# Improved Standard-Model prediction for the dilepton decay of the neutral pion

based on arXiv:2105.04563 [hep-ph]

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# Outline

# Introduction & motivation

Neutral pion main decay modes: Zyla et al., 2020

| Decay modes | $\pi^0 \to \gamma\gamma$ | $\pi^0 \to e^+e^-\gamma$ | $\pi^0 \to e^+e^-e^+e^-$ | $\pi^0 \to e^+e^-$ |
|---|---|---|---|---|
| Branching ratios | $98.823\%$ | $1.174\%$ | $3.34 \times 10^{-5}$ | $6.46 \times 10^{-8}$ |

- $\pi^0 \to \gamma\gamma$: Adler–Bell–Jackiw anomaly talk by Kampf
- $\pi^0 \to e^+e^-\gamma$: Dalitz decay
- $\pi^0 \to e^+e^-e^+e^-$: double Dalitz decay
- $\pi^0 \to e^+e^-$: rare decay, loop- and helicity-suppressed

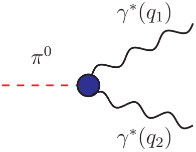
$\Rightarrow$ All listed decays described by pion transition form factor $F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$

# Introduction & motivation

Pion transition form factor (TFF) $F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$:

- Defined by the matrix element of two electromagnetic currents $j_\mu(x)$

$$i\int \mathrm{d}^4x\, e^{iq_1\cdot x}\, \langle 0 | T\{j_\mu(x) j_\nu(0)\} | \pi^0(q_1+q_2)\rangle$$
$$= \epsilon_{\mu\nu\rho\sigma}\, q_1^\rho q_2^\sigma F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$$




- Normalization fixed by the Adler–Bell–Jackiw anomaly:

$$F_{\pi^0\gamma^*\gamma^*}(0,0) = \frac{1}{4\pi^2 F_\pi} \equiv F_{\pi\gamma\gamma}$$

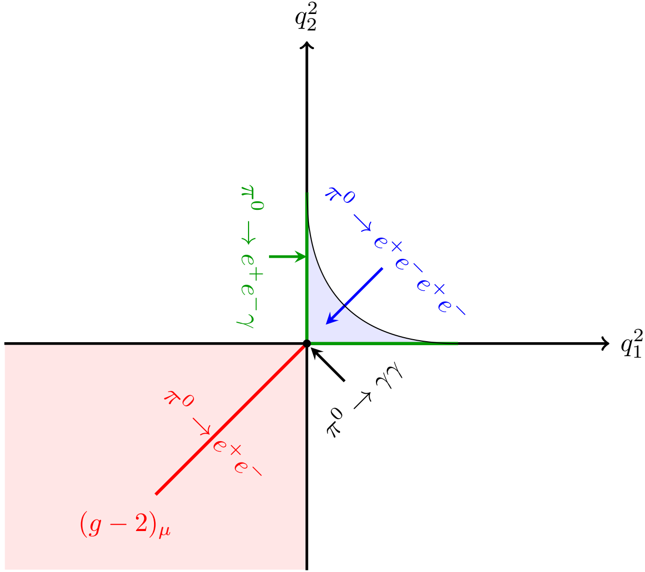
$F_\pi = 92.28(10)\,\mathrm{MeV}$: pion decay constant

Zyla et al., 2020

talk by Gasparian

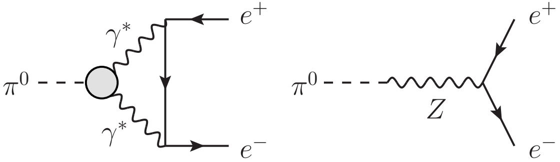
# Introduction & motivation

$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2)$ kinematic regions:



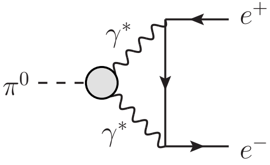
# Introduction & motivation

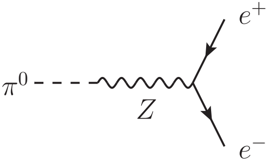
Leading Standard-Model contributions to $\pi^0 \to e^+e^-$:



# Introduction & motivation

Leading Standard-Model contributions to $\pi^0 \to e^+e^-$:




- QED loop contribution dominates

$$\frac{\mathrm{BR}[\pi^0 \to e^+e^-]}{\mathrm{BR}[\pi^0 \to \gamma\gamma]} \sim \left(\frac{\alpha}{\pi}\right)^2 \frac{m_e^2}{M_{\pi^0}^2} \pi^2 \log^2 \frac{m_e}{M_{\pi^0}} \sim \mathcal{O}\left(10^{-8}\right)$$

Drell, 1959

# Introduction & motivation

Experiment: Abouzaid et al., 2006

$$\mathsf{BR}[\pi^0 \to e^+e^-(\gamma), x_D > 0.95]\big|_{\mathsf{KTeV}} = 6.44(25)(22) \times 10^{-8}$$

$$x_D = \frac{m^2_{e^+e^-}}{M^2_{\pi^0}} = 1 - 2\frac{E_\gamma}{M_{\pi^0}}$$

- With old radiative corrections Bergström, 1982

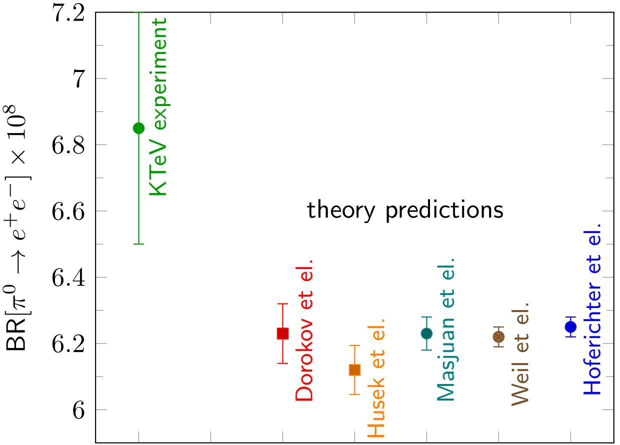
$$\mathsf{BR}[\pi^0 \to e^+e^-]\big|_{\mathsf{KTeV}} = 7.48(29)(25) \times 10^{-8}$$

- With reexamined radiative corrections Vaško, Novotný, 2011, Husek et el., 2014

$$\boxed{\mathsf{BR}[\pi^0 \to e^+e^-]\big|_{\mathsf{KTeV}} = 6.85(27)(23) \times 10^{-8}}$$

# Introduction & motivation

Experiment vs theory:




- $\sim 2\sigma$ discrepancy between experiment and theory

# Pion transition form factor

We build the form factor double-spectral representation:

Hoferichter et al., 2018

$$\boxed{F_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = F^{\mathsf{disp}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\mathsf{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) + F^{\mathsf{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2)}$$

- Reconstructed from the lowest-lying singularities $2\pi$ and $3\pi$
- Fulfills the asymptotic constraints at $\mathcal{O}\big(1/Q^2\big)$
- Suitable for $a_\mu$ & $\pi^0 \to e^+e^-$ loop-integral evaluation

# Pion transition form factor

Dispersive form factor:

$$F^{\mathsf{disp}}_{\pi^0\gamma^*\gamma^*}(-Q_1^2,-Q_2^2) = \frac{1}{\pi^2}\int_{4M_\pi^2}^{s_{\mathsf{iv}}} \mathrm{d}x \int_{s_{\mathsf{thr}}}^{s_{\mathsf{is}}} \frac{\rho^{\mathsf{disp}}(x,y)\,\mathrm{d}y}{(x+Q_1^2)(y+Q_2^2)},$$

$$\rho^{\mathsf{disp}}(x,y) = \frac{q_\pi^3(x)}{12\pi\sqrt{x}}\mathrm{Im}\left[\left(F_\pi^V(x)\right)^* f_1(x,y)\right] + [x \leftrightarrow y]$$

$F_\pi^V(s)$ pion vector form factor, $f_1(s,q^2) = \gamma_s^*(q) \to 3\pi$ $P$-wave amplitude

Effective pole term:

$$F^{\mathsf{eff}}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{g_{\mathsf{eff}}}{4\pi^2 F_\pi}\frac{M_{\mathsf{eff}}^4}{(M_{\mathsf{eff}}^2-q_1^2)(M_{\mathsf{eff}}^2-q_2^2)}$$

$g_{\mathsf{eff}}$ fixed by fulfilling the chiral anomaly

$M_{\mathsf{eff}}$ fit to singly-virtual data excluding BaBar above $5\,\mathrm{GeV}^2$ Gronberg et al., 1998, Aubert et al., 2009, Uehara et al., 2012

# Pion transition form factor

Asymptotically, $F_{\pi^0\gamma^*\gamma^*}$ should fulfill Brodsky, Lepage, 1979-1981

$$F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = -\frac{2F_\pi}{3}\int_0^1 \mathrm{d}x \frac{\phi_\pi(x)}{xq_1^2 + (1-x)q_2^2} + \mathcal{O}\left(\frac{1}{q_i^4}\right),$$

Pion distribution amplitude $\phi_\pi(x) = 6x(1-x) + \cdots$

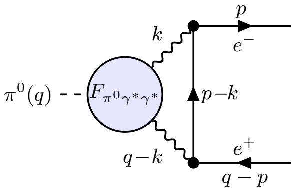
This defines the asymptotic contribution

$$F^{\mathsf{asym}}_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) = 2F_\pi \int_{s_\mathrm{m}}^{\infty} \mathrm{d}x \frac{q_1^2 q_2^2}{(x-q_1^2)^2(x-q_2^2)^2}$$

- Restores the asympotics for singly/doubly-virtual kinematics

# Results

Reduced amplitude $\mathcal{A}(q^2)$:




$$\frac{\mathsf{BR}[\pi^0 \to e^+e^-]}{\mathsf{BR}[\pi^0 \to \gamma\gamma]} = 2\sigma_e(q^2)\left(\frac{\alpha}{\pi}\right)^2 \frac{m_e^2}{M_{\pi^0}^2}\left|\mathcal{A}(q^2)\right|^2$$

$$q^2 = M_{\pi^0}^2,\ \sigma_e(q^2) = \sqrt{1 - \tfrac{4m_e^2}{q^2}}$$

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2 q^2}\int \mathsf{d}^4k \frac{q^2k^2 - (q\cdot k)^2}{k^2(q-k)^2[(p-k)^2 - m_e^2]} \times \tilde{F}_{\pi^0\gamma^*\gamma^*}\left(k^2,(q-k)^2\right)$$

$\tilde{F}_{\pi^0\gamma^*\gamma^*}\left(k^2,(q-k)^2\right) = F_{\pi^0\gamma^*\gamma^*}\left(k^2,(q-k)^2\right)/F_{\pi\gamma\gamma}$, normalized pion TFF

# Results

Imaginary part from the $\gamma\gamma$ cut:

$$\mathrm{Im}\,\mathcal{A}(q^2) = \frac{\pi}{2\sigma_e(q^2)} \log\left[\frac{1-\sigma_e(q^2)}{1+\sigma_e(q^2)}\right] = -17.52$$

$\mathrm{Re}\,\mathcal{A}(q^2)$? $\Rightarrow$ need to perform the integral with the form factor

- $\mathcal{A}^{\text{eff}}$: standard reduction — Passarino, Veltman, 1979, 't Hooft, Veltman, 1979
- $\mathcal{A}^{\text{asym}}$: integration by parts — Chetyrkin, Tkachov, 1981

# Results

For the dispersive part we can write

Masjuan, Sánchez-Puertas, 2016

$$\mathcal{A}^{\mathsf{disp}}(q^2) = \frac{2}{\pi^2}\int_{4M_\pi^2}^{s_{\mathsf{iv}}} \mathsf{d}x \int_{s_{\mathsf{thr}}}^{s_{\mathsf{is}}} \mathsf{d}y \frac{\tilde{\rho}^{\mathsf{disp}}(x,y)}{xy} K(x,y)$$

Integration kernel

$$K(x,y) = \frac{2i}{\pi^2 q^2}\int \mathsf{d}^4 k \frac{q^2k^2 - (q\cdot k)^2}{k^2(q-k)^2[(p-k)^2 - m_e^2]} \times \frac{xy}{(k^2 - x)[(q-k)^2 - y]}$$

- Checked with several standard techniques

# Results

Long-range contribution from the final representation:

$$\text{Re}\,\mathcal{A}(q^2)\big|_{\gamma^*\gamma^*} = 10.16(5)_{\text{disp}}(8)_{\text{BL}}(2)_{\text{asym}}$$

$Z$-boson contribution:

$$\text{Re}\,\mathcal{A}(q^2)\big|_{Z} = -\frac{F_\pi G_F}{\sqrt{2}\,\alpha^2 F_{\pi\gamma\gamma}} = -0.05(0)$$

Final Standard-Model prediction:

$$\boxed{\begin{aligned} \text{Re}\,\mathcal{A}(q^2)\big|_{\text{SM}} &= 10.11(10) \\ \text{BR}[\pi^0 \to e^+e^-]\big|_{\text{SM}} &= 6.25(3)\times 10^{-8} \end{aligned}}$$

- Fully controlled uncertainty estimates
- Mild $1.8\sigma$ tension with experiment

# Results

Beyond the Standard-Model scenerios with light axial-vector $Z'$ or pseudoscalar $a$:

$$\mathcal{L}_{\mathsf{BSM}} = \sum_{f=e,u,d} \bar{f}\left(c_A^f \gamma^\mu \gamma_5 Z'_\mu + c_P^f i\gamma_5 a\right) f$$

Limit from $\pi^0 \to e^+e^-$:

$$-\frac{(c_A^u - c_A^d)c_A^e}{M_{Z'}^2} = (-280)_{-150}^{+160}\,\mathsf{TeV}^{-2}, \quad \frac{(c_P^u - c_P^d)c_P^e}{m_a^2 - q^2} = (-0.108)_{-0.057}^{+0.062}\,\mathsf{TeV}^{-2}$$

Contributions to electron $g-2$:

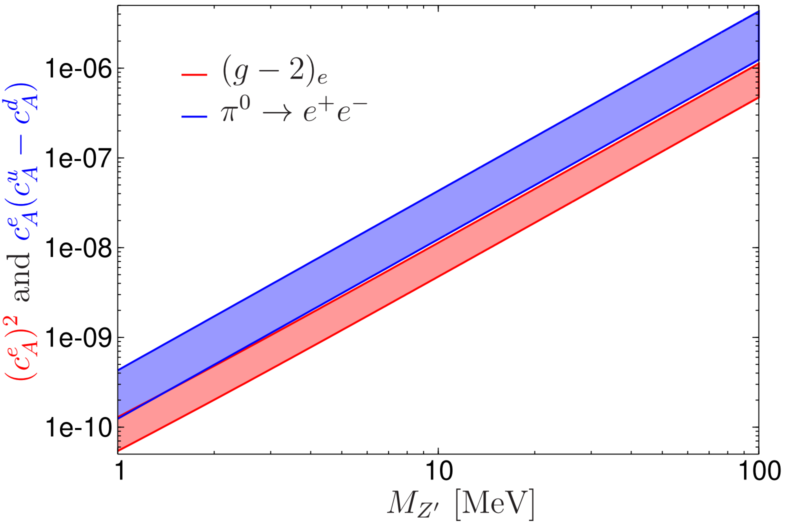
$$a_e^A = -\frac{(c_A^e)^2 m_e^2}{4\pi^2 M_{Z'}^2}\int_0^1 \mathsf{d}x\,\frac{2x^3 m_e^2 + x(1-x)(4-x)M_{Z'}^2}{m_e^2 x^2 + M_{Z'}^2(1-x)}$$
$$a_e^P = -\frac{(c_P^e)^2 m_e^2}{8\pi^2}\int_0^1 \mathsf{d}x\,\frac{x^3}{m_e^2 x^2 + m_a^2(1-x)}$$

Constraints from Cs atom interferometry ( $a_e^A \& a_e^P < 0$): Parker et el., 2018

$$\Delta a_e[\mathsf{Cs}] = a_e^{\mathsf{exp}} - a_e^{\mathsf{SM}}[\mathsf{Cs}] = -0.88(36)\times 10^{-12}$$

# Results

Constraints for axial-vector $Z'$:

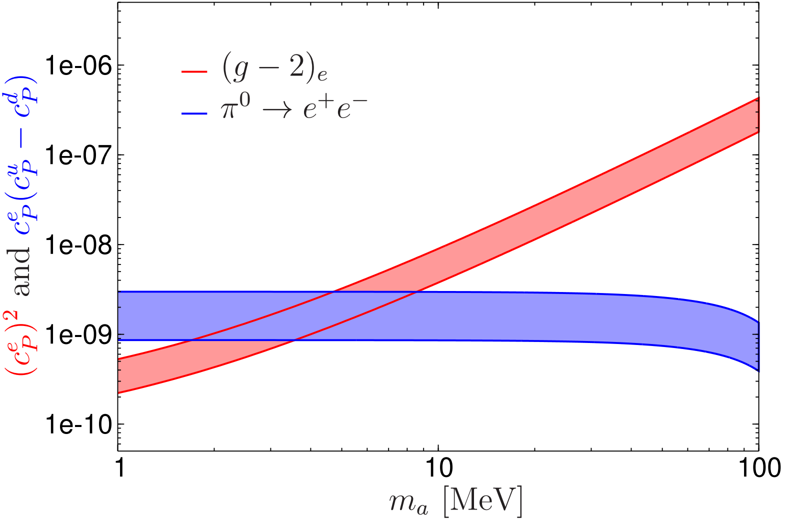



- Revises the previously preferred region

Parker et el., 2018

# Results

Constraints for axion-like particle $a$:



# Conclusion and outlook

- $\pi^0 \to e^+e^-$ decay
  - Pion transition form factor
  - Reduced amplitude with form factor representation
  - Standard-Model prediction with 0.5% precision
  - Constraints on physics beyond the Standard Model
- Experiment and lattice progress NA62, Christ et el.
- Similar analysis for other pseudoscalar decays

# Much obliged for your attention!

"Rare is the union of beauty and purity."

Juvenal