On the QCD contribution to vacuum energy

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Outline

- EFT of gravitational field interacting with matter fields;
- One- and two-loop contributions to VE, the vacuum expectation value of the gravitational field and the graviton self-energy. Conditions of consistency;
- QCD contribution to VE and the problem of the fine-tuning;
- Summary;

Talk based on
J. G. and U.-G. Meißner, Phys. Rev. D **100**, no. 4, 046021 (2019);
J. G. and U.-G. Meißner, Phys. Rev. D **100**, no.12, 124002 (2019);

EFT of gravitational field interacting with matter fields

At low energies the physics of the fundamental particles is described by EFTs, SM being its LO approximation
S. Weinberg, *The Quantum Theory Of Fields. Vol. 1: Foundations* (Cambridge University Press, Cambridge, England, 1995).

Gravitation can also be included in this framework by considering the effective Lagrangian of metric fields interacting with matter fields
J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).

Within this approach the metric field is represented as the Minkowski background plus the graviton field and the cosmological constant Λ is usually set equal to zero.

For a non-vanishing Λ the graviton propagator has a pole corresponding to a massive ghost mode.
M. J. G. Veltman, Conf. Proc. C **7507281**, 265 (1975).

As the cosmological constant term is not suppressed by any symmetry of the EFT, setting it to zero does not solve the problem, because the radiative corrections re-generate the massive ghost.

Representing Λ as a power series in $\hbar$ the coefficients can be adjusted such that the unphysical mass of the graviton is cancelled D. Burns and A. Pilaftsis, Phys. Rev. D **91**, no. 6, 064047 (2015).

Thus, to take into account Λ other than obtained in above work it is necessary to consider an EFT in a curved background field.

To the best of our knowledge, such an EFT is not available yet.

VE in an EFT of matter interacting with gravitons

EFT of general relativity is described by the most general effective Lagrangian of gravitational and matter fields, invariant under general coordinate transformations and other symmetries,

$$\begin{aligned} S &= \int d^4x \sqrt{-g}\left\{\frac{2}{\kappa^2}(R-2\Lambda)+\mathcal{L}_{\rm gr,ho}(g)+\mathcal{L}_{\rm m}(g,\psi)\right\} \\ &= S_{\rm gr}(g)+S_{\rm m}(g,\psi), \end{aligned}$$

$\kappa^2 = 32\pi G$, Newton's constant $G = 6.70881 \cdot 10^{-39}\,\mathrm{GeV}^{-2}$, ψ and $g^{\mu\nu}$ denote the matter and metric fields, respectively, $g = \det g^{\mu\nu}$, and R is the scalar curvature.

$\mathcal{L}_{\rm m}(g,\psi)$ is the Lagrangian of the matter fields. Interaction terms with higher orders of derivatives in $\mathcal{L}_{\rm gr,ho}(g)$ and $\mathcal{L}_{\rm m}(g,\psi)$ are heavily suppressed for energies accessible by current accelerators.

Vielbein tetrad fields have to be introduced for an EFT with fermions.

On EFT of general relativity in Minkowski background we impose the condition:

Energy of the vacuum has to be zero!

This uniquely fixes the cosmological constant!

Does this value of Λ correspond to a consistent perturbative EFT?

Consider a model of fermion, scalar and a vector fields interacting with gravity which for Minkowski metric coincides to a model of spontaneously broken Abelian gauge symmetry in unitary gauge.

The action of the matter part of the model is given by

$$
\begin{aligned}
S_{\mathrm{m}} &= \int d^4x\sqrt{-g}\,\Big\{\frac{1}{2}\,\bar\psi\, ie_a^\mu\gamma^a\nabla_\mu\psi - \frac{1}{2}\,\nabla_\mu\bar\psi\, ie_a^\mu\gamma^a\psi - m_F\bar\psi\psi \\
&- \frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{M^2}{2}\,g^{\mu\nu}A_\mu A_\nu + \frac{g^{\mu\nu}}{2}\,\partial_\mu H\partial_\nu H - \frac{m^2}{2}\,H^2 \\
&+ \mathcal{L}_{\mathrm{MI}} + \mathcal{L}_{\mathrm{HO}}\Big\}\,,
\end{aligned}
$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, A_μ is vector field, H is the scalar field, $\mathcal{L}_{\mathrm{MI}}$ denotes the renormalizable interactions between matter fields and $\mathcal{L}_{\mathrm{HO}}$ are interactions of higher order.

The covariant derivative acting on the fermion field has the form

$$\begin{aligned}\nabla_\mu \psi &= \partial_\mu \psi - \omega_\mu^{ab} \sigma_{ab} \psi, \\ \nabla_\mu \bar{\psi} &= \partial_\mu \bar{\psi} + \bar{\psi}\, \sigma_{ab}\, \omega_\mu^{ab},\end{aligned}$$

where $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ and

$$\begin{aligned}\omega_\mu^{ab} &= -g^{\nu\lambda} e_\lambda^a \left(\partial_\mu e_\nu^b - e_\sigma^b \Gamma_{\mu\nu}^\sigma \right), \\ \Gamma_{\alpha\beta}^\lambda &= \frac{1}{2} g^{\lambda\sigma} \left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta} \right) .\end{aligned}$$

The vielbein fields satisfy the following relations:

$$\begin{aligned} e_\mu^a e_\nu^b \eta_{ab} = g_{\mu\nu}, \quad e_a^\mu e_b^\nu \eta^{ab} = g^{\mu\nu}, \\ e_\mu^a e_\nu^b g^{\mu\nu} = g^{ab}, \quad e_a^\mu e_b^\nu g_{\mu\nu} = g_{ab}.\end{aligned}$$

The energy-momentum tensor corresponding to matter fields:

$$
\begin{aligned}
T_m^{\mu\nu} &= -g^{\mu\alpha} g^{\nu\rho} g^{\beta\sigma} F_{\alpha\beta} F_{\rho\sigma} + M^2 g^{\mu\alpha} g^{\nu\beta} A_\alpha A_\beta + \partial_\mu H \partial_\nu H \\
&- g^{\mu\nu} \left\{ -\frac{1}{4} g^{\alpha\rho} g^{\beta\sigma} F_{\alpha\beta} F_{\rho\sigma} + \frac{M^2}{2} g^{\alpha\beta} A_\alpha A_\beta + \frac{g^{\alpha\beta}}{2} \partial_\alpha H \partial_\beta H \right. \\
&- \left. \frac{m^2}{2} H^2 \right\} + \frac{i}{4} \left(\bar{\psi} e_{a\mu} \gamma^a \nabla_\nu \psi + \bar{\psi} e_{a\nu} \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e_{a\nu} \gamma^a \psi \right. \\
&- \left. \nabla_\nu \bar{\psi} e_{a\mu} \gamma^a \psi \right) + T^{\mu\nu}_{\mathrm{HO}} + T^{\mu\nu}_{\mathrm{MI}} ,
\end{aligned}
$$

where $T^{\mu\nu}_{\mathrm{MI}}$ and $T^{\mu\nu}_{\mathrm{HO}}$ correspond to $\mathcal{L}_{\mathrm{MI}}$ and $\mathcal{L}_{\mathrm{HO}}$, respectively.

For the gravitational field we have:

$$
\begin{aligned}
&T^{\mu\nu}_{\mathrm{gr}}(g) = \frac{4}{\kappa^2}\,\Lambda\, g^{\mu\nu} + T^{\mu\nu}_{LL}(g)\,,\\
&(-g)\,T^{\mu\nu}_{LL}(g) = \frac{2}{\kappa^2}\Bigg(\frac{1}{8}\, g^{\lambda\sigma} g^{\mu\nu} g_{\alpha\gamma} g_{\beta\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda}\\
&-\frac{1}{4}\, g^{\mu\lambda} g^{\nu\sigma} g_{\alpha,\gamma} g_{\beta\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} - \frac{1}{4}\, g^{\lambda\sigma} g^{\mu\nu} g_{\beta\alpha} g_{\gamma\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda}\\
&+\frac{1}{2}\, g^{\mu\lambda} g^{\nu\sigma} g_{\beta\alpha} g_{\gamma\delta}\, \mathfrak{g}^{\alpha\gamma}{}_{,\sigma}\ \mathfrak{g}^{\beta\delta}{}_{,\lambda} + g^{\beta\alpha} g_{\lambda\sigma}\, \mathfrak{g}^{\nu\sigma}{}_{,\alpha}\ \mathfrak{g}^{\mu\lambda}{}_{,\beta}\\
&+\frac{1}{2}\, g^{\mu\nu} g_{\lambda\sigma}\, \mathfrak{g}^{\lambda\beta}{}_{,\alpha}\ \mathfrak{g}^{\alpha\sigma}{}_{,\beta} - g^{\mu\lambda} g_{\sigma\beta}\, \mathfrak{g}^{\nu\beta}{}_{,\alpha}\ \mathfrak{g}^{\sigma\alpha}{}_{,\lambda}\\
&- g^{\nu\lambda} g_{\sigma\beta}\, \mathfrak{g}^{\mu\beta}{}_{,\alpha}\ \mathfrak{g}^{\sigma\alpha}{}_{,\lambda} + \mathfrak{g}^{\lambda\sigma}{}_{,\sigma}\ \mathfrak{g}^{\mu\nu}{}_{,\lambda} - \mathfrak{g}^{\mu\lambda}{}_{,\lambda}\ \mathfrak{g}^{\nu\sigma}{}_{,\sigma}\Bigg)\,,
\end{aligned}
$$

with $\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$ and $\mathfrak{g}^{\mu\nu}{}_{,\lambda} = \partial\mathfrak{g}^{\mu\nu}/\partial x^\lambda$.

L. D. Landau and E. M. Lifschits, “The Classical Theory of Fields,” Oxford: Pergamon Press (1975).
The full energy-momentum tensor $T^{\mu\nu} = T^{\mu\nu}_{\rm m}(g,\psi) + T^{\mu\nu}_{\rm gr}(g)$ defines the conserved full four-momentum

$$P^\mu = \int (-g)\, T^{\mu\nu} dS_\nu \,,$$

where the integration is carried out over any hypersurface containing the whole three-dimensional space.

The energy of the vacuum (VE) will be zero if the vacuum expectation value of EMT$\times(-g)$ vanishes.
This quantity is given by the following path integral:

$$\langle 0|(-g)T^{\mu\nu}|0\rangle = \int \mathcal{D}g\,\mathcal{D}\psi\,(-g)\left[T^{\mu\nu}_{\rm gr}(g) + T^{\mu\nu}_{\rm m}(g,\psi)\right]$$
$$\times \exp\left\{i\int d^4x\,\sqrt{-g}\,[\mathcal{L}(g,\psi) + \mathcal{L}_{\rm GF}]\right\},$$

where $\mathcal{L}_{\rm GF}$ is the gauge fixing term and the Faddeev-Popov determinant is included in the integration measure.

The condition of vanishing of VE uniquely fixes all coefficients in the power series expansion of the cosmological constant in terms of $\hbar$:

$$\Lambda = \sum_{i=0}^{\infty} \hbar^i \Lambda_i \,.$$

For perturbative calculations we split the metric and vielbein fields as sums of the Minkowskian background and the quantum fields

$$\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + \kappa h_{\mu\nu}, \\
g^{\mu\nu} &= \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} - \kappa^3 h^{\mu}_{\lambda} h^{\lambda}_{\sigma} h^{\sigma\nu} + \cdots \,, \\
e^{a}_{\mu} &= \delta^{a}_{\mu} + \frac{\kappa}{2} h^{a}_{\mu} - \frac{\kappa^2}{8} h_{\mu\rho} h^{a\rho} + \cdots \,, \\
e^{\mu}_{a} &= \delta^{\mu}_{a} - \frac{\kappa}{2} h^{\mu}_{a} + \frac{3\kappa^2}{8} h_{a\rho} h^{\mu\rho} + \ldots \,,
\end{aligned}$$

and applying standard QFT techniques obtain the Feynman rules.

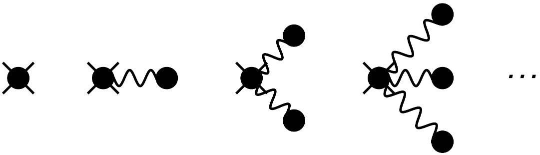

Figure: Tree diagrams contributing to the vacuum expectation value of $\mathrm{EMT}\times(-g)$. Filled circle corresponds to the cosmological constant term. The cross stands for $\mathrm{EMT}\times(-g)$, wiggly lines represent gravitons.

An infinite number of diagrams contribute to the vacuum expectation value of $\mathrm{EMT}\times(-g)$ at tree order, however, all of them vanish if we take $\Lambda_0 = 0$.

This also removes the mass term from the graviton propagator, corresponding to a ghost degree of freedom, at tree order.

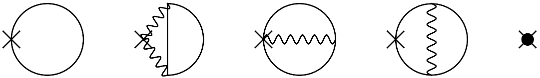

Figure: Loop diagrams contributing to the vacuum expectation value of EMT $\times$ $(-g)$. Filled circle corresponds to the cosmological constant term. The cross stands for EMT$\times(-g)$, wiggly and solid lines represent graviton and matter field propagators, respectively.

To obtain one-loop contributions to the vacuum expectation value of EMT$\times(-g)$, we calculated the corresponding diagrams and by demanding that Λ_1 cancels this contribution obtained

$$\Lambda_1 = \frac{\kappa^2 \mu^{4-d} m_F^d \Gamma\left(1-\frac{d}{2}\right)}{2^d \pi^{\frac{d}{2}} d} - \frac{\kappa^2 \Gamma\left(1-\frac{d}{2}\right)\left(m^d + (d-1)M^d\right)}{2^{d+6}\pi^{\frac{d}{2}+4}\, d}.$$

It is a trivial consequence of the definition of EMT of the matter fields that Λ_1 also cancels the one-loop contribution to the vacuum expectation value of $h_{\mu\nu}$, and consequently, the graviton self-energy at zero momentum, i.e. graviton mass, as a result of a Ward identity.
D. Burns and A. Pilaftsis, Phys. Rev. D **91**, no. 6, 064047 (2015).

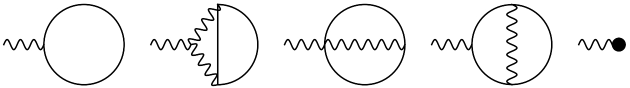

Figure: Diagrams contributing to the graviton tadpole. The filled circle corresponds to the cosmological constant term. Wiggly and solid lines represent the graviton and the matter fields, respectively.

The first non-trivial result is obtained by calculating two-loop diagrams contributing to the vacuum expectation value of EMT$\times(-g)$ and to the vacuum expectation value of $h_{\mu\nu}$.

The same value

$$\Lambda_2 = -\frac{d^3\kappa^4\mu^{8-2d}m_F^{2d-2}\csc\left(\frac{\pi d}{2}\right)\Gamma\left(-\frac{d}{2}\right)}{2^{2d+7}\pi^{d-1}(d-2)\Gamma\left(\frac{d}{2}\right)} - \frac{d(d+1)\kappa^4 M^{2d-2}\csc\left(\frac{\pi d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)}{2^{2(d+3)}\pi^{d-1}\Gamma\left(\frac{d}{2}\right)}$$

cancels both quantities.

To check the reliability of the obtained results we also calculated the two-loop contributions to the graviton self-energy and veryfied that the same value of Λ_2 ensures that the graviton remains massless in agreement with the Ward identity.

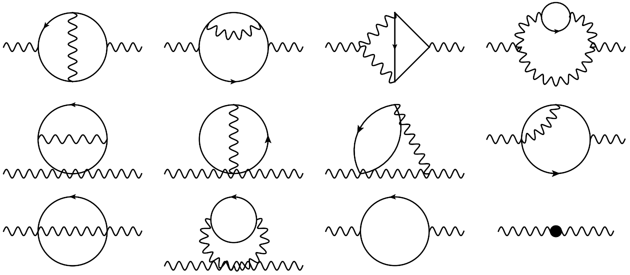

Figure: Diagrams contributing to the graviton self-energy. The filled circle corresponds to the cosmological constant term. Wiggly and solid lines represent the graviton and the matter field propagators, respectively.

QCD contribution to the vacuum energy

The QCD contribution to VE due to the explicit breaking of chiral symmetry can be calculated with great accuracy.

Two-flavour QCD Lagrangian of massless up and down quarks with external scalar and pseudoscalar currents $s(x)$ and $p(x)$:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\,\bar{\psi}\gamma_\alpha D^\alpha \psi - \bar{\psi}(s - i\,\gamma_5 p)\psi\,,$$

where $\psi = (\psi_u, \psi_d)^T$ is a doublet of up and down quark fields. This Lagrangian is invariant under $SU(2)_L \times SU(2)_R$ chiral symmetry transformations

$$\frac{1}{2}(1-\gamma_5)\psi \to L\,\frac{1}{2}(1-\gamma_5)\psi\,, \quad \frac{1}{2}(1+\gamma_5)\psi \to R\,\frac{1}{2}(1+\gamma_5)\psi\,,$$
$$(s + i\,p) \to L\,(s + i\,p)R^\dagger\,,$$

with L and R elements of $SU(2)_L$ and $SU(2)_R$, respectively.

Massless QCD undergoes spontaneous symmetry breaking with pions appearing as Goldstone bosons.

Low-energy effective Lagrangian at lowest order has the form

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\,\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi^2}{4}\,\mathrm{Tr}(\chi U^\dagger + U\chi^\dagger)\ ,$$

where the field U is given in terms of the pion fields π^a $(a = 1, 2, 3)$ $U = \exp\left(\frac{i\tau^a\pi^a}{F_\pi}\right)$, $\chi = 2B_0(s + ip)$ with B_0 related to the vacuum expectation value of the quark condensate and F_π is the pion decay constant.

The above Lagrangian is invariant under chiral transformations

$$U \to LUR^\dagger\,,\ \ (s + i\,p) \to L(s + i\,p)R^\dagger\,.$$

EFT corresponding to QCD without external sources is obtained by substituting the external sources as follows

$$s = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix}, \quad p = 0\,.$$

As the quark masses explicitly break the chiral symmetry, the pions obtain a small mass to leading order in the chiral expansion

$$M_\pi^2 = B_0(m_u + m_d) + \mathcal{O}(m_q^2)\ ,$$

where m_q denotes the light quark mass.

The effective Lagrangian generates a tree-order contribution to the vacuum energy

$$\Lambda_m = -\langle 0|\mathcal{L}_2|0\rangle = -F_\pi^2 B_0(m_u + m_d) = -F_\pi^2 M_\pi^2\ .$$

There is no other term which could compensate this contribution, e.g. $\mathrm{Tr}(\chi + \chi^\dagger)$ would contribute to VE but it violates the chiral symmetry.

It is argued in
J. F. Donoghue, Ann. Rev. Nucl. Part. Sci. **66**, 1 (2016)
that to cancel the QCD contribution to VE one needs to adjust numerically Λ in EFT of pions interacting with gravitation where it is one of the parameters.

QCD gives a large contribution to the vacuum energy

$$\Lambda_m = 1.5 \times 10^8\,\mathrm{MeV}^4 = 0.63 \times 10^{43}\Lambda_{\mathrm{exp}},$$

where $\Lambda_{\mathrm{exp}} = 2.4 \times 10^{-47}\ \mathrm{GeV}^4$ is the observed value.
It is stated in the above paper that:
"Because of the large multiplier, if one holds all the other parameters of the Standard Model fixed, a change of the up quark mass in its forty-first digit would produce a change in Λ outside the anthropically allowed range. ... because the calculation is so well controlled, it illustrates the degree of fine-tuning required as well as the futility of thinking that some feature of the Standard Model could lead to a vanishing contribution to Λ."

Below we critically examine this statement.

It is straightforward to construct an EFT action of pions including the interaction with the gravitational field:

$$S_{\rm gr}(g) + \int d^4x \sqrt{-g}\left[-F_\pi^2 M_\pi^2 + \frac{g^{\mu\nu}}{2}\partial_\mu \pi^a \partial_\nu \pi^a - \frac{M_\pi^2}{2}\pi^a \pi^a + \mathcal{O}(\pi^4)\right].$$

To cancel the tree order contribution to VE we need to take

$$\Lambda_0 = F_\pi^2 M_\pi^2 \,.$$

This value of Λ_0 exactly cancels also the graviton mass at tree order.

At one-loop order there are two diagrams, contributing to the graviton self-energy:

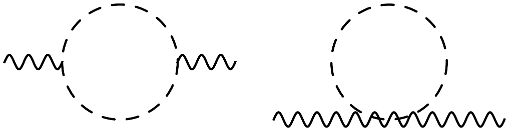

Figure: Wiggly and dashed lines represent gravitons and pions, respectively.

By demanding that the order $\hbar$ term exactly cancels the contribution of these two one-loop diagrams for $p^2 = 0$ we obtain

$$\Lambda_1 = \frac{3\kappa^2\left(2M_\pi^2 A_0\left(M_\pi^2\right) + M_\pi^4\right)}{512\pi^2}\,, A_0(M^2) = \frac{-i\mu^{4-d}}{\pi^2(2\pi)^{d-4}}\int \frac{d^d k}{k^2 - M^2 + i0^+}.$$

Thus, Λ has to be a fixed function of the light quark masses (pion mass) for any values of the masses and couplings.

Such a condition invalidates arguments about the numerical fine tuning.

However, chiral invariance of the effective Lagrangian of pions does not allow a quark mass dependent cosmological constant term.

Thus, on the one hand the consistency condition of the EFT of general relativity requires that Λ is a given fixed function of the light quark masses and on the other hand the chiral symmetry of QCD does not allow such a term.

The solution to this problem is that the chiral symmetry of the QCD Lagrangian with external sources is not an exact symmetry of the full theory including the gravity.

In EFT of general relativity the cosmological constant term has to be a fixed function of other parameters, i.e. $\Lambda \equiv \Lambda(m_u, m_d, g, e, \ldots)$.

The Lagrangian $\mathcal{L}_{\rm m}(g, \psi)$ at LO coincides with the Lagrangian of the SM taken in a non-flat metric field.

To obtain the LO Lagrangian of the strong interaction we drop "non-renormalizable" terms and “switch off” EW interactions.

To “switch off" gravity we approximate the metric field $g^{\mu\nu}$ by the constant Minkowski metric.

Resulting Lagrangian for two flavours of quarks:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \bar{\psi}\, (i\, \gamma_\alpha D^\alpha - \mathcal{M})\, \psi + L_0(m_u, m_d, g)\,,$$

where $L_0(m_u, m_d, g) = -4\Lambda(m_u, m_d, g, 0, 0, \cdots)/\kappa^2$.

This term does not contradict to any *physical symmetries*, however, it does not contribute to physical quantities when gravity is not taken into account.

The above Lagrangian leads to the following contribution to the VE

$$\Lambda_m = \langle 0|m_u\bar{\psi}_u\psi_u + m_d\bar{\psi}_d\psi_d|0\rangle - \langle 0|L_0(m_u, m_d, g)|0\rangle$$

$$= -F_\pi^2 M_\pi^2 - L_0(m_u, m_d, g).$$

By taking
$L_0(m_u, m_d, g) = -F_\pi^2 M_\pi^2 + \mathcal{O}(M_\pi^4) = -F_\pi^2 B_0(m_u + m_d) + \mathcal{O}(m_q^2)$
we obtain for the contribution to the VE

$$\Lambda_m = 0 + \mathcal{O}(m_q^2).$$

By adjusting the terms of higher orders in light quark masses m_q in $L_0(m_u, m_d, g)$ we can achieve that the QCD contribution to the vacuum energy Λ_m exactly vanishes for any values of the quark masses.

While cancelling the standard QCD contribution to VE, the addition of L_0 term to the Lagrangian does not affect the construction of the low-energy EFT, in exact analogy to

J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984)

by starting with the following Lagrangian with external sources

$$\begin{aligned}\mathcal{L}_{\text{ext}} &= -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\,\bar{\psi}\gamma_\alpha D^\alpha\psi + \bar{\psi}\gamma^\mu(v_\mu + a_\mu\gamma_5)\psi \\ &\quad -\bar{\psi}(s - i\,\gamma_5 p)\psi + L_0(m_u, m_d, g),\end{aligned}$$

where $v_\mu(x)$, $a_\mu(x)$, $s(x)$ and $p(x)$ are Hermitian, colour neutral matrices in flavour space and $s(x) = \mathcal{M} + \cdots$ incorporates the quark mass term.

Greens functions of scalar, pseudo-scalar, vector and axial vector currents are generated by

$$\langle 0_{\rm out}|0_{\rm in}\rangle_{v,a,s,p} = e^{iZ[v,a,s,p]} = \frac{\int \mathcal{D}A\mathcal{D}q\, e^{i\int d^4x \mathcal{L}_{\rm ext}(x)}}{\int \mathcal{D}A\mathcal{D}q\, e^{i\int d^4x \mathcal{L}(x)}}.$$

$Z[v, a, s, p]$ does not depend on L_0 and therefore the construction of the low-energy EFT, namely ChPT, is exactly the same as for the Lagrangian without the L_0 term, which is invariant under

$$\psi(x) \to \left[\frac{1}{2}(1+\gamma_5)R(x) + \frac{1}{2}(1-\gamma_5)L(x)\right]\psi(x)$$

provided that the external sources transform as follows,

$$\begin{aligned} v'_\mu + a'_\mu &= R(v_\mu + a_\mu)R^\dagger + iR\partial_\mu R^\dagger, \\ v'_\mu - a'_\mu &= L(v_\mu - a_\mu)L^\dagger + iL\partial_\mu L^\dagger, \\ s' + ip' &= R(s+ip)L. \end{aligned}$$

We conclude that the presence in the QCD Lagrangian of $L_0(m_u, m_d, g)$, which is nothing else then a cosmological constant, does not contradict to any physically relevant symmetries of QCD.

Summary

- If there is any physical reason for choosing a fixed value of Λ then it must be the condition of vanishing of VE.
- We calculated the vacuum expectation value of the four-momentum of the matter and gravitational fields at two-loop order in an Abelian model.
- The value of Λ canceling the contributions to VE also eliminates the vacuum expectation value of the graviton field and the massive ghost, thus leading to a consistent EFT of general relativity in Minkowski background.
- Our results resolve the issue of the numerical fine-tuning of the QCD contribution in VE:
  $\Lambda(m_q, g)$ exactly cancels the QCD contribution to VE.

  While this solution seems to be incompatible with the chiral symmetry of QCD, we observe that there is no contradiction with any symmetries of QCD with observable physical consequences.