Deuteron VVCS and Nuclear Structure Effects in Muonic Deuterium in Pionless EFT

Vadim Lensky

The 10th International Workshop on Chiral Dynamics

Beijing November 15-19, 2021

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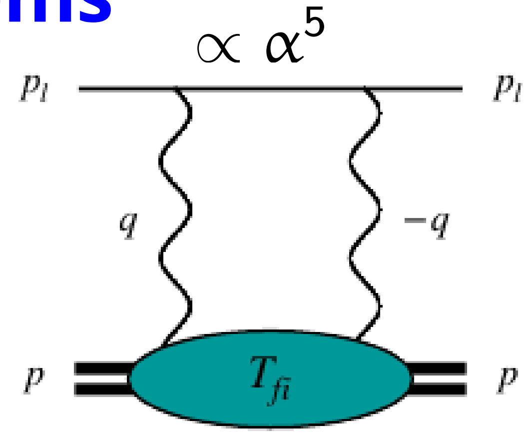
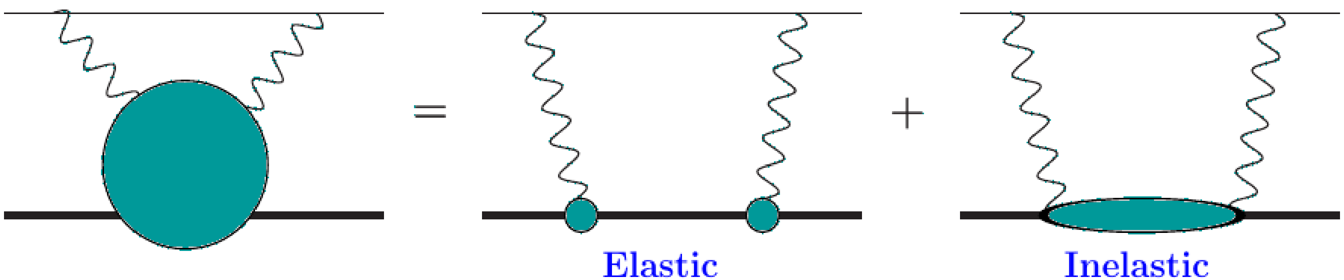
Two-Photon Exchange in (Muonic) Atoms



- → Talks by S. Li Muli and V. Pascalutsa
- Instead of response functions, use the forward VVCS amplitude: a (slightly) different view
- Unpolarised: longitudinal and transverse amplitudes

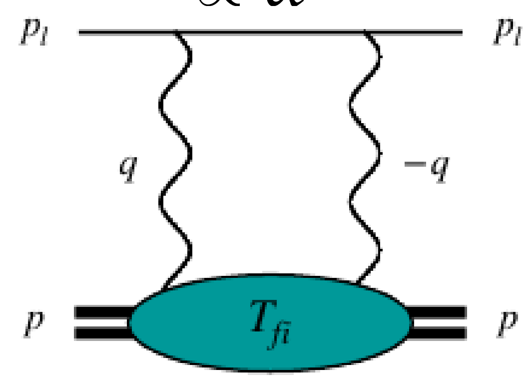
$$\Delta E_{nl} = -8i\pi m\,[\phi_{nl}(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

- Elastic ($\nu = \pm Q^2/2M_{\text{target}}$, elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)



- Calculate VVCS and get a handle on everything (elastic and inelastic) at once!

Two-Photon Exchange in (Muonic) Atoms



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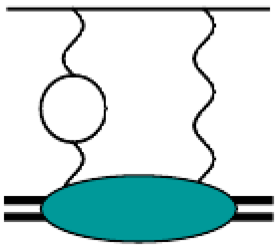
- Higher-order contributions are important in μD
  - Coulomb (non-forward) distortions $\alpha^6 \log \alpha$
  - eVP and three-photon contributions α^6 Kalinowski (2018)



- Will not be covered here; are included and (partly) revisited in our work

Theory Framework: Pionless EFT

- Nucleons are non-relativistic → $E \simeq p^2/M = O(p^2)$
- Loop integrals $dE\, d^3p = O(p^5)$
- Nucleon propagators $(E - p^2/2M)^{-1} = O(p^{-2})$
- Typical momenta $p \sim \gamma = \sqrt{ME_d} \simeq 45$ MeV
- Expansion parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
- NN system has a low-lying bound/virtual state → enhance S-wave coupling constants, resum the LO NN S-wave scattering amplitude

- Easier to solve than χEFT (analytic results for NN)
- Easier to analyse (e.g., discover correlations between various quantities)
- Explicit gauge invariance and renormalisability
- Slower convergence (~larger uncertainty) and (potentially) a narrower range of applicability than χEFT

Counting for VVCS and TPE

- Transverse contribution starts at N4LO in TPE

$$f_L(\nu, Q^2) = 4\pi\alpha_{E1}Q^2 + \dots$$

$$f_T(\nu, Q^2) = -\frac{e^2}{M_d} + 4\pi\beta_{M1}Q^2 + 4\pi(\alpha_{E1} + \beta_{M1})\nu^2 + \dots$$

$$\alpha_{E1} = \frac{\alpha M}{32\pi\gamma^4} + \dots$$

$$\beta_{M1} = -\frac{\alpha}{32M\gamma^2}\left[1 - \frac{16}{3}\mu_1^2 + \frac{32}{3}\mu_1^2\frac{\gamma}{\gamma_s - \gamma}\right] + \dots$$

$$\Delta E_{nl} = -8i\pi m\,[\phi_{nl}(0)]^2 \int \frac{d^4q}{(2\pi)^4}\frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2)f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

$$f_L = O(p^{-2}), \qquad f_T = O(p^0)$$ in VVCS

$$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$$ in TPE

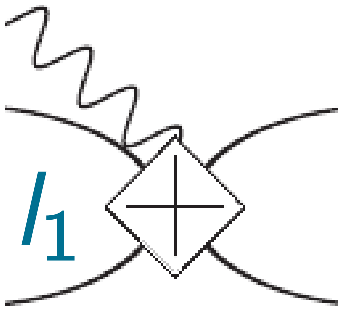
- We go up to N3LO in f_L, and up to (relative) NLO in f_T

Lagrangian

$$
\begin{aligned}
\mathcal{L} = {} & N^\dagger \left[i D_0 + \frac{D^2}{2M} \right] N + \frac{e}{2M} N^\dagger \hat{\mu} \left(\sigma \cdot B \right) N + \frac{e}{6} N^\dagger \hat{r}_E^2 N \left(\nabla \cdot E \right) \\
& - C_0 \; N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{P}_i N - C_0^s \; N^\dagger \mathcal{T}_a N_c \; N_c^\dagger \mathcal{T}_a N + \frac{1}{2} C_2 \left[N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{O}_i^{(2)} N + \text{H.c.} \right] \\
& + \frac{1}{2} C_2^s \left[N^\dagger \mathcal{T}_a N_c \; N_c^\dagger \mathcal{O}_a^{(2,s)} N + \text{H.c.} \right] - C_4 N^\dagger \mathcal{O}_i^{(2)} N_c \; N_c^\dagger \mathcal{O}_i^{(2)} N \\
& - \frac{1}{2} \tilde{C}_4 \left[N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{O}_i^{(4)} N + \text{H.c.} \right] + \frac{1}{2} C_6 \left[N^\dagger \mathcal{O}_i^{(2)} N_c \; N_c^\dagger \mathcal{O}_i^{(4)} N + \text{H.c.} \right] \\
& - \frac{1}{4} \left[C_{^3P_0} \delta_{ia} \delta_{jb} + C_{^3P_1} \left(\delta_{ij} \delta_{ab} - \delta_{ib} \delta_{ja} \right) + 2 C_{^3P_2} \left(\delta_{ij} \delta_{ab} + \delta_{ib} \delta_{ja} - \frac{2}{3} \delta_{ia} \delta_{jb} \right) \right] \\
& \times N^\dagger \mathcal{O}_{ia}^{(1,P)} N_c \; N_c^\dagger \mathcal{O}_{jb}^{(1,P)} N \\
& + e L_1^{M1_V} \left[N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{T}_3 N + \mathrm{H.c.} \right] B_i - 2 i e L_2^{M1_S} \, \epsilon_{ijk} N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{P}_j N \, B_k \\
& - \frac{e}{2} L_1^{E1_V} \left[N^\dagger \mathcal{O}_{ij}^{(1,P)} N_c \; N_c^\dagger \mathcal{P}_j N + \mathrm{H.c.} \right] E_i + \frac{e}{2} L_3^{E1_V} \left[N^\dagger \mathcal{O}_{ij}^{(1,P)} N_c \; N_c^\dagger \mathcal{O}_j^{(2)} N + \mathrm{H.c.} \right] E_i \\
& + e L_1^{C0_S} N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{P}_i N \left(\nabla \cdot E \right) - \frac{e}{2} L_3^{C0_S} \left[N^\dagger \mathcal{P}_i N_c \; N_c^\dagger \mathcal{O}_i^{(2)} N + \mathrm{H.c.} \right] \left(\nabla \cdot E \right)
\end{aligned}
$$

Lagrangian

- … is rather lengthy and has many coupling constants
- All of which are known from one-nucleon and two-nucleon sectors with a high precision
- … apart from one combination that corresponds to a longitudinal photon coupling to two nucleons
- l_1 fitted from the deuteron elastic charge form factor (charge radius) at N3LO → correlation with TPE!
- Not included (all start at N4LO):
  - relativistic effects and SD mixing
  - nucleon polarisabilities and form factors beyond the charge radii (can be included beyond strict pionless EFT expansion, plugging in, respectively, the nucleon VVCS/nucleon form factors; are important)



VVCS Amplitude

- LSZ reduction

$$T_{fi} = \frac{\mathcal{M}(q,p,q',p')}{\Sigma'(E_d)}$$

- We use:
  - the Z-parametrisation of the (EFT) expansion — Phillips, Rupak, Savage (1999)
    new formal expansion parameter $(Z-1)$

$$Z = \frac{1}{1-\gamma\rho_d} = 1.6893(30)$$

Epelbaum, Krebs, Reinert (2020)

NN T matrix residue at the deuteron pole

deuteron pole residue of NN scattering amplitude reproduced at NLO

$$[\Sigma'(E_d)]^{-1} = \frac{8\pi\gamma}{M^2}\,[1+(Z-1)+0+0+\ldots]$$

  - dimensonal regularisation with power divergence subtraction (PDS)

Kaplan, Savage, Wise (1998)

VVCS Amplitude

- Expansion for the VVCS amplitude

$$f_L(\nu, Q^2) = \frac{8\pi}{M^2}\Big[\underbrace{\gamma\mathcal{M}_L^{(-3)}}_{O(P^{-2})} + \underbrace{\gamma\mathcal{M}_L^{(-2)} + \gamma(Z-1)\mathcal{M}_L^{(-3)}}_{O(P^{-1})} + \dots\Big]$$

$$f_T(\nu, Q^2) = \frac{8\pi}{M^2}\Big[\underbrace{\gamma\mathcal{M}_T^{(-1)}}_{O(P^{0})} + \underbrace{\gamma\mathcal{M}_L^{(0)} + \gamma(Z-1)\mathcal{M}_L^{(-1)}}_{O(P^{1})} + \dots\Big]$$

- $Z = 1.6893(30)$, obtained from the asymptotic deuteron S-wave normalisation, is the least precise parameter of the calculation
- However, its contribution to uncertainty is negligible compared to the dominant source: truncation at N3LO
  - Bayesian quantification → talks by D. Phillips and R. Furnstahl

Deuteron Charge Radius: Fitting N3LO Coupling

- Form factors obtained from residues of the amplitude at the elastic pole

$$
\begin{aligned}
G_C(Q^2) = {}& \frac{4\gamma}{Q}\arctan\frac{Q}{4\gamma} \\
& - (Z-1)\left(1 - \frac{4\gamma}{Q}\arctan\frac{Q}{4\gamma}\right) \\
& - \frac{4}{3} r_0^2\,\gamma\,Q \arctan\frac{Q}{4\gamma} \\
& + \frac{1}{3}(Z-1)\,r_0^2\,Q^2\left(1 - \frac{4\gamma}{Q}\arctan\frac{Q}{4\gamma}\right) - \frac{(Z-1)^3\,l_1}{2\gamma^2}Q^2
\end{aligned}
$$

$$
\begin{aligned}
r_d^2 = -6\frac{\mathrm{d}G_C(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0} &= \frac{1}{8\gamma^2} + \frac{Z-1}{8\gamma^2} + 2r_0^2 + \frac{3(Z-1)^3}{\gamma^2}\,l_1 \\
&= [2.3303 + 1.6063 + 0.6241 + 18.3166\,l_1]\,\mathrm{fm}^2
\end{aligned}
$$

- At NNLO $r_d^2 = 4.5607(76)\ \mathrm{fm}^2$, very close to, e.g., μD: $r_d^2 = 4.5183(33)\ \mathrm{fm}^2$ CREMA (2016)
- l_1 is therefore rather small but important in the charge radius:

$$
l_1 = -2.32(18)(37)\times 10^{-3}
$$

Fitting N3LO Coupling and Correlations

- Looking at the TPE correction, l_1 gives a rather small contribution:

$$\Delta E_{2S}^{\text{TPE,N3LO}} = -1.956(20)\ \text{meV},$$

where less than 10^{-4} meV is due to l_1 .

- Luckily, the NNLO value of r_d is already close to the empirical value, and the contribution of l_1 to TPE can be safely neglected
- However, its contribution to elastic and inelastic TPE separately is more sizeable and can be comparable to the precision of the elastic TPE, e.g., $\Delta E_{2S}^{\text{TPE,elastic}} = -0.417(2)$ meV obtained with empirical form factors

Carlson et al (2013)

- To be safe, use the H-D isotope shift. It also contains TPE, but the contribution of the N3LO couping is much less important:

$$\Delta f_{\text{th}} = \left[671\,000\,535.325(431)(898) + 0.838\, l_1 - 1369.346\, r_d^2\right]\ \text{kHz},$$

l_1 gives at most 10^{-2} kHz to the total or elastic/inelastic separately

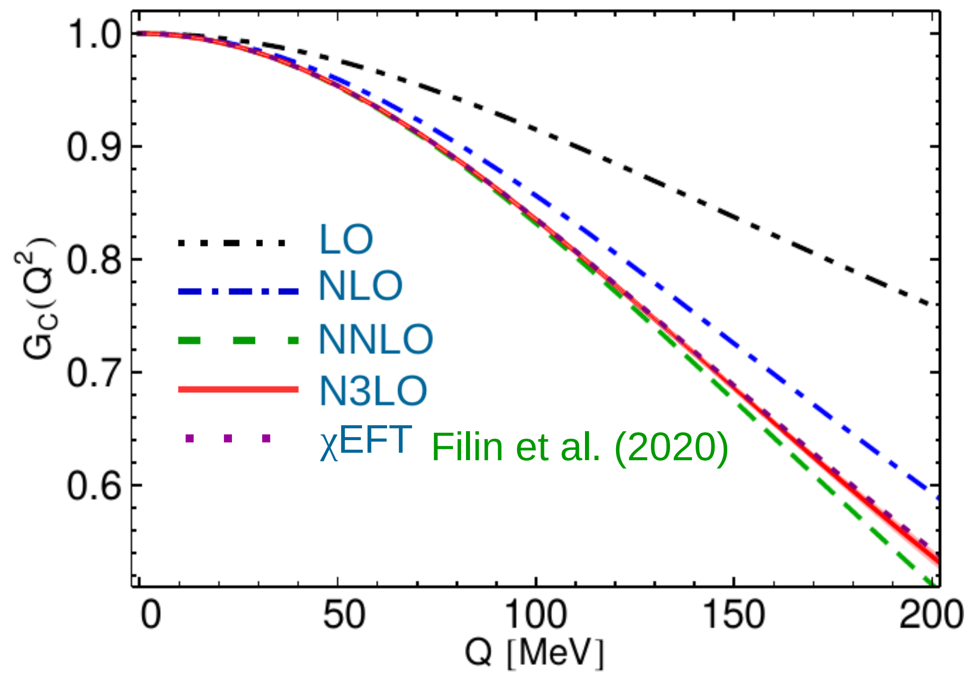
- Extraction of r_d and l_1 using our TPE results:

$$r_d(\mu\text{H}\ \&\ \text{iso}) = 2.12796(17)\ \text{fm}, \quad l_1 = -1.77(38) \times 10^{-3}$$

Deuteron Charge Form Factor and Elastic TPE

- The charge form factor at N3LO coincides with the χEFT result
- Vindicates both theories
- Empirical FFs would be very close to these curves
- What about elastic TPE?

→ look at different form factors



$$\Delta E_{nl}^{\text{elastic}} = \frac{m\alpha^2}{M_d(M_d^2 - m^2)}[\phi_{nl}(0)]^2 \int_0^\infty \frac{\mathrm{d}Q^2}{Q^2} \left\{ \frac{2}{3} G_M^2(Q^2)(1+\tau_d)\hat{\gamma}_1(\tau_d,\tau_l) \right.$$

$$\left. - \left[\frac{G_C^2(Q^2)-1}{\tau_d} + \frac{2}{3} G_M^2(Q^2) + \frac{8}{9}\tau_d G_Q^2(Q^2) \right] \hat{\gamma}_2(\tau_d,\tau_l) + 16 M_d^2 \frac{M_d - m}{Q} G_C'(0) \right\}$$

- Magnetic and quadrupole contributions can be neglected

Deuteron Charge Form Factor and Elastic TPE

- Elastic contribution (charge FF only)
  - a huge difference between the t20 parametrisation and others
  - t20 form factor used in dispersive TPE calculations

    Carlson et al. (2013), Acharya et al. (2020)
  - Charge radii are different, but curvatures are more important!

t20 Abbott et al. (2000)	-0.417(2)
Sick, Trautmann (1998)	-0.451
This work	-0.4463(77)
χEFT Filin et al (2020)	-0.4456(18)

- Approximate expression for elastic TPE

$$\Delta E_{nl}^{\text{elastic}} = -\frac{m\alpha^2}{M_d(M_d^2 - m^2)}[\phi_{nl}(0)]^2 \int_0^\infty \frac{\mathrm{d}Q^2}{Q^2}\left[\frac{G_C^2(Q^2)-1}{\tau_d}\hat{\gamma}_2(\tau_d,\tau_l) + 16M_d^2\frac{M_d - m}{Q}G_C'(0)\right]$$

$$\simeq -\frac{m_r^4\alpha^5}{24}r_{\mathrm{F}d}^3$$

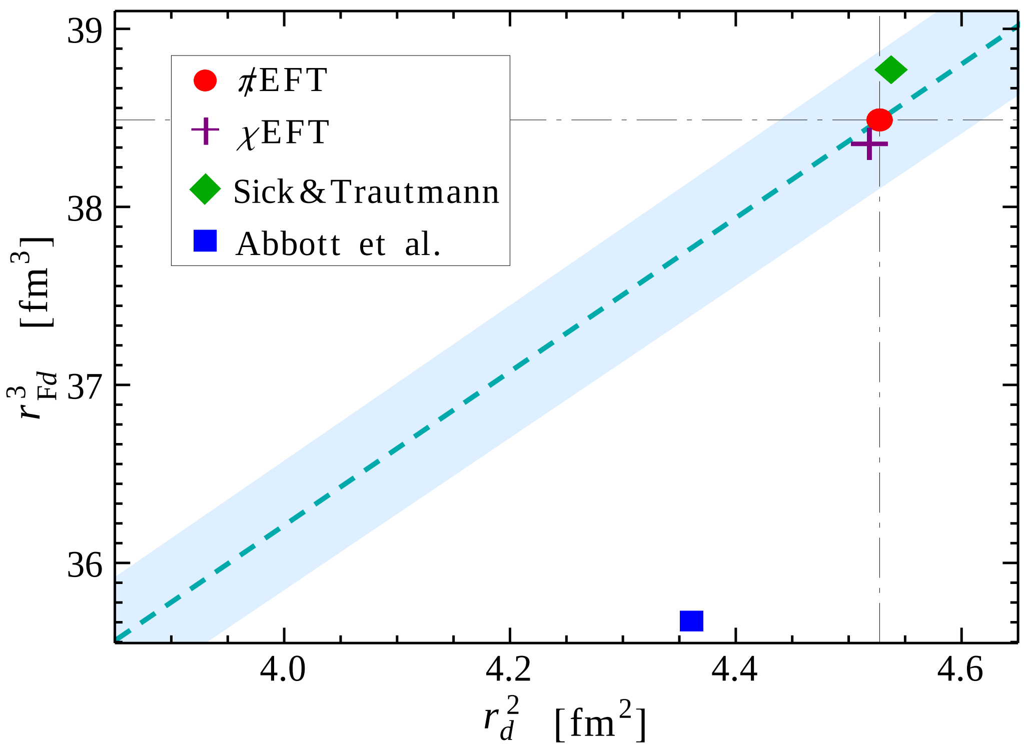
$$\mathrm{r}_{\mathrm{F}d}^3 = \frac{48}{\pi}\int_0^\infty \frac{dQ}{Q^4}\left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

- Can be calculated analytically in pionless EFT

$$r_{\mathrm{F}d}^3 = \frac{3}{80\gamma^3}\left\{Z\left[5 - 2Z(1-2\ln 2)\right] - \frac{320}{9}r_0^2\gamma^2\left[Z(1-4\ln 2) - 2 + 2\ln 2\right] + 80(Z-1)^3\,l_1\right\}$$

Correlation: Friar radius vs charge radius

- N3LO correlation band
- Width estimated due to higher-order terms
- t20 parametrisation is very different from others
  - should not be used for TPE and other precise low-Q properties



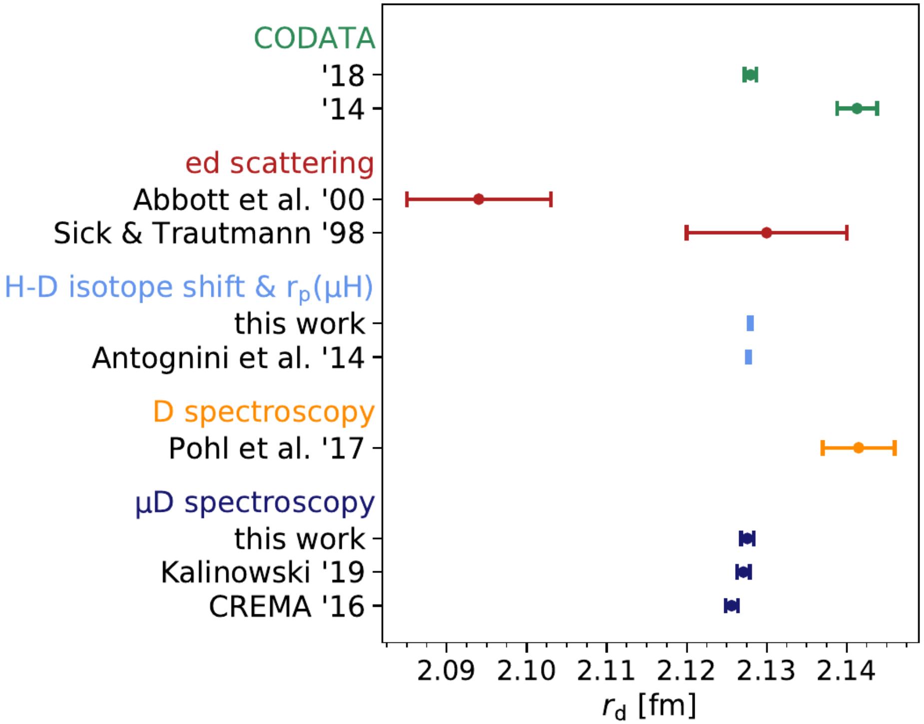
- The correlation can be used to test low-Q properties of form factor parametrisations
- It would be interesting to see if it can be reproduced in χEFT
- Heavier nuclei? → A. Filin's talk

Summary and Outlook

- Inelastic TPE is consistent with other calculations
- The most important higher-order terms come from individual nucleons
  → included in the analysis, not covered here
- Pionless EFT result for the nuclear structure (TPE + ...) correction:

$$\Delta E_{2S}^{\mu\mathrm{D},\,\mathrm{structure}} = -1.760(20)\ \mathrm{meV}$$

- Revisited extraction of r_d from H-D isotope shift and from µD, using the same theory to calculate TPE
- Resulting radii consistent with each other and other (µ) extractions

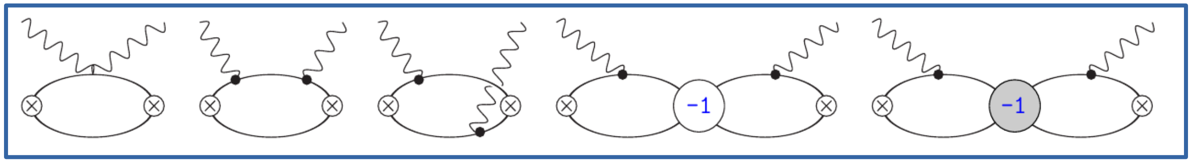


- It is safe(r) to use the isotope shift to fit the currents/potential
- Check the form factors!

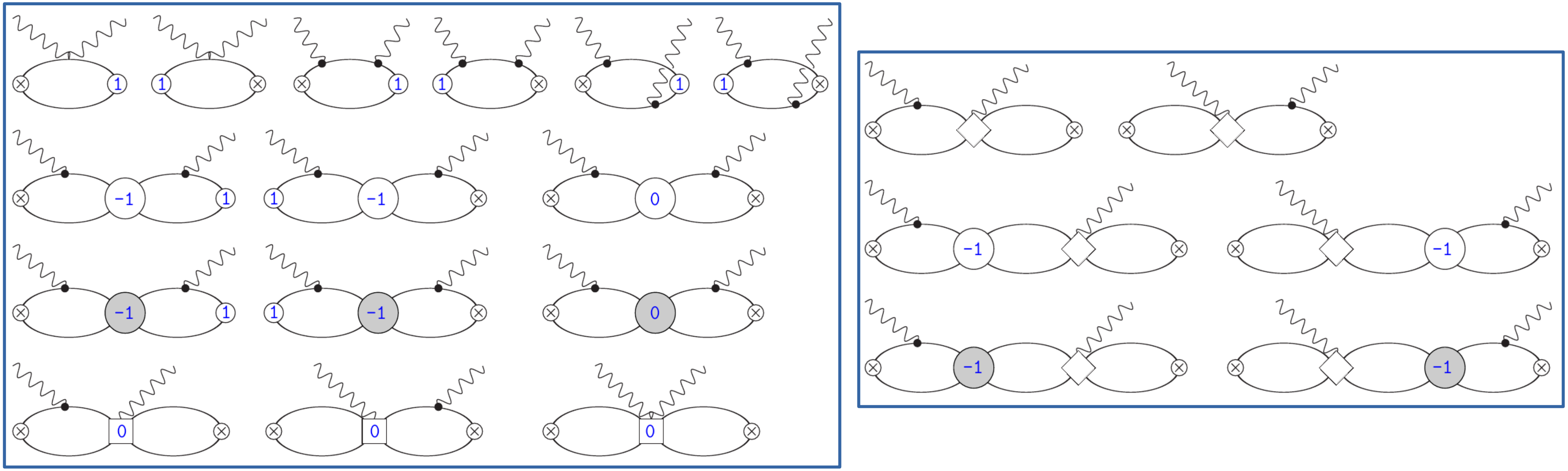
Backup Slides

Feynman Graphs

LO



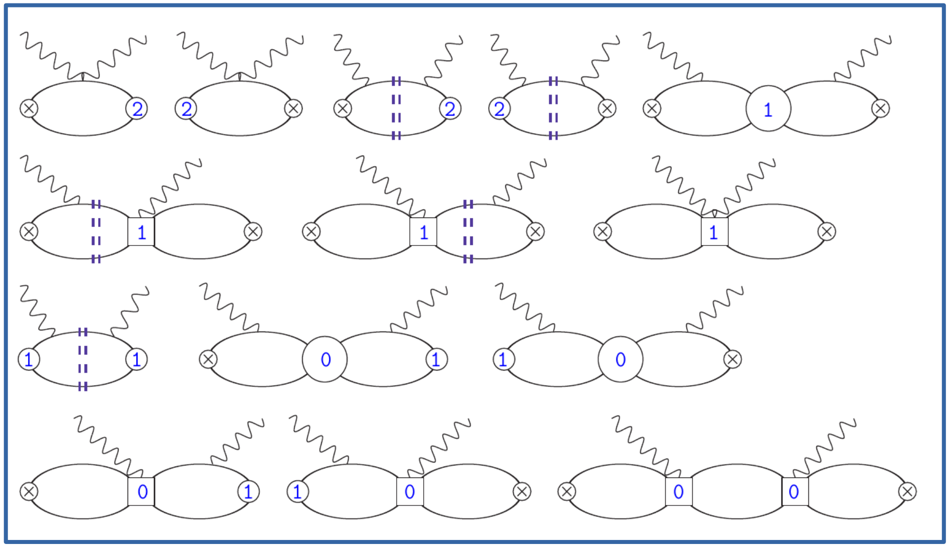
NLO

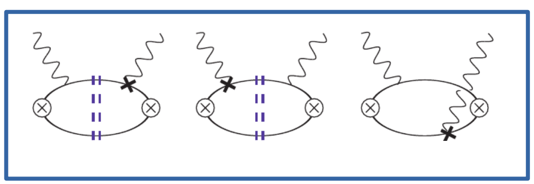


- Amplitudes are calculated analytically
- Checks:
  - ➔ the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - ➔ regularisation scale dependence has to vanish

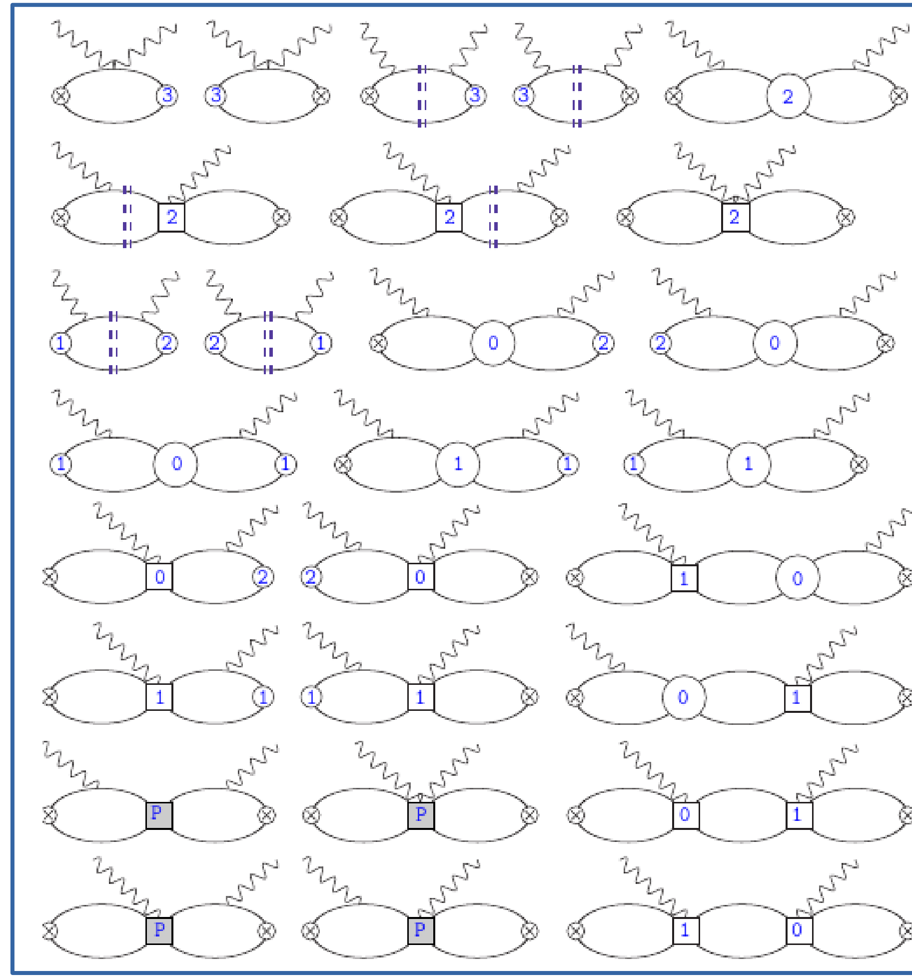
Feynman Graphs

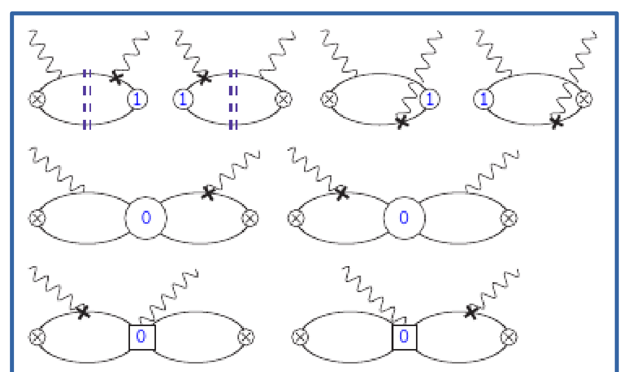
NNLO

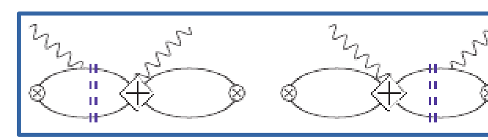




N3LO







Deuteron Generalised Polarisabilities