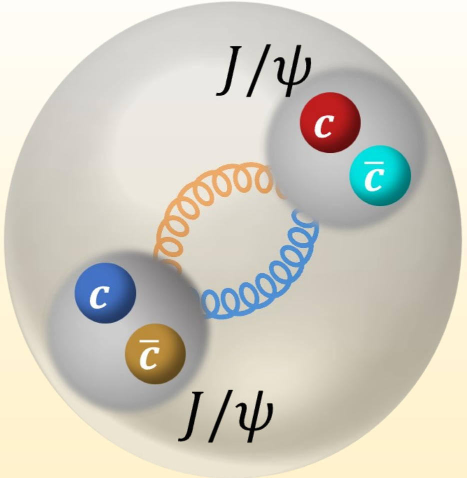
中国科学院理论物理研究所
Institute of Theoretical Physics, Chinese Academy of Sciences

Possible Bound States of $J/\psi J/\psi$ from dispersion relation

Xiang-Kun Dong

Nov. 16, 2021



- X.K. Dong, V. Baru, F.K. Guo, C. Hanhart, and A. Nefediev, Phys.Rev.Lett. 126 (2021) 13, 132001
- X.K. Dong, V. Baru, F.K. Guo, C. Hanhart, A. Nefediev and B.S. Zou, Sci. Bull. 66 (2021) 24, 2462-2470

Outline

- Coupled-channel interpretation of LHCb data
- Interactions between $J/\psi J/\psi$ from dispersive relation:

  soft gluon exchange → two pion exchange + …

- Possible bound states of $J/\psi J/\psi$

$X(6900)$ in LHCb measurement

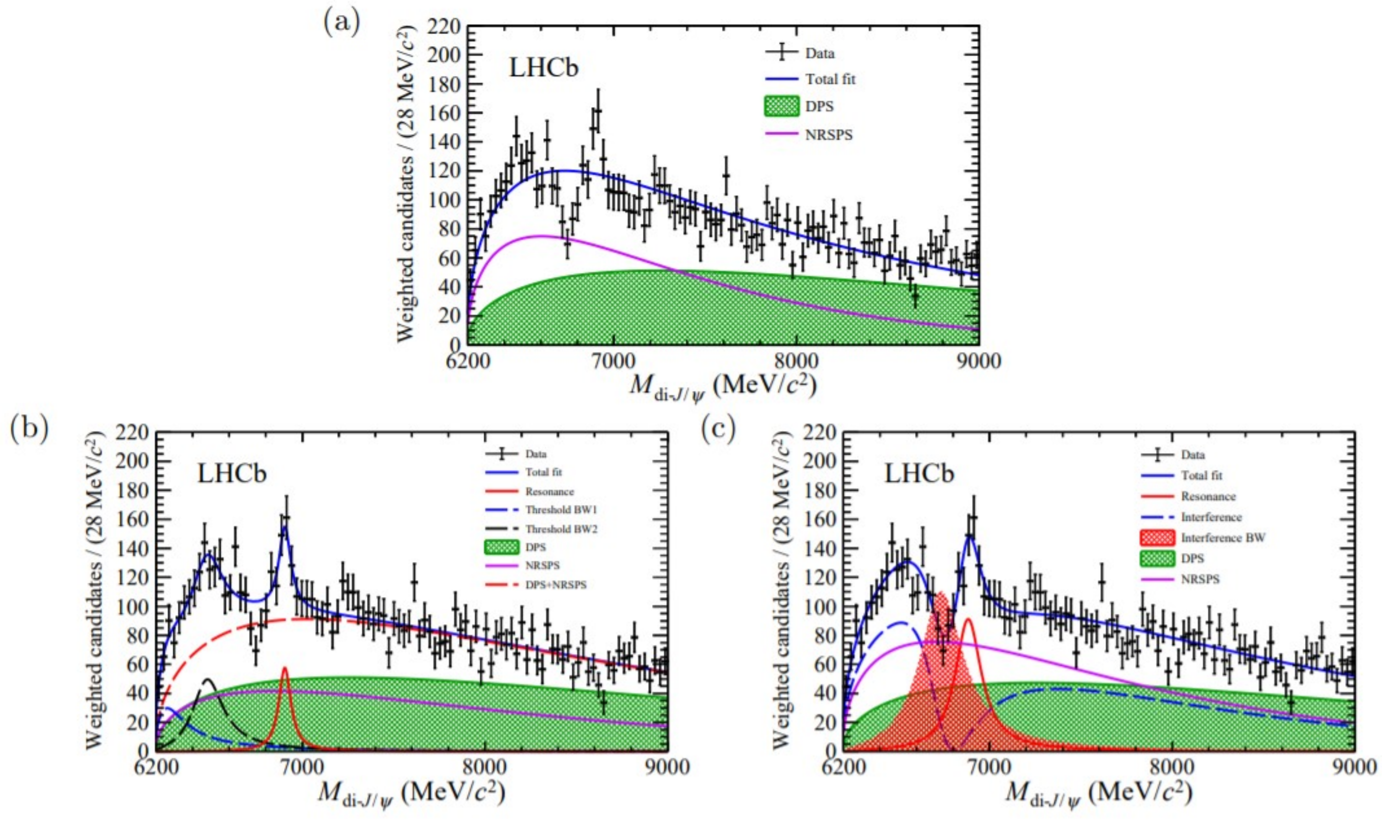


Figure 3: Invariant mass spectra of weighted di-J/ψ candidates with $p_{\mathrm{T}}^{\text{di-}J/\psi} > 5.2\,\mathrm{GeV}/c$ and overlaid projections of the $p_{\mathrm{T}}^{\text{di-}J/\psi}$-threshold fit using (a) the NRSPS plus DPS model, (b) model I, and (c) model II.

LHCb, Sci.Bull.65,1983(2020)

- ✓ A **narrow** resonance-like structure at 6.9 GeV, $\boldsymbol{X(6900)}$
- ✓ A **broad** structure just above double-J/ψ threshold
- ✓ A **dip** near 6.7 GeV

➢ $X(6900)$ compact tetraquark?

Predictions dated back to 1970s

Y. Iwasaki, Prog.Theor.Phys. 1975;54:492

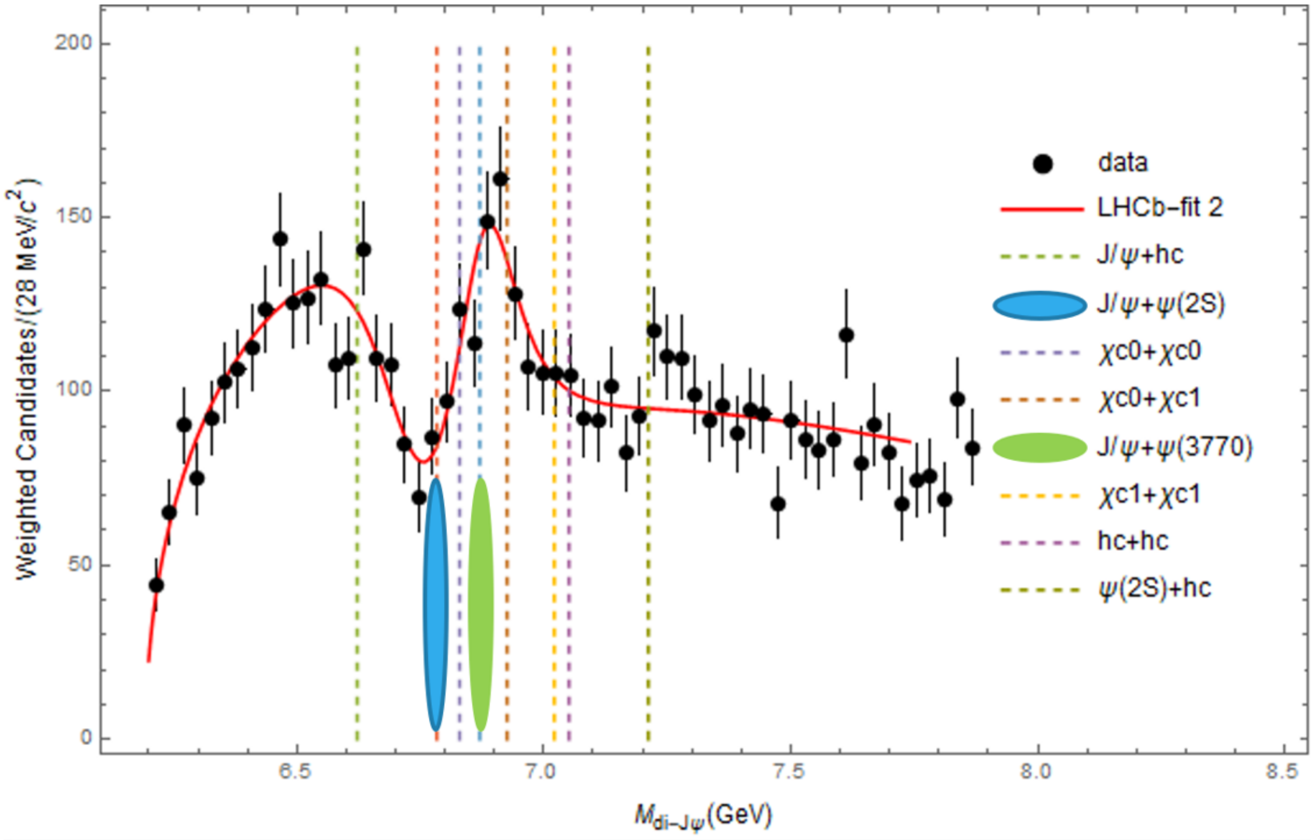
K.-T. Chao, Z.Phys.C7, 317(1981),

A.M. Badalian et al., PRD25,2370(1982), …

Many theoretical investigations

…

$X(6900)$ in LHCb measurement



Discussions of general threshold behaviors, see X.-K. Dong, F.-K. Guo and B.-S. Zou PRL126, 152001(2021)

- Fully charmed tetraquark state?
  - 6.9 GeV too high for ground state
  - The gap 700 MeV too large for ground and 1st excited states
  - No lighter states (easier) observed
- Threshold effects. Near threshold,
  - **Breit-Wigner** fits mislead rather than educate
  - **Breit-Wigner** parameters (M and Γ) hide nature of states
  - Threshold effects sometimes play critical role
- Coupled-channel approach, Minimal models with X.-K. Dong et al, PRL126,132001(2021)
  - most relevant channels (2 models)
  - minimal necessary orders in interactions

Coupled-channel models

Two channel model

$$J/\psi J/\psi \ \& \ \psi(2S)J/\psi$$

$$V_{2\text{ch}}(E) = \begin{pmatrix} a_1 + b_1 k_1^2 & c \\ c & a_2 + b_2 k_2^2 \end{pmatrix}$$

Three channel model

$$J/\psi J/\psi, \ \psi(2S)J/\psi \ \& \ \psi(3770)J/\psi$$

$$V_{3\text{ch}}(E) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

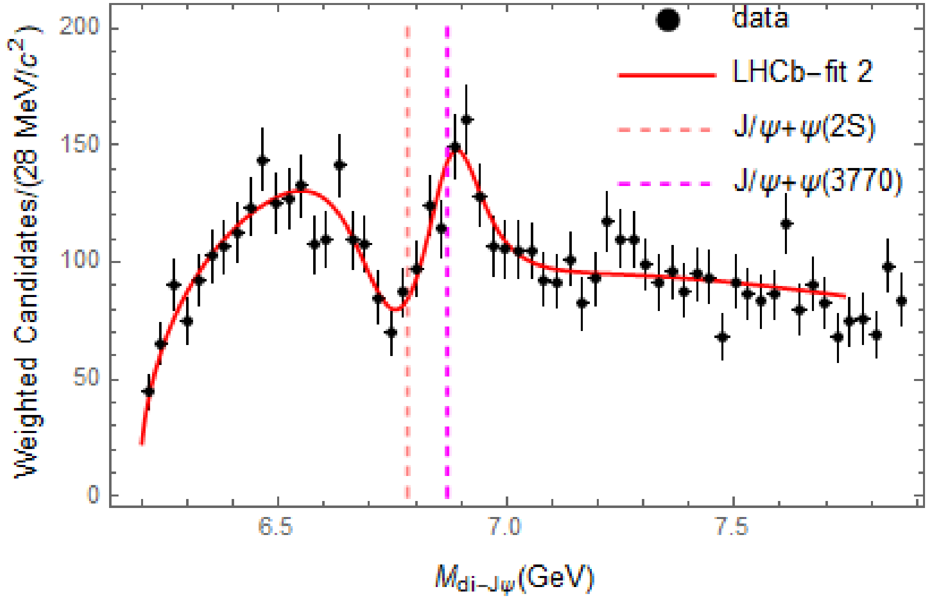
- η_c and h_c spin-0, HQSS
- $\chi_{cJ} \to \psi$ by ω exchange

Lippmann-Schwinger equation

$$T(E) = V(E) \cdot [1 - G(E)V(E)]^{-1}$$

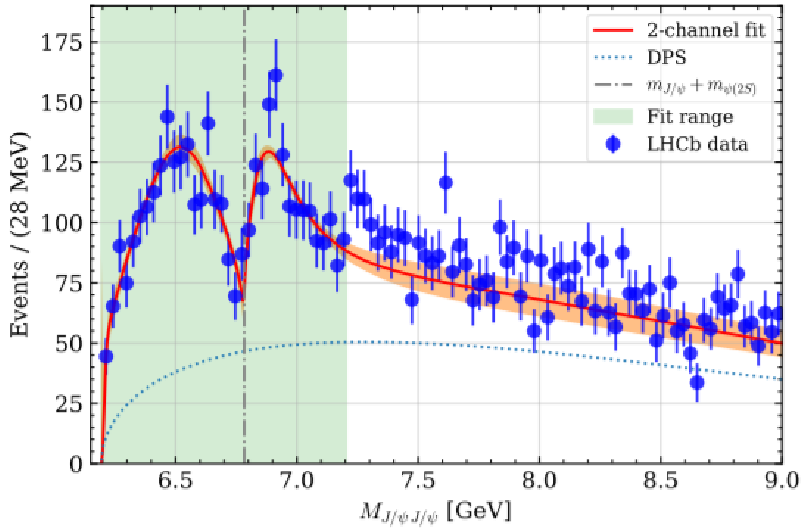
Production amplitude in J/ψJ/ψ channel (channel 1)

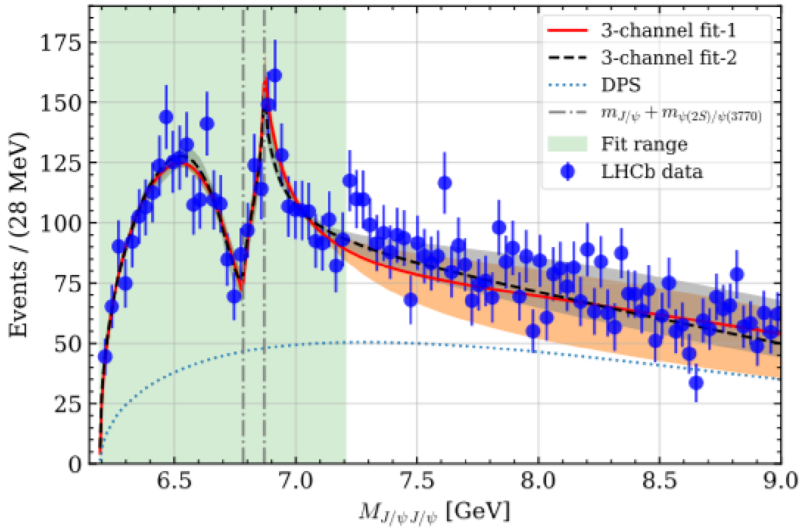
$$\mathcal{M}_1 = \alpha e^{-\beta E^2} \left[b + G_1(E)T_{11}(E) + \textstyle\sum_{i=2,3} r_i G_i(E) T_{i1}(E) \right]$$



Hints of a $J/\psi J/\psi$ molecule

X.-K. Dong et al, PRL126,132001(2021)

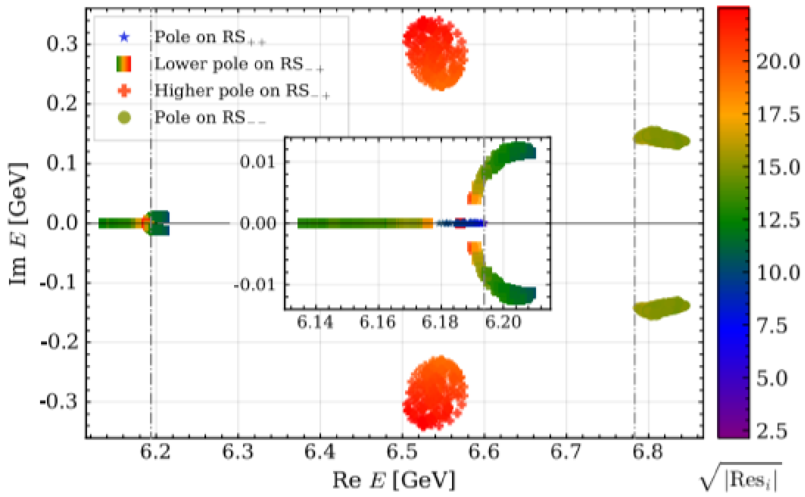


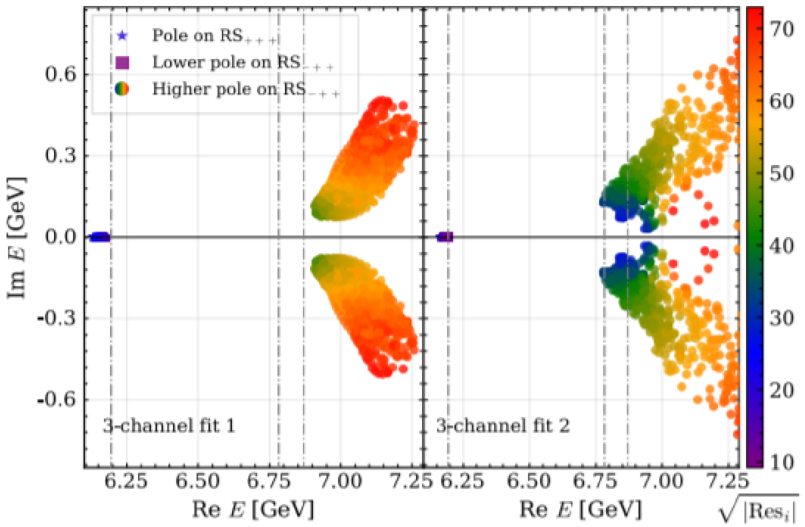


	2-ch. fit	3-ch. fit 1	3-ch. fit 2
a_0(fm)	≤ -0.49 or ≥ 0.48	$-0.61^{+0.29}_{-0.32}$	≤ -0.60 or ≥ 0.99
r_0(fm)	$-2.18^{+0.66}_{-0.81}$	$-0.06^{+0.03}_{-0.04}$	$-0.09^{+0.08}_{-0.05}$
$\bar{X}_A$	$0.39^{+0.58}_{-0.12}$	$0.91^{+0.04}_{-0.07}$	$0.95^{+0.04}_{-0.06}$

$X(6900)$ is uncertain

$X(6200)$ is robust





- Compositeness of $X(6200)$

$$\bar{X}_A = (1 + 2|r_0/a_0|)^{-1/2}$$

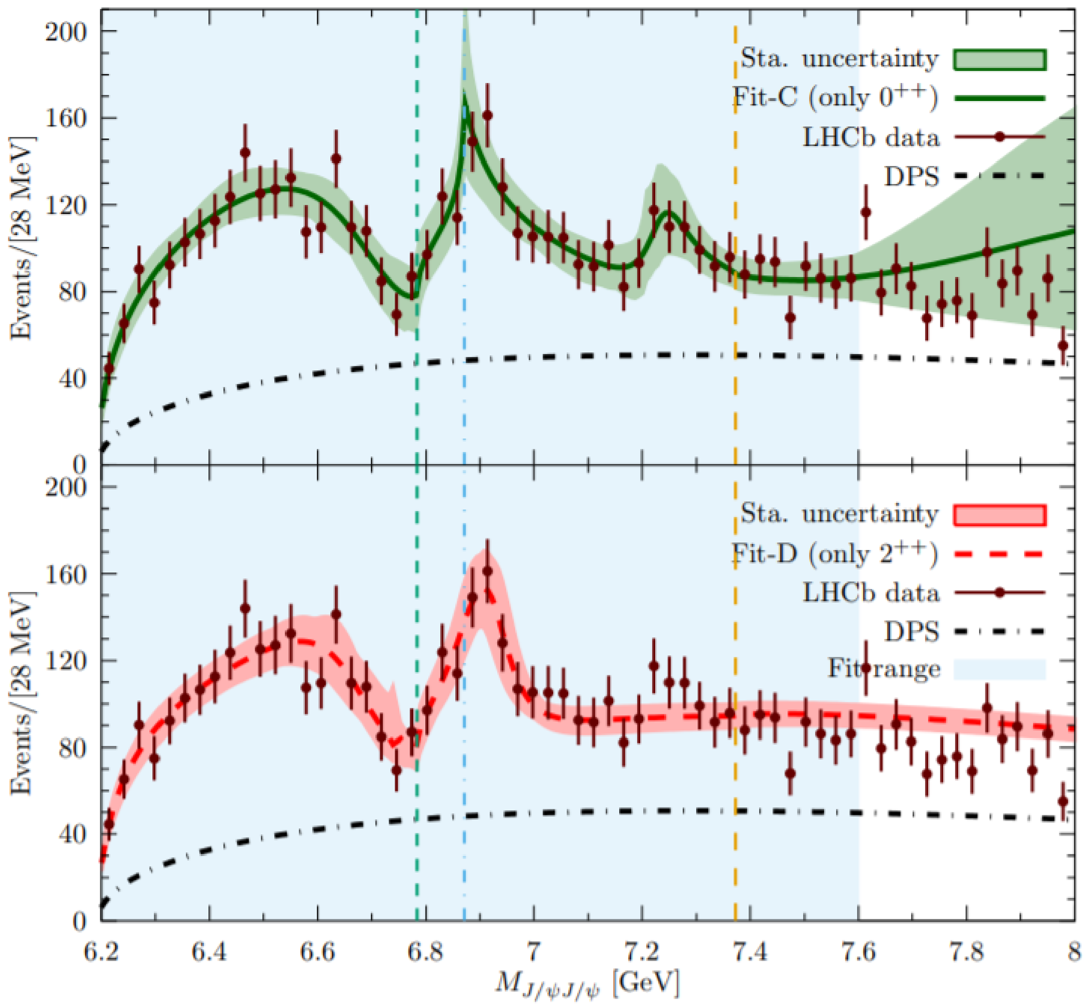
$\bar{X}_A = 1$ for molecule and

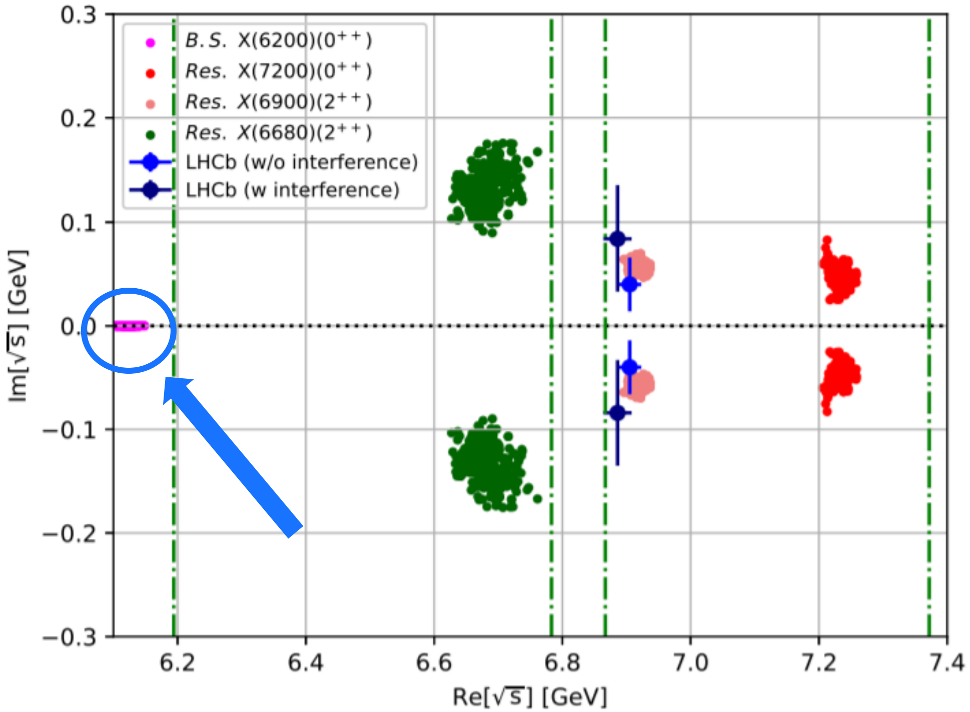
0 for compact state

I. Matusche et al, EPJA57(2021)3,101

- Large molecular component in $X(6200)$ in 3-channel fit.

Hints of a $J/\psi J/\psi$ molecule



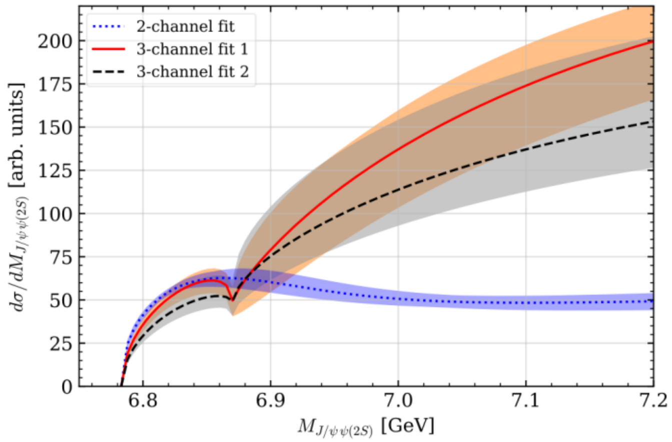


- Support of existence of $X(6200)$ from independent analysis

Z.-R. Liang et al, Phys.Rev.D 104 (2021) 3, 034034

Further Tests

X.-K. Dong et al, PRL126,132001(2021)



- ✓ Data in the $\psi(2S)J/\psi$ channel ⇒ distinguish between the models
- ✓ Data in the $\eta_c\eta_c$ channel ⇒ verify predictive power of the models
- ✓ Data on $\Upsilon\Upsilon$ production ⇒ check in complementary sector
- ✓ Lattice simulation of double-J/ψ (η_c) scattering ⇒ independent test

Binding mechanism??? ⇒ (dis)prove X(6200) nature!!!

- ➢ Van der Waals interaction between color dipoles

  H. Fujii and D. Kharzeev, Phys.Rev.D 60, 114039 (1999).

- ➢ Long-range potential from two pion exchange between 2 S-wave bottomonia

  N. Brambilla et al, Phys.Rev.D 93, 054002 (2016)

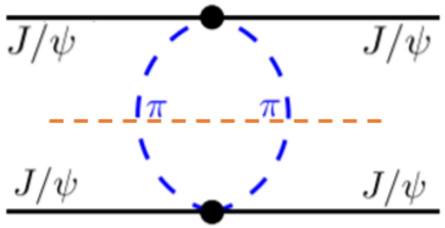
Evidence of an $\Omega_{ccc}\Omega_{ccc}$ bound state from HAL QCD method

Yan Lyu et al, Phys.Rev.Lett. 127 (2021) 7, 072003

At long-range, soft gluon exchange → two pion exchange + heavier…

OPE highly suppressed by isospin

Two pion/kaon exchange potential

- $J/\psi J/\psi\pi\pi$ coupling $\mathcal{M}_\pi(t) =$ 

- $\psi\psi\pi\pi$ couplings related to the chromopolarisabilities, α_{AB}, satisfying A. Sibirtsev et al, PRD71, 076005(2005)

$$\alpha_{J/\psi J/\psi}\alpha_{\psi(2S)\psi(2S)} \geq \alpha^2_{\psi(2S)J/\psi}.$$

- Chiral Lagrangian (& HQSS)

$J/\psi,\ \psi(2S), \ldots$

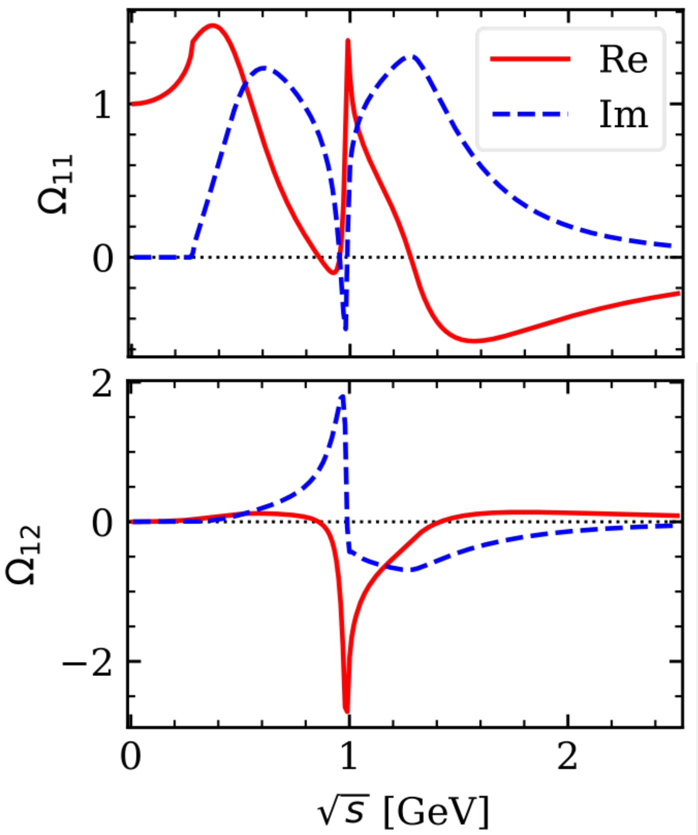
$\pi, K, \ldots$

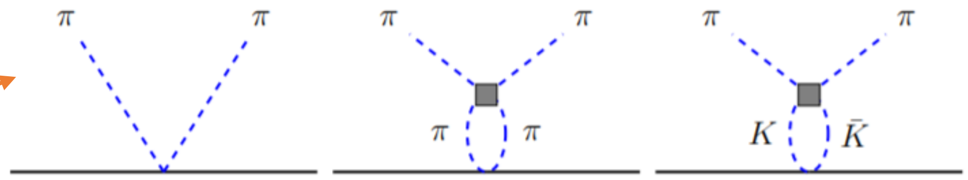
$$\mathcal{L}_{\psi_\alpha\psi_\beta\Phi\Phi} = \frac{c_1^{(\alpha\beta)}}{2}\left\langle J_\beta^\dagger J_\alpha\right\rangle \mathrm{Tr}[u_\mu u^\mu] + \frac{c_2^{(\alpha\beta)}}{2}\left\langle J_\beta^\dagger J_\alpha\right\rangle \mathrm{Tr}[u_\mu u_\nu v^\mu v^\nu] + \mathrm{H.c.},$$

LECs

- Non-perturbative interaction of scalar-isoscalar $\pi\pi$ and $K\bar{K}$ channel is necessary.
  - ✓ A polynomial times Omnès function
    Omnès, Nuovo Cim. 8 (1958) 316

Unitarity

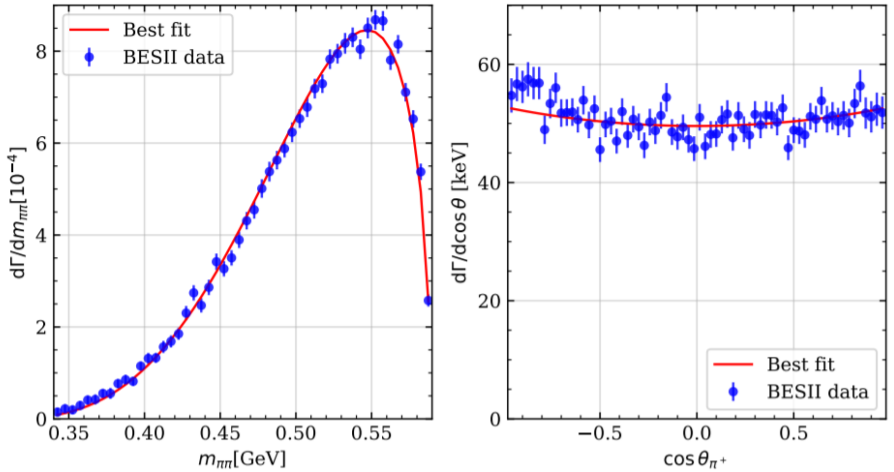




Two pion/kaon exchange potential

- $c_i^{(12)}$ from fits to BESII data on $\psi(2S) \to J/\psi\pi\pi$ decay.

  [BES Collaboration], PLB645, 19 (2007)



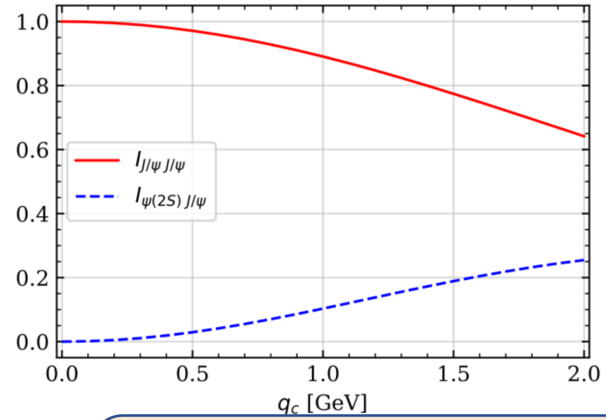
- Difference between c_i^{11} and c_i^{12}

$$\xi \equiv \frac{\alpha_{J/\psi J/\psi}}{\alpha_{\psi(2S)J/\psi}} \approx 1 \sim 3$$

estimated by

- overlap of quark model wavefunctions

$$\alpha_{AB} \propto \int \mathrm{d}^3 r\ \psi_A^*(\vec{r}) e^{-i\vec{q}_c\cdot\vec{r}/2} \psi_B(\vec{r}) \equiv I_{AB}(q_c)$$



- S-Wave $J/\psi\pi$ scattering length

$$a_0 = \frac{(c_1^{(11)} + c_2^{(11)}) m_\pi^2}{2\pi F_\pi^2 (M_{J/\psi} + m_\pi)} \approx 0.0036\ \xi\ \text{fm}$$

$$|a_0^{\text{lat}}| \sim 0.01\ \text{fm}$$

K. Yokokaw et al. PRD74, 034504 (2006)
L. Liu et al. PoS LATTICE2008, 112 (2008)
X.-H. Liu et al. EPJC73, 2284(2013)

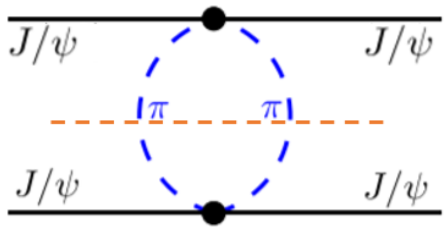
P-Wave $J/\psi\pi$ Scattering Lattice QCD

$$\mathcal{M}_1[J/\psi\pi \to J/\psi\pi] \approx 8\pi \left(M_{J/\psi} + m_\pi\right) k^2 a_1$$

$$a_1 = -\frac{M_{J/\psi} c_1 - m_\pi c_2}{6\pi F_\pi^2 M_{J/\psi}\left(M_{J/\psi} + m_\pi\right)} \approx -\frac{c_1}{6\pi F_\pi^2 M_{J/\psi}}$$

Potentials from dispersive relation

- Dispersive relation $\mathcal{M}_\pi(t) =$



$$= \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}\mathcal{M}_\pi(t')}{t'-t-i\epsilon}$$

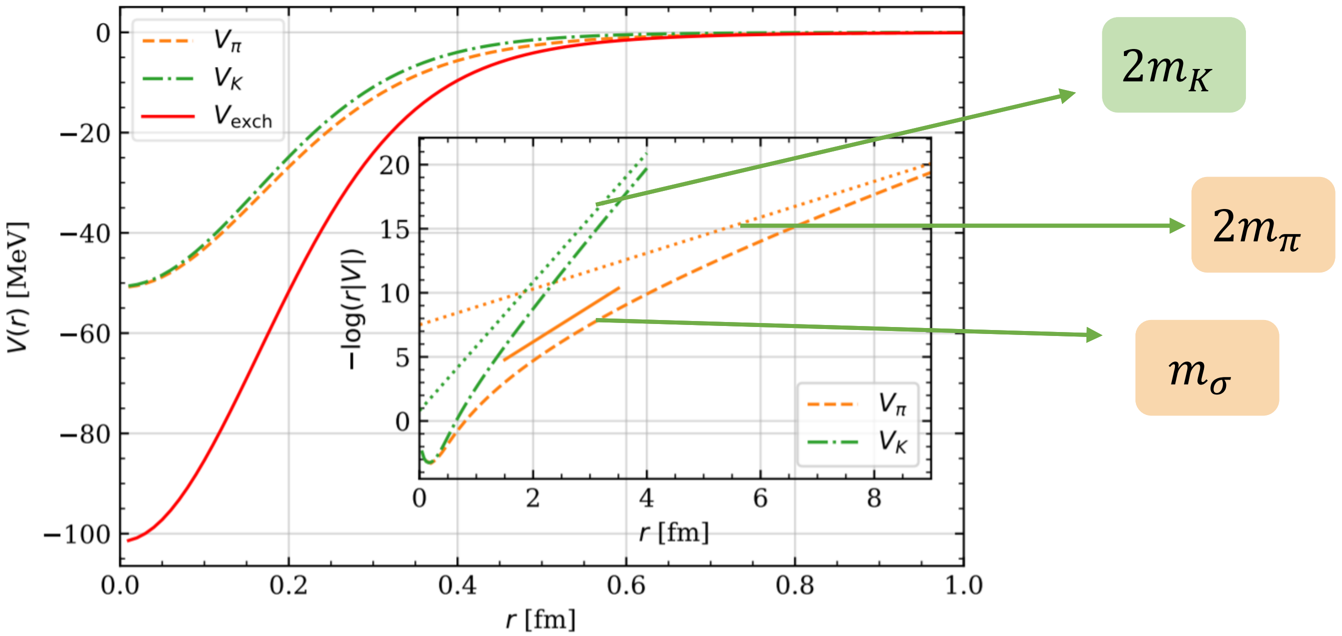
- Two pion (kaon) exchange potential

$$\mathcal{M}_{J/\psi J/\psi}(t) = \mathcal{M}_\pi(t) + \mathcal{M}_K(t)$$

$$V_{\text{exch}}(q,\Lambda) = \frac{-1}{4\pi M_{J/\psi}^2}$$

$$\times \int_{4m_\pi^2}^{\infty} \mathrm{d}\mu^2 \frac{\mathrm{Im}\mathcal{M}_{J/\psi J/\psi}(\mu^2)}{\mu^2+q^2} \boxed{e^{-\frac{q^2+\mu^2}{\Lambda^2}}}$$

Form factor to regularize the UV divergence in both q and μ and keep the long-range potential unchanged.



$J/\psi J/\psi$ molecular state

- Renormalization with a contact term $V_{\mathrm{CT}}(q,\Lambda) = C\ e^{-\frac{q^2}{\Lambda^2}}$

$$V_{\mathrm{tot}}(q,\Lambda) = V_{\mathrm{CT}}(q,\Lambda) + V_{\mathrm{exch}}(q,\Lambda)$$

- Lippmann Schwinger equation

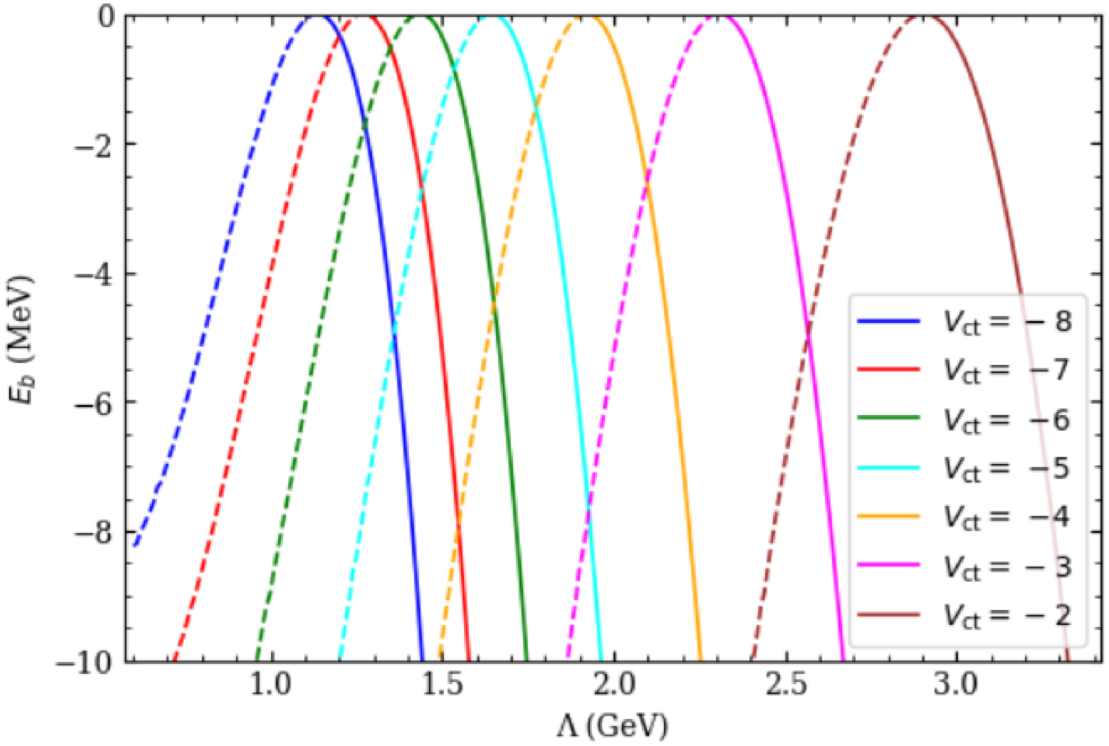
$$T(E;k',k) = V_{\mathrm{tot}}^{S}(k',k,\Lambda) + \int \frac{\mathrm{d}^3 l}{(2\pi)^3} \frac{V_{\mathrm{tot}}^{S}(k',l,\Lambda)\,T(E;l,k)}{E - l^2/M_{J/\psi} + i\epsilon}$$

- Parameters:

$$\Lambda(1\sim 3\ \mathrm{GeV}),\ \xi(1\sim 3)\ \text{and}\ V_{\mathrm{ct}}\ \text{in}\ \mathrm{GeV}^{-2}$$

- Poles for $\xi = 1$.

Solid for bound states and dashed for virtual states

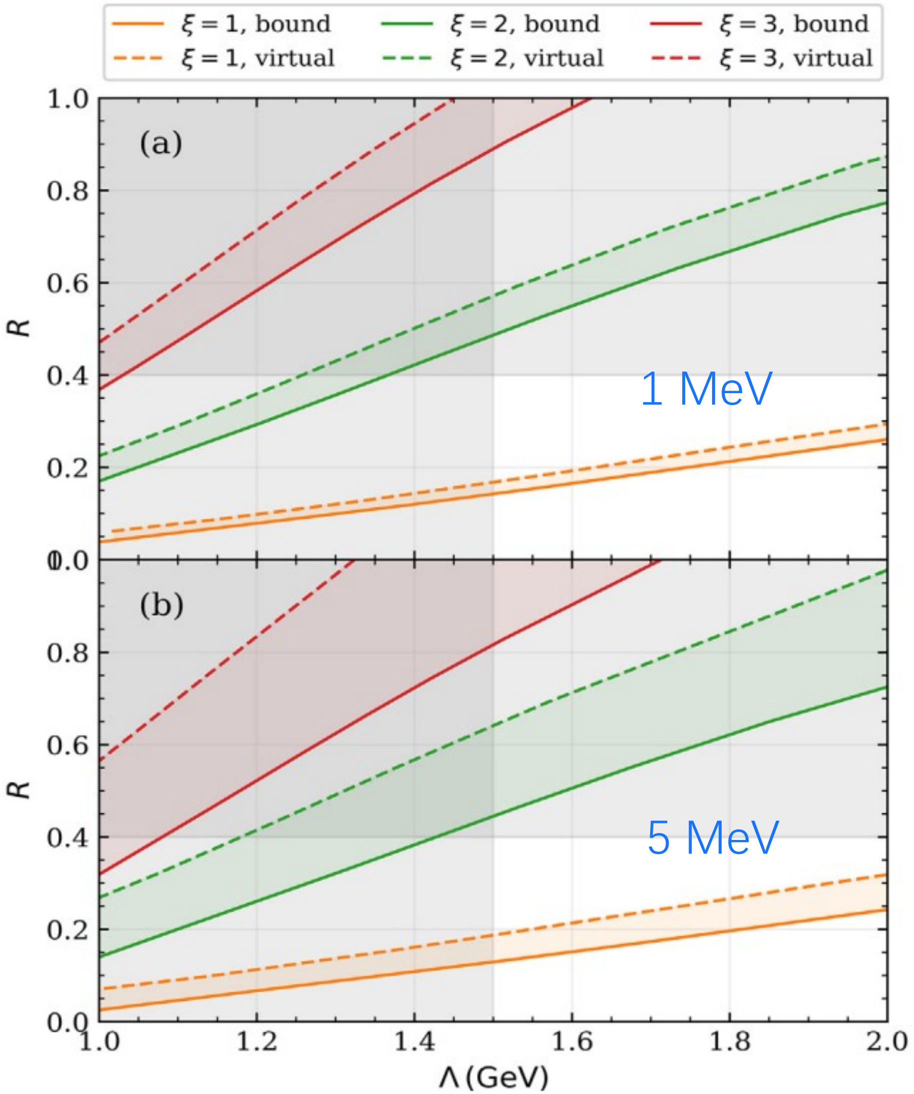


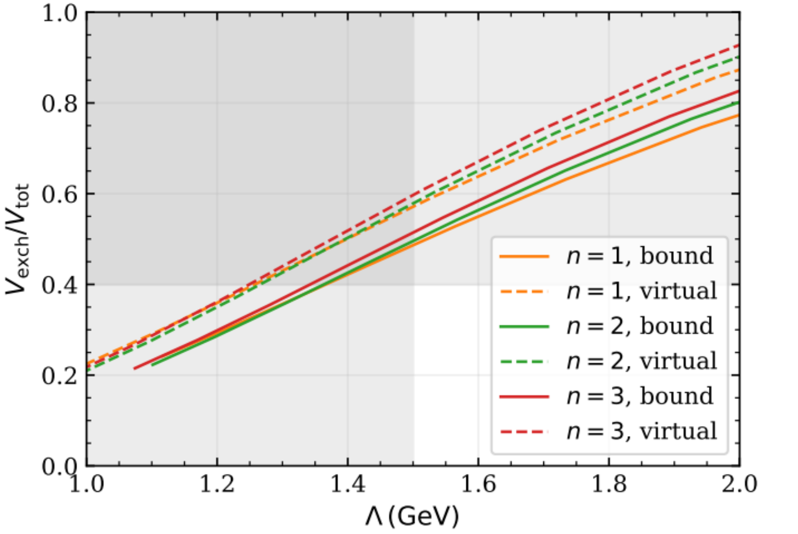
$J/\psi J/\psi$ molecular state

Strategy:

- ✓ Near threshold poles (bound or virtual), E_B = 1 or 5 MeV fixed (hints from LHCb data)
- ✓ ξ (=1,2,3) fixed
- ✓ Contribution of V_{exch} to the binding, R

$$R \equiv \frac{V^S_{\text{exch}}\left(k'=0,k=0,\Lambda\right)}{V^S_{\text{tot}}\left(k'=0,k=0,\Lambda\right)}$$





$e^{-(q^2+\mu^2)^n/\Lambda^{2n}}$ for meson exchanges

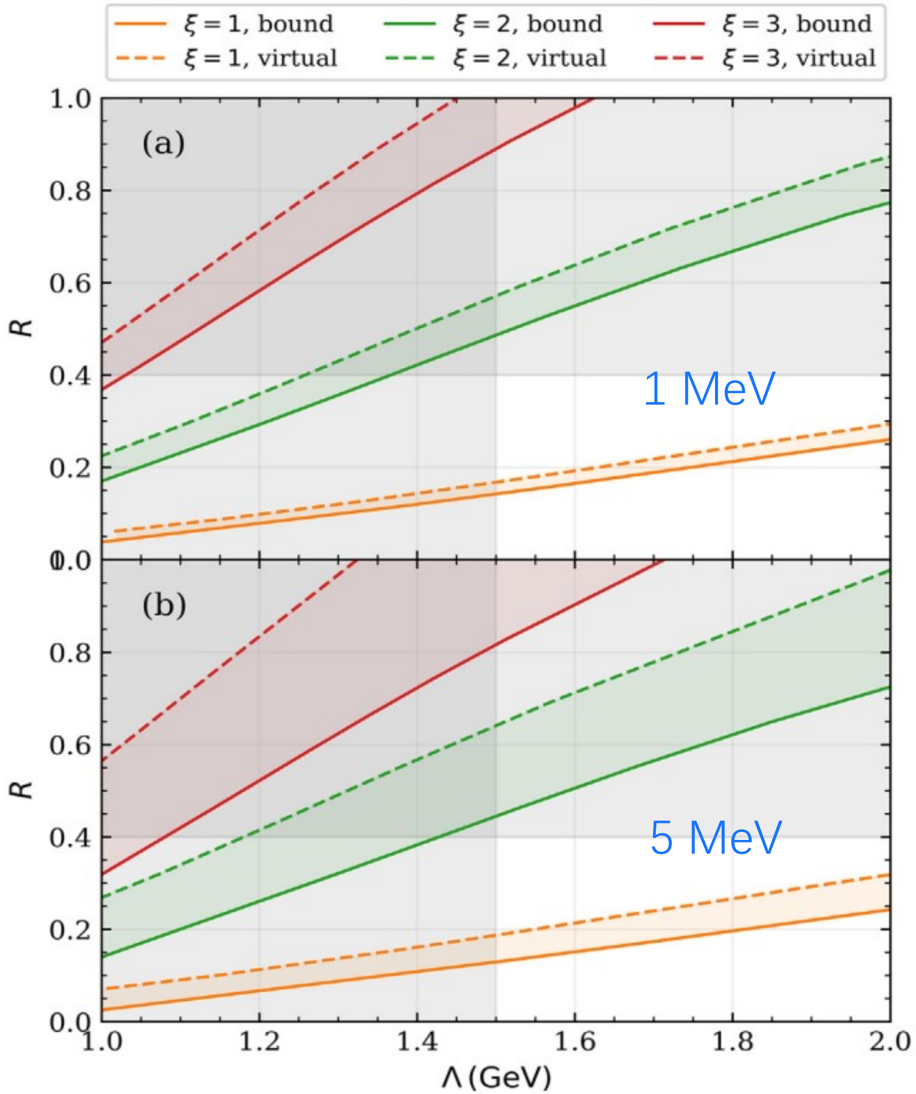
$e^{-q^{2n}/\Lambda^{2n}}$ for the contact term

Weak dependence on n

Plausible?

- Contact term + two pion/kaon exchange leads to a molecule of $J/\psi J/\psi$
- $J/\psi J/\psi$ interaction suppressed by OZI or $\Lambda^2_{\rm QCD}/m_c^2$, no reason for $V_{\rm ct} \gg V_{\rm exch}$
- Naturalness: contact term is of the same order as two pion/kaon exchange
- Reasonable cutoff, $1 \sim 1.5$ GeV
- We take it plausible if two pion/kaon exchange has sizeable contributions to the binding of $J/\psi J/\psi$, characterized by the ratio

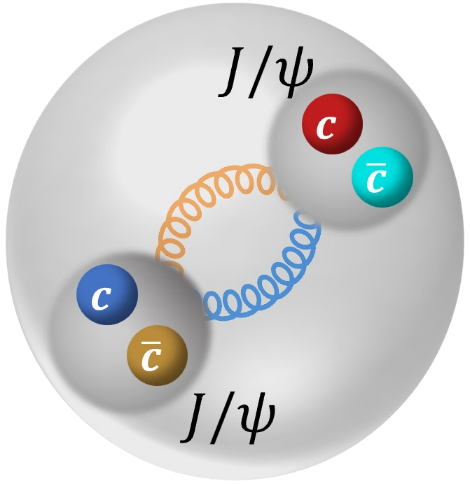
$$R \equiv \frac{V^S_{\rm exch}\left(k'=0,k=0,\Lambda\right)}{V^S_{\rm tot}\left(k'=0,k=0,\Lambda\right)} \gtrsim 1/2$$



Summary

- LHCb $J/\psi J/\psi$ data can be well described by the coupled-channel method.
- Hints of a state near $J/\psi J/\psi$ threshold, with large molecular component.
- Soft gluon exchange between $J/\psi J/\psi$ described by two pion and kaon exchange.
- Coupling constants of $\psi(2S)J/\psi\pi\pi$ coupling extracted from BESII data on $\psi(2S) \to J/\psi\pi\pi$ decay.
- $J/\psi J/\psi\pi\pi$ coupling is argued to be larger than $\psi(2S)J/\psi\pi\pi$ coupling.
- With reasonable cutoff Λ, two pion and kaon exchanges provide sizeable contribution to the $J/\psi J/\psi$ attraction.
- The binding of $J/\psi J/\psi$ system is plausible, given our current understanding of hadron-hadron interaction.

✓ Data in the $\psi(2S)J/\psi$ channel ⇒ distinguish between the models
✓ Data in the $\eta_c\eta_c$ channel ⇒ verify predictive power of the models
✓ Data on $\Upsilon\Upsilon$ production ⇒ check in complementary sector
✓ Lattice simulation of double-J/ψ (η_c) scattering ⇒ independent test
✓ Take a look at $J/\psi e^+e^-$ or $J/\psi\mu^+\mu^-$ channels
✓ Lattice simulations of S- and P-wave $J/\psi\pi$ scattering ⇒ c_1 and c_2




THANK YOU .