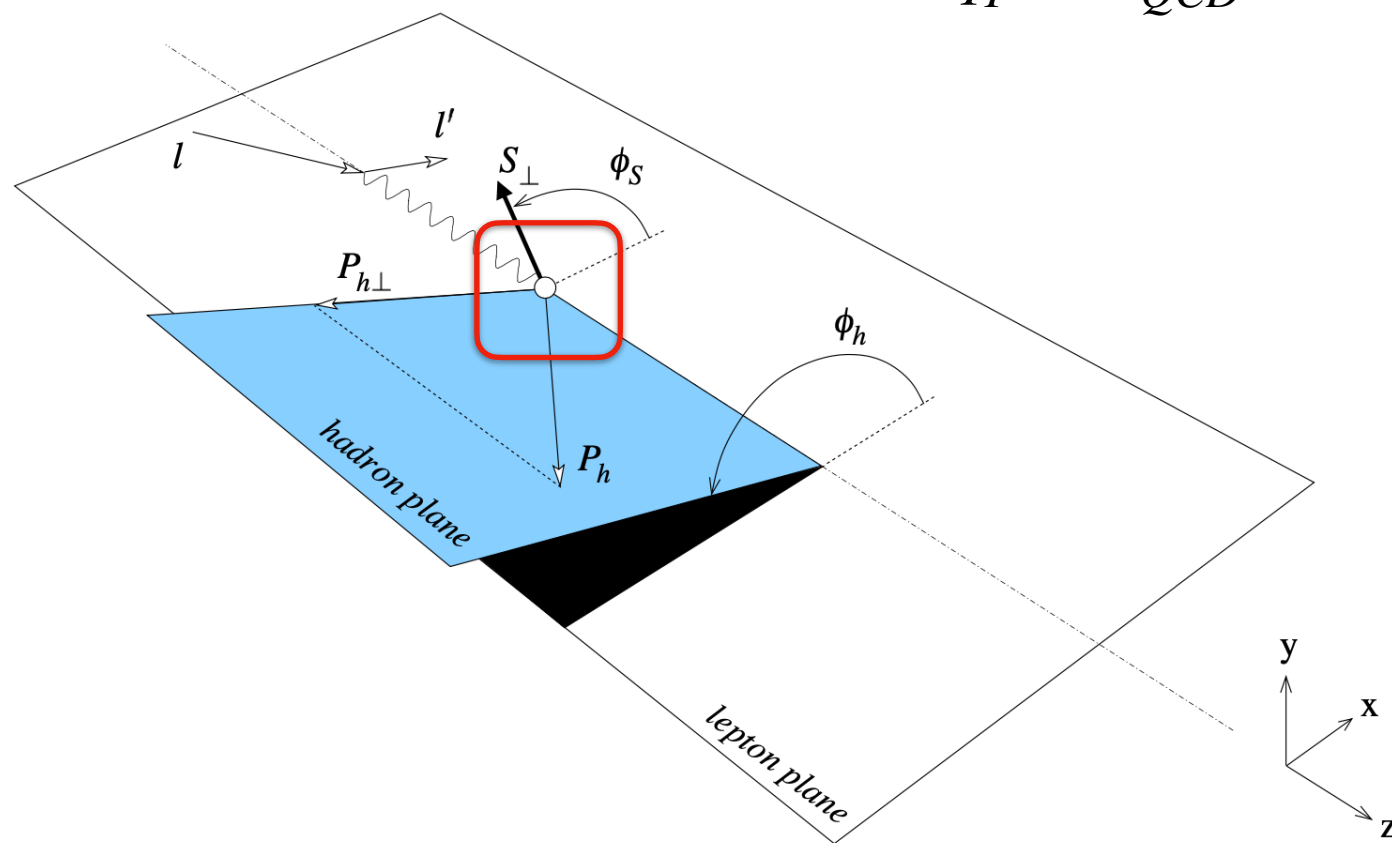
- A lot to answer @ **EicC, EIC**
 - Spin components
 - Mass decomposition
 - Proton radius
- 3D Parton distribution functions
 - Longitudinal + transverse
 - Polarization of partons in a (un-) polarized proton



Proton Structure Studies

- Semi Inclusive DIS
 - Tag one hadron, inclusive over others, require $P_{h,\perp}$ (the q_T of the system) small

$$q_T \sim \Lambda_{QCD}$$



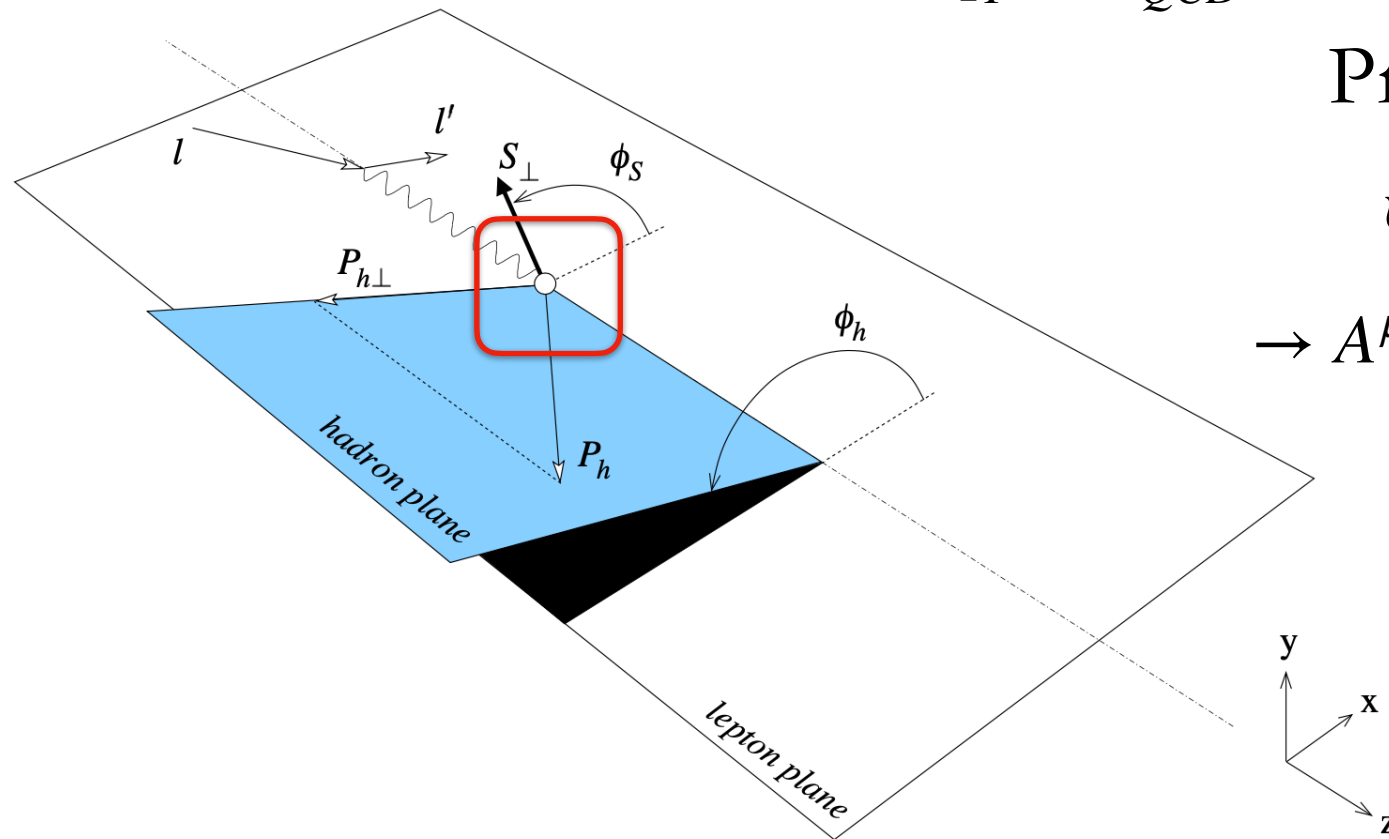
- Usually performed in the GNS frame, Breit frame
- TMD factorization

$$\sigma = \hat{\sigma}_{i \rightarrow j} f_{i/P}(x, k_T, S) \otimes_{q_T} D_{j/H}(z, k'_T)$$

Proton Structure Studies

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$$q_T \sim \Lambda_{QCD}$$



$$\sigma(\phi, \phi_S) \equiv \frac{d^6\sigma}{dx dy dz d\phi d\phi_S dP_{hT}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \times$$

$$\left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} + \right. \right.$$

$$\left. + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right] \right.$$

$$\left. + |S_T| \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \right.$$

$$\left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \left. + |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \right\},$$

Diagram labels: Cahn-effect + BM ⊗ Collins, Worm-gear (Kotzinian-Mulders) ⊗ Collins, BM ⊗ Collins, Sivers ⊗ D1, Worm-gear ⊗ D1, Transversity ⊗ Collins, Pretzelosity ⊗ Collins.

Pro: Abundant structures

$$\bar{\psi}_n \gamma^\mu \psi_{\bar{n}} \bar{\psi}_{\bar{n}} \gamma^\nu \psi_n$$

$$\rightarrow A^{\mu\nu} \bar{\psi}_n \not{n} \psi_n \bar{\psi}_{\bar{n}} \not{n} \psi_{\bar{n}} + B_{\alpha\beta}^{\mu\nu} \bar{\psi}_n \not{n} \gamma_\perp^\alpha \gamma^5 \psi_n \bar{\psi}_{\bar{n}} \not{n} \gamma_\perp^\beta \gamma^5 \psi_{\bar{n}} + \dots$$

Possible since non-pert.

$$\sigma = \hat{\sigma}_{i \rightarrow j} f_{i/P}(x, k_T, S) \otimes_{q_T} D_{j/H}(z, k'_T)$$

Proton Structure Studies

- Semi Inclusive DIS
 - Tag one hadron, inclusive over others, require $P_{h,\perp}$ (the q_T of the system) small

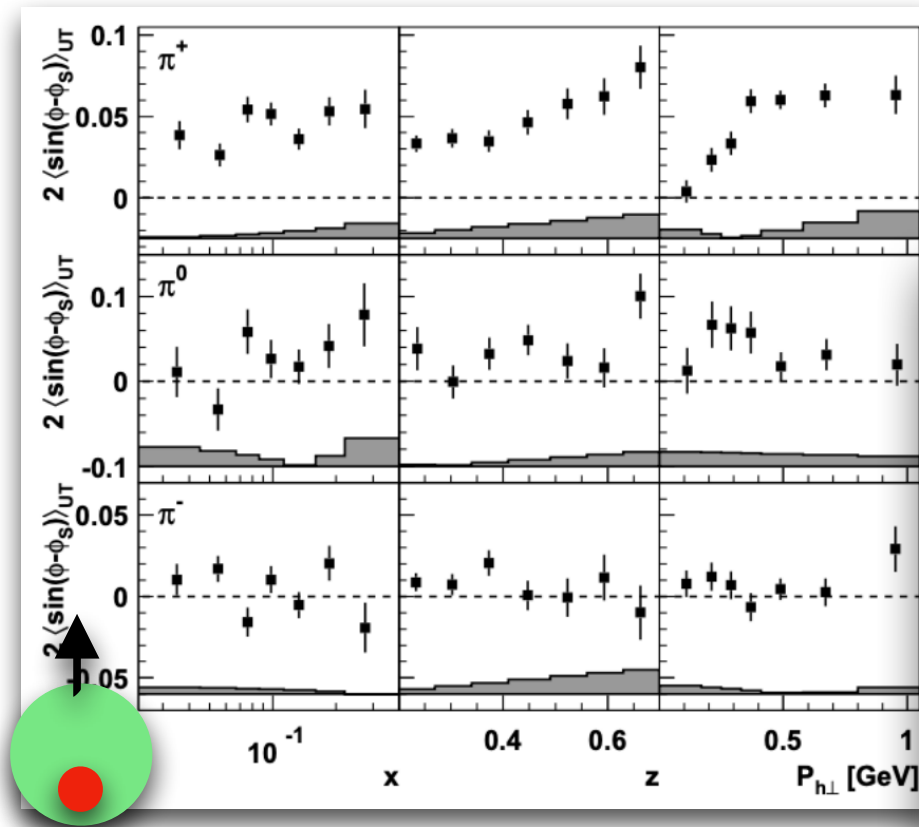
$$\sigma(\phi, \phi_S) \equiv \frac{d^6\sigma}{dx dy dz d\phi d\phi_S dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right)$$

$$\begin{aligned} & \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \right] + \right. \\ & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right] \\ & + |S_T| \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right. \\ & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right] \\ & \left. + |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \right\}, \end{aligned}$$

Diagram illustrating the decomposition of the cross-section into various structure functions and their associated physical effects:

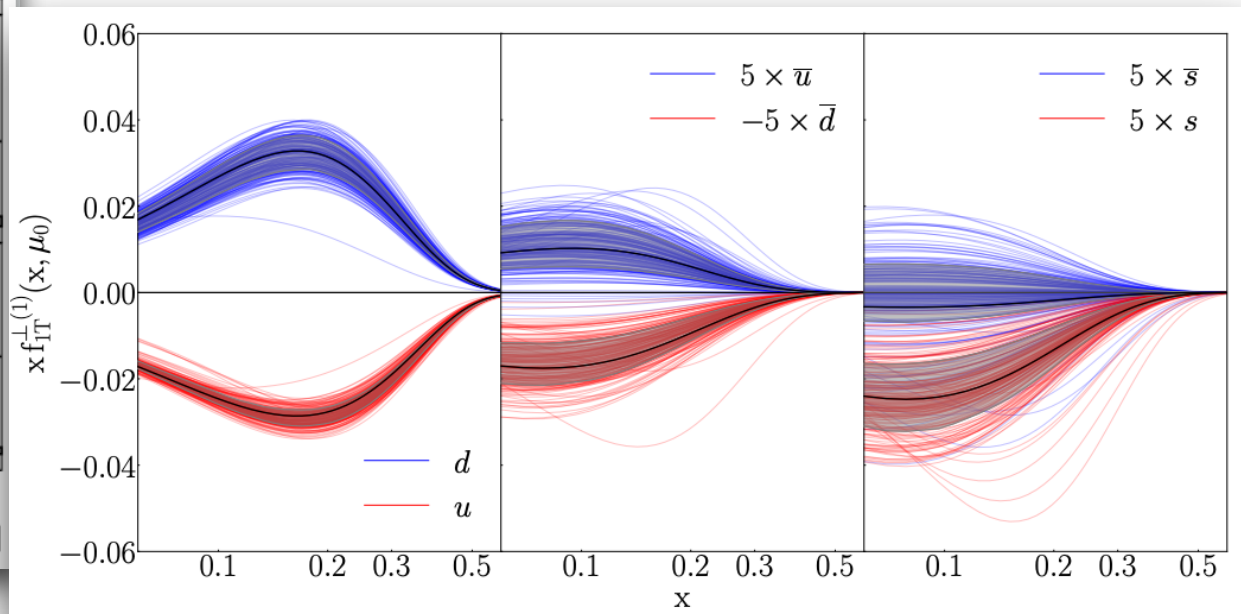
- Cahn-effect + BM $\otimes$ Collins** (points to $F_{UU,T}$ and $F_{UU,L}$)
- Worm-gear (Kotzinian-Mulders) $\otimes$ Collins** (points to $F_{UU}^{\cos\phi}$ and $F_{UU}^{\cos(2\phi)}$)
- BM $\otimes$ Collins** (points to $F_{LU}^{\sin\phi}$ and $F_{UL}^{\sin\phi}$)
- Sivers $\otimes$ D1** (points to $F_{UT,T}^{\sin(\phi-\phi_S)}$ and $F_{UT,L}^{\sin(\phi-\phi_S)}$)
- Worm-gear $\otimes$ D1** (points to $F_{UT}^{\sin(\phi+\phi_S)}$)
- Transversity $\otimes$ Collins** (points to F_{LL} and $F_{LL}^{\cos\phi}$)
- Pretzelosity $\otimes$ Collins** (points to $F_{UT}^{\sin(3\phi-\phi_S)}$)

Pro: Abundant structures



HERMES collaboration, 2009

$\sim \Lambda_{QCD}$



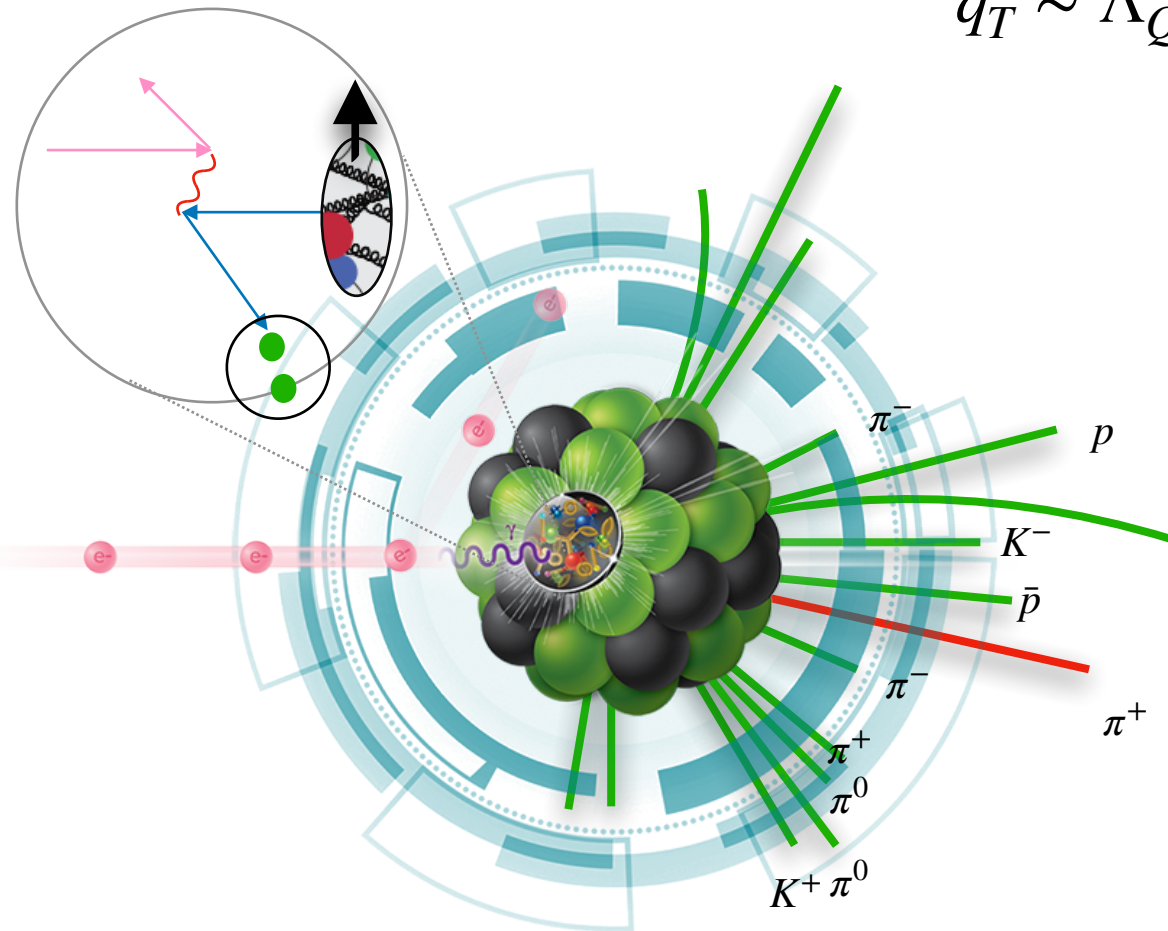
Kang et al., JHEP 2021

$q_T D_{j/H}(z, k'_T)$

Proton Structure Studies

- Semi Inclusive DIS
 - Tag one hadron, inclusive over others, require $P_{h,\perp}$ (the q_T of the system) small

$$q_T \sim \Lambda_{QCD}$$



$$\sigma(\phi, \phi_S) \equiv \frac{d^6\sigma}{dx dy dz d\phi d\phi_S dP_{hT}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \times$$

$$\left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \right] + \right.$$

$$+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right]$$

$$+ |S_T| \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right.$$

$$\left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right]$$

$$+ |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \Big\} ,$$

Contributions from various effects are indicated by boxes:

- Cahn-effect + BM $\otimes$ Collins** (points to $F_{UU,T}$ and $F_{UU,L}$)
- Worm-gear (Kotzinian-Mulders) $\otimes$ Collins** (points to $F_{UU}^{\cos\phi}$ and $F_{UU}^{\cos(2\phi)}$)
- BM $\otimes$ Collins** (points to $F_{LU}^{\sin\phi}$ and $F_{UL}^{\sin\phi}$)
- Sivers $\otimes$ D1** (points to $F_{UT,T}^{\sin(\phi - \phi_S)}$ and $F_{UT,L}^{\sin(\phi - \phi_S)}$)
- Worm-gear $\otimes$ D1** (points to $F_{UT}^{\sin(\phi + \phi_S)}$)
- Transversity $\otimes$ Collins** (points to F_{LL} and $F_{LL}^{\cos\phi}$)
- Pretzelosity $\otimes$ Collins** (points to $F_{UT}^{\sin(3\phi - \phi_S)}$)

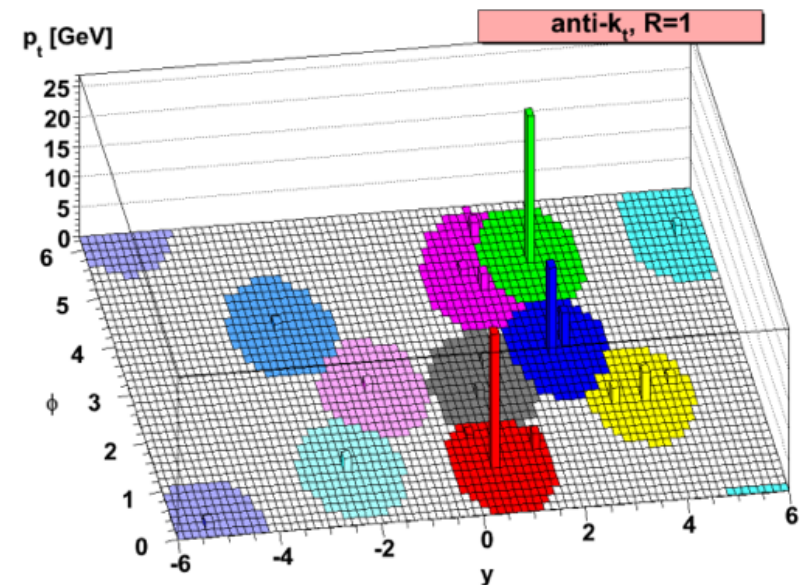
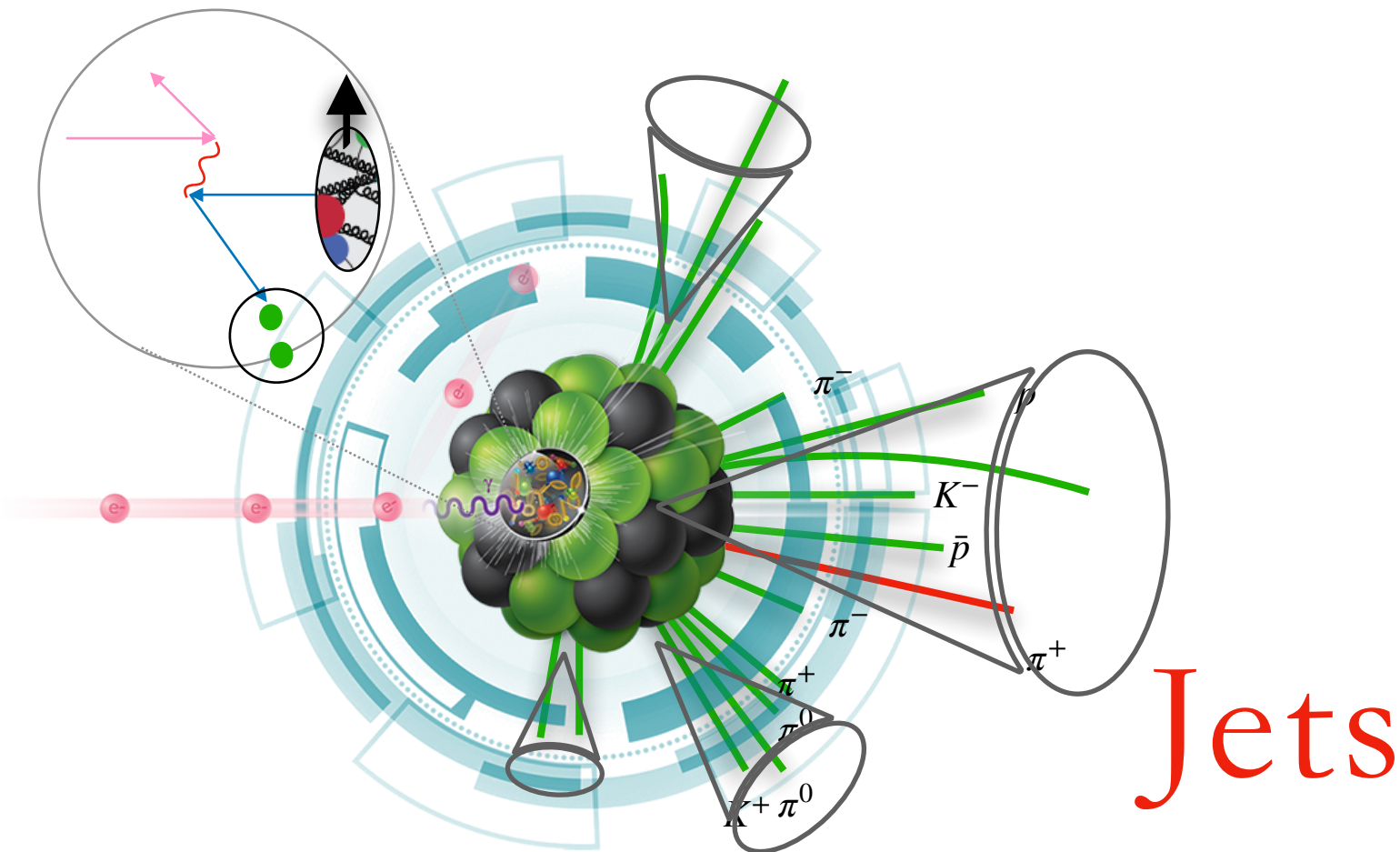
Pro: Abundant structures

Con: $D_{j/H}(z, k_T)$ is non-perturbative, hard to extract. **Alternatives ?**

$$\sigma = \hat{\sigma}_{i \rightarrow j} f_{i/P}(x, k_T, S) \otimes_{q_T} D_{j/H}(z, k'_T)$$

Jet Probes

- Jets
 - Sprays of hadrons,
 - Man-made objects, by algorithms
 - perturbative (?)



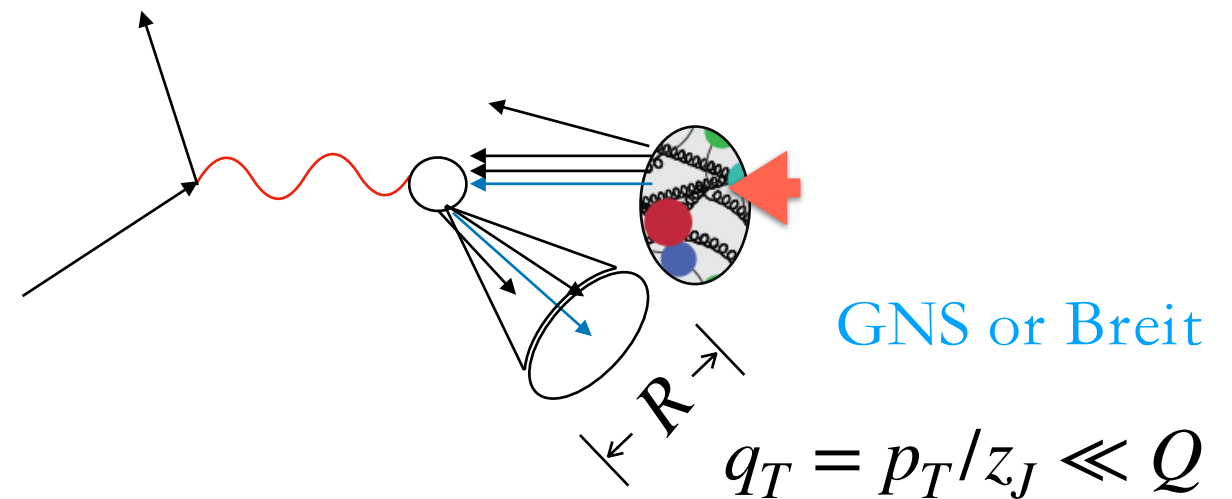
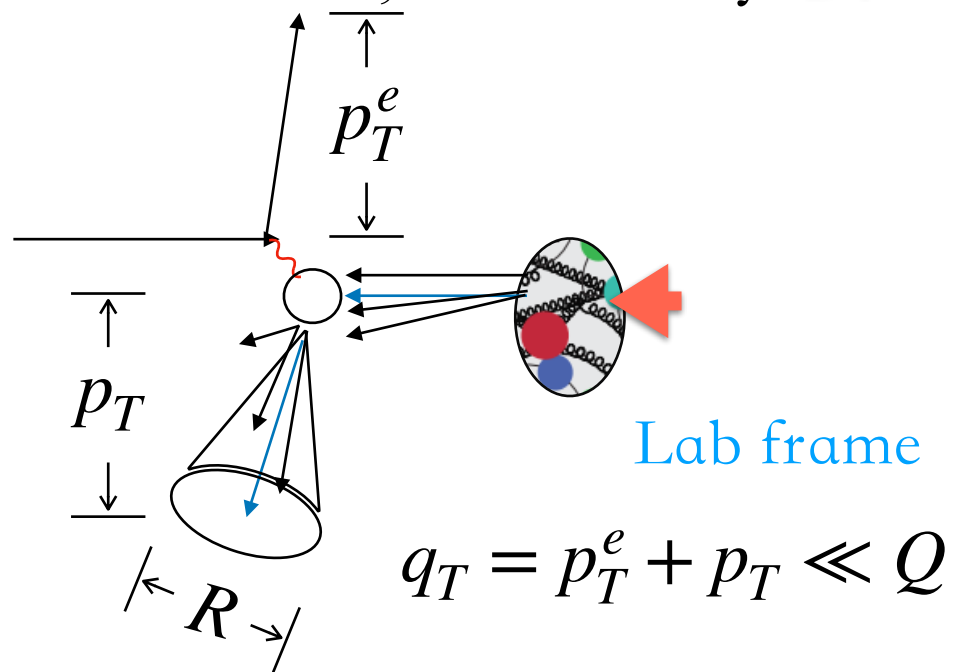
$$d_{ij} = \min(p_{ti}^{-2}, p_{tj}^{-2}) \frac{\Delta R_{ij}^2}{R^2}, \quad d_{iB} = p_{ti}^{-2}$$

$$\Delta R_{ij}^2 = \Delta^2 \phi_{ij} + \Delta^2 \eta_{ij}$$

- Find the minimum of d_{iB} and d_{ij}
- If d_{ij} is the smallest, combine i, j
- If d_{iB} is the smallest, then i is a jet and remove from the list
- Iterate till all partons fall into jets

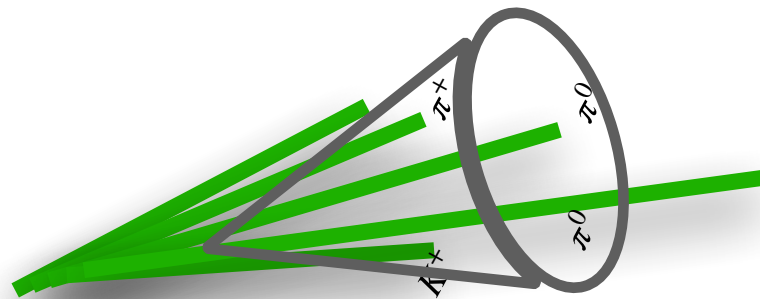
Jet Probes

- Frame choice, flexibility 🍀



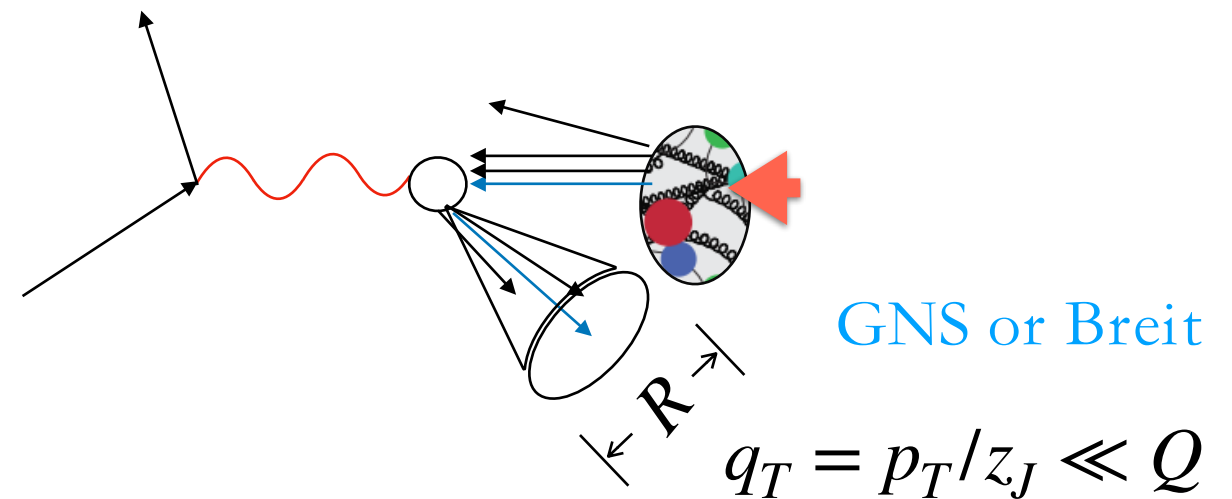
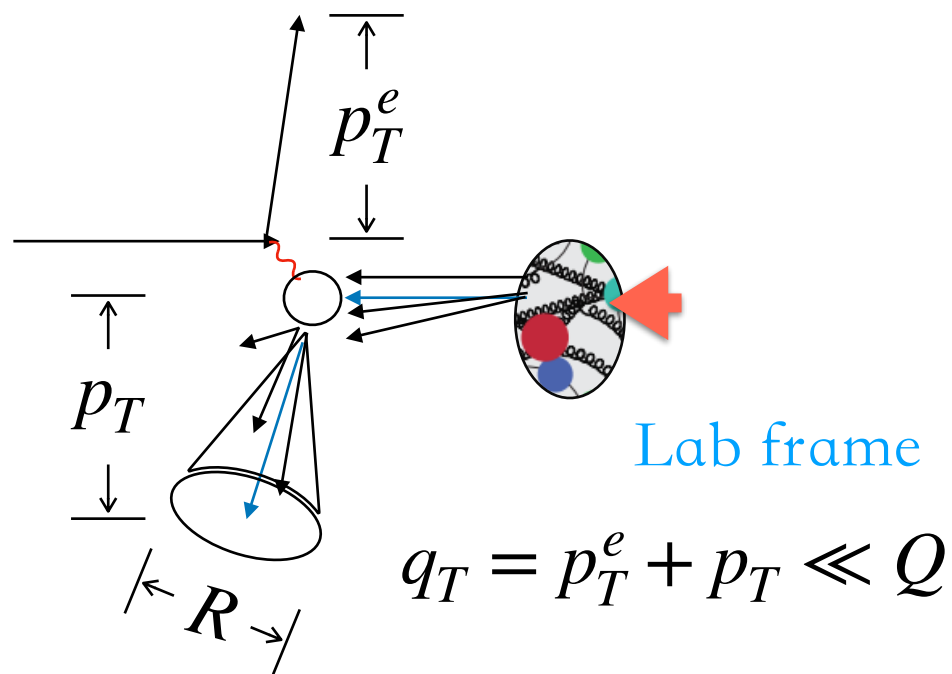
- LHC-like
 - anti- k_T
 - Migrate LHC techniques

- New jet play-ground ...
 - Centauro jet, WTA jet ...
 - Jet physics possible for low energy machine?



Jet Probes

- Precision 



- LHC-like

$$\sigma(q_T) \sim f(x, k_T) \otimes_{q_T} S_C(k_T'', R) \otimes_{\Omega} J(P_T R)$$

Known to NLL *XL et al., PRL 2019*

Efforts to go beyond NLL:

$J(p_T R)$ @ 2-loops *Liu, XL, Moch, PRD 2021*

- New jet play-ground ...

$$\sigma(q_T) \sim f(x, k_T) \otimes_{q_T} J_{WTA}(P_T R, k_T')$$

$$\rightarrow_{R \gg q_T/p_T} f(x, k_T) \otimes_{q_T} J_{WTA}(k_T')$$

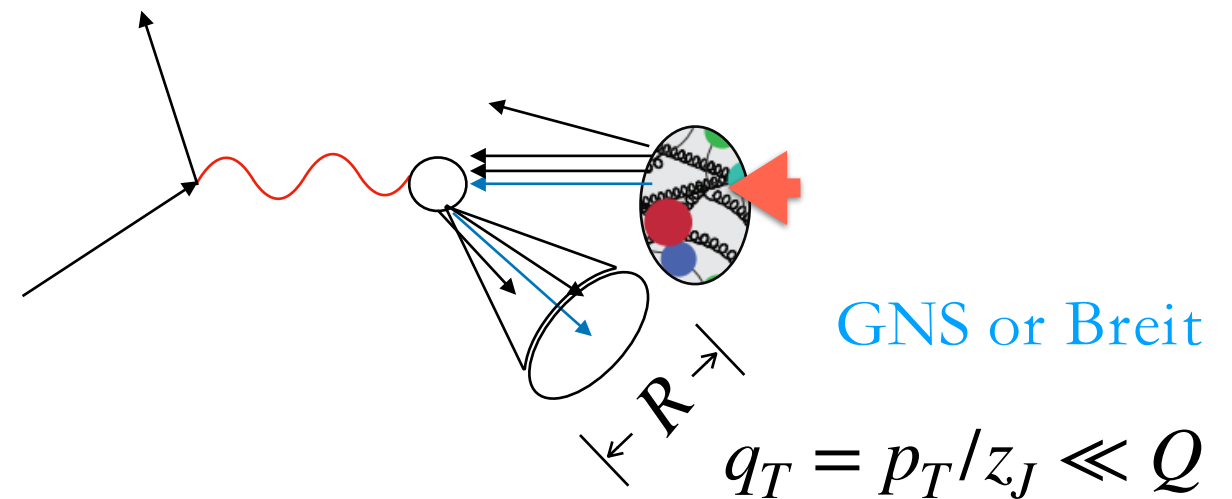
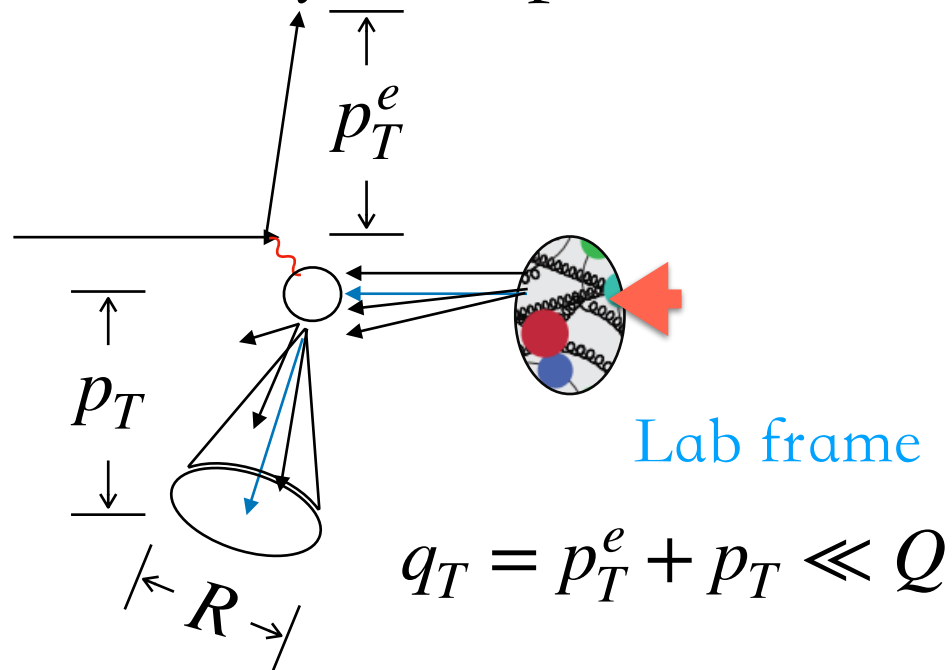
Known to NNLL

Gutierrez-Reyes et al., PRL 2018

Challenging due to complicated
clustering procedure

Jet Probes

- Intrinsically non-perturbative when $q_T \sim \Lambda_{QCD}$ ☢



- LHC-like

$$\sigma(q_T) \sim f(x, k_T) \otimes_{q_T} S_C(k_T'', R) \otimes_{\Omega} J(P_T R)$$

- New jet play-ground ...

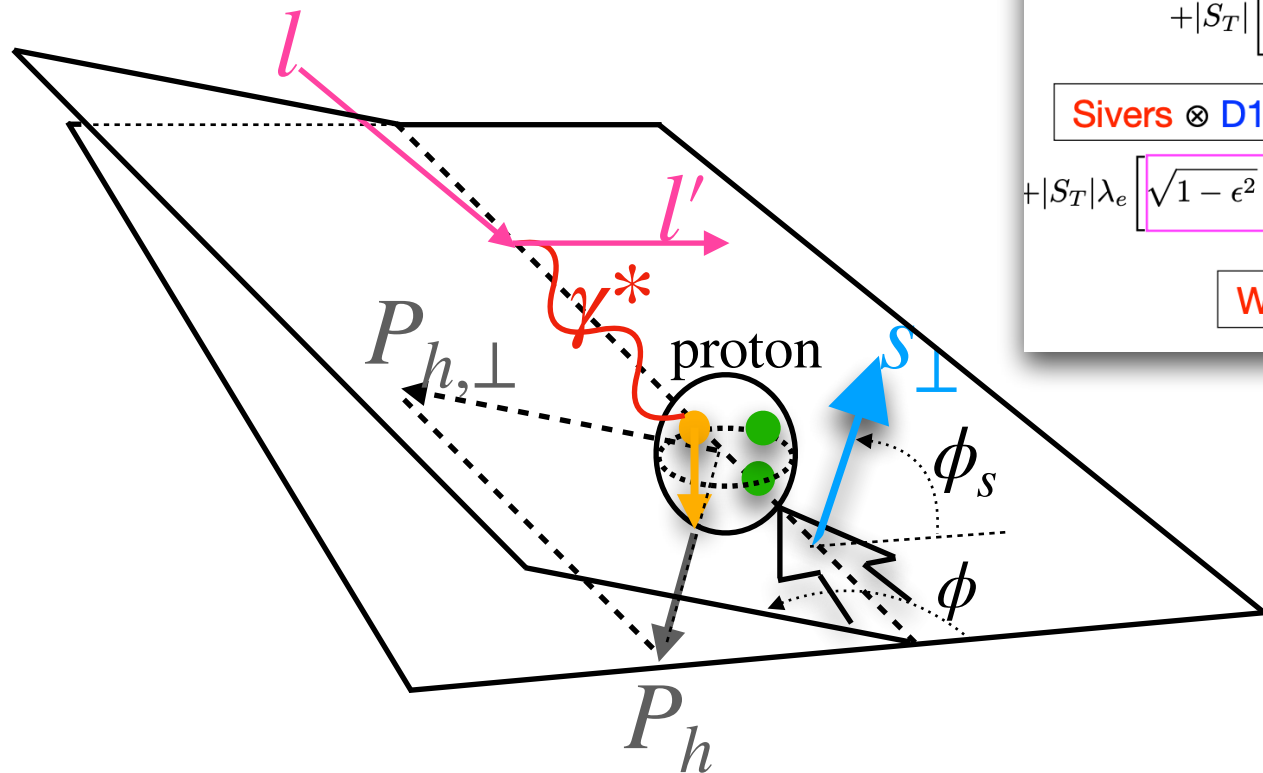
$$\sigma(q_T) \sim f(x, k_T) \otimes_{q_T} J_{WTA}(P_T R, k_T')$$

$$\rightarrow_{R \gg q_T/p_T} f(x, k_T) \otimes_{q_T} J_{WTA}(k_T')$$

- Well control via operator studies, reduced parameters
- Interesting phenomenologies different from the LHC jets could arise

Jet Probes

- Abundant structures for hadron probes



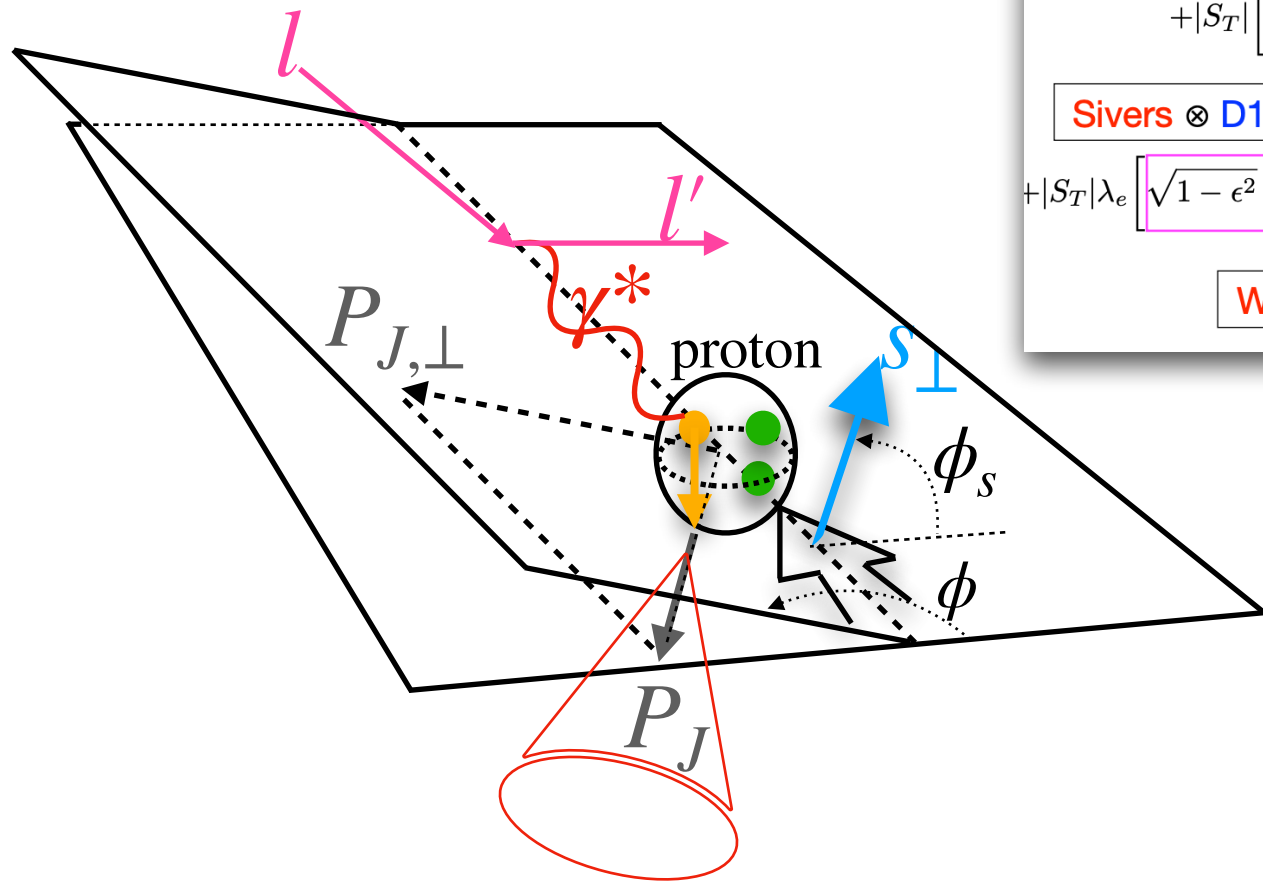
$$\begin{aligned}
 \sigma(\phi, \phi_S) &\equiv \frac{d^6 \sigma}{dx dy dz d\phi d\phi_S dP_{\perp T}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \\
 &\left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \right] + \right. \\
 &+ S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right] \\
 &+ |S_T| \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \\
 &\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right] \\
 &\left. + |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \right\} ,
 \end{aligned}$$

Diagram illustrating the decomposition of the cross-section $\sigma(\phi, \phi_S)$ into various helicity amplitudes and their corresponding theoretical models:

- Worm-gear (Kotzinian-Mulders) $\otimes$ Collins** (Top Center)
- BM $\otimes$ Collins** (Top Right)
- Cahn-effect + BM $\otimes$ Collins** (Top Left)
- Sivers $\otimes$ D1** (Middle Left)
- Worm-gear $\otimes$ D1** (Bottom Left)
- Transversity $\otimes$ Collins** (Bottom Center)
- Pretzelosity $\otimes$ Collins** (Bottom Right)

Jet Probes

- Extremely limited structures for jet probes



$$\begin{aligned}
 & \sigma(\phi, \phi_s) \equiv \frac{d^6\sigma}{dx dy dz d\phi d\phi_s dP_{JT}^2} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \\
 & \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\sin(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \right] + \right. \\
 & + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right] \\
 & + |S_T| \left[\sin(\phi - \phi_s) \left(F_{UT,T}^{\sin(\phi-\phi_s)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_s)} \right) + \epsilon \sin(\phi + \phi_s) F_{UT}^{\sin(\phi+\phi_s)} + \epsilon \sin(3\phi - \phi_s) F_{UT}^{\sin(3\phi-\phi_s)} \right. \\
 & \left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_s F_{UT}^{\sin\phi_s} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_s) F_{UT}^{\sin(2\phi-\phi_s)} \right] \\
 & + |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_s) F_{LT}^{\cos(\phi-\phi_s)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_s F_{LT}^{\cos\phi_s} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_s) F_{LT}^{\cos(2\phi-\phi_s)} \right] \Big\} ,
 \end{aligned}$$

Diagram illustrating the structure functions and their combinations for jet probes. The combinations are categorized into boxes:

- Cahn-effect + BM $\otimes$ Collins** (includes $F_{UU,T}$ and $F_{UU,L}$)
- Worm-gear (Kotzinian-Mulders) $\otimes$ Collins** (includes $F_{UU}^{\cos\phi}$ and $F_{UU}^{\sin(2\phi)}$)
- BM $\otimes$ Collins** (includes $F_{LU}^{\sin\phi}$)
- Sivers $\otimes$ D1** (includes $F_{UL}^{\sin\phi}$ and $F_{UL}^{\sin(2\phi)}$)
- Worm-gear $\otimes$ D1** (includes $F_{UT,T}^{\sin(\phi-\phi_s)}$ and $F_{UT,L}^{\sin(\phi-\phi_s)}$)
- Transversity $\otimes$ Collins** (includes $F_{UT}^{\sin(\phi+\phi_s)}$ and $F_{UT}^{\sin(3\phi-\phi_s)}$)
- Pretzelosity $\otimes$ Collins** (includes $F_{UT}^{\sin\phi_s}$ and $F_{UT}^{\sin(2\phi-\phi_s)}$)

Red 'X' marks indicate combinations that are not allowed or are zero. Blue checkmarks indicate allowed combinations.

Needs 2 polarized beams for jet probes, highly inefficient

Jet Probes

- Non-perturbative phase

○ Chiral-odd TMDs

(Collins) Chiral-odd FFs

$$\propto \int e^{ik^+x^-} e^{-k_\perp \cdot x_\perp} \gamma^5 \gamma^+ s_\perp \cdot \gamma_\perp \langle 0 | \psi(x^-, x_\perp) | hX \rangle \langle hX | \bar{\psi} | 0 \rangle$$

$$\sigma(\phi, \phi_S) \equiv \frac{d^6 \sigma}{dx dy dz d\phi d\phi_S dP_{\perp T}^2} = \frac{1}{xyQ^2 2(1-\epsilon)} \left(1 + \frac{\gamma^2}{2x} \right) \times$$

$$\left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos\phi F_{UU}^{\cos\phi} + \epsilon \cos(2\phi) F_{UU}^{\cos(2\phi)} + \lambda_e \left[\sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \right] + \right.$$

$$\left. + S_L \left[\sqrt{2\epsilon(1+\epsilon)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin(2\phi) F_{UL}^{\sin(2\phi)} \right] + S_L \lambda_e \left[\sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi F_{LL}^{\cos\phi} \right] \right.$$

$$\left. + |S_T| \left[\sin(\phi - \phi_S) \left(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \right) + \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \right] \right.$$

$$\left. + \sqrt{2\epsilon(1+\epsilon)} \sin\phi_S F_{UT}^{\sin\phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \right]$$

$$+ |S_T| \lambda_e \left[\sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi_S F_{LT}^{\cos\phi_S} + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \right] \Big\} ,$$

Diagram labels and connections:

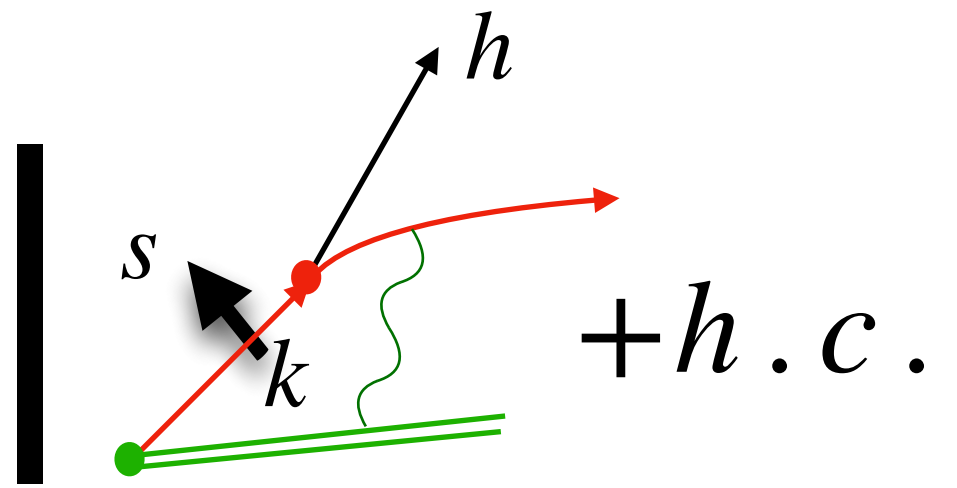
- Worm-gear (Kotzinian-Mulders) ⊗ Collins** (orange oval) points to the $F_{UU}^{\cos\phi}$ and $F_{UU}^{\cos(2\phi)}$ terms.
- Cahn-effect + BM ⊗ Collins** (orange oval) points to the $F_{UU,T}$ and $F_{UU,L}$ terms.
- BM ⊗ Collins** (orange oval) points to the $F_{LU}^{\sin\phi}$ term.
- Sivers ⊗ D1** (blue box) points to the $F_{UL}^{\sin\phi}$ and $F_{UL}^{\sin(2\phi)}$ terms.
- Worm-gear ⊗ D1** (blue box) points to the $F_{UT,T}^{\sin(\phi-\phi_S)}$ and $F_{UT,L}^{\sin(\phi-\phi_S)}$ terms.
- Transversity ⊗ Collins** (orange oval) points to the $F_{UT}^{\sin(\phi+\phi_S)}$ term.
- Pretzelosity ⊗ Collins** (orange oval) points to the $F_{UT}^{\sin(3\phi-\phi_S)}$ term.

$$\bar{\psi}_n \gamma^\mu \psi_{\bar{n}} \bar{\psi}_{\bar{n}} \gamma^\nu \psi_n$$

$$\rightarrow A^{\mu\nu} \bar{\psi}_n \not{n} \psi_n \bar{\psi}_{\bar{n}} \not{n} \psi_{\bar{n}} + B_{\alpha\beta}^{\mu\nu} \bar{\psi}_n \not{n} \gamma_\perp^\alpha \gamma^5 \psi_n \bar{\psi}_{\bar{n}} \not{n} \gamma_\perp^\beta \gamma^5 \psi_{\bar{n}} + \dots$$

Jet Probes

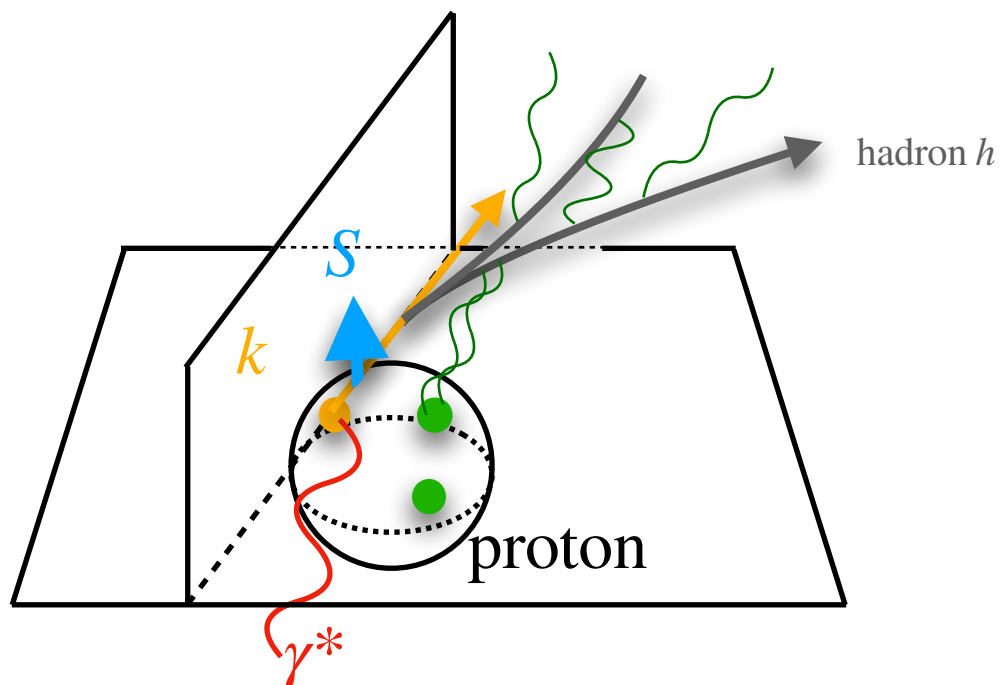
- Non-perturbative phase



$$+ h.c. \left| \right|^2 \propto M \operatorname{Im}(\sigma) \epsilon_{\alpha\beta\mu\nu} k_{\perp}^{\alpha} s_{\perp}^{\beta} P_h^{\mu} n^{\nu}$$

Collins, Metz + ...

Final state interactions
generate an asymmetry

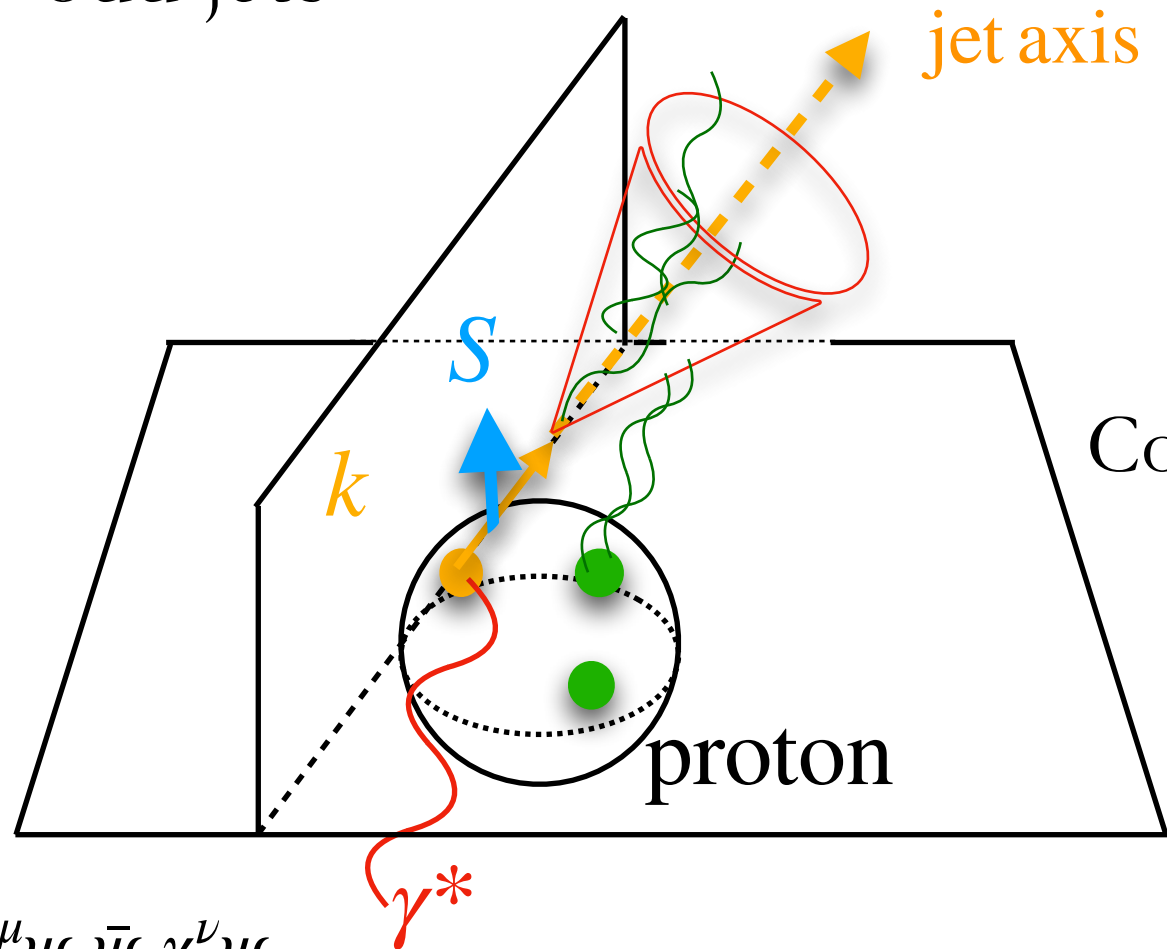


To observe the asymmetry :

- h not in the k - S plane
- Non-perturbative

Jet Probes

- The T-odd jets



Conventional wisdom:

- Jet is perturbative
- Jet axis is in the k-S plane

$$\bar{\psi}_n \gamma^\mu \psi_{\bar{n}} \bar{\psi}_{\bar{n}} \gamma^\nu \psi_n$$

$$\rightarrow A^{\mu\nu} \bar{\psi}_n \not{n} \psi_n \bar{\psi}_{\bar{n}} \not{n} \psi_{\bar{n}} + B^{\mu\nu}_{\alpha\beta} \bar{\psi}_n \not{n} \gamma^\alpha_\perp \gamma^5 \psi_n \boxed{\bar{\psi}_{\bar{n}} \not{n} \gamma^\beta_\perp \gamma^5 \psi_{\bar{n}}} + \dots$$

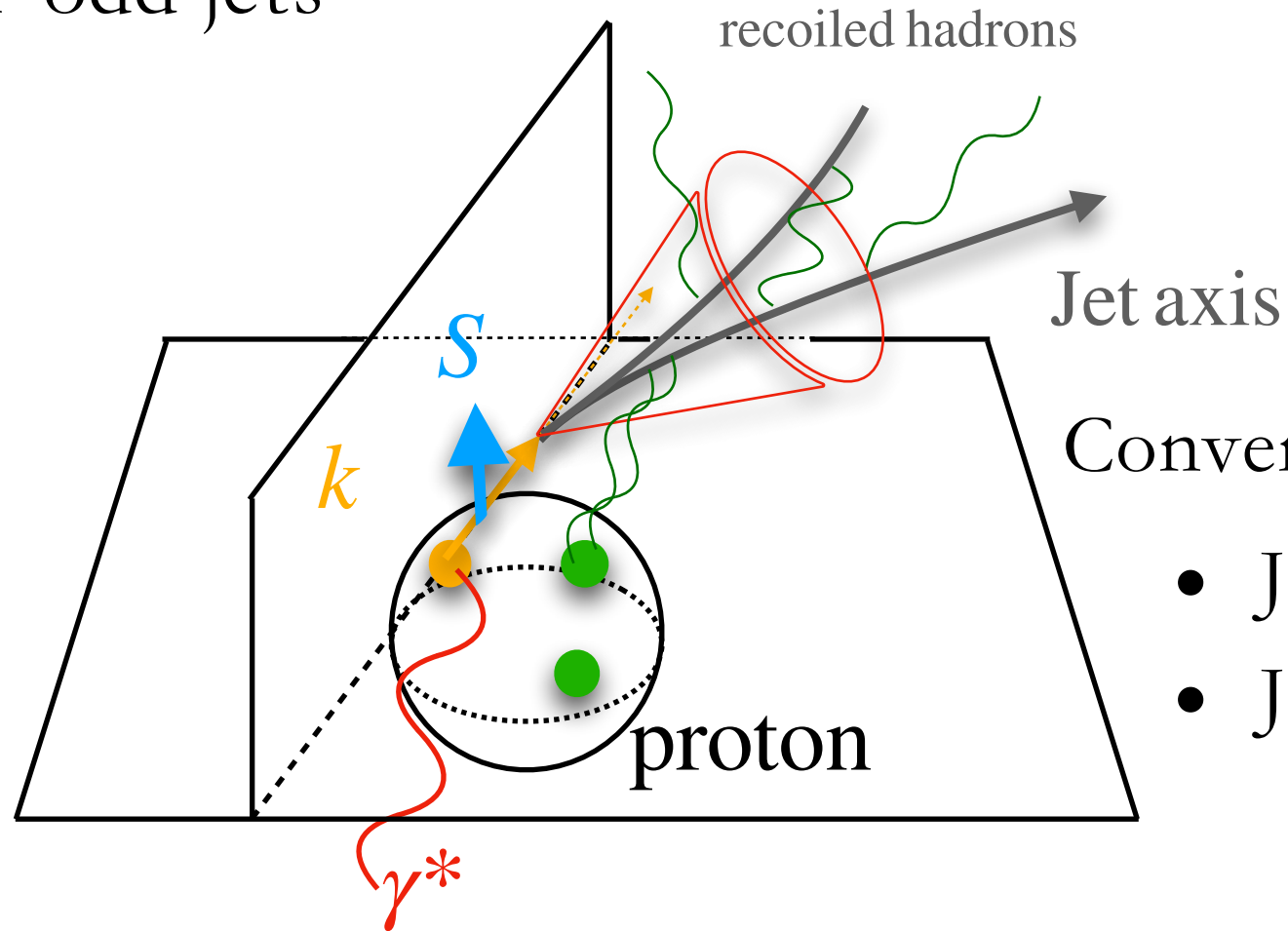
$$\langle 0 | \bar{\psi}(x) | JX \rangle \langle JX | \psi | 0 \rangle \propto \frac{\not{n}}{2} J + i \frac{k \not{n} \gamma_5}{2} J_T$$

Conventional
jet function



Jet Probes

- The T-odd jets



- Jet is perturbative
- Jet axis is in the k - S plane

Depends on
observable!

Not true

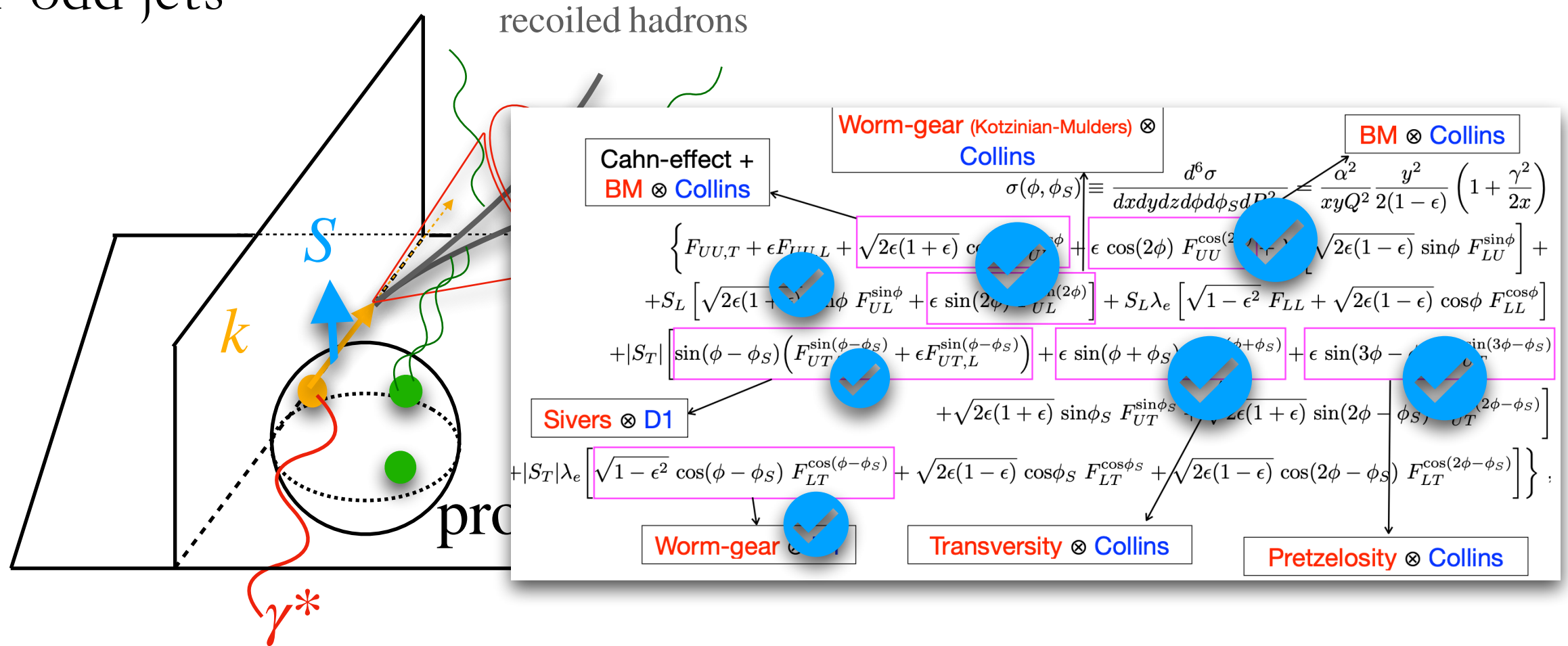
$$\langle 0 | \bar{\xi}_\alpha(x) | JX \rangle \langle JX | \xi_\beta | 0 \rangle \propto \frac{\hbar}{2} J + i \frac{k \hbar \gamma_5}{2} J_T$$



XL, Xing, 2104.03328, 2021

Jet Probes

- The T-odd jets



$$\langle 0 | \bar{\xi}_\alpha(x) | JX \rangle \langle JX | \xi_\beta | 0 \rangle \propto \frac{\hbar}{2} J + i \frac{k \hbar \gamma_5}{2} J_T$$

Unlock almost all possibilities

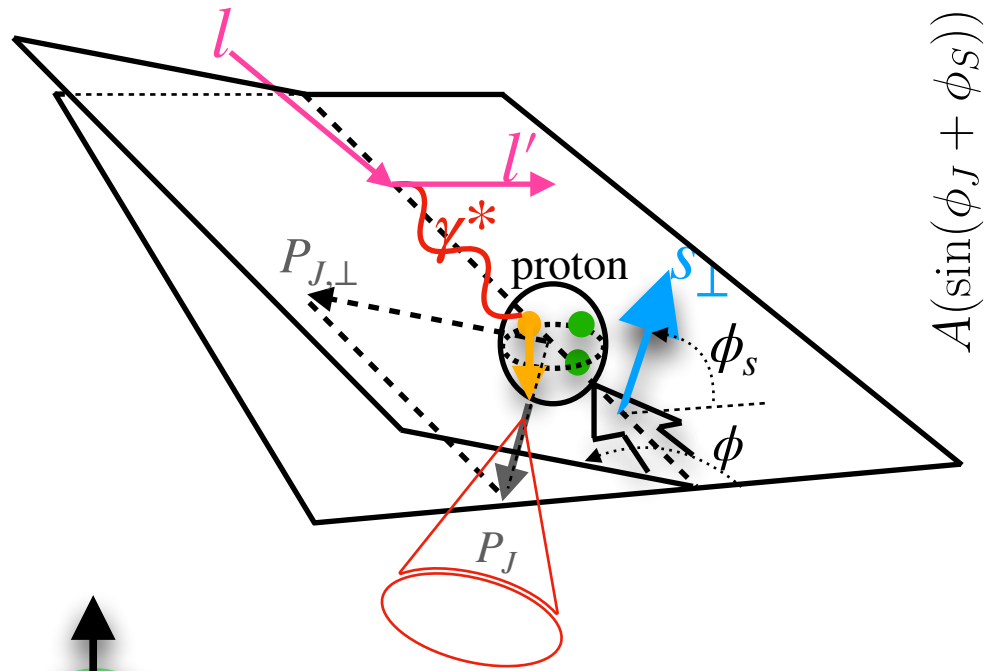
Only need one polarized beam

With reduced d.o.f.s and more flexibility

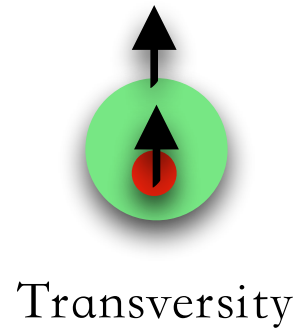
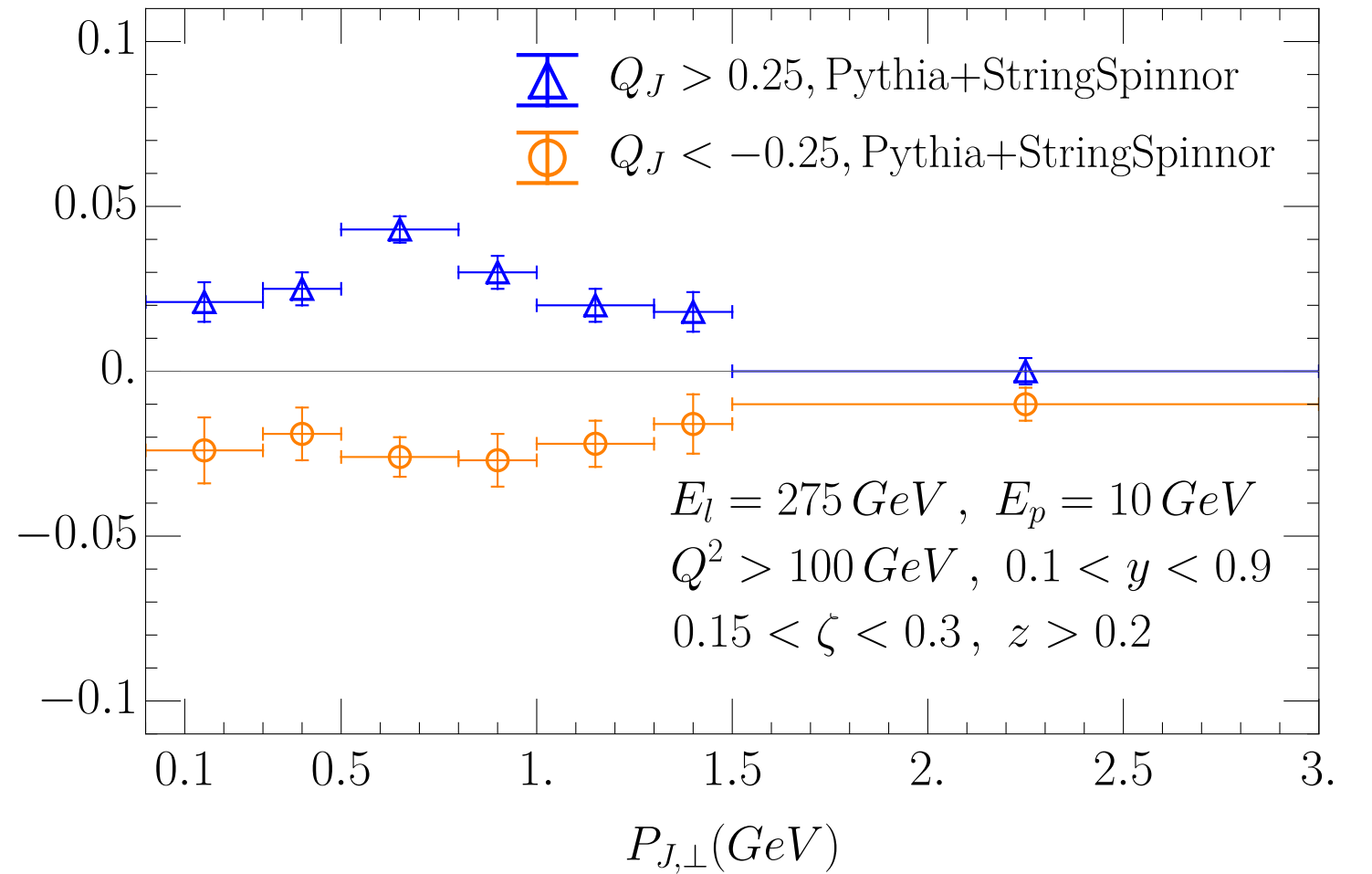
XL, Xing, 2104.03328, 2021

Jet Probes

- The T-odd jets



$$l + p(P, s_T) \rightarrow J_{anti-l, R=1}^{\text{WTA}}(P_J) + l' + X$$

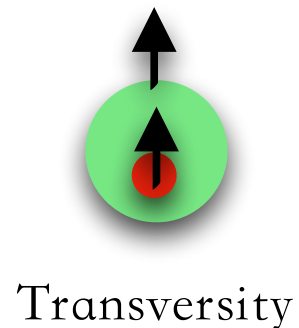
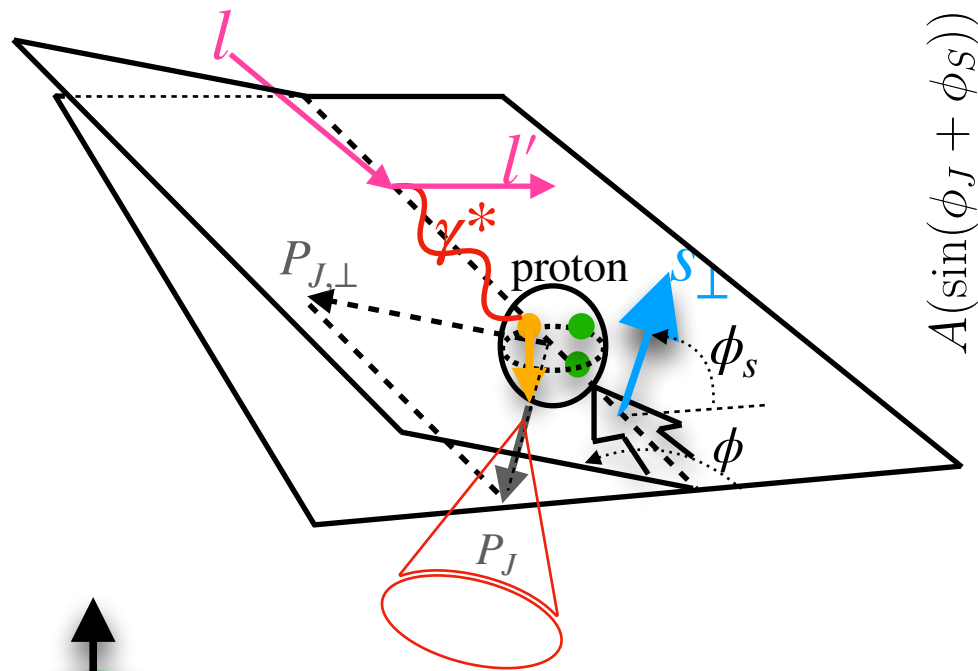


$$A = 1 + \epsilon |s_{\perp}| \sin(\phi_J + \phi_s) \frac{F_{UT}}{F_{UU}}$$

$$F_{UT} = \sum_q e_q^2 \int \frac{d^2 b}{4\pi^2} e^{-iP_{J\perp} \cdot b_{\perp}} i \frac{P_{J\perp}^{\alpha}}{P_{J\perp}} \zeta h_1^q(\zeta, b) \partial_{b^{\alpha}} J_T^q(b)$$

Jet Probes

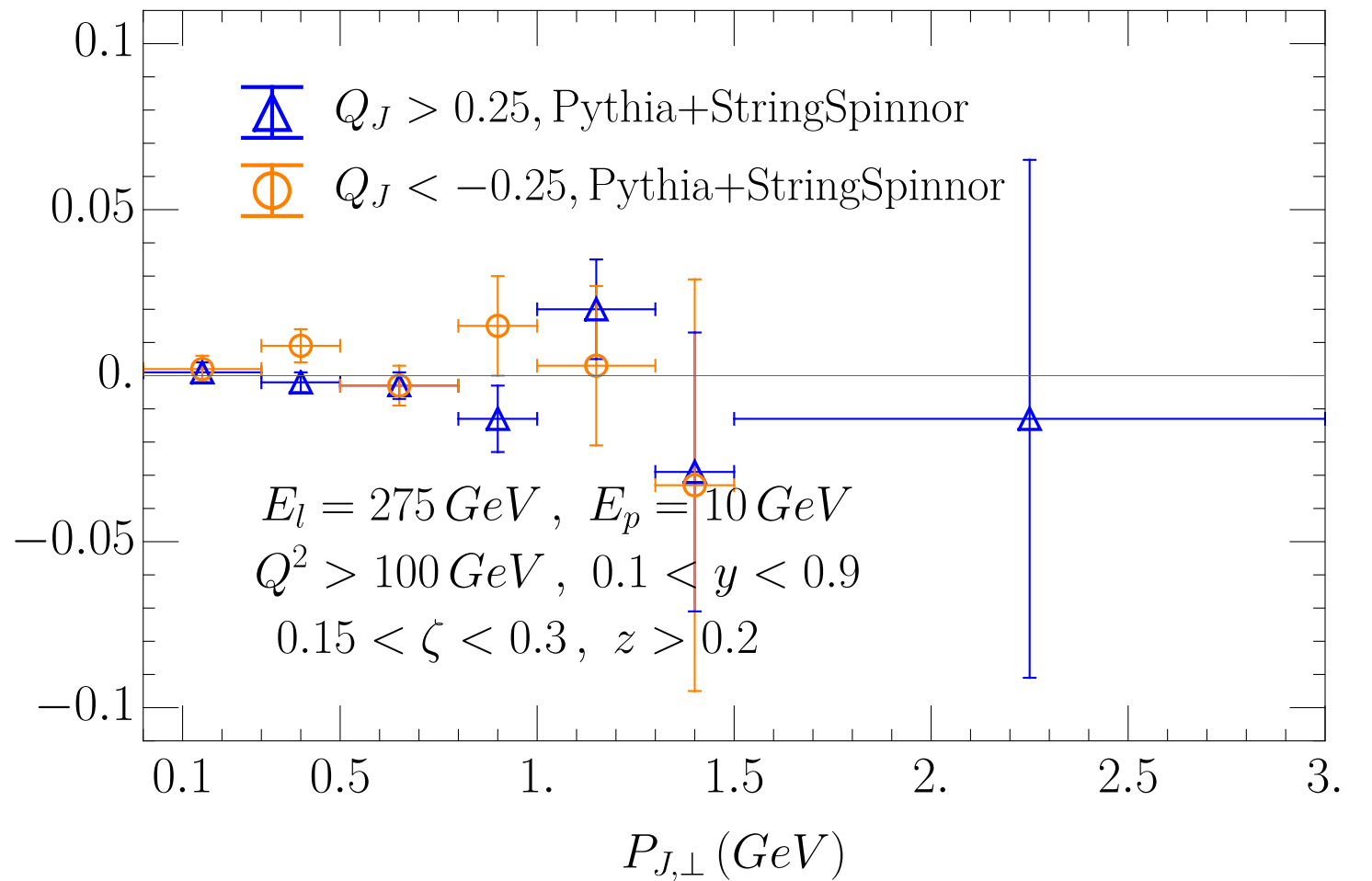
- The T-odd jets



$$A = 1 + \epsilon |s_{\perp}| \sin(\phi_J + \phi_s) \frac{F_{UT}}{F_{UU}}$$

$$F_{UT} = \sum_q e_q^2 \int \frac{d^2b}{4\pi^2} e^{-iP_{J\perp} \cdot b_{\perp}} i \frac{P_{J\perp}^{\alpha}}{P_{J\perp}} \zeta h_1^q(\zeta, b) \partial_{b^{\alpha}} J_T^q(b)$$

$$l + p(P, s_T) \rightarrow J_{anti-l, R=1}^{E \text{ scheme}}(P_J) + l' + X$$

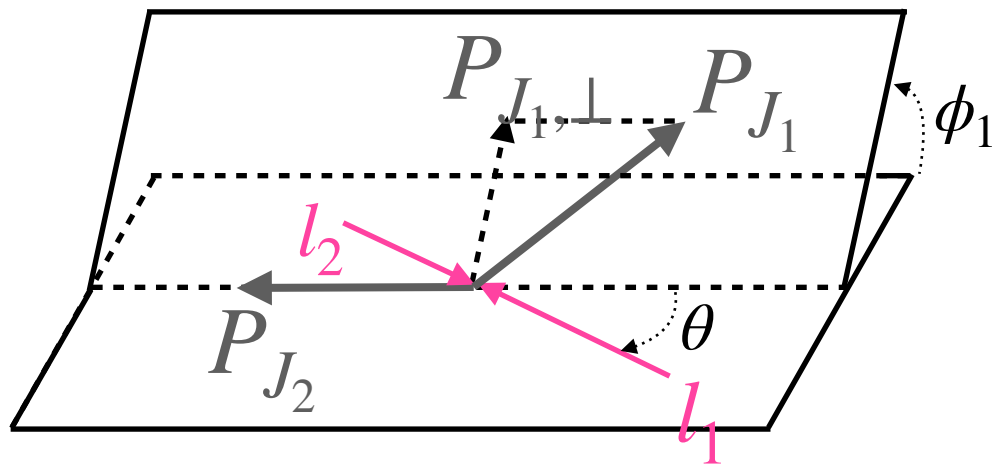


Different jet axis leads to different sensitivity

Lai, XL, Xing, Wang in preparation

Jet Probes

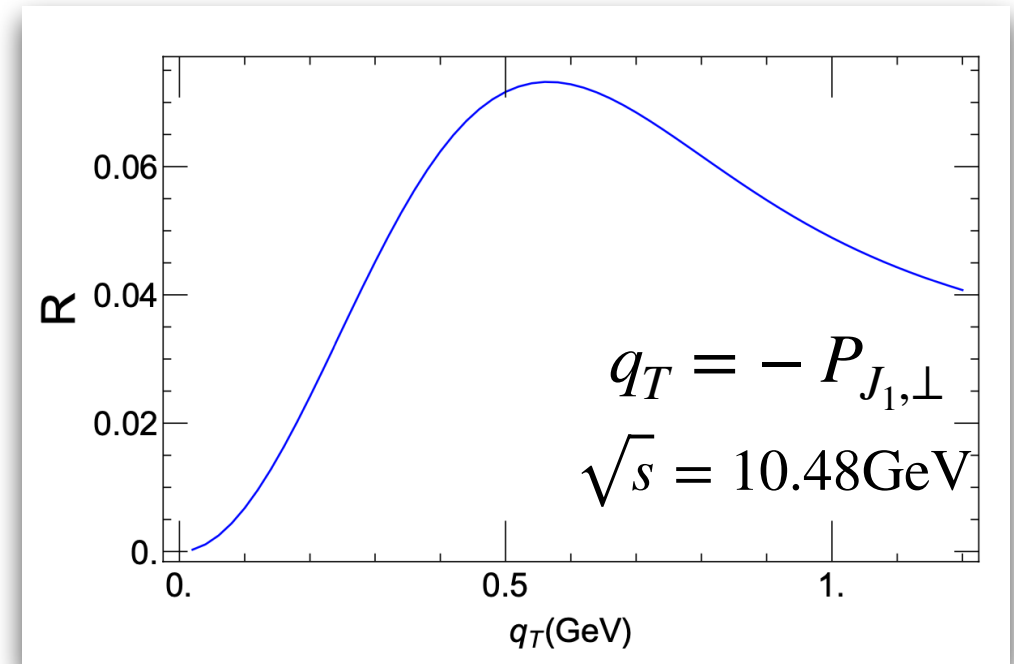
- The T-odd jets



$$R = 1 + \cos(2\phi_1) \frac{\sin^2 \theta}{1 + \cos^2 \theta} \frac{F_T}{F_U}$$

$$F_T = q_T \sum_q e_q^2 \int \frac{d^2 b}{4\pi^2} e^{-iq_T \cdot b} \left(2 \frac{q_T^\alpha q_T^\beta}{q_T^2} + g^{\alpha\beta} \right) \partial_{b^\alpha} J_T^\alpha(b) \partial_{b^\beta} J_T^{\bar{q}}(b)$$

$$\partial_{b^\alpha} J_T^q \partial_{b^\beta} J_T^{\bar{q}} = e^{-S_{pert.} - S_{NP}^T} \frac{b^\alpha b^\beta}{4} \mathcal{N}_q(b) \mathcal{N}_{\bar{q}}(b)$$



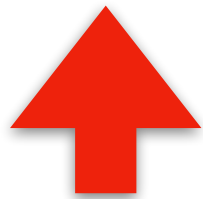
Can be extracted directly from BELLE or BaBar data
The lower the jet energy the better (if statistics guaranteed)

XL, Xing, 2104.03328, 2021

Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

$$f(x) = \int dz^- e^{-ixM_h x z^-} \langle h | \bar{\psi}(z^-) \gamma^+ \psi(0) | h \rangle \quad z^- = (z, 0, 0, -z)$$



Same IR

$$f(x) = C \otimes f_{quasi} + \mathcal{O}\left(\frac{1}{P_z}\right)$$



$$f_{quasi}(x) = \int dz^- e^{-ixM_h x z^-} \langle h | \bar{\psi}(0, -z) \gamma^+ \psi(0) | h \rangle$$



$$\langle \Omega | \mathcal{O}_h \bar{\psi} \gamma^+ \psi \mathcal{O}_h | \Omega \rangle$$

Path integral on Lattice
virtual time

Now it can be calculated by lattice QCD

See Wei's talk in this series.

Ji, PRL, 2013, Ma and Qiu, PRL 2018, Lian et al., PRD 2020, Lin, Chen, Wang, Yang, Yong, J. Zhang, + a long list

Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

$$f(x) = \int dz^- e^{-ixM_h x z^-} \langle h | \bar{\psi}(z^-) \gamma^+ \psi(0) | h \rangle \quad z^- = (z, 0, 0, -z)$$

$$= \int dz^- e^{-ixM_h x z^-} \langle h | e^{iH_z} \bar{\psi}(0, -z) e^{-iH_z} \gamma^+ \psi(0) | h \rangle$$

$$e^{-iH_z} \approx \lim_{\delta z \rightarrow 0, N \rightarrow \infty} \left[e^{-iH \delta z} \right]_N \quad \text{Trotter, 1959}$$

Classically: diagonalize Costy, hard

Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

$$f(x) = \int dz^- e^{-ixM_h x z^-} \langle h | \bar{\psi}(z^-) \gamma^+ \psi(0) | h \rangle \quad z^- = (z, 0, 0, -z)$$

$$= \int dz^- e^{-ixM_h x z^-} \langle h | e^{iH_z} \bar{\psi}(0, -z) e^{-iH_z} \gamma^+ \psi(0) | h \rangle$$

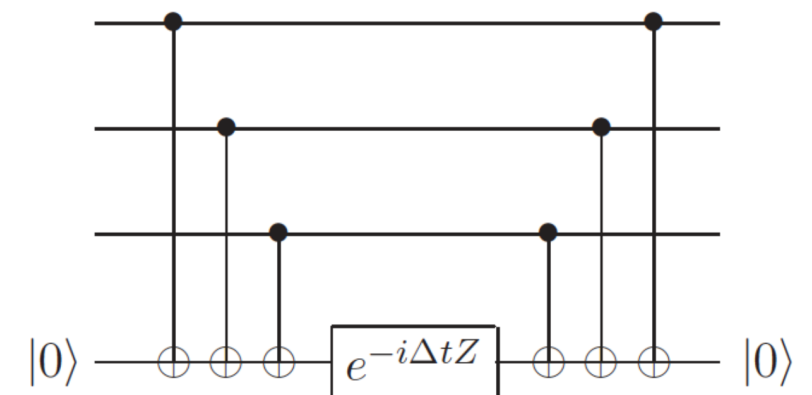
$$e^{-iH_z} \approx \lim_{\delta z \rightarrow 0, N \rightarrow \infty} [e^{-iH \delta z}]_N \quad \text{Trotter, 1959}$$

Quantum: decompose to set of gates

e.g. Local $e^{-i\sigma_x t} = H X R_z(-t) X R_z(t) H$

non-Local $H = Z_1 \otimes Z_2 \otimes Z_3$

$$e^{-iH\Delta t} =$$



Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

- A toy model

$$\mathcal{L} = \bar{\psi}(i\partial - m)\psi + g(\bar{\psi}\psi)^2 \quad (\text{no gauge, } 1+1) \quad \text{Gross, Neveu, 1974}$$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \begin{pmatrix} \phi_{2n} \\ \phi_{2n+1} \end{pmatrix} \quad \text{Staggered fermion} \quad \phi_n = \prod_{i < n} \sigma_i^3 \sigma_n^+ \quad \text{Jordan-Wigner}$$

$$f(x) \rightarrow \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi_{-2z+i}^\dagger e^{-iHz} \phi_j | h \rangle$$

$$H_1 = \sum_{n=\text{even}} \frac{1}{4} [\sigma_n^1 \sigma_{n+1}^2 - \sigma_n^2 \sigma_{n+1}^1] + H_2 + H_3 + H_4$$

Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

- A toy model

Prepare the state by QAOA

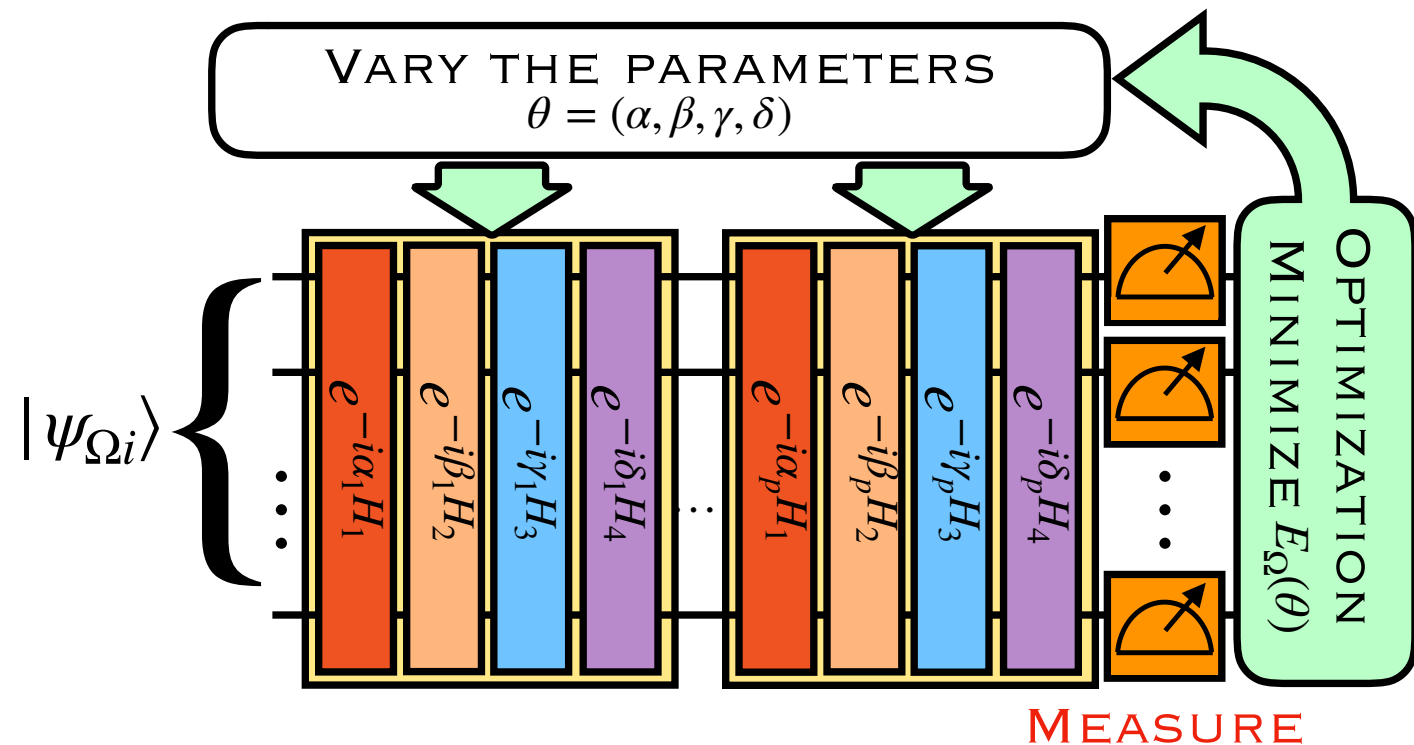
Farhi et al., 411.4028, 2014

$$|\psi_i(\theta)\rangle = \prod e^{iH_i\theta_i} |\psi_i\rangle \quad [H_i, H_{i+1}] \neq 0$$

$$E(\theta) = \sum_i^k w_i \langle \psi_i(\theta) | H | \psi_i(\theta) \rangle \quad \text{Minimize}$$

$$|\psi_\Omega\rangle = |0101\dots 01\rangle$$

$$|\psi_h\rangle = |1001\dots 01\rangle + |0110\dots 01\rangle + \dots$$



$$f(x) \rightarrow \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi_{-2z+i}^\dagger e^{-iHz} \phi_j | h \rangle$$

$$H_1 = \sum_{n=\text{even}} \frac{1}{4} [\sigma_n^1 \sigma_{n+1}^2 - \sigma_n^2 \sigma_{n+1}^1] + H_2 + H_3 + H_4$$

Quantum Simulation

Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

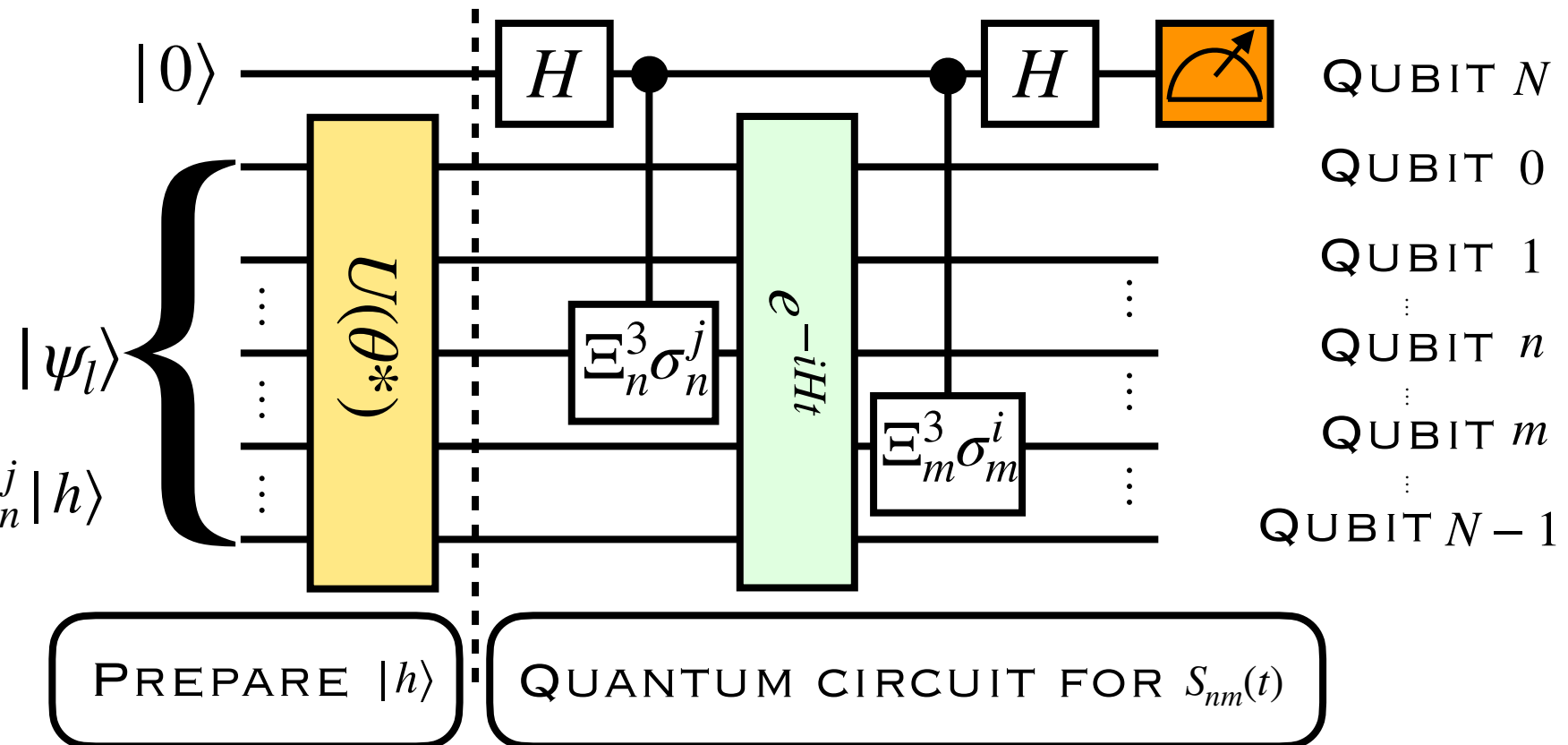
- A toy model

Evaluate evolution

$$S_{mn}(t) = \langle h | e^{iHt} \Xi_m^3 \sigma_m^i e^{-iHt} \Xi_n^3 \sigma_n^j | h \rangle$$

$$S_{mn}(t) = \frac{1}{2} p_{mn}(t) - \frac{1}{2}$$

Pedernales et al., PRL, 2014



$$f(x) \rightarrow \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iH_z} \phi_{-2z+i}^\dagger e^{-iH_z} \phi_j | h \rangle$$

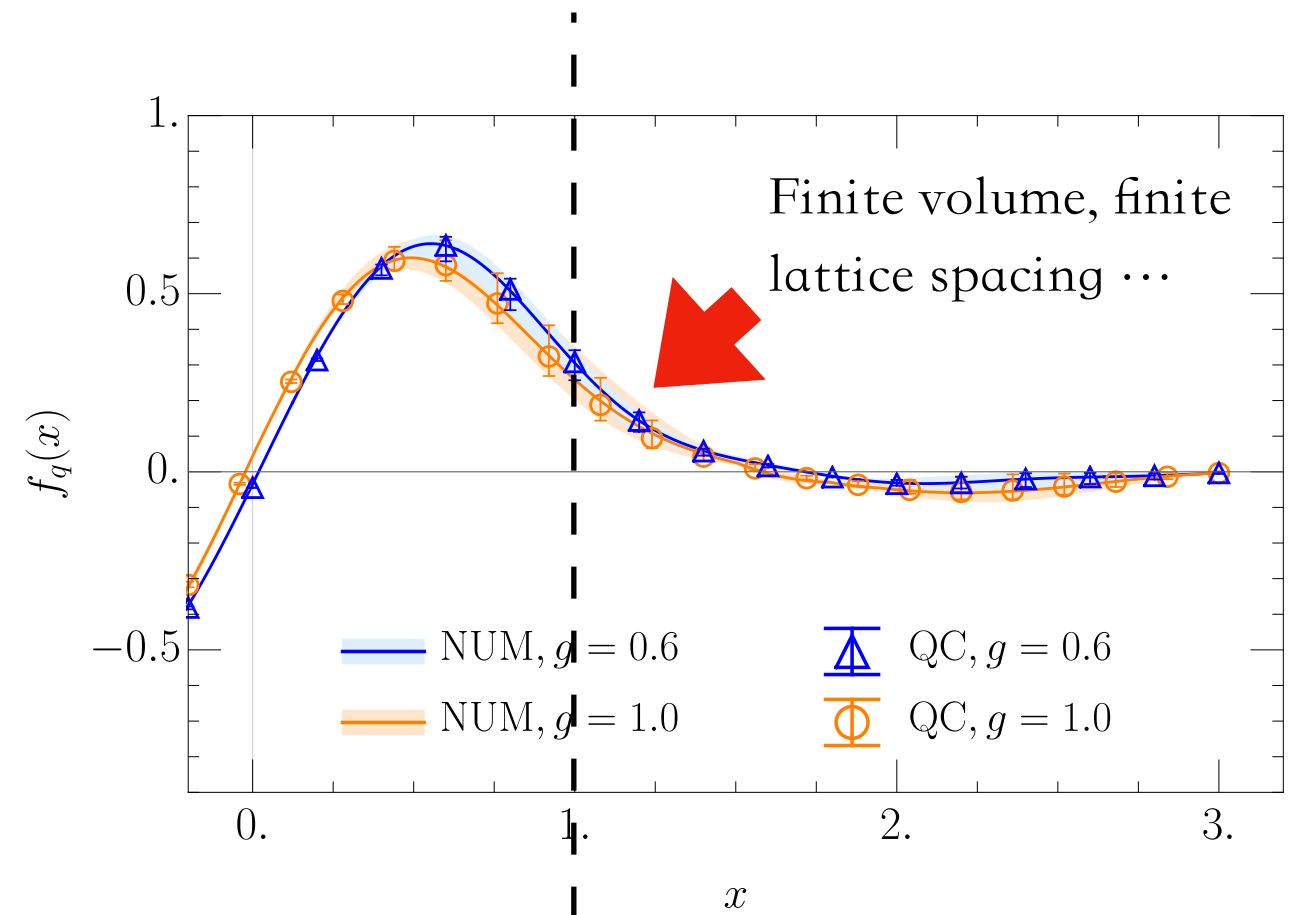
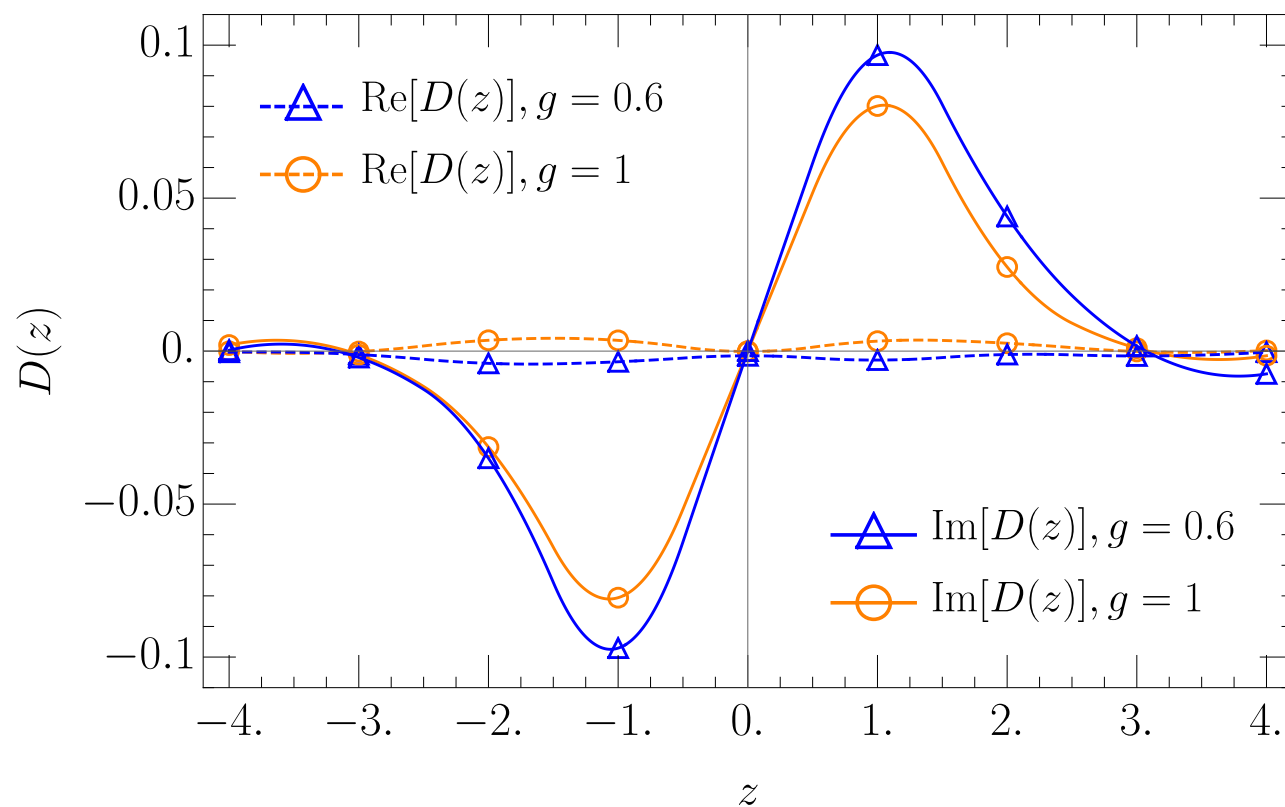
$$H_1 = \sum_{n=\text{even}} \frac{1}{4} [\sigma_n^1 \sigma_{n+1}^2 - \sigma_n^2 \sigma_{n+1}^1] + H_2 + H_3 + H_4$$

Quantum Simulation

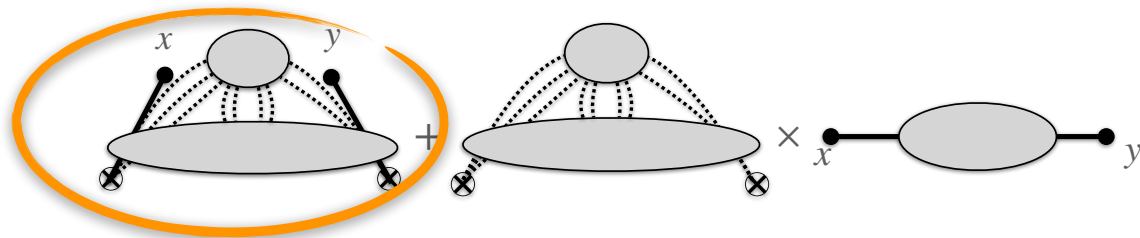
Li, Guo, Lai, XL, Wang, Xing, Zhang, Zhu, 2021

- A toy model

Results



$${}_H\langle h | T[\psi_H(x) \psi_H(y)] | h \rangle_H =$$



$$f(x) \rightarrow \sum_{i,j} \sum_z \frac{1}{4\pi} e^{-ixM_h z} \langle h | e^{iHz} \phi_{-2z+i}^\dagger e^{-iHz} \phi_j | h \rangle$$

Summary

- Jet probe
 - an active direction for hard probes of the nucleon intrinsic information.
 - New avenue to QCD strong dynamics
- Quantum Computation
 - Future direction to explore
 - The system and applications are still limited

Thanks