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Scalar resonance dynamics and its relevance in the determiatnion of light-quark mass

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Outline:

1. Background

2. QCD sum rule

3. Spectral functions from chiral EFT

4. Results and discussions

5. Summary

m_q: fundamental parameter in SM, but cannot be directly measured, and can be only determined in an indirect way!

PDG Live

2020 Review of Particle Physics.
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. **2020**, 083C01 (2020)

$$\overline{m} = (m_u + m_d)/2$$

VALUE (MeV)		*DOCUMENT ID*		*TECN*
$\mathbf{3.45^{+0.55}_{-0.15}}$		**OUR EVALUATION**		
3.9 ± 0.3	1	DOMINGUEZ	2019	THEO
$4.7\ ^{+0.8}_{-0.7}$	2	YUAN	2017	THEO
3.70 ± 0.17	3	CARRASCO	2014	LATT
3.45 ± 0.12	4	ARTHUR	2013	LATT
$3.469 \pm 0.047 \pm 0.048$	5	DURR	2011	LATT
3.6 ± 0.2	6	BLOSSIER	2010	LATT
3.39 ± 0.06	7	MCNEILE	2010	LATT

1 DOMINGUEZ 2019: determine the quark mass from a QCD finite energy sum rule for the divergence of the axial current.

2 YUAN 2017: determine m_q using QCD sum rules in the isospin I=0 scalar channel. The result is clearly larger than other determinations !

QCD sum rule

$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle\Omega|T\{j_S(x), j_S^\dagger(0)\}|\Omega\rangle$$

QCD： OPE, instanton (isoscalar scalar case) , nonperturbative condensates

Phenomenology: resonance (mass, width), background

We focus on

$$j_S(x) = m_q\frac{1}{\sqrt{2}}\Big(\bar{u}(x)u(x) + \bar{d}(x)d(x)\Big)$$

$$m_q = \frac{1}{2}(m_u + m_d)$$

OPE + instanton

$$
\begin{aligned}
R^{(\mathrm{QCD})}(\tau,\hat{m}_q) =& R^{(\mathrm{OPE})}(\tau,\hat{m}_q) + R^{(\mathrm{Inst})}(\tau,\hat{m}_q) \\
=& m_q^2(1/\sqrt{\tau})\left\{\frac{3}{8\pi^2}\frac{1}{\tau^2}\left[1+4.821098\frac{\alpha_s(1/\tau)}{\pi}+21.97646\left(\frac{\alpha_s(1/\tau)}{\pi}\right)^2\right.\right. \\
&\left. + 53.14197\left(\frac{\alpha_s(1/\tau)}{\pi}\right)^3\right] + \frac{\langle\alpha_s G^2\rangle}{8\pi}\left(1+\frac{11}{2}\frac{\alpha_s(1/\tau)}{\pi}\right) \\
&+ 3\langle m_q\bar{q}q\rangle\left(1+\frac{13}{3}\frac{\alpha_s(1/\tau)}{\pi}\right) - \frac{176}{27}\pi\kappa\alpha_s\langle\bar{q}q\rangle^2\left[\frac{\alpha_s(1/\tau)}{\alpha_s(\mu_0^2)}\right]^{1/9}\tau \\
&\left. + \frac{3}{8\pi^2}\frac{e^{\frac{-\rho^2}{2\tau}}\rho^2}{\tau^3}\left[K_0\left(\frac{\rho^2}{2\tau}\right)+K_1\left(\frac{\rho^2}{2\tau}\right)\right]\right\},
\end{aligned}
$$

[Shuryak, NPB'83] [Elias et al., JPG'98] [Shi et al., NPA'00] [Yuan et al., PRD'17]

Running of α_s and m_q upto four-loop order

$$\alpha_s(1/\tau) = \pi \left\{ \frac{4}{9}\frac{1}{L} - \frac{256 \ln L}{729 L^2} + \frac{1}{L^3}\left[\frac{16384 \ln^2 L}{59049} - \frac{16384 \ln L}{59049} + \frac{6794}{59049}\right]\right\}$$

$$m_q(1/\sqrt{\tau}) = \hat{m}_q \frac{1}{\left(\frac{1}{2}L\right)^{4/9}} \left\{ 1 + \frac{290}{729}\frac{1}{L} - \frac{256}{729}\frac{\ln L}{L} + \left(\frac{550435}{1062882} - \frac{80\zeta(3)}{729}\right)\frac{1}{L^2} \right.$$
$$- \frac{388736}{531441}\frac{\ln L}{L^2} + \frac{106496}{531441}\frac{\ln^2 L}{L^2} + \left(\frac{2121723161}{2324522934} + \frac{8}{6561}\pi^4 - \frac{119840}{531441}\zeta(3)\right.$$
$$\left. - \frac{8000}{59049}\zeta(5)\right)\frac{1}{L^3} + \left(-\frac{611418176}{387420489} + \frac{112640}{531441}\zeta(3)\right)\frac{\ln L}{L^3}$$
$$\left. + \frac{335011840}{387420489}\frac{\ln^2 L}{L^3} - \frac{149946386}{1162261467}\frac{\ln^3 L}{L^3} \right\},$$

[Chetyrkin et al., PLB'97] [Shi et al., NPA'00]

Phenomenological spectral functions

$$R^{(\text{Phen})}(\tau, s_0, \hat{m}_q) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} e^{-s\tau}\,\text{Im}\Pi^{(\text{Phen})}(s)\,ds$$
$$= R^{(\text{Res})}(\tau, s_0) + R^{(\text{ESC})}(\tau, s_0, \hat{m}_q)$$

$\downarrow$

$$\text{Im}\,\Pi_{S^a}(s) = \sum_i \rho_i(s)|F_i^a(s)|^2\theta(s - s_i^{\text{th}})$$

$$F_{PQ}^a(s) = \langle 0|\bar{q}\lambda_a q|PQ\rangle \qquad \rho_i(s) = \frac{q_i}{8\pi\sqrt{s}}$$

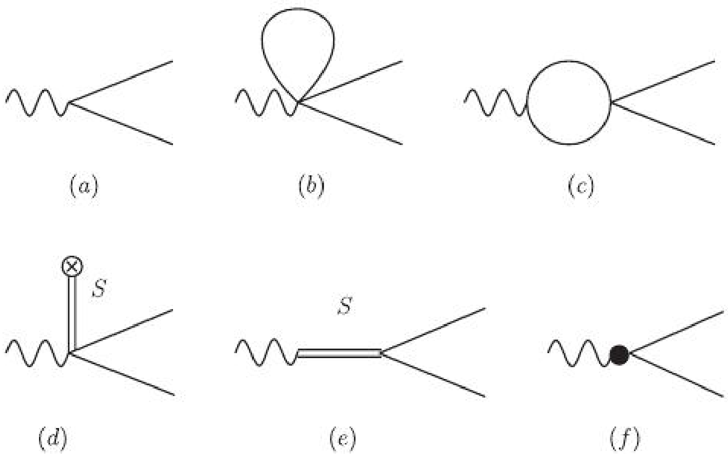
(scalar form factors of the PQ mesons)

- The most important theoretcial input in our study is the scalar form factor of $\pi\pi$

Resonance dynamics and spectral functions from Chiral EFT

Chiral EFT calculation

[ZHG et al., PLB'12, PRD'12]



$$\mathcal{L}_\chi = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{3} M_0^2 \ln^2 \det u$$

$$\mathcal{L}_S = c_d\langle S_8 u_\mu u^\mu\rangle + c_m\langle S_8 \chi_+\rangle + \tilde{c}_d S_1\langle u_\mu u^\mu\rangle + \tilde{c}_m S_1\langle \chi_+\rangle$$

[Gasser, Leutwyler, NPB'85] [Ecker et al., NPB'89]

Unitarity

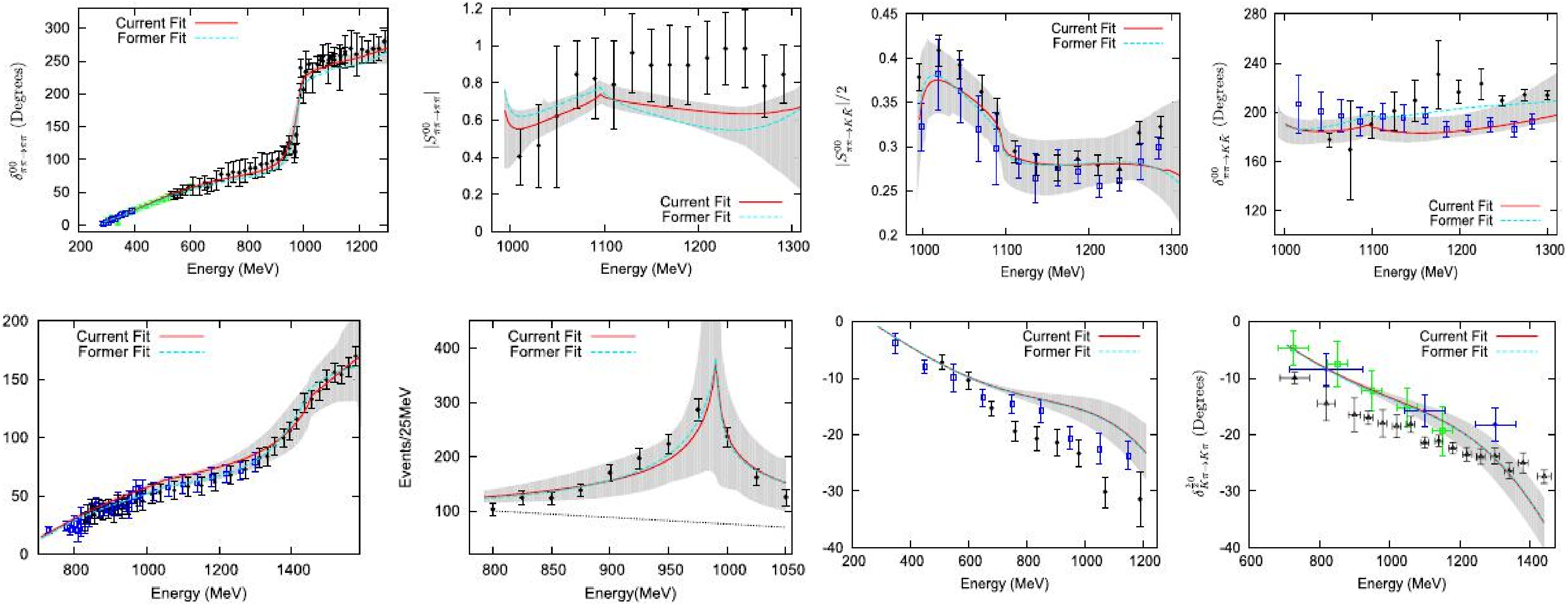
$$\mathrm{Im}\, F_j^I(s) = \sum_{k=1}^{Z} T_{jk}^{IJ}(s)^* \rho_k(s) F_k^I(s)$$

$$F^I(s) = [1 + N^{IJ}(s) g^{IJ}(s)]^{-1} R^I(s)$$

$$N^{IJ}(s) = T^{IJ}(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + T^{IJ}(s)^{(2)} g^{IJ}(s) T^{IJ}(s)^{(2)},$$

$$R^I(s) = F^I(s)^{(2)+\mathrm{Res}+\mathrm{Loop}} + N^{IJ}(s)^{(2)} g^{IJ}(s) F^I(s)^{(2)}.$$

All the unknown FF parameters are fixed by scattering data

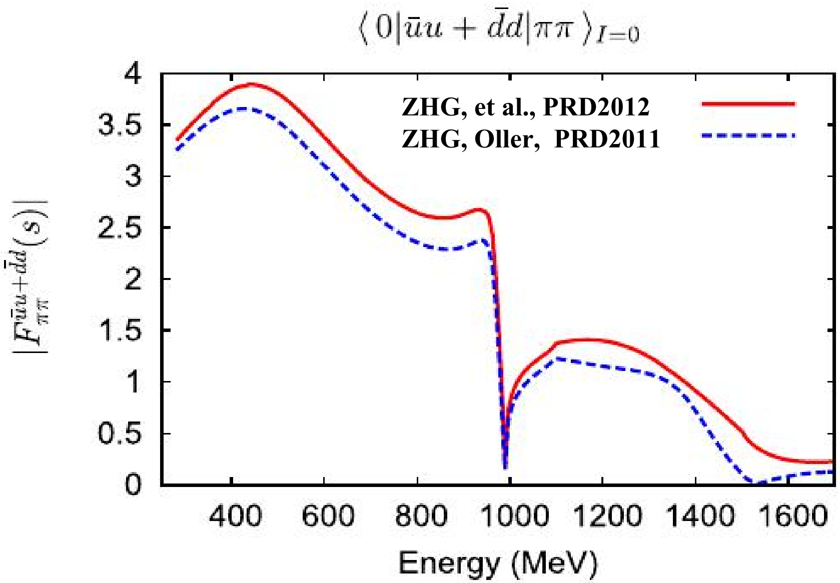


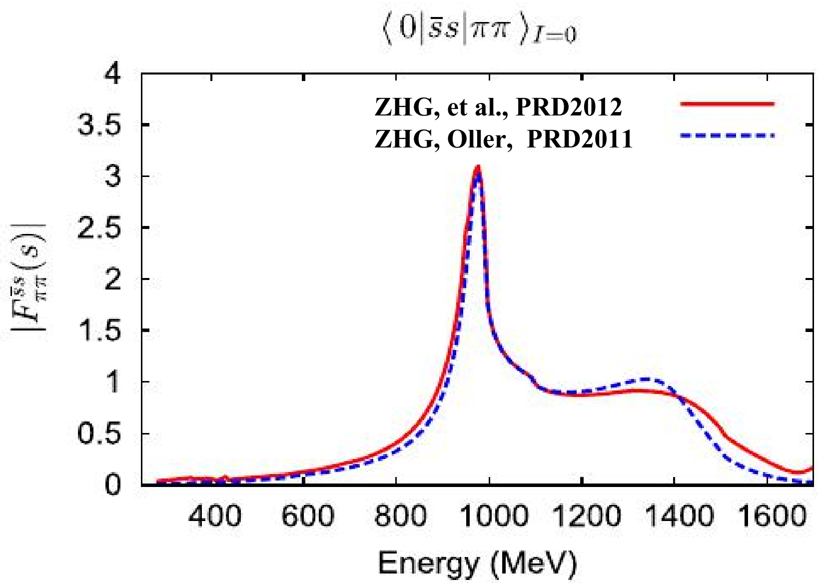
Resonance poles and their couplings in various channels

R	M (MeV)	$\Gamma/2$ (MeV)	$\lvert\text{Residues}\rvert^{1/2}$ (GeV)	Ratios	
$f_0(500)$	442^{+4}_{-4}	246^{+7}_{-5}	$3.02^{+0.03}_{-0.04}(\pi\pi)$	$0.50^{+0.04}_{-0.08}(K\bar{K}/\pi\pi)$	$0.17^{+0.09}_{-0.09}(\eta\eta/\pi\pi)$
				$0.33^{+0.06}_{-0.10}(\eta\eta'/\pi\pi)$	$0.11^{+0.05}_{-0.06}(\eta'\eta'/\pi\pi)$
$f_0(980)$	978^{+17}_{-11}	29^{+9}_{-11}	$1.8^{+0.2}_{-0.3}(\pi\pi)$	$2.6^{+0.2}_{-0.3}(K\bar{K}/\pi\pi)$	$1.6^{+0.4}_{-0.2}(\eta\eta/\pi\pi)$
				$1.0^{+0.3}_{-0.2}(\eta\eta'/\pi\pi)$	$0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$f_0(1370)$	1360^{+80}_{-60}	170^{+55}_{-55}	$3.2^{+0.6}_{-0.5}(\pi\pi)$	$1.0^{+0.7}_{-0.3}(K\bar{K}/\pi\pi)$	$1.2^{+0.7}_{-0.3}(\eta\eta/\pi\pi)$
				$1.5^{+0.4}_{-0.5}(\eta\eta'/\pi\pi)$	$0.7^{+0.2}_{-0.3}(\eta'\eta'/\pi\pi)$
$K_0^*(800)$	643^{+75}_{-30}	303^{+25}_{-75}	$4.8^{+0.5}_{-1.0}(K\pi)$	$0.9^{+0.2}_{-0.3}(K\eta/K\pi)$	$0.7^{+0.2}_{-0.3}(K\eta'/K\pi)$
$K_0^*(1430)$	1482^{+55}_{-110}	132^{+40}_{-90}	$4.4^{+0.2}_{-1.1}(K\pi)$	$0.3^{+0.3}_{-0.3}(K\eta/K\pi)$	$1.2^{+0.2}_{-0.2}(K\eta'/K\pi)$
$a_0(980)$	1007^{+75}_{-10}	22^{+90}_{-10}	$2.4^{+3.2}_{-0.4}(\pi\eta)$	$1.9^{+0.2}_{-0.5}(K\bar{K}/\pi\eta)$	$0.03^{+0.10}_{-0.03}(\pi\eta'/\pi\eta)$
$a_0(1450)$	1459^{+70}_{-95}	174^{+110}_{-100}	$4.5^{+0.6}_{-1.7}(\pi\eta)$	$0.4^{+1.2}_{-0.2}(K\bar{K}/\pi\eta)$	$1.0^{+0.8}_{-0.3}(\pi\eta'/\pi\eta)$
$\rho(770)$	760^{+7}_{-5}	71^{+4}_{-5}	$2.4^{+0.1}_{-0.1}(\pi\pi)$	$0.64^{+0.01}_{-0.02}(K\bar{K}/\pi\pi)$	
$K^*(892)$	892^{+5}_{-7}	25^{+2}_{-2}	$1.85^{+0.07}_{-0.07}(K\pi)$	$0.91^{+0.03}_{-0.02}(K\eta/K\pi)$	$0.41^{+0.07}_{-0.06}(K\eta'/K\pi)$
$\phi(1020)$	$1019.1^{+0.5}_{-0.6}$	$1.9^{+0.1}_{-0.1}$	$0.85^{+0.01}_{-0.02}(K\bar{K})$		

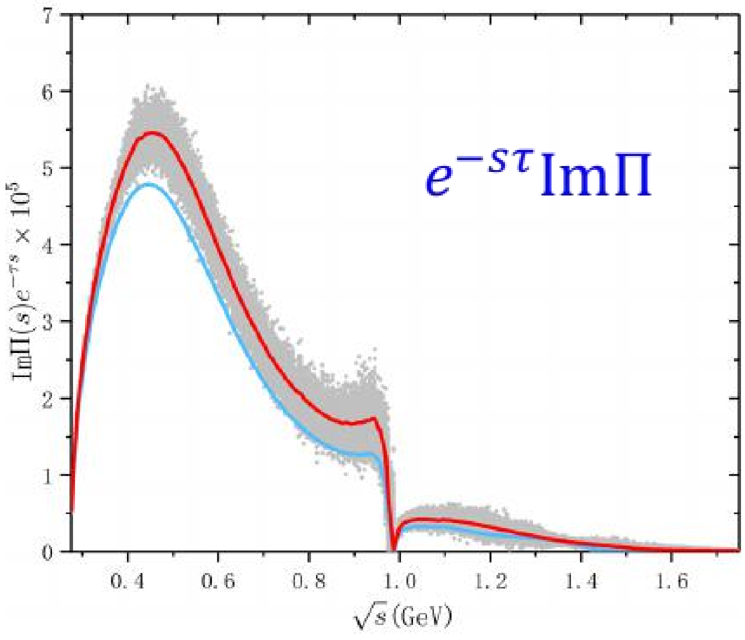
Parameter-free predictions of the scalar form factors

[ZHG et al., PLB'12, PRD'12]





$$\mathrm{Im}\,\Pi$$
$$= \sum_i \rho_i(s)|F_i^a(s)|^2\theta(s - s_i^{\rm th})$$



[Yin, et al., EPJC'21]

Results from SVZ QCD sum rule

Inputs

$$\langle \alpha_s G^2 \rangle = (0.070 \pm 0.007)\ \mathrm{GeV}^4$$

$$\langle m_q \bar{q} q \rangle = -(0.10 \pm 0.01)\ \mathrm{GeV}^4,$$

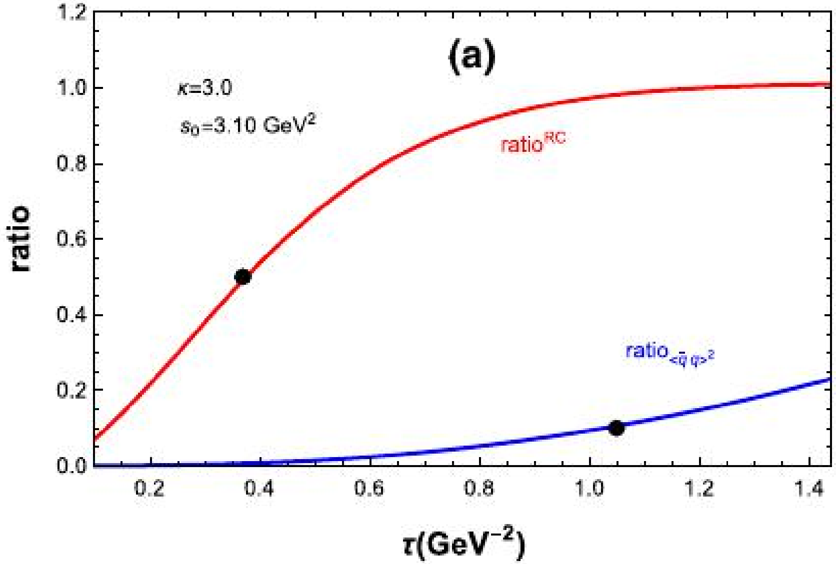
$$\kappa \alpha_s \langle \bar{q} q \rangle^2 = (1.49 \pm 0.15) \times 10^{-4} \kappa\ \mathrm{GeV}^6$$

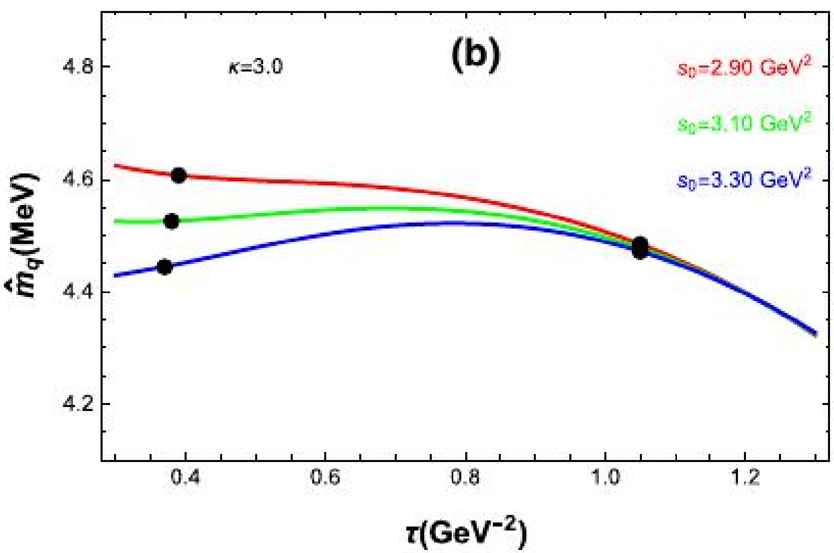
$$\rho = (1.67 \pm 0.17)\ \mathrm{GeV}^{-1},$$

Two criteria

$$\mathrm{ratio}_{\langle \bar{q} q \rangle^2} = \frac{R^{\mathrm{OPE}}_{\langle \bar{q} q \rangle^2}(\tau, \hat{m}_q)}{R^{\mathrm{OPE}}(\tau, \hat{m}_q)} < 10\%$$

$$\mathrm{ratio}^{\mathrm{RC}} = \frac{R^{\mathrm{QCD}}(\tau, \hat{m}_q, s_0)}{R^{\mathrm{QCD}}(\tau, \hat{m}_q)} > 50\%$$





$$m_q(2\ \mathrm{GeV}) = 3.46^{+0.16}_{-0.22} \pm 0.33\ \mathrm{MeV}$$

[Yin, Tian, Tang, ZHG, EPJC'21]

Monte-Carlo QCD sum rule

Key objects of QCD Sum rule:

$$R^{(\mathrm{QCD})}(\tau,\hat{m}_q) = R^{(\mathrm{Phen})}(\tau, s_0, \hat{m}_q)$$

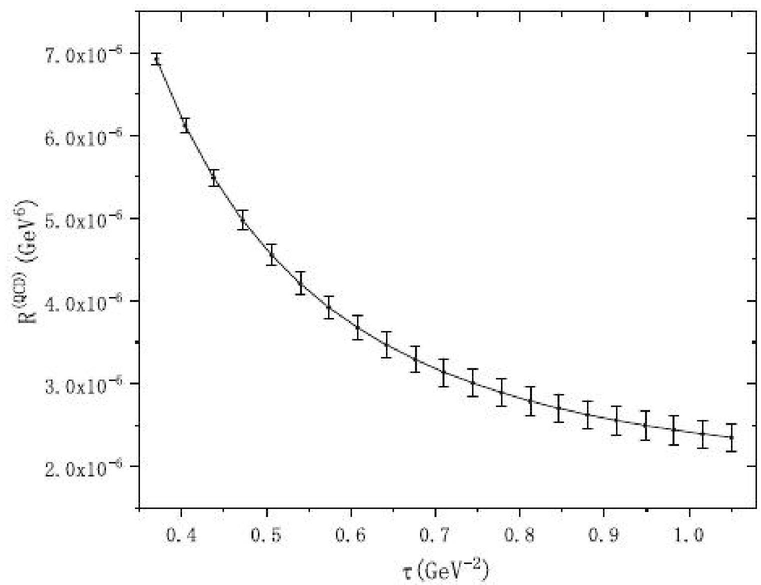
To quantitatively impose the equivalent condition between QCD and phenomenology:

$$\chi^2_{k,\xi} = \sum_{j=1}^{n_B} \frac{[R_k^{(\mathrm{QCD})}(\tau_j,\hat{m}_q^{k,\xi}) - R_\xi^{(\mathrm{phen})}(\tau_j, s_0^{k,\xi},\hat{m}_q^{k,\xi})]^2}{\sigma^2_{\mathrm{QCD}}(\tau_j)}$$

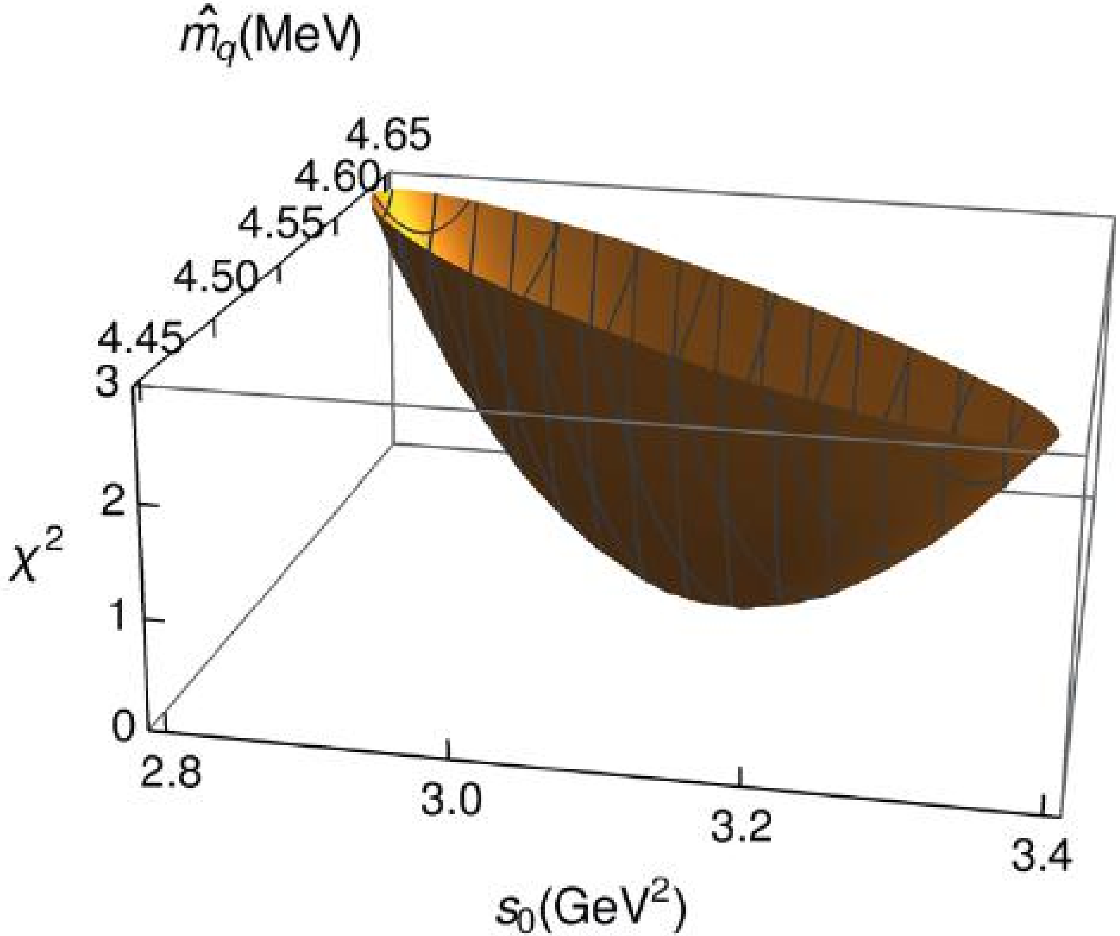
- **Random samples of QCD parameters with 10% uncertainties**
- **Large samples are also generated for the hadron spectral functions**
- **Estimate of the σ_{QCD}**

$$\sigma^2_{\mathrm{QCD}}(\tau_j) =$$

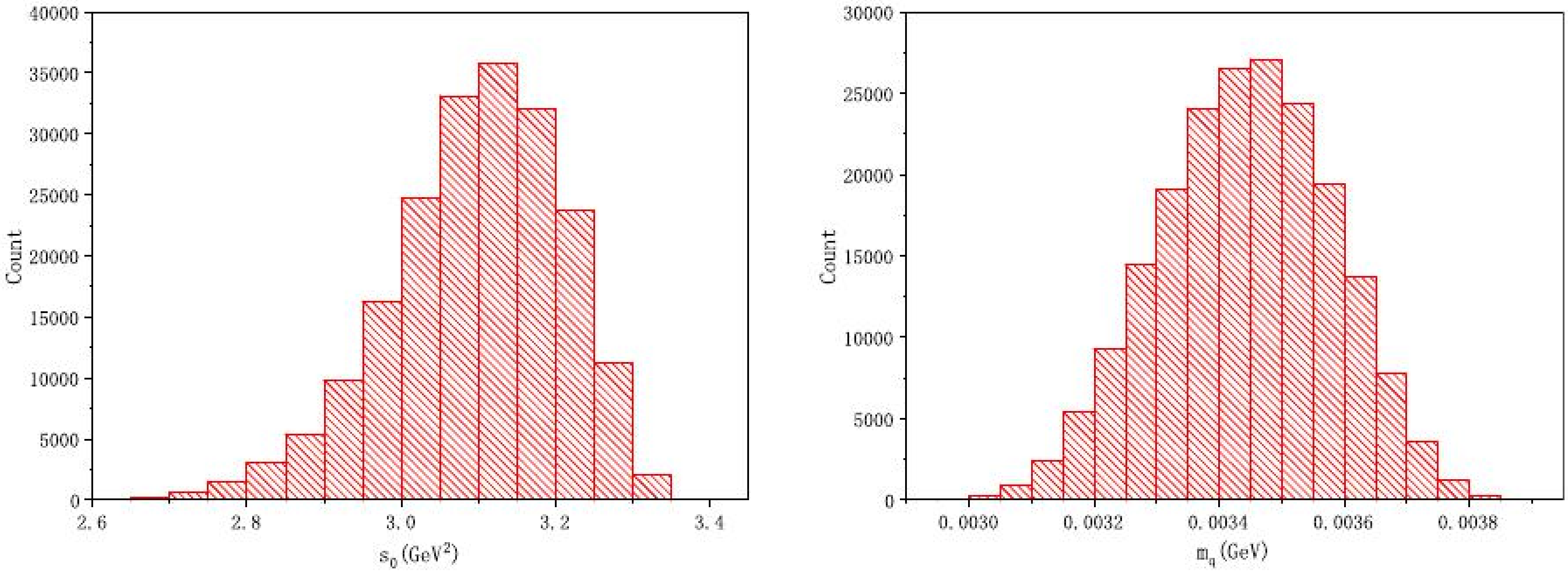
$$\frac{1}{(n_S-1)}\sum_{k=1}^{n_S}[R_k^{(\mathrm{QCD})}(\tau_j) - \overline{R^{(\mathrm{QCD})}}(\tau_j)]^2$$



A typical result from a single minimization procedure



We repeat the minimization procedrues hundreds of thousands of times with the large random samples of the QCD parameters and the hadron spectral functions.



s_0/GeV^2	$\hat{m}_q/\mathrm{MeV}$	m_q (2 GeV)/MeV
3.10 ± 0.11	4.52 ± 0.18	3.44 ± 0.14

Systematical uncertainties are estimated by taking a different hadron spectral function. Our final result is

[Yin,Tian,Tang, ZHG, EPJC'21]

$$s_0 = 3.10 \pm 0.11 \pm 0.16\ \mathrm{GeV}^2$$

$$m_q = 3.44 \pm 0.14 \pm 0.32\ \mathrm{MeV}$$

(SVZ: $m_q(2\ \mathrm{GeV}) = 3.46^{+0.16}_{-0.22} \pm 0.33\ \mathrm{MeV}$)

Both results are nicely compatible with PDG average

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Summary

- **We demonstrate that the light-quark mass resulting from the isoscalar and scalar QCD sum rules is compatible with the determinations from other approaches.**
- **Chiral EFT provides a more sophisticated way to calculate the hadron spectral functions, beyond the simple pole + background method.**
- **We foresee that the strange quark mass can be determined in a similar way.**

Thanks for your attention!