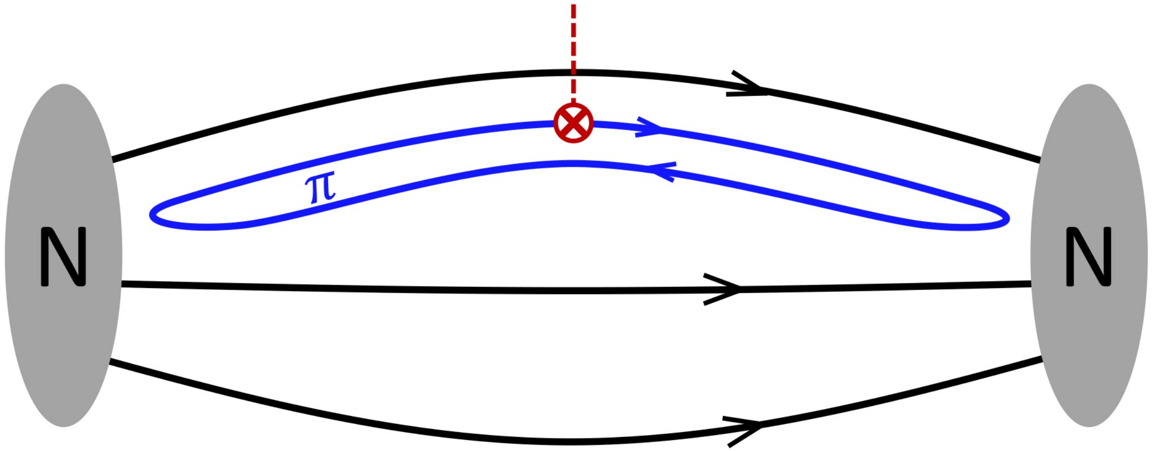
The pion–nucleon sigma term from lattice QCD

Rajan Gupta[1], Sungwoo Park[2], Martin Hoferichter[3],
Emanuelle Mereghetti[1], Boram Yoon[1], Tanmoy Bhattacharya[1]

[1] Los Alamos National Laboratory, NM
[2] Jefferson Lab, VA
[3] University of Bern



LA-UR-21-31181

Reference: arXiv:2105.12095, to appear in PRL

CD2021, November 16, 2021

$\sigma_{\pi N}$: pion–nucleon sigma term

$$\boxed{\sigma_{\pi N} \equiv m_{ud}\, g_S^{u+d} \equiv m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

- Fundamental parameter of QCD that quantifies the amount of the nucleon mass generated by u and d quarks.
- g_S^2: enters in cross-section of dark matter with nucleons
- Important input in the search of BSM physics

Three methods to calculate $\sigma_{\pi N}$

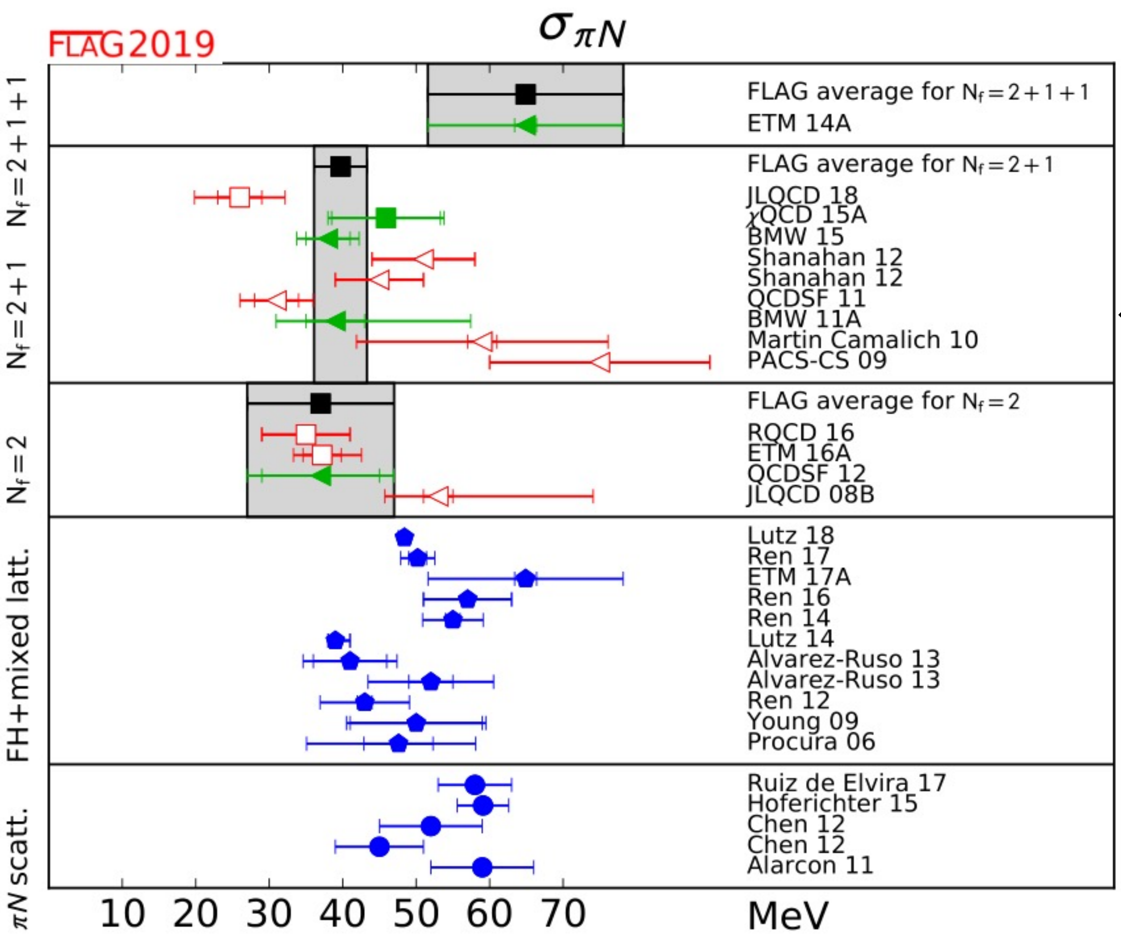
$$\boxed{\sigma_{\pi N} \equiv m_{ud}\, g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle}$$

- Lattice: direct calculation of $\langle N | \bar{u}u + \bar{d}d | N \rangle$
- Lattice: Feynman-Hellmann relation $\left(\frac{g_S^q}{Z_S} = \frac{\partial M_N}{\partial m_q}\right)$
- Phenomenology: connection to πN-scattering amplitude via Cheng-Dashen low-energy theorem

All discussion here assumes isospin symmetry. Effect ~1 MeV

Status of results for $\sigma_{\pi N}$ as of Dec 2020

- $\sigma_{\pi N} \equiv m_{ud} g_S^{u+d} \equiv m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$ in isospin limit



- Lattice
  - With 2+1+1, 2+1, 2-flavors
  - Squares: Direct
  - Triangles: Feynman-Hellmann

- Mixed Lattice: Feynman-Hellmann but taking data from multiple calculations

- Phenomenology: connection to πN-scattering amplitude via Cheng-Dashen low-energy theorem

Tension between lattice and phenomenology

- Lattice results favor ~40 MeV
- Phenomenology favors ~60 MeV

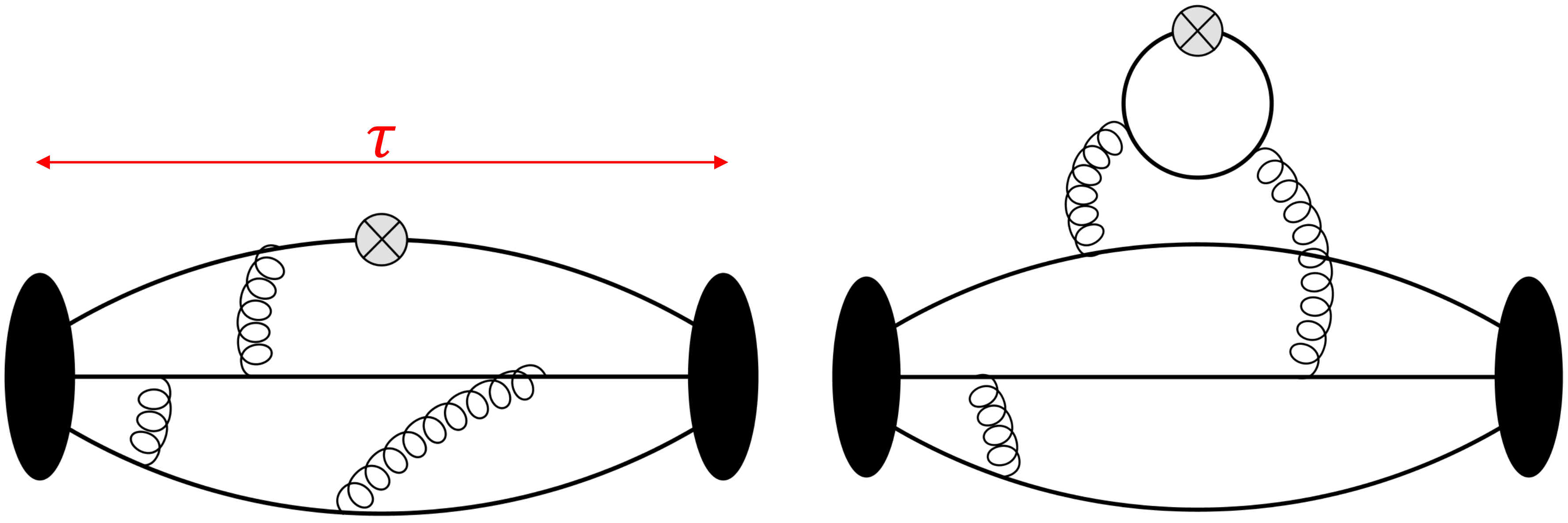
New results post FLAG 2019:

BMW (arXiv:2007.03319) $\sigma_{\pi N} = 37.4(5.1)$ MeV (FH)
ETM (PRD **102**, 054517) $\sigma_{\pi N} = 41.6(3.8)$ MeV (Direct method)

Lattice Methodology well established

Direct: "connected" and "disconnected" 3-point correlation functions

Nucleon charges g_A, g_T, and $\boldsymbol{g_S}$ obtained from ME of local quark bilinear operators $\bar{q}_i\,\Gamma\,q_j$ within ground state nucleons: $\langle N|\bar{q}_i\,\Gamma\,q_j|N\rangle$

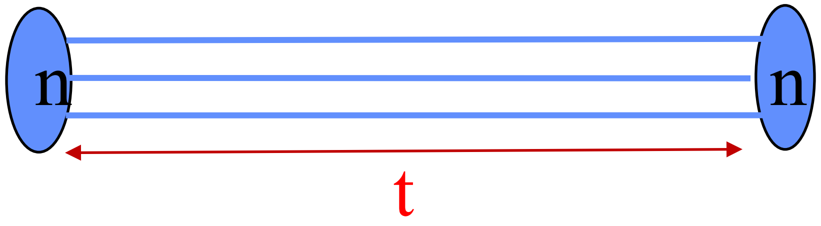


Connected **Disconnected**

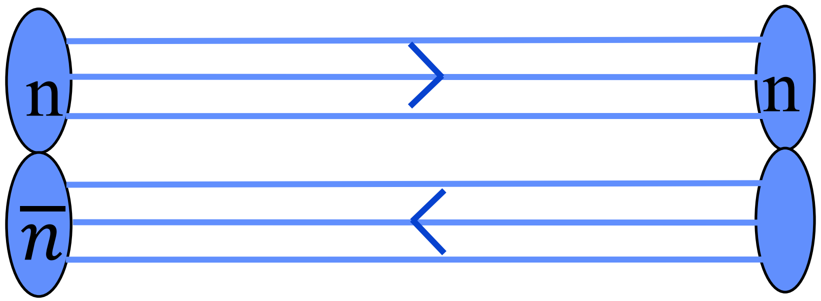
$$g_S^{u+d} = g_S^{u+d,\mathrm{conn}} + 2g_S^{l,disc}$$

Main Systematics in LQCD calculations of nucleons

1. Statistics
   - Signal in nucleon correlators falls as $e^{-(M_N-1.5M_\pi)\tau}$
   - Signal in "disconnected" more noisy than in "connected"



Signal in C_{2pt}: $e^{-E_N t}$



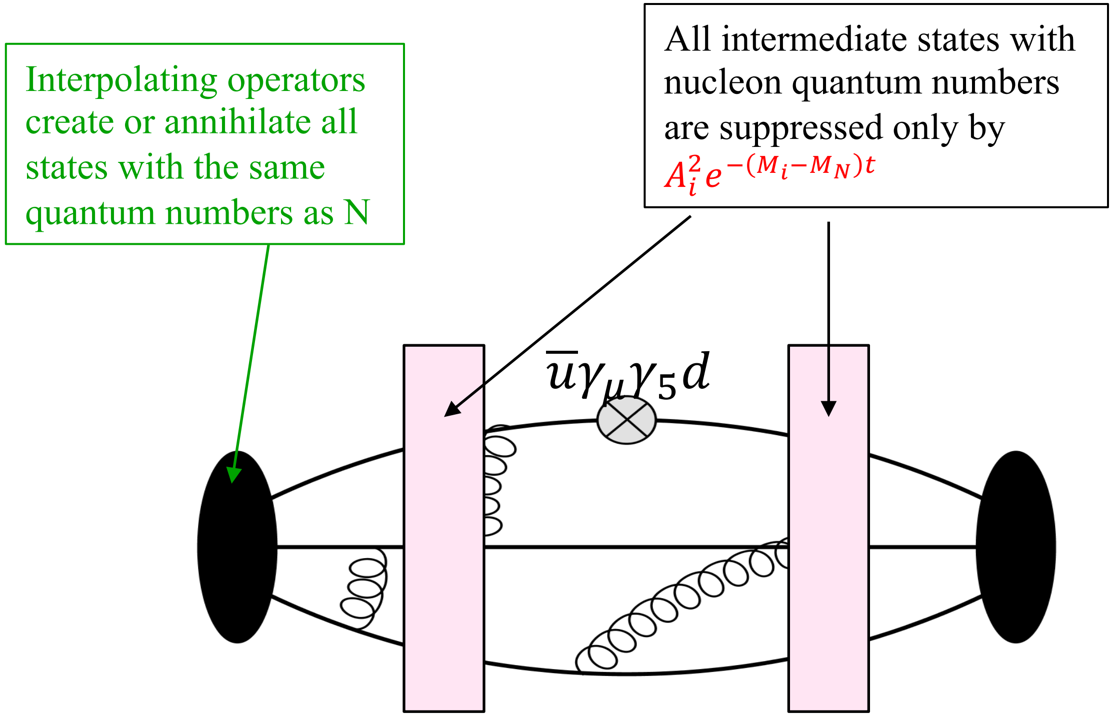
Variance: $e^{-3E_\pi t}$

Main Systematics in lattice calculations

2. Excited state contributions (ESC)
   a) Excited states are a challenge for nucleons versus pions
   b) Which excited states make a significant contribution?
   c) Towers of multihadron states starting at ~1200 MeV
   d) Do $N\pi$ / $N\pi\pi$ states contribute to a given ME?
   e) Fits using the spectral decomposition to connected plus disconnected contributions keeping up to 3 states

Excited states in correlation functions

Challenge: To get the matrix elements in the ground state of hadrons (nucleons), the contributions of all excited states have to be removed.



Towers of multihadron states

$$N(\vec{p})\pi(-\vec{p})$$
$$N(0)\pi(\vec{p})\pi(-\vec{p})$$
$$N(\vec{p})2\pi(-\vec{p})$$
$$\cdots$$

+ radial excitations

- Which excited states make significant contributions to a given matrix element?
- What are their energies in a finite box?

Spectral decomposition of 3-point function

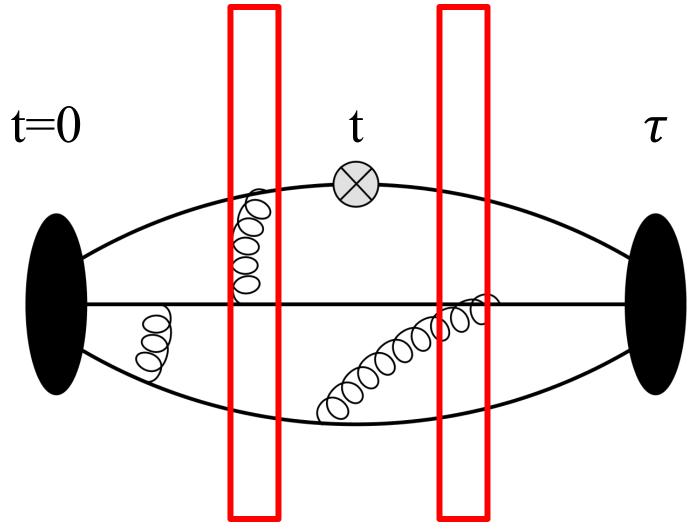
$$C^{3pt} = \langle 0|\mathcal{O}|0\rangle |A_0|^2 e^{-M_0\tau} \times \Big[1 + \frac{\langle 1|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|^2}{|A_0|^2}e^{-\Delta M_1\tau}$$

$$+ \frac{\langle 2|\mathcal{O}|2\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_2|^2}{|A_0|^2}e^{-(\Delta M_2+\Delta M_1)\tau}$$

$$+ \frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|}{|A_0|}e^{-\Delta M_1\frac{\tau}{2}} \times 2\cosh\left(\Delta M_1(t-\frac{\tau}{2})\right)$$

$$+(\cdots)$$

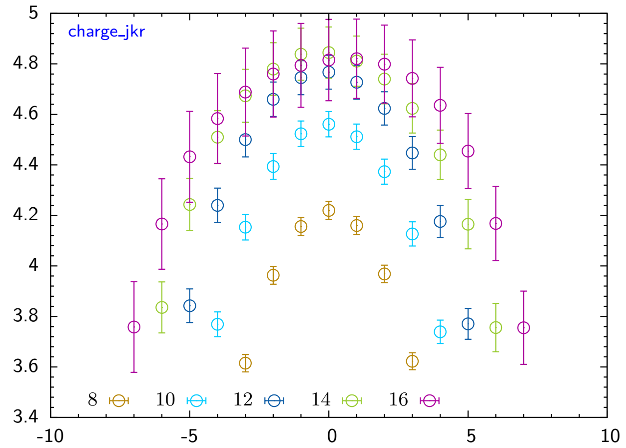
Ground-state matrix element → g_S



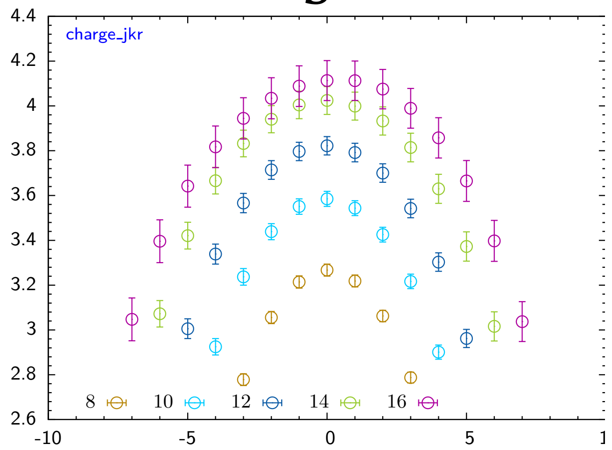
- All states with same quantum numbers as the nucleon are allowed
- A_i are large in some cases!
- Their mass gaps ΔM_i are small!

Data from physical $M_\pi \approx 135\ MeV$ ensemble @ $a \approx 0.09 fm,\ M_\pi L = 3.9$

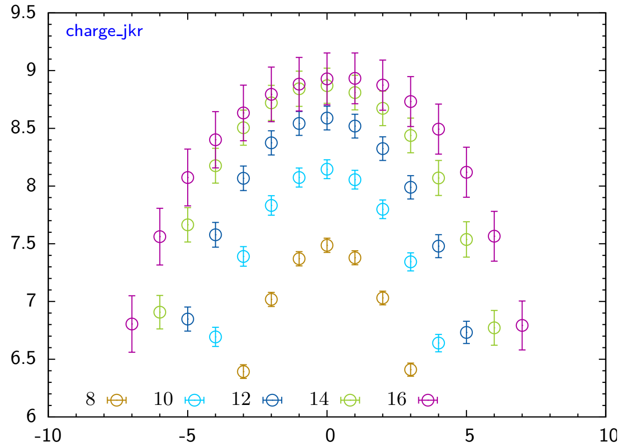
$g_S^{u,conn}$



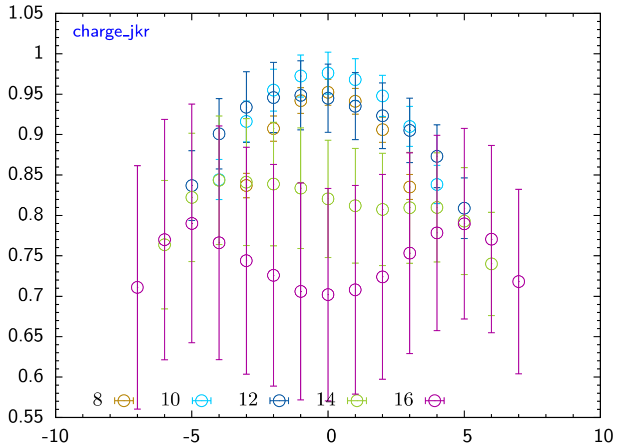
$g_S^{d,conn}$



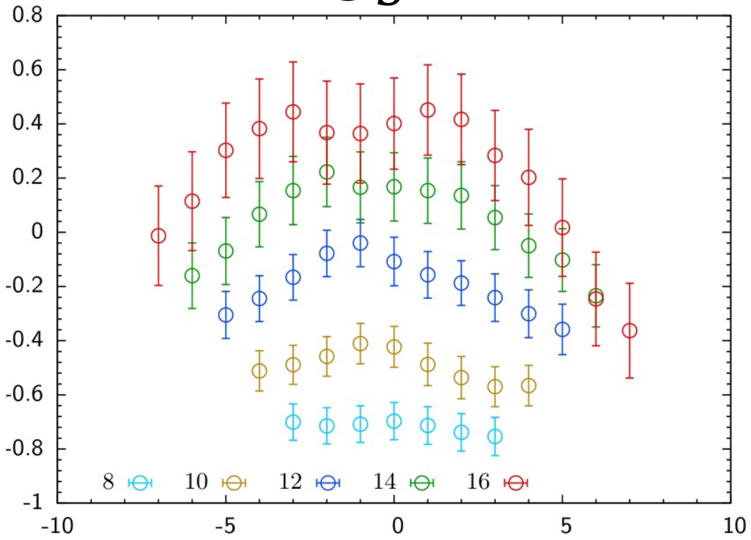
$g_S^{u+d,conn}$



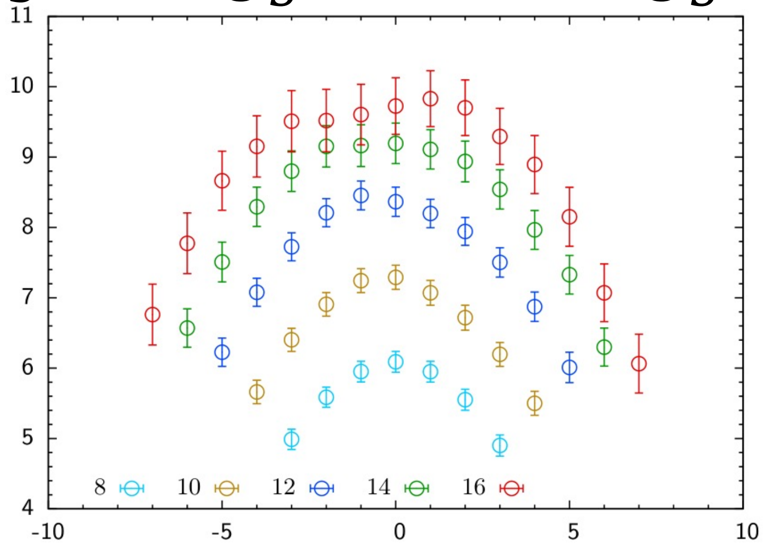
g_S^{u-d}



$g_S^{l,disc}$



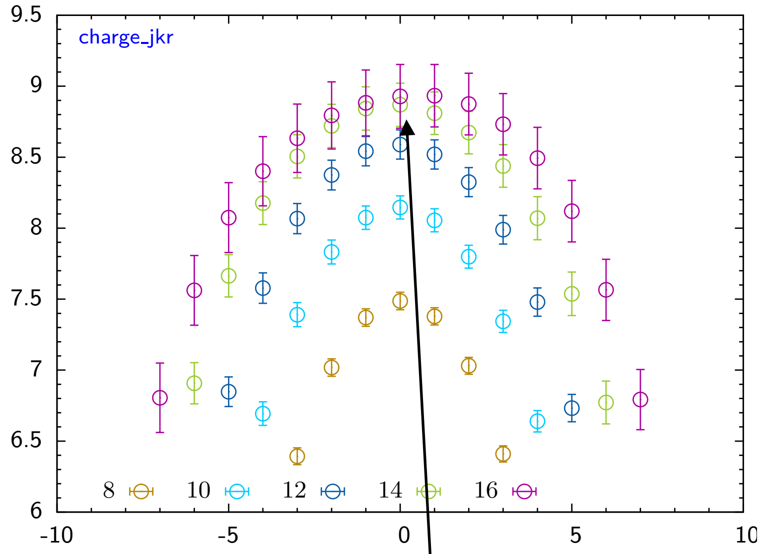
$g_S^{u+d} = g_S^{u+d,\mathrm{conn}} + 2 g_S^{l,\mathrm{disc}}$



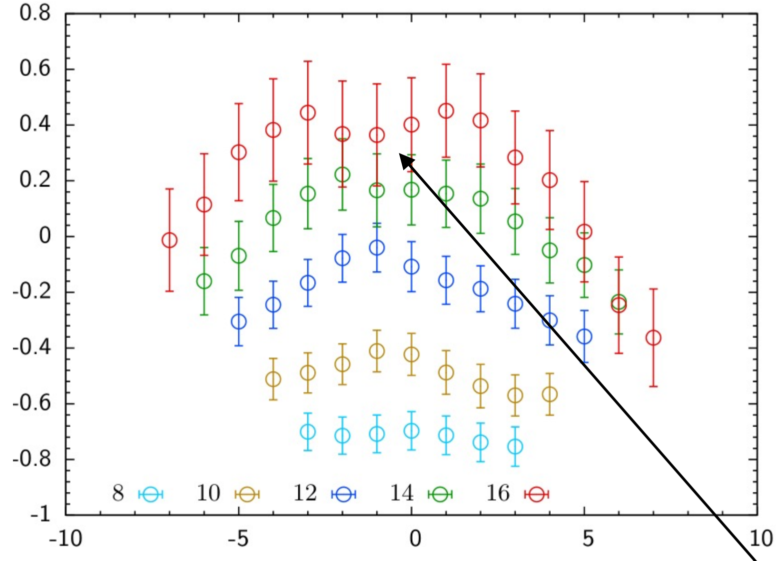
Also See S. Park, *et al.*, 2103.05599

Physical M_π Ensemble: $a \approx 0.09 fm,\ M_\pi = 135\ MeV,\ M_\pi L = 3.9$

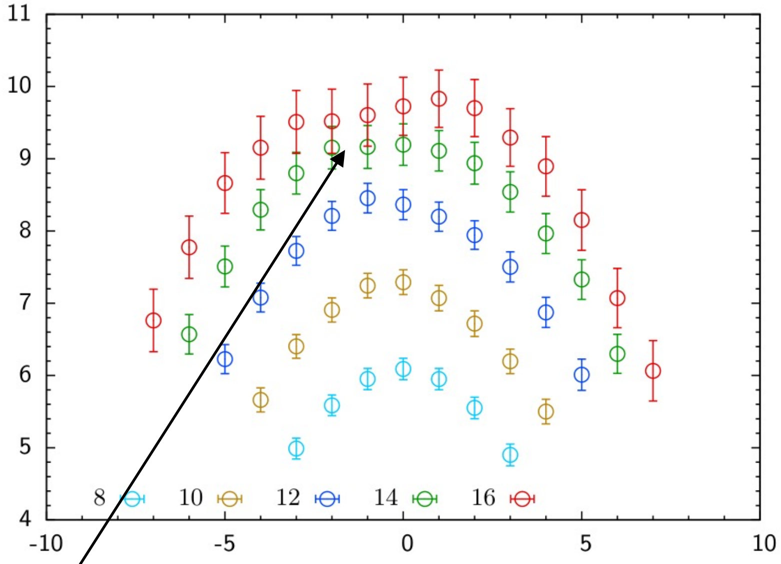
$g_S^{u+d,conn}$



$g_S^{l,disc}$



$g_S^{u+d} = g_S^{u+d,\text{conn}} + 2 g_S^{l,\text{disc}}$



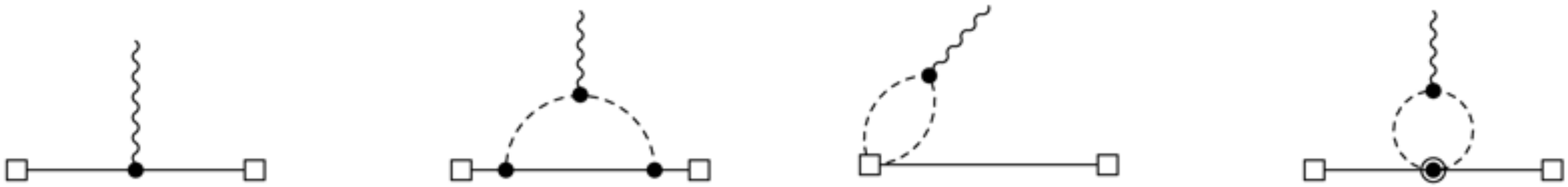
"Appears" to converge as $\tau \to \infty$

"No" convergence as $\tau \to \infty$

Excited-state contribution is large at realizable $\tau \approx 1.5$ fm

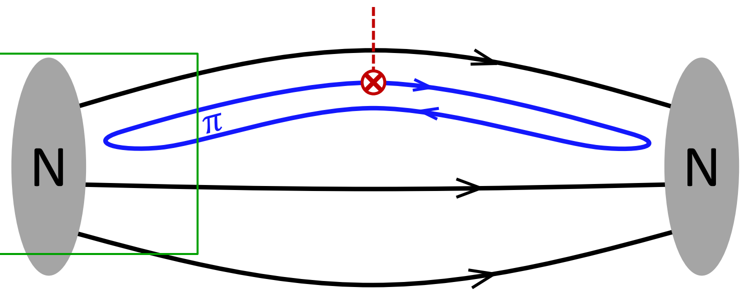
χPT analysis shows $\boldsymbol{N(\vec{k})\pi(-\vec{k})}$ and $\boldsymbol{N(0)\pi(\vec{k})\pi(-\vec{k})}$ states give significant contributions.

Coupling of S to $\boldsymbol{\pi\pi}$ is large

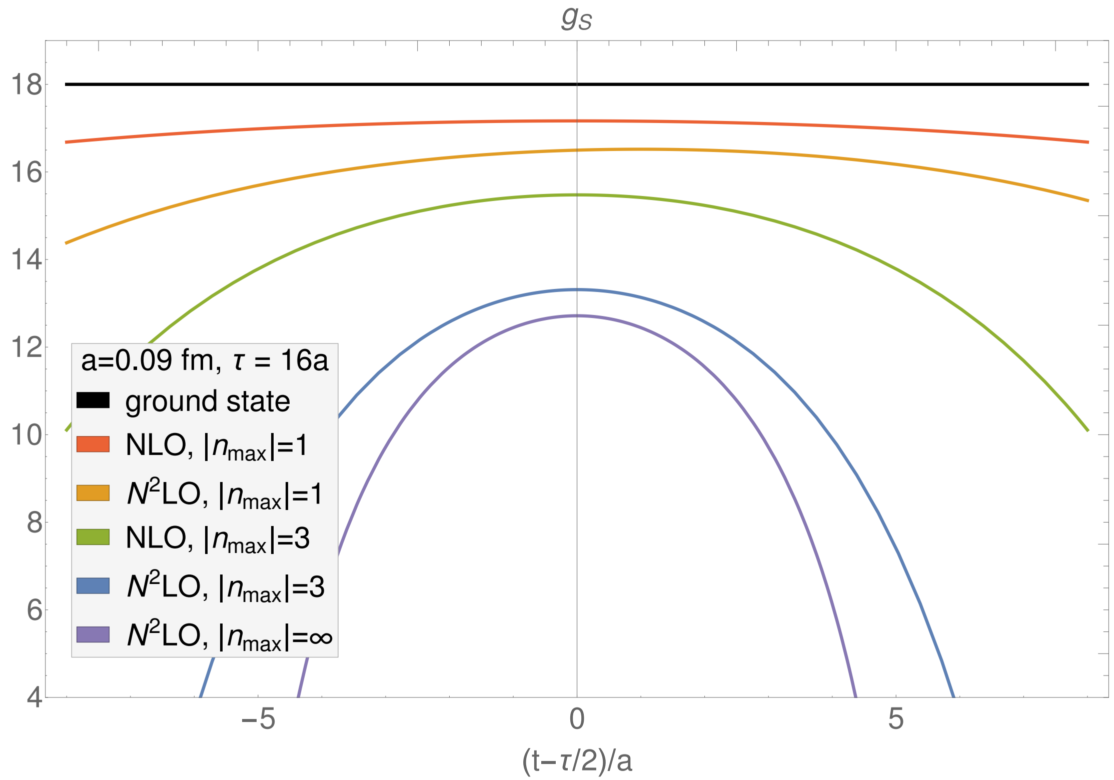


LO NLO NLO N²LO

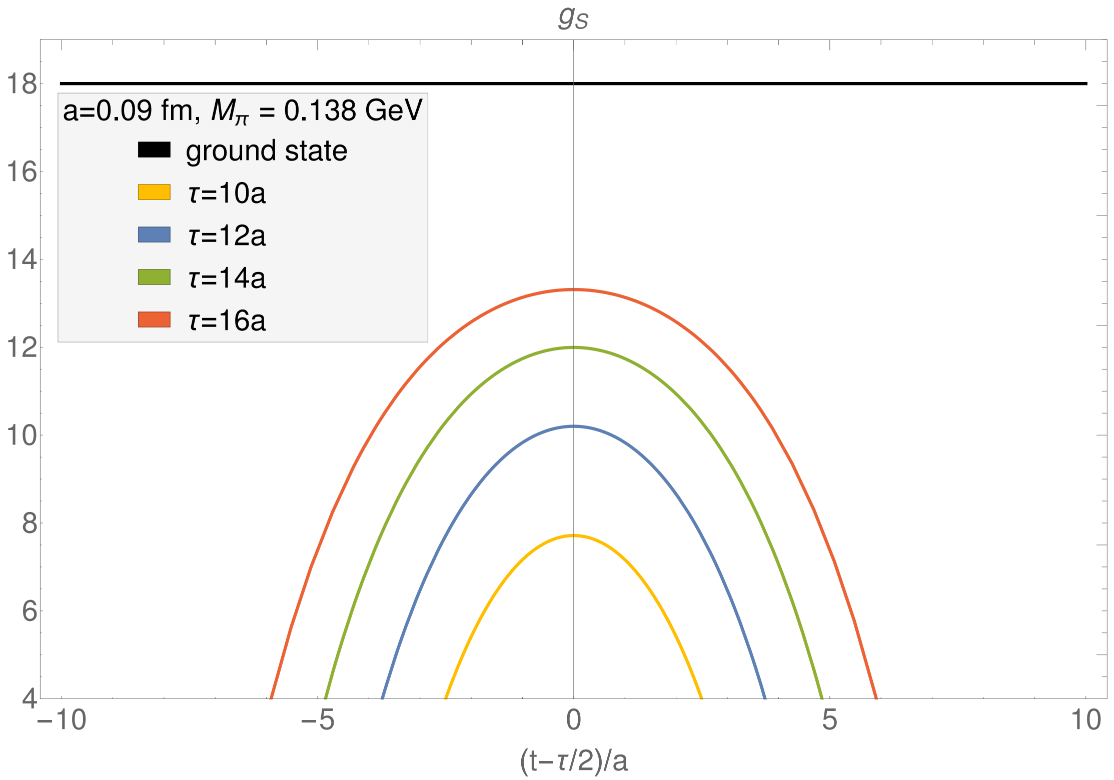
Why disconnected contribution is large



g_S: ESC from $N\pi$ / $N\pi\pi$ in N^{2}LO χPT



Different truncations (χPT order and $\vec{p}$)

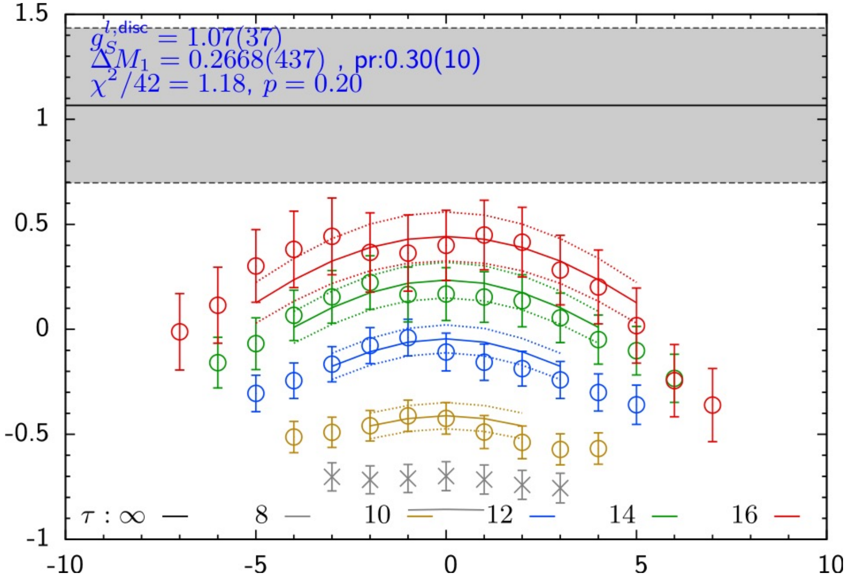


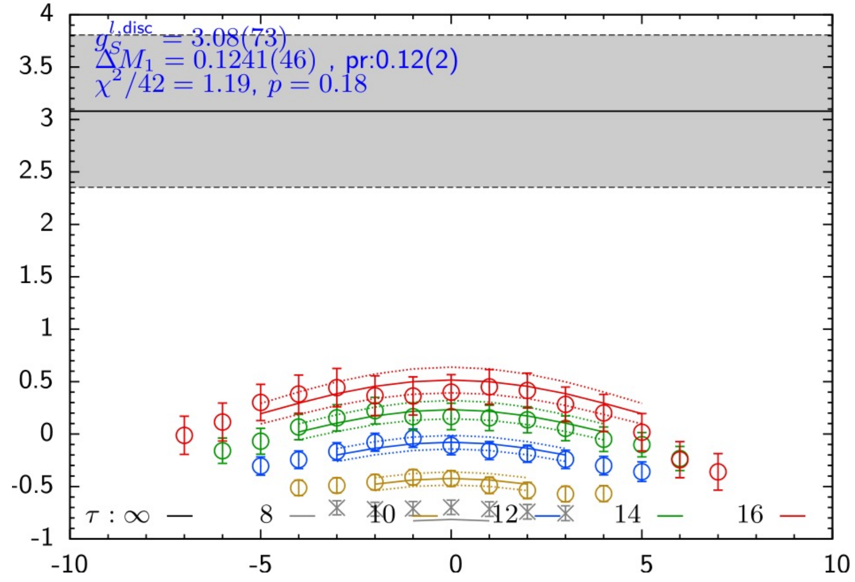
N^{2}LO χPT estimates for $\tau = 10,12,14,16$

The NLO and N^{2}LO ESC can each reduce $\sigma_{\pi N}$ at a level of 10 MeV

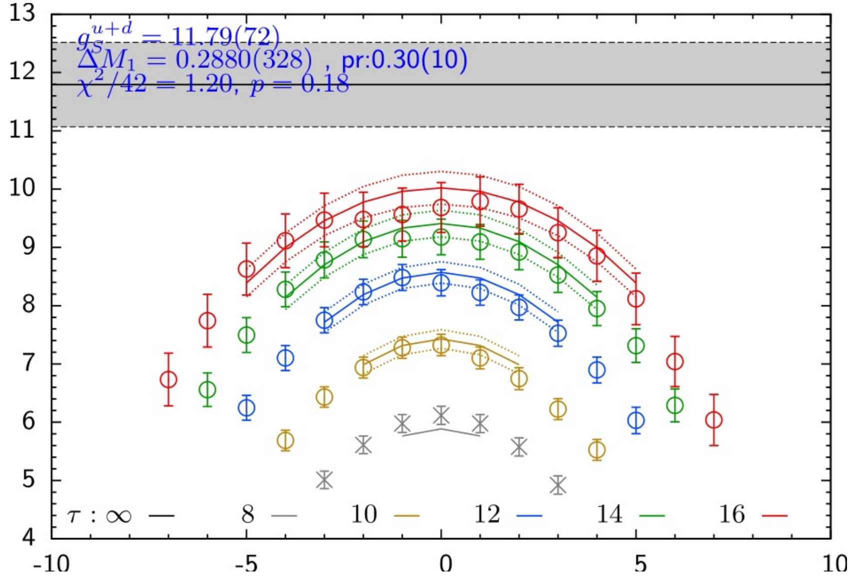
Estimates for the $a \approx 0.09 fm$; $M_\pi \approx 135 MeV$ ensemble assuming the asymptotic value is 18 without $N\pi/N\pi\pi$ $(M_1 \approx 1.6\ GeV)$ with $N\pi$ / $N\pi\pi$ $(M_1 \approx 1.2\ GeV)$

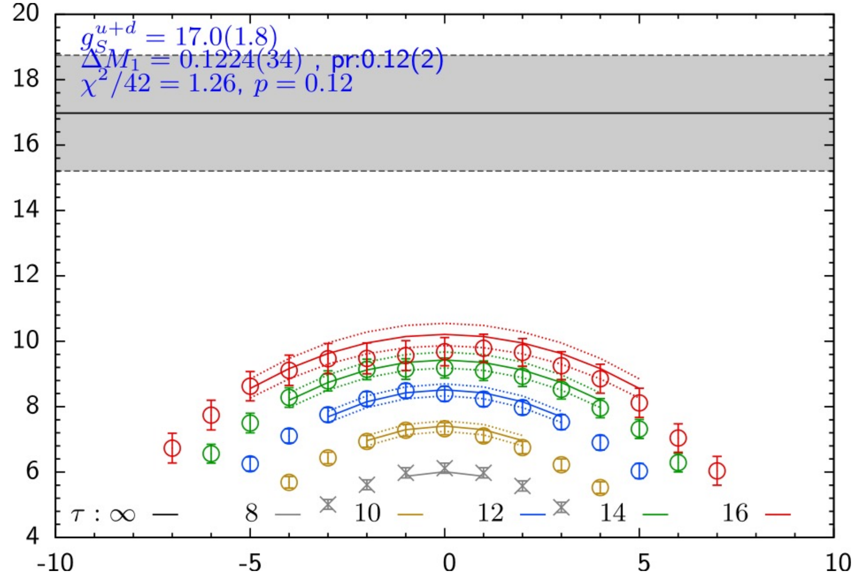
$g_S^{l,\mathrm{disc}}$





$g_S^{u+d} = g_S^{u+d,\mathrm{conn}} + 2g_S^{l,\mathrm{disc}}$





$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV $\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV

List of Lattice Parameters & Statistics

- All discussion based on Clover-on-HISQ data

Ensemble ID	a (fm)	M_π (MeV)	$(L/a)^3 \times T/a$	$M_\pi L$	$N_{\rm conf}^{\rm 2pt}$	$N_{\rm conf}^{\rm conn}$	$N_{\rm LP}$	$N_{\rm HP}$	$N_{\rm conf}^{l}$	$N_{\rm src}^{l}$	$N_{\rm LP}^{l}/N_{\rm HP}^{l}$
$a12m310$	0.1207(11)	310(3)	$24^3 \times 64$	4.55	1013	1013	64	8	1013	5000	30
$a12m220$	0.1184(10)	228(2)	$32^3 \times 64$	4.38	959	744	64	4	958	11000	30
$a09m310$	0.0888(08)	313(3)	$32^3 \times 96$	4.51	2263	2263	64	4	–	–	–
$a09m220$	0.0872(07)	226(2)	$48^3 \times 96$	4.79	964	964	128	8	712	8000	30
$a09m130$	0.0871(06)	138(1)	$64^3 \times 96$	3.90	1274	1290	128	4	1270	10000	50
$a06m310$	0.0582(04)	320(2)	$48^3 \times 144$	4.52	977	500	128	4	808	12000	50
$a06m220$	0.0578(04)	235(2)	$64^3 \times 144$	4.41	1010	649	64	4	1001	10000	50

Physical pion mass ensemble

2pt and connected 3pt

disconnected 3pt

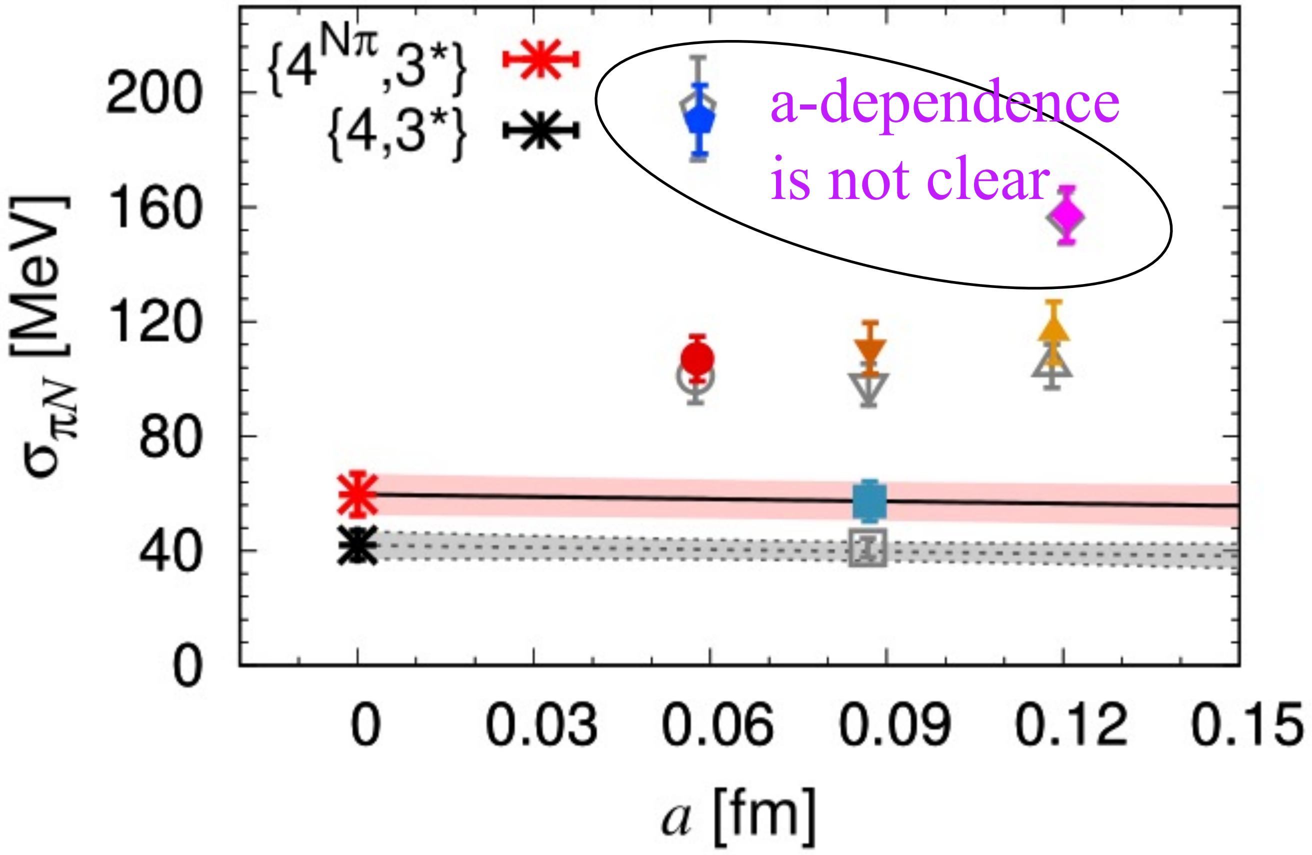
Reference: arXiv:2105.12095, to appear in PRL

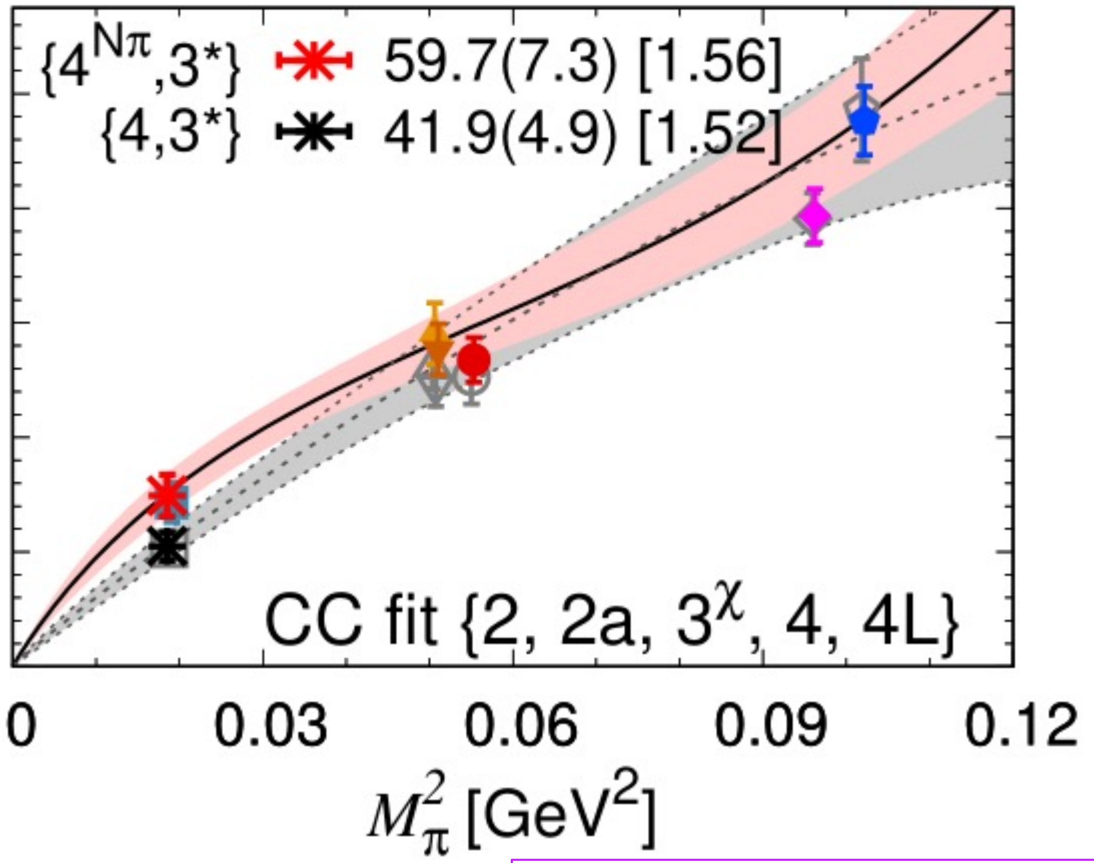
Main Systematics in lattice calculations

3. Chiral-Continuum-Finite-Volume extrapolation

$$\sigma_{\pi N}(\mathrm{a}, \mathrm{M}_\pi, \mathrm{M}_\pi \mathrm{L}) = \sigma_{\pi N}(0, \mathrm{M}_\pi = 135\mathrm{MeV}, \infty) + \cdots$$

Chiral-Continuum fits to 6 points



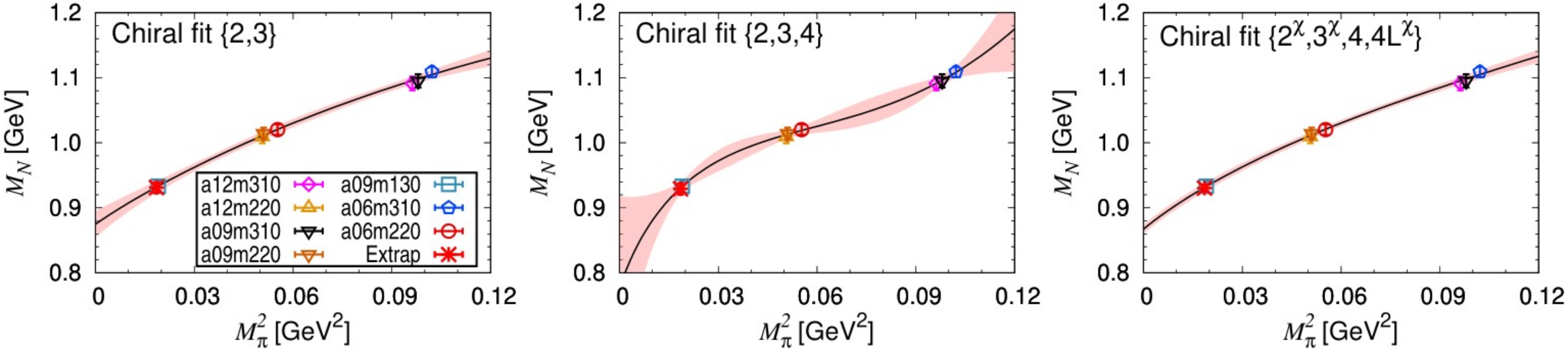


$$\sigma_{\pi N} = d_2 M_\pi^2 + d_2^a M_\pi^2 a + d_3^\chi M_\pi^3 + d_4 M_\pi^4 + d_{4L} M_\pi^4 \log M_\pi^2$$

d_3^χ: fixed to χPT prediction

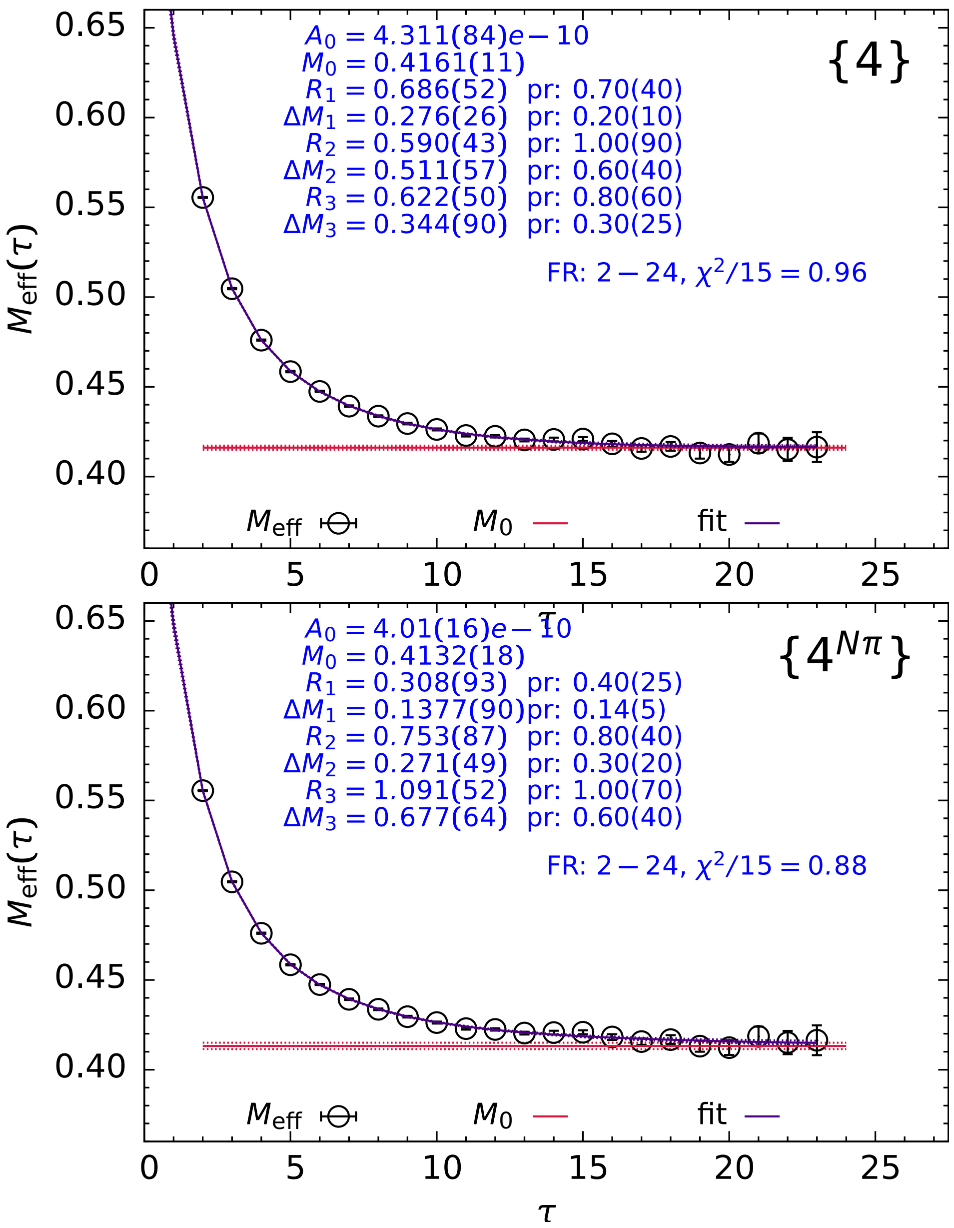
Other colors
$\{4^{N\pi},3*\}$: with $N\pi$
Light gray
$\{4,3*\}$: without $N\pi$

F-H method: Chiral fit to M_N



- χPT ansatz for M_N similar to that for $\sigma_{\pi N}$
- The most constrained fit does yield a good description of the data
- The resulting uncertainty in $\sigma_{\pi N}$ via FH method is too large with our data to compete with the direct method

Need to resolve excited states in F-H method also



$M_\pi = 170$ MeV ensemble
$a \approx 0.091$ fm
3000×320 measurements

- No plateau (ground state domination) even at ≈ 2 fm
- 4-state fits with $\Delta M_1 \approx 320$ and 640 MeV give similar χ^2
- For a state with $\Delta M_1 \approx 300$ MeV, the suppression $e^{-\Delta M_1 \tau}$=0.1 only at 1.5 fm

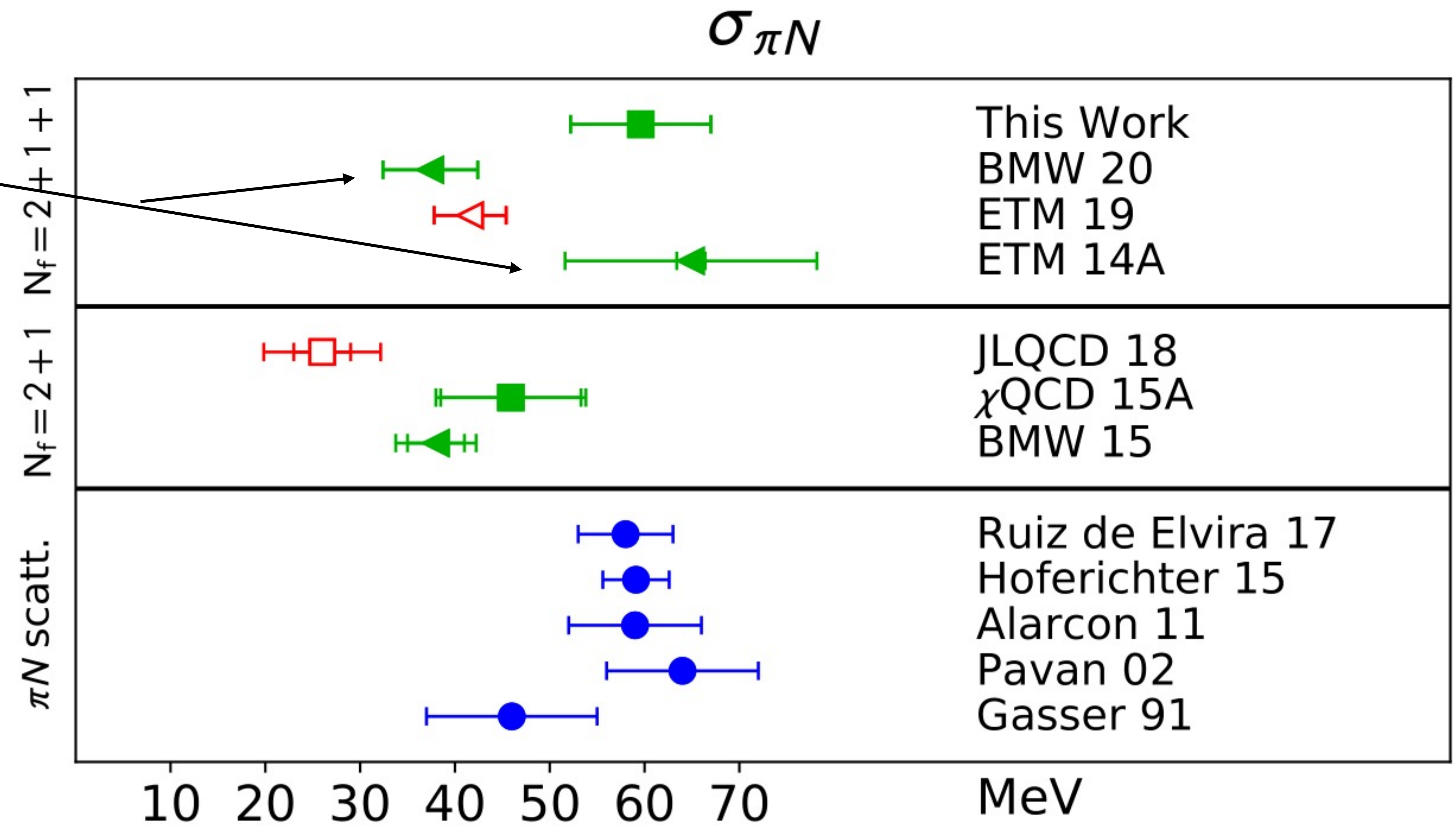
Update of FLAG Summary in **arXiv:2105.12095**

Tension in
F-H method.

Both with $M_\pi \gtrsim$ 200 MeV data

Chiral extrapolation is different!

→ Need data at $M_\pi \approx 135$ MeV

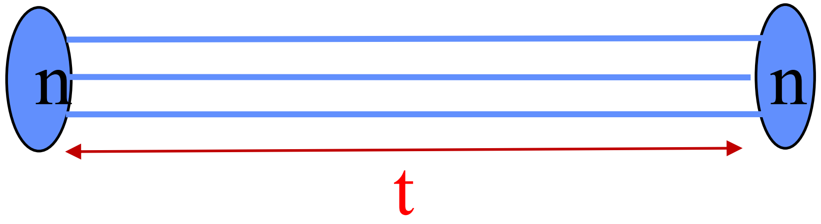


Summary

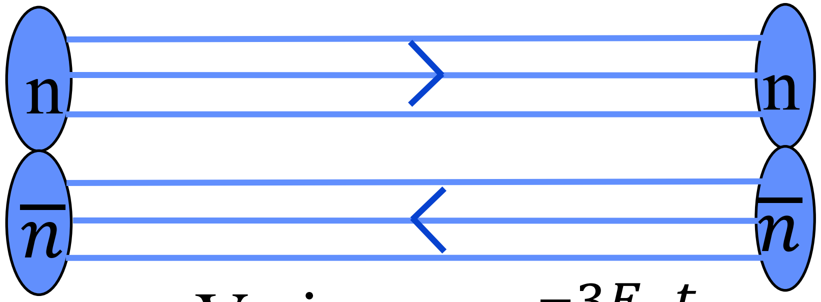
- ESC large in g_S^{u+d} : Data show large τ dependence at $\tau \approx 1.5$ fm
- χPT suggests large contribution of $N\pi$ & $N\pi\pi$ states to g_S^{u+d}
- Contribution increases as $M_\pi \to 135$ MeV and $\Delta\vec{p} \to 0$
- Fits are consistent with coefficients predicted by χPT
- $\sigma_{\pi N}$ changes from ~40 MeV to ~60 MeV on including the $N\pi$ and $N\pi\pi$ excited states
- Need more data to improve the chiral-continuum-finite-volume extrapolation of $\sigma_{\pi N}$
- Large $N\pi$ contribution also seen in axial/pseudoscalar form factors and required for them to satisfy the PCAC relation

Extras

Spectrum from the 2-point function $\boldsymbol{C_{2pt}}$



Signal in C_{2pt}: $e^{-E_N t}$



Variance: $e^{-3E_\pi t}$

$$C_{2pt}(t) \equiv \Gamma^2(t) = |A_0|^2 e^{-M_0 t} + |A_1|^2 e^{-M_1 t} + |A_2|^2 e^{-M_2 t} + |A_3|^2 e^{-M_3 t} + \ldots.$$

Fit the data for $C_{2pt}(t)$ versus t to extract

M_0, M_1, … masses of the ground & excited states
A_0, A_1, … corresponding amplitudes

- The signal degrades exponentially $e^{-(M_N - 1.5 M_\pi)t}$
- To resolve a *small* mass gap $(M_1 - M_0)$ *requires* large t