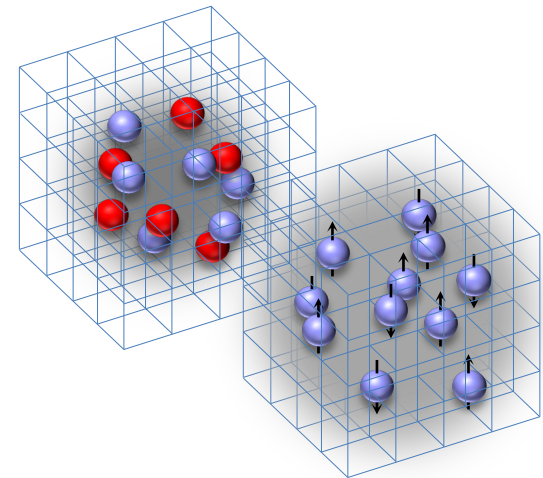


An Overview of Recent and Ongoing Developments in Lattice Simulations of Nuclei

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Nuclear Lattice EFT Collaboration

10th International Workshop on Chiral Dynamics
Institute of High Energy Physics, CAS
November 18, 2021



Outline

Lattice effective field theory

Hidden spin-isospin exchange symmetry

Essential elements for nuclear binding

Nuclear thermodynamics

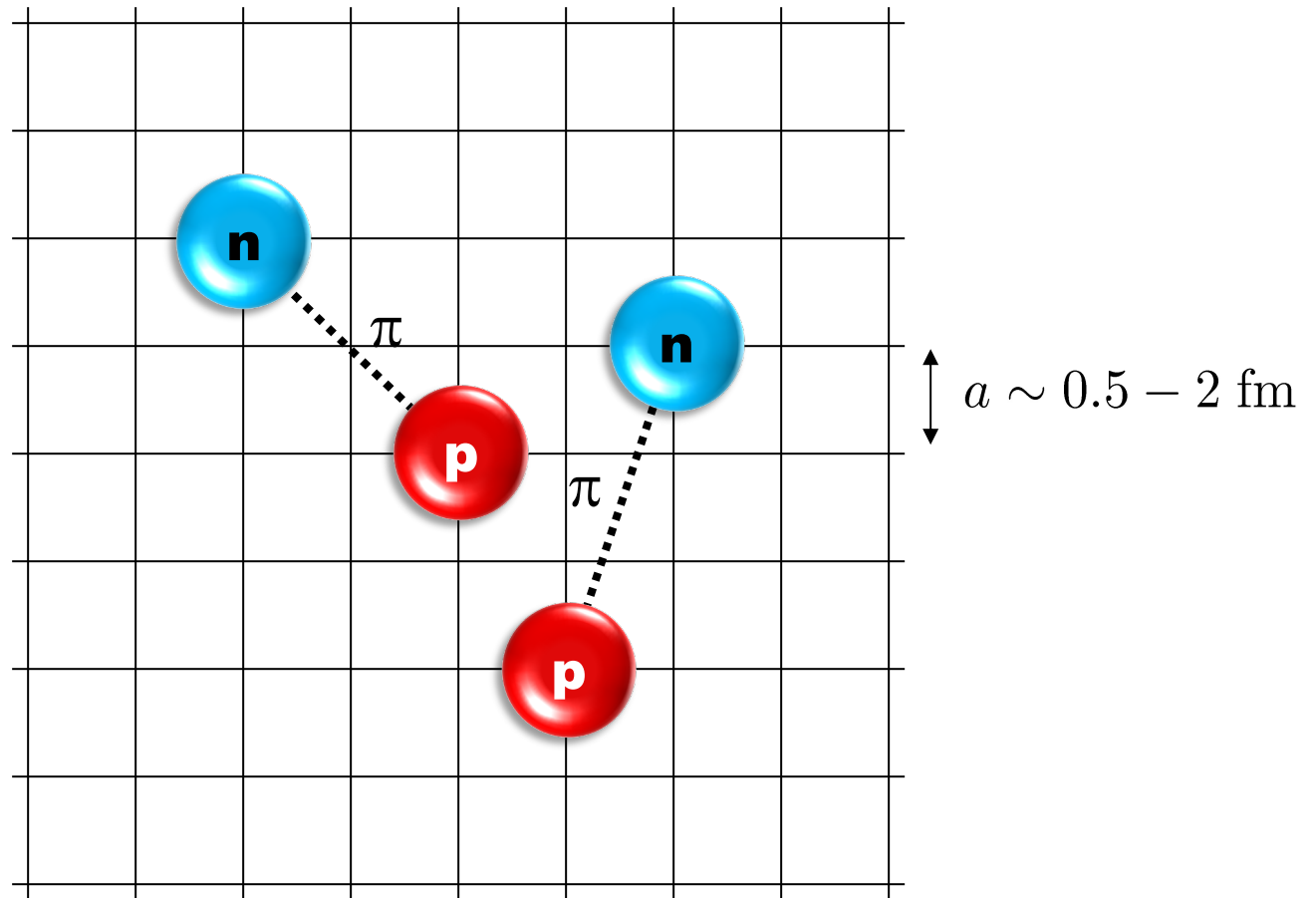
Structure and clustering in carbon-12

Hypernuclei with impurity lattice Monte Carlo

Wave function matching

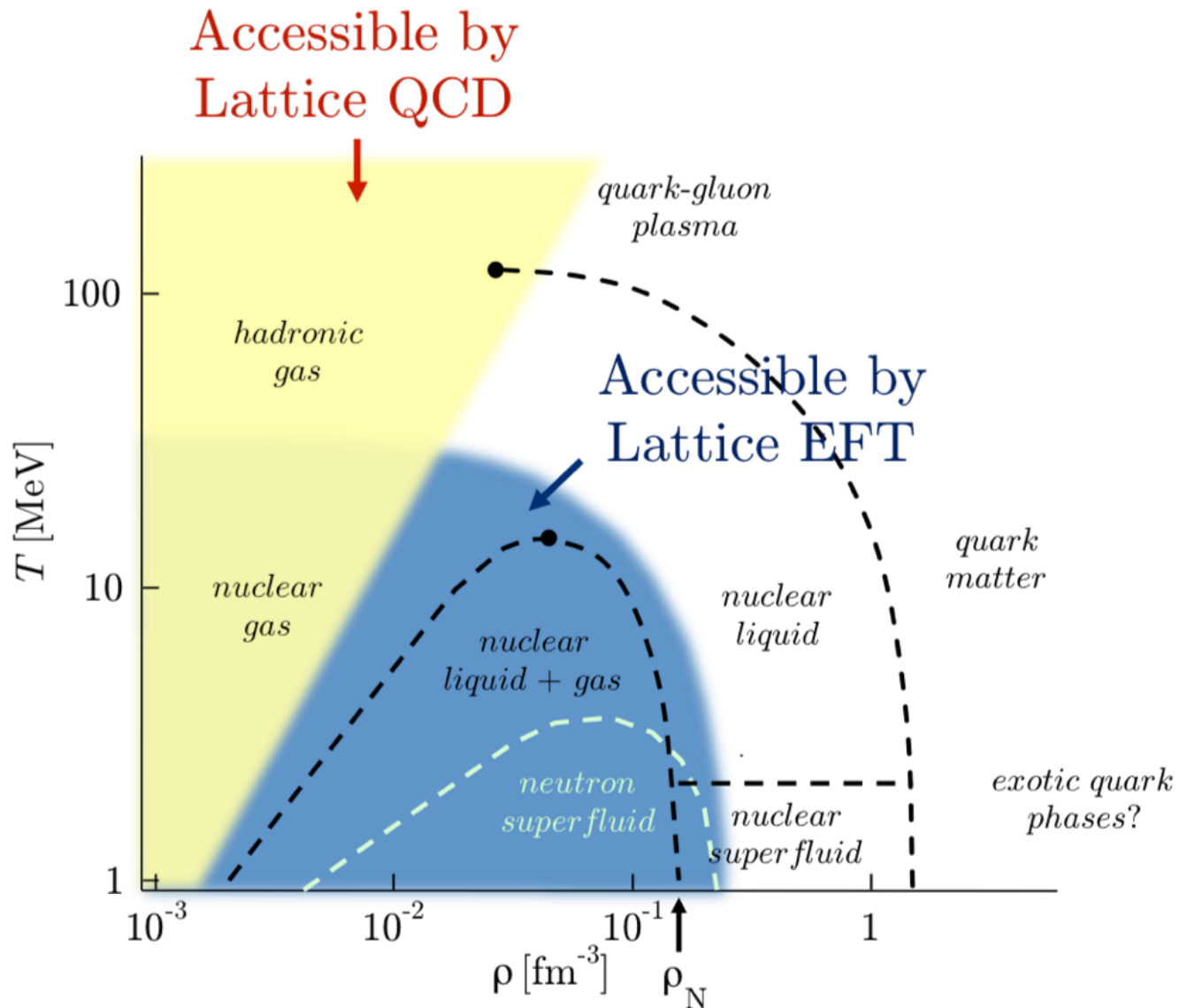
Summary

Lattice effective field theory



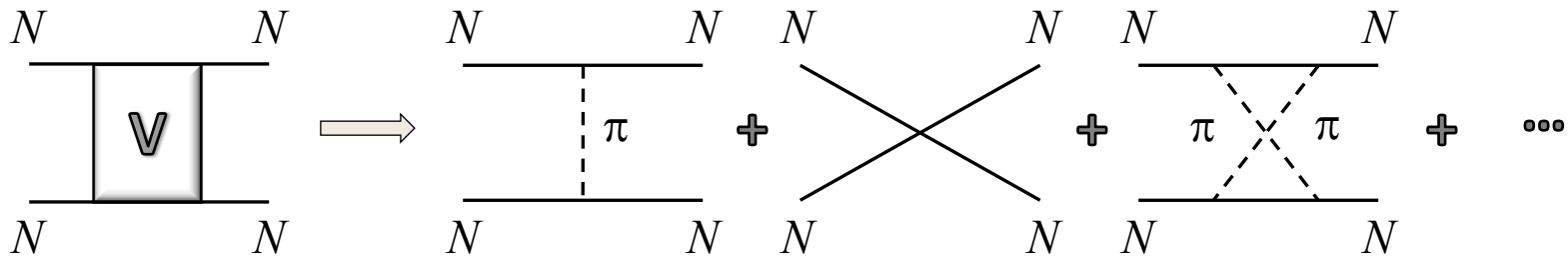
D.L, Prog. Part. Nucl. Phys. 63 117-154 (2009)

Lähde, Meißner, Nuclear Lattice Effective Field Theory (2019), Springer



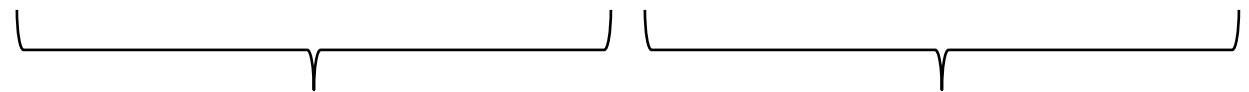
Chiral effective field theory

Construct the effective potential order by order



V^{OPEP} Contact interactions

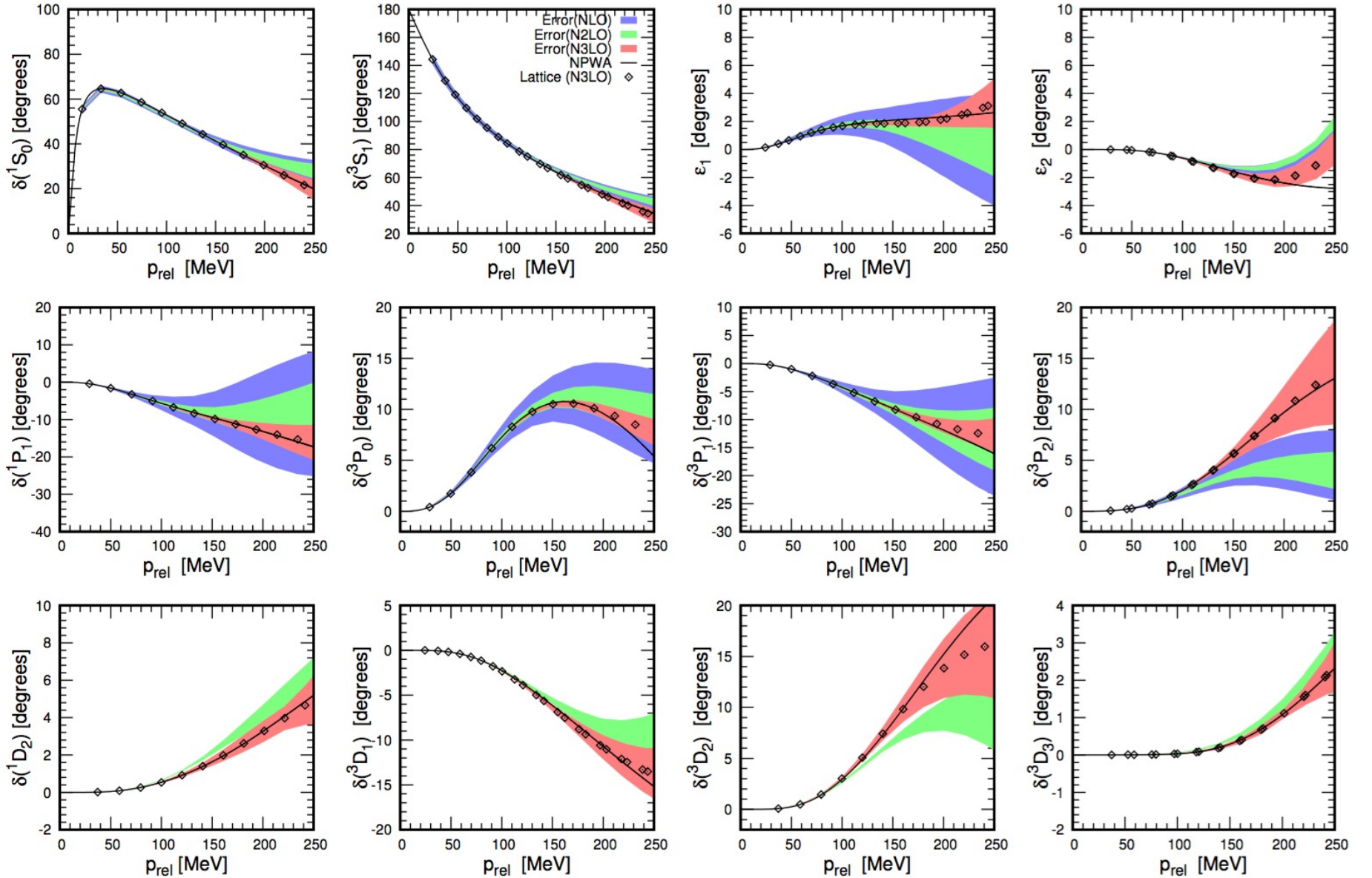
V^{TPEP}



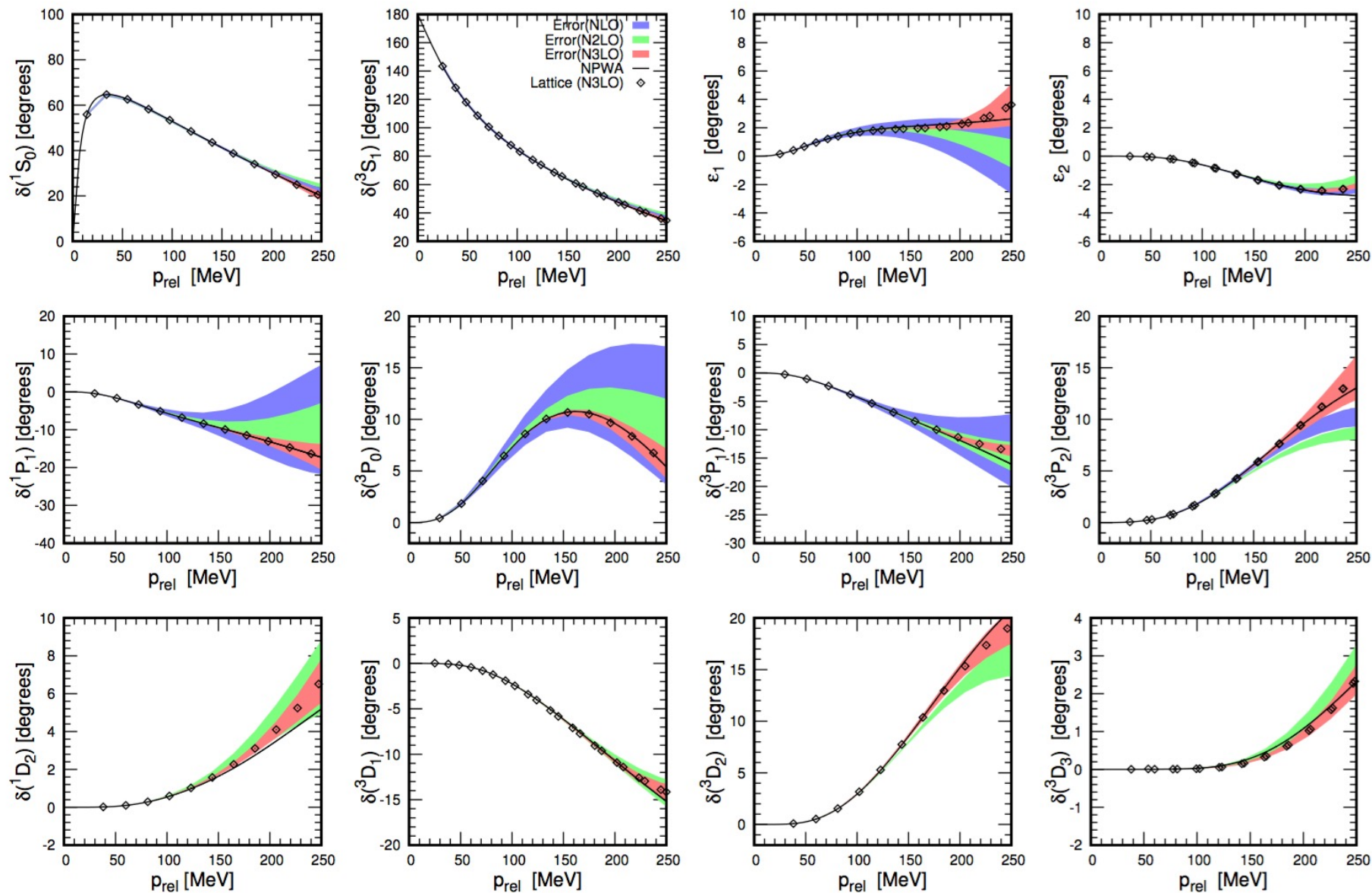
Leading order (LO)

Next-to-leading order (NLO)

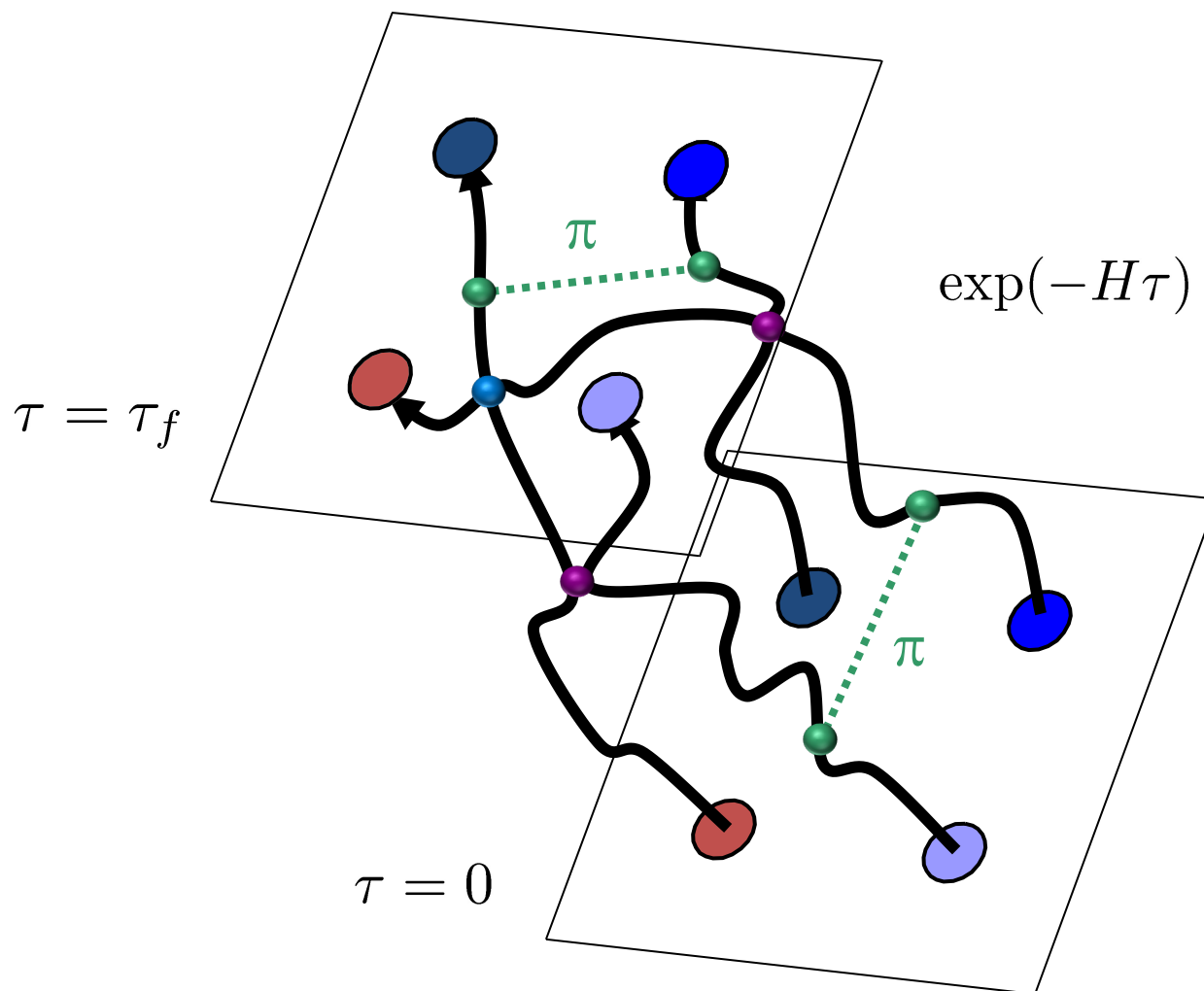
$$a = 1.315 \text{ fm}$$



$$a = 0.987 \text{ fm}$$



Euclidean time projection

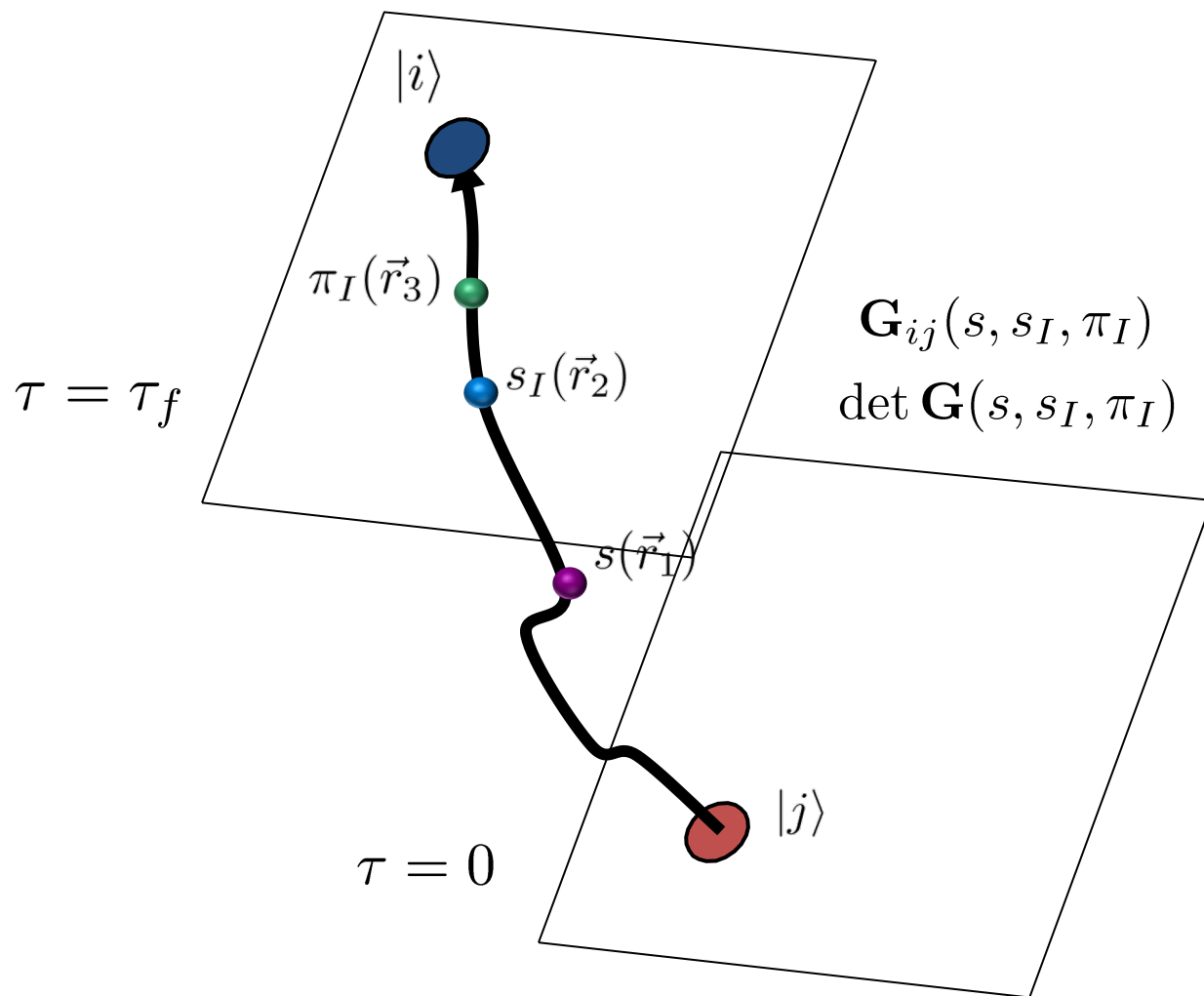


Auxiliary field method

We can write exponentials of the interaction using a Gaussian integral identity

$$\exp \left[-\frac{C}{2} (N^\dagger N)^2 \right] \quad \text{X} \quad (N^\dagger N)^2$$
$$= \sqrt{\frac{1}{2\pi}} \int_{-\infty}^{\infty} ds \exp \left[-\frac{1}{2} s^2 + \sqrt{-C} s (N^\dagger N) \right] \quad \text{Y} \quad s N^\dagger N$$

We remove the interaction between nucleons and replace it with the interactions of each nucleon with a background field.



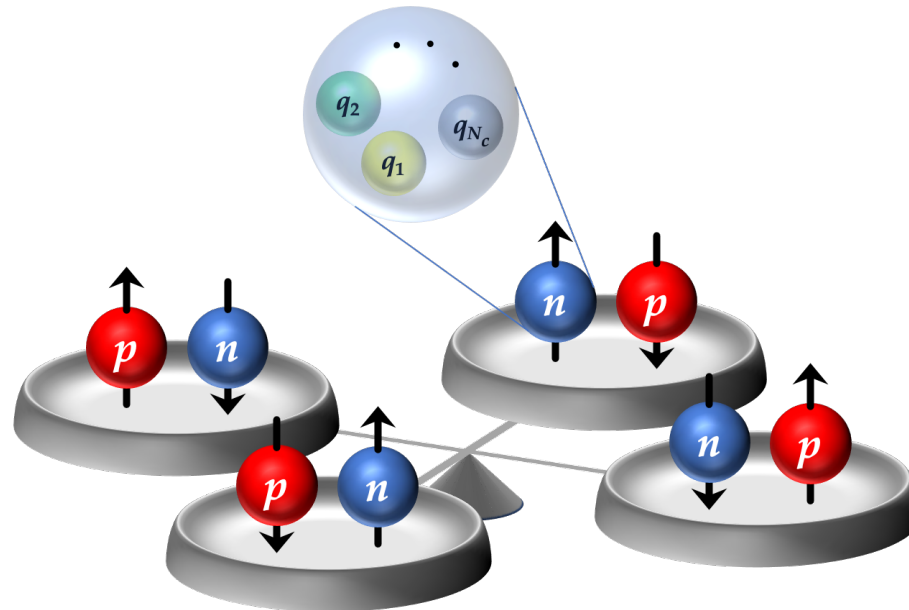
Hidden spin-isospin exchange symmetry

Kaplan, Savage, PLB 365, 244 (1996)

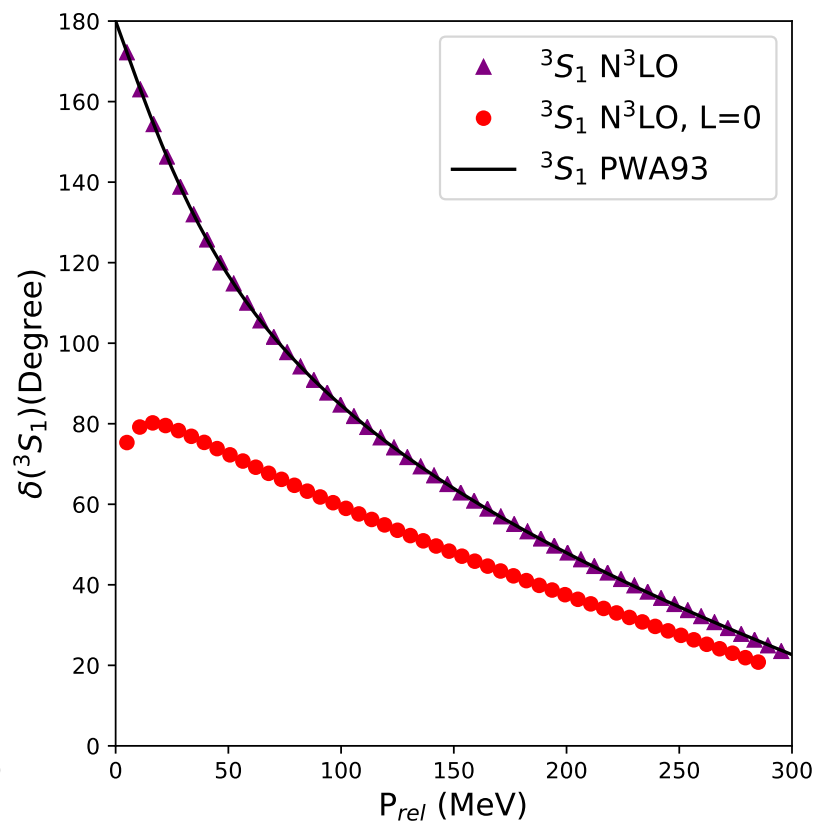
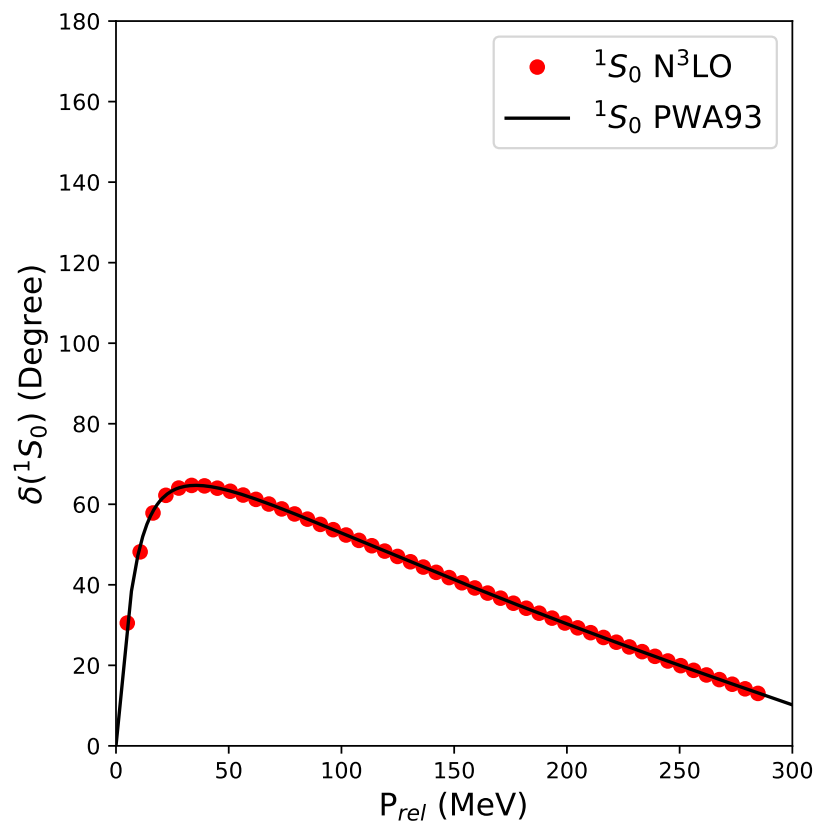
Calle Gordon, Arriola, PRC 80, 014002 (2009)

Kaplan, Manohar, PRC 56, 76 (1997)

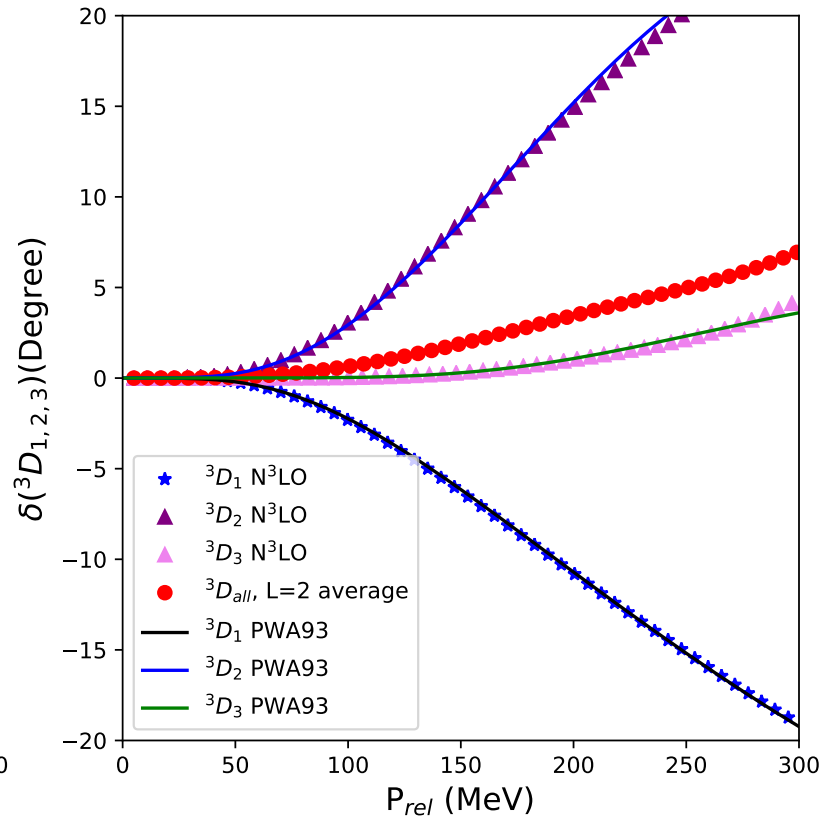
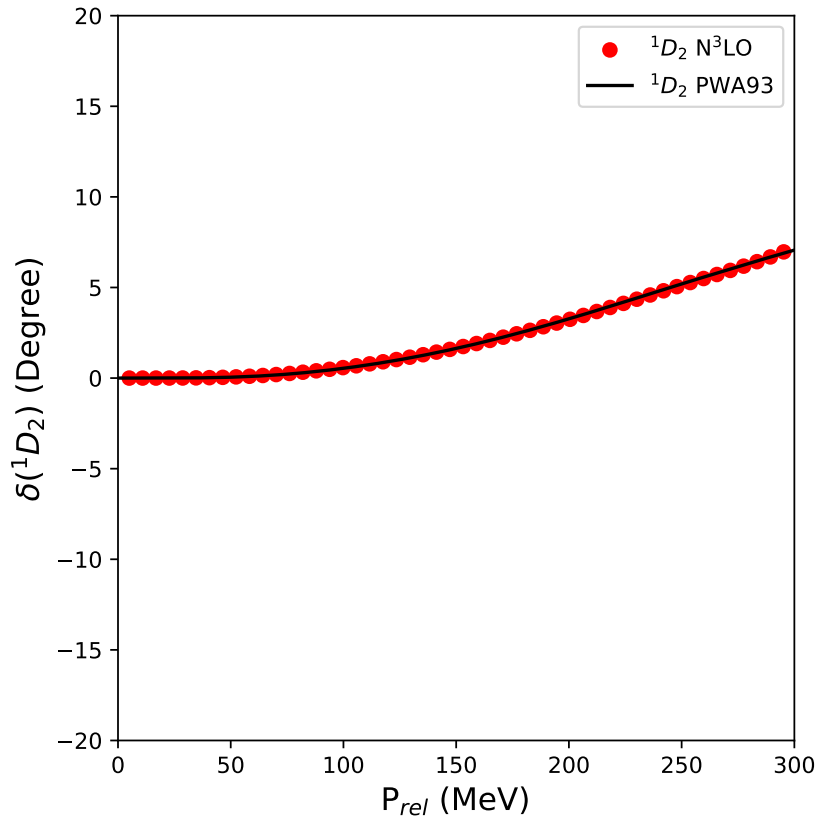
$$V_{\text{large-}N_c}^{2N} = V_C + \vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\tau}_1 \cdot \vec{\tau}_2 W_S + (3\hat{r} \cdot \vec{\sigma}_1 \hat{r} \cdot \vec{\sigma}_2 - \vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 W_T$$



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner,
PRL 127, 062501 (2021)



D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner,
PRL 127, 062501 (2021)



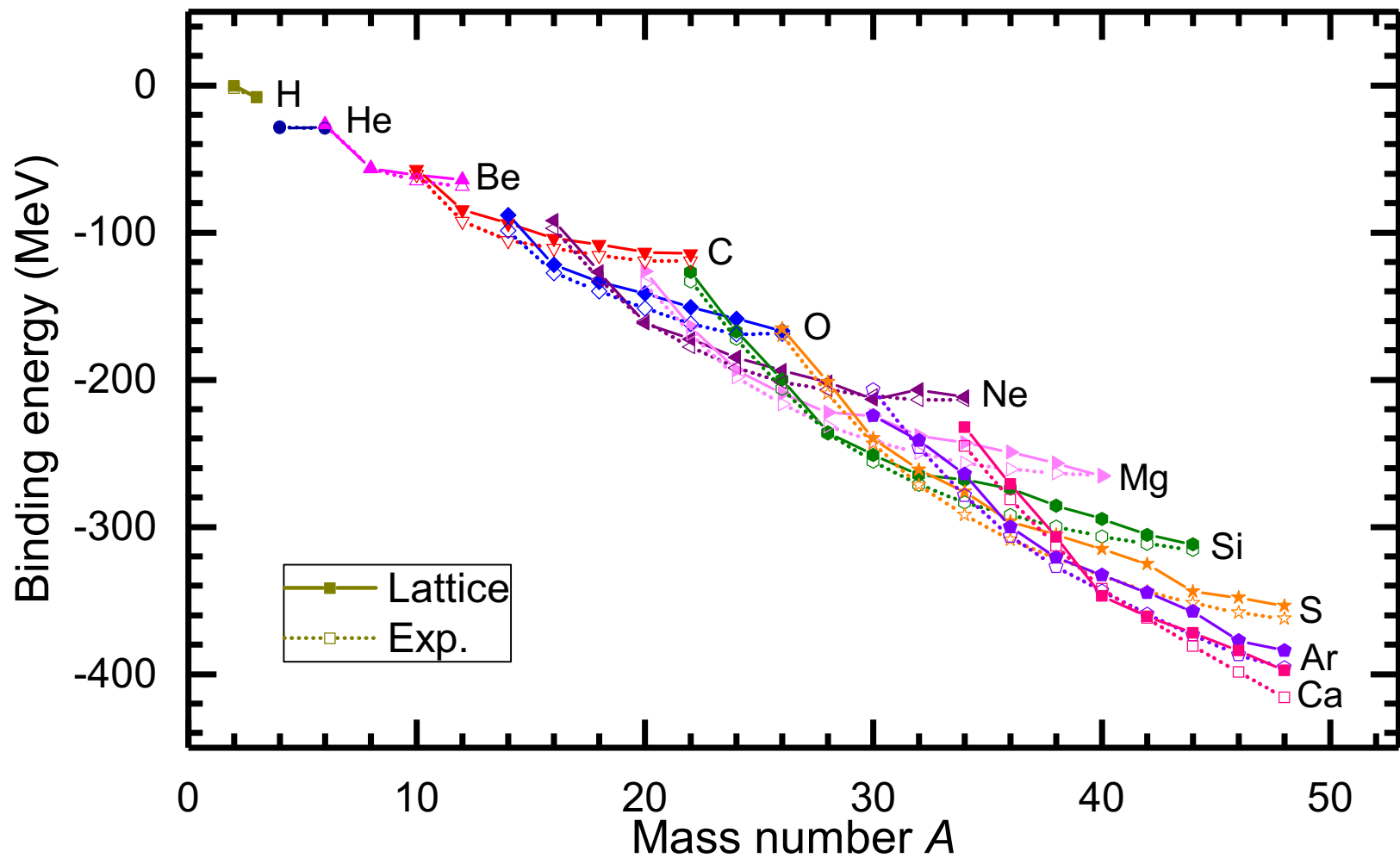
D.L., Bogner, Brown, Elhatisari, Epelbaum, Hergert, Hjorth-Jensen, Krebs, Li, Lu, Meißner,
PRL 127, 062501 (2021)

Essential elements for nuclear binding

What is the minimal nuclear interaction that can reproduce the ground state properties of light nuclei, medium-mass nuclei, and neutron matter simultaneously with no more than a few percent error in the energies and charge radii?

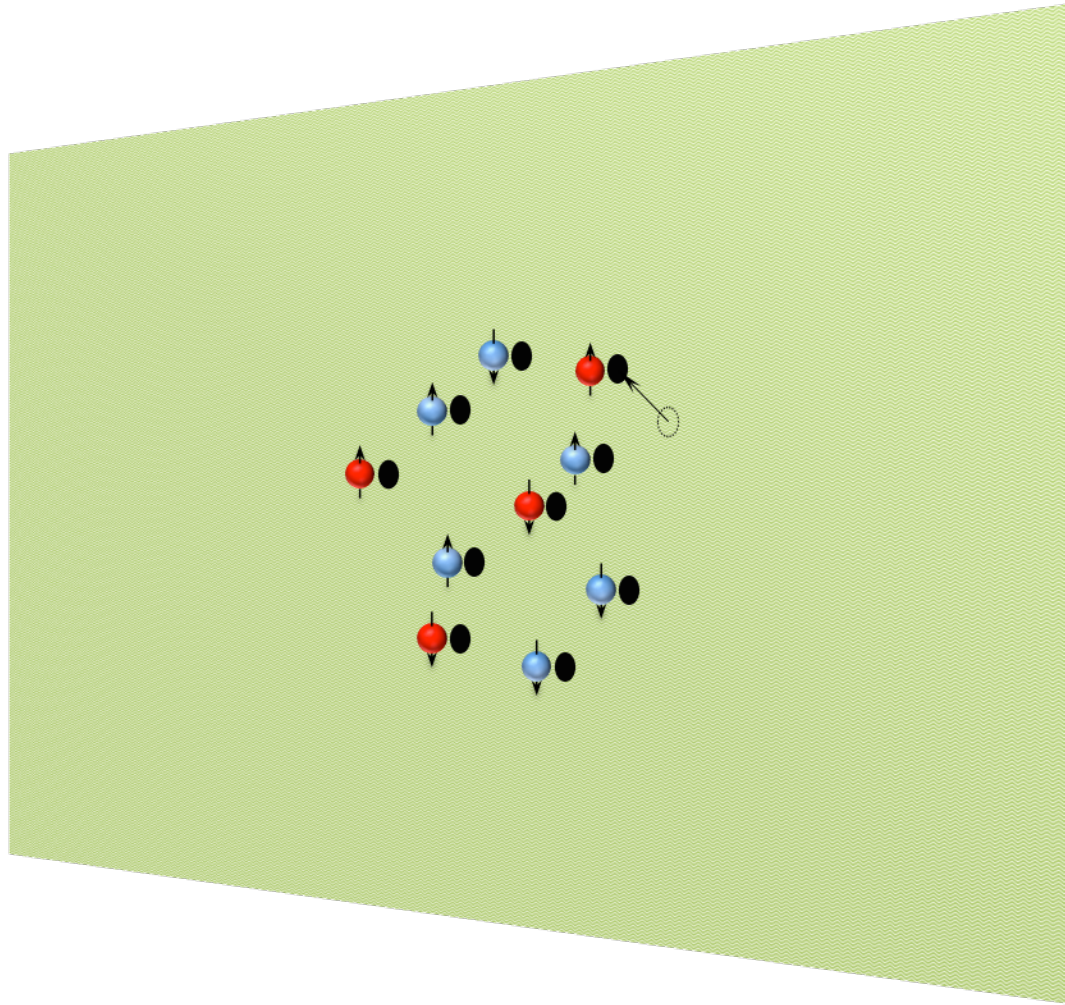
We construct an interaction with only four parameters.

1. Strength of the two-nucleon S -wave interaction
2. Range of the two-nucleon S -wave interaction
3. Strength of three-nucleon contact interaction
4. Range of the local part of the two-nucleon interaction



	B	Exp.	R_{ch}	Exp.
^3H	8.48(2)(0)	8.48	1.90(1)(1)	1.76
^3He	7.75(2)(0)	7.72	1.99(1)(1)	1.97
^4He	28.89(1)(1)	28.3	1.72(1)(3)	1.68
^{16}O	121.9(1)(3)	127.6	2.74(1)(1)	2.70
^{20}Ne	161.6(1)(1)	160.6	2.95(1)(1)	3.01
^{24}Mg	193.5(02)(17)	198.3	3.13(1)(2)	3.06
^{28}Si	235.8(04)(17)	236.5	3.26(1)(1)	3.12
^{40}Ca	346.8(6)(5)	342.1	3.42(1)(3)	3.48

Pinhole algorithm



Seeing Structure with Pinholes

Consider the density operator for nucleon with spin i and isospin j

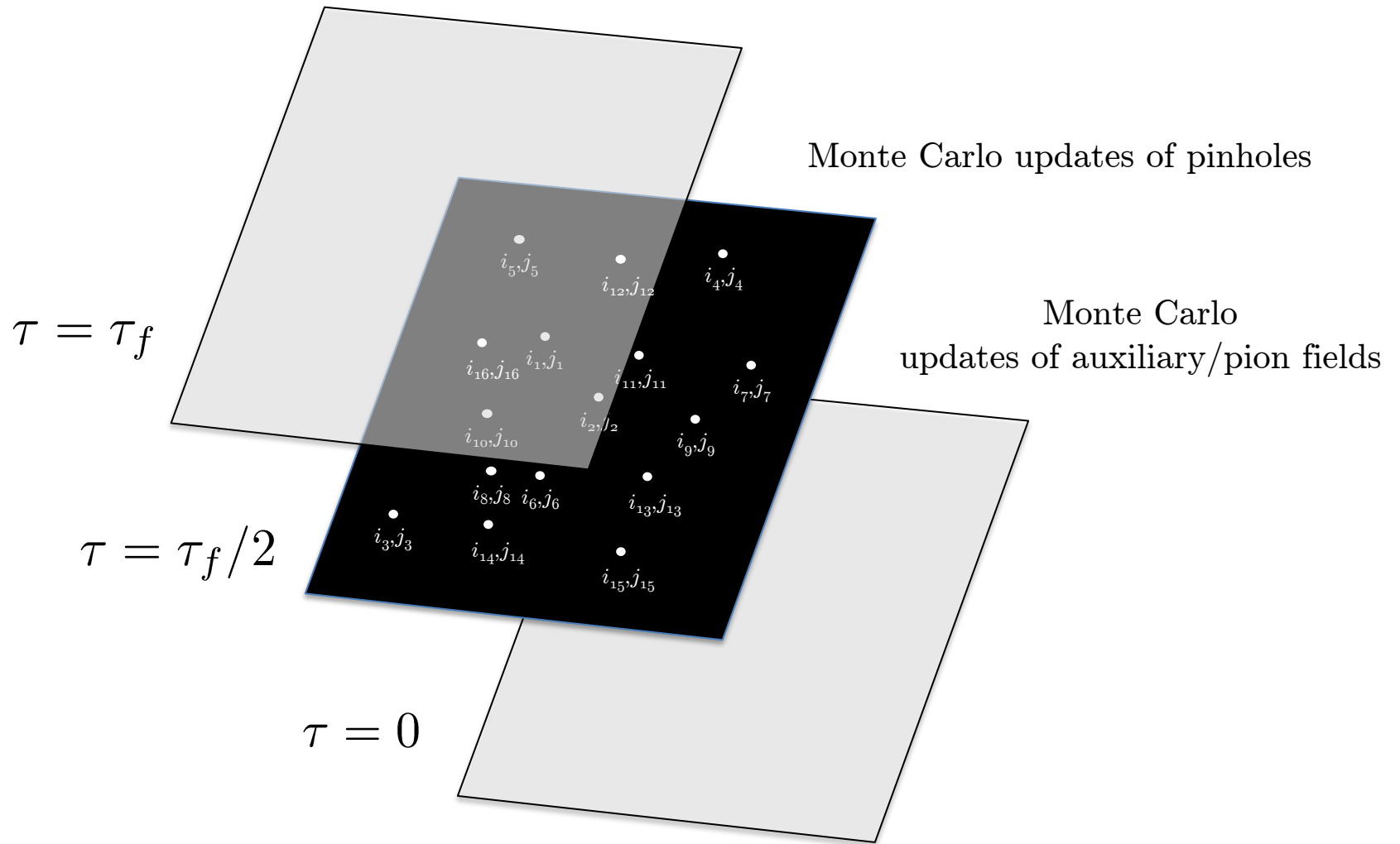
$$\rho_{i,j}(\mathbf{n}) = a_{i,j}^\dagger(\mathbf{n})a_{i,j}(\mathbf{n})$$

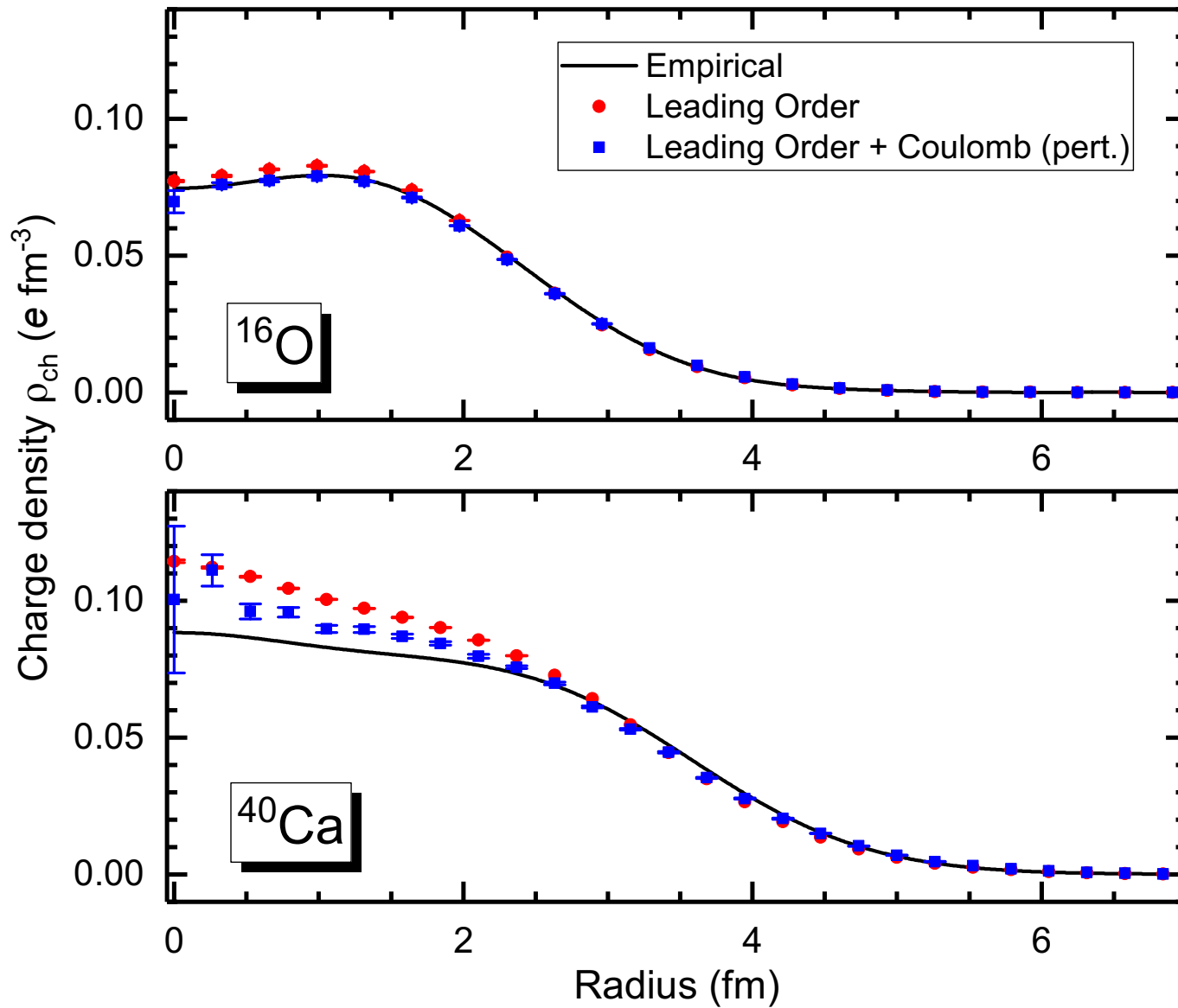
We construct the normal-ordered A -body density operator

$$\rho_{i_1,j_1,\dots,i_A,j_A}(\mathbf{n}_1,\dots,\mathbf{n}_A) = : \rho_{i_1,j_1}(\mathbf{n}_1) \cdots \rho_{i_A,j_A}(\mathbf{n}_A) :$$

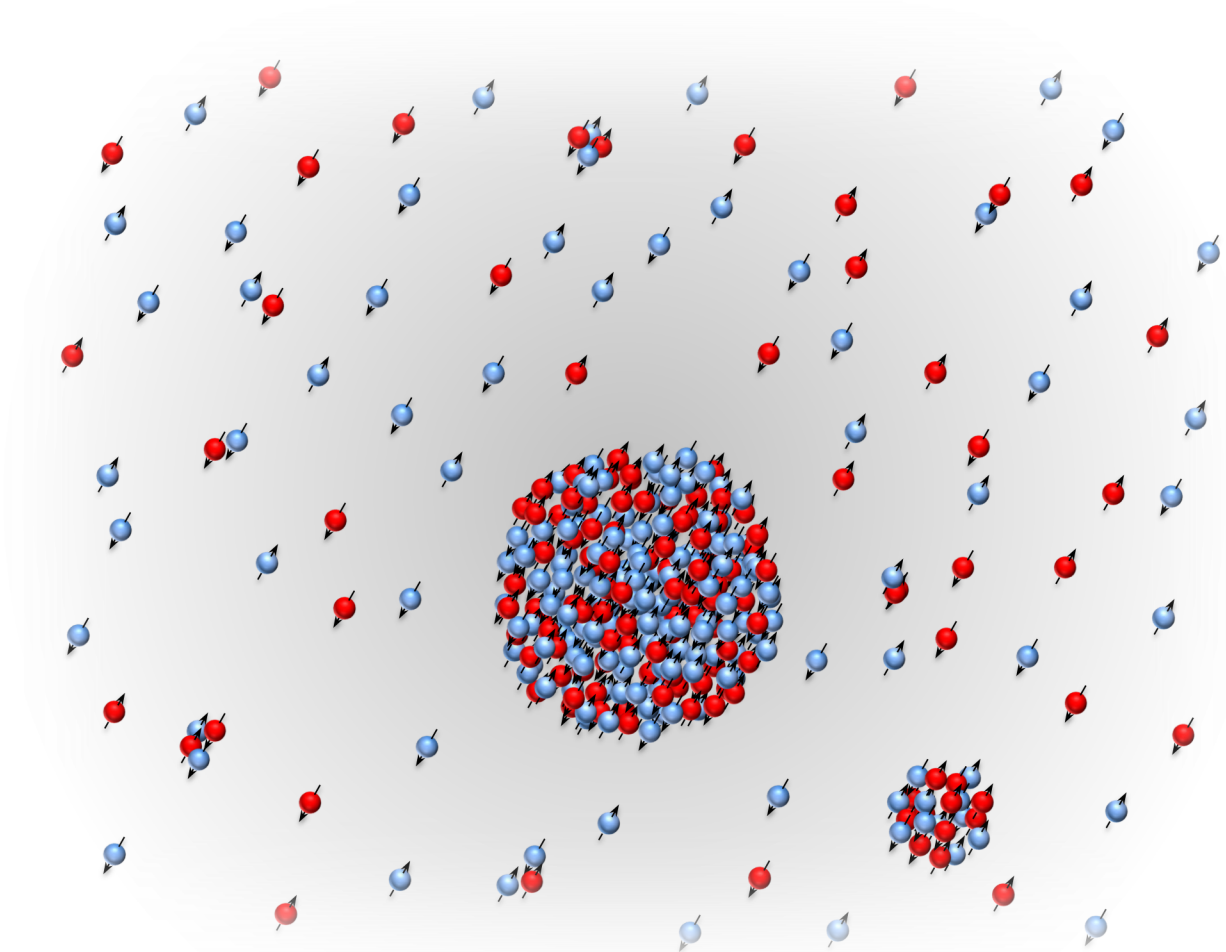
In the simulations we do Monte Carlo sampling of the amplitude

$$A_{i_1,j_1,\dots,i_A,j_A}(\mathbf{n}_1,\dots,\mathbf{n}_A,t) = \langle \Psi_I | e^{-Ht/2} \rho_{i_1,j_1,\dots,i_A,j_A}(\mathbf{n}_1,\dots,\mathbf{n}_A) e^{-Ht/2} | \Psi_I \rangle$$





Ab initio nuclear thermodynamics



Lu, Li, Elhatisari, D.L., Drut, Lähde, Epelbaum, Meißner, PRL 125, 192502 (2020)

Ab initio nuclear thermodynamics

In order to compute thermodynamic properties of finite nuclei, nuclear matter, and neutron matter, we need to compute the partition function

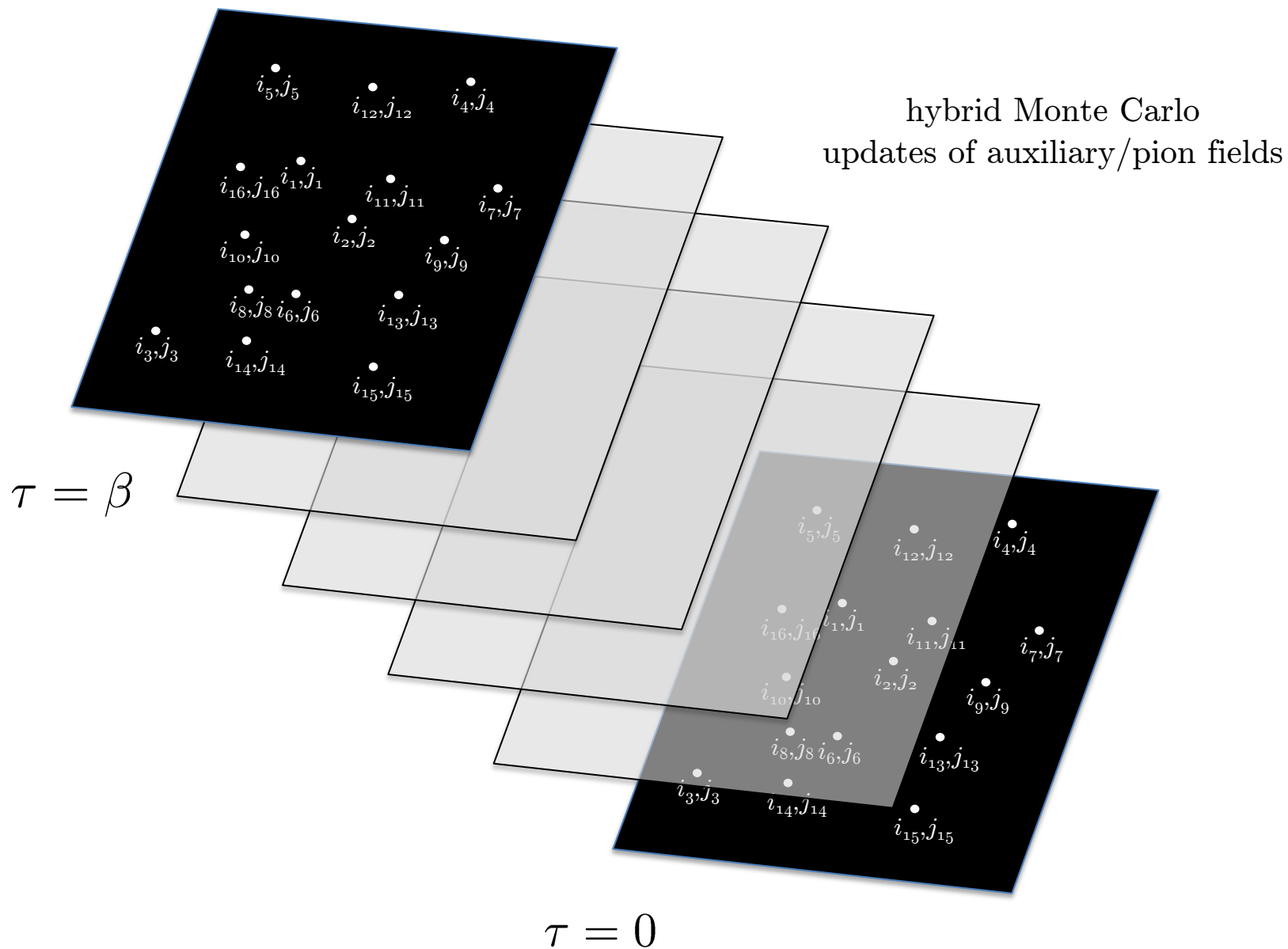
$$\text{Tr} \exp(-\beta H)$$

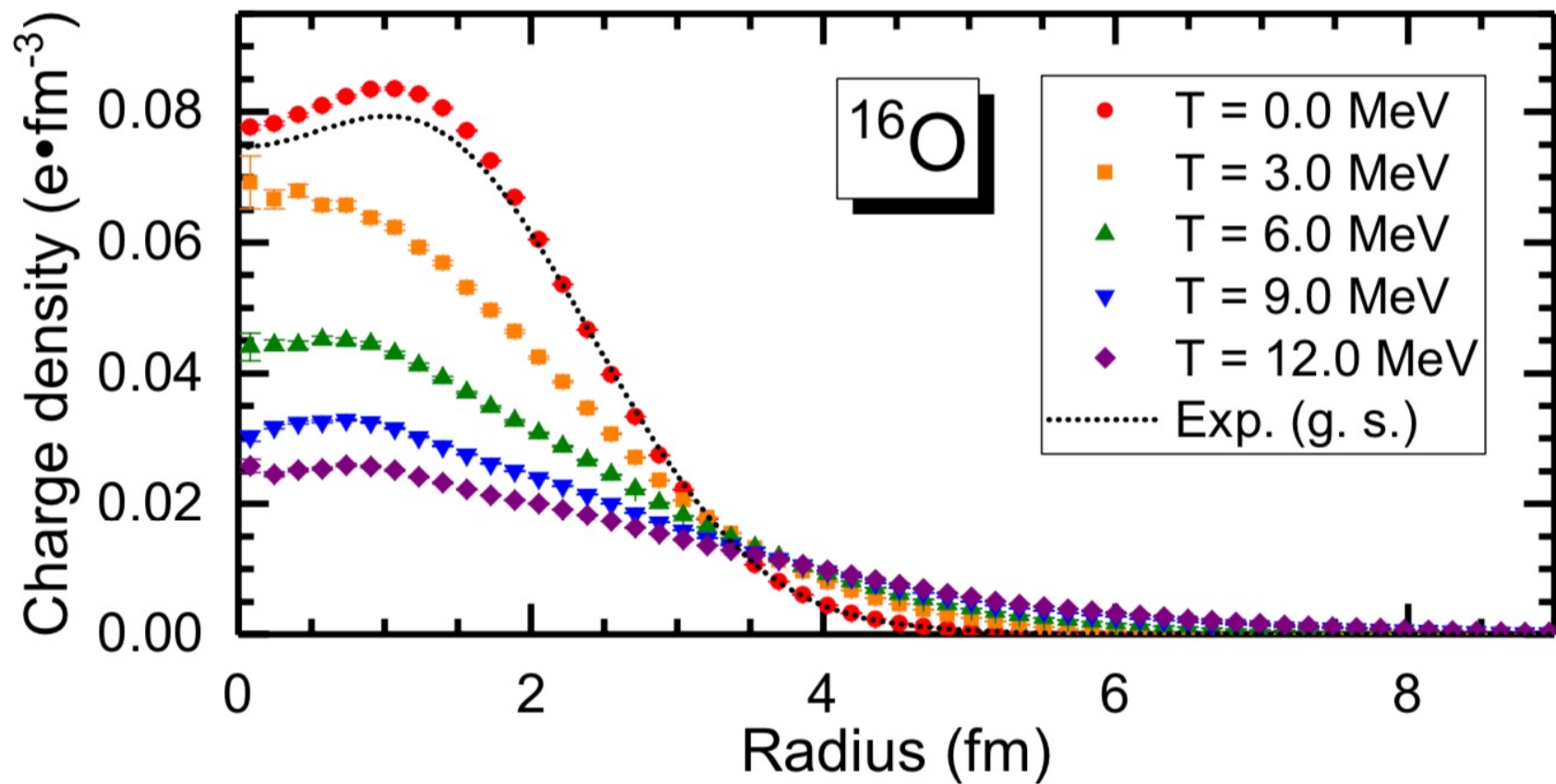
We compute the quantum mechanical trace over A -nucleon states by summing over pinholes (position eigenstates) for the initial and final states

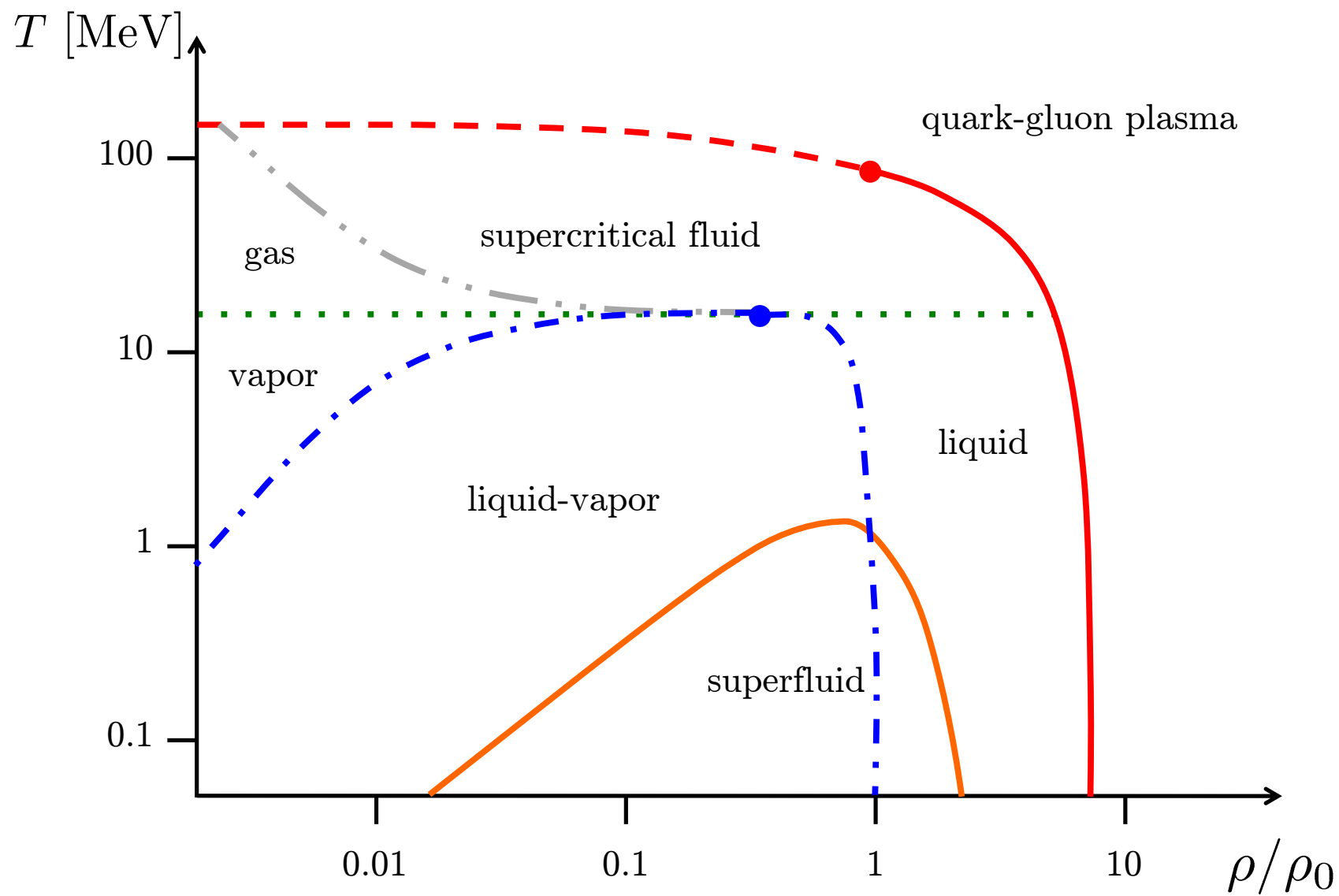
$$\begin{aligned} & \text{Tr } O \\ &= \frac{1}{A!} \sum_{i_1 \cdots i_A, j_1 \cdots j_A, \mathbf{n}_1 \cdots \mathbf{n}_A} \langle 0 | a_{i_A, j_A}(\mathbf{n}_A) \cdots a_{i_1, j_1}(\mathbf{n}_1) O a_{i_1, j_1}^\dagger(\mathbf{n}_1) \cdots a_{i_A, j_A}^\dagger(\mathbf{n}_A) | 0 \rangle \end{aligned}$$

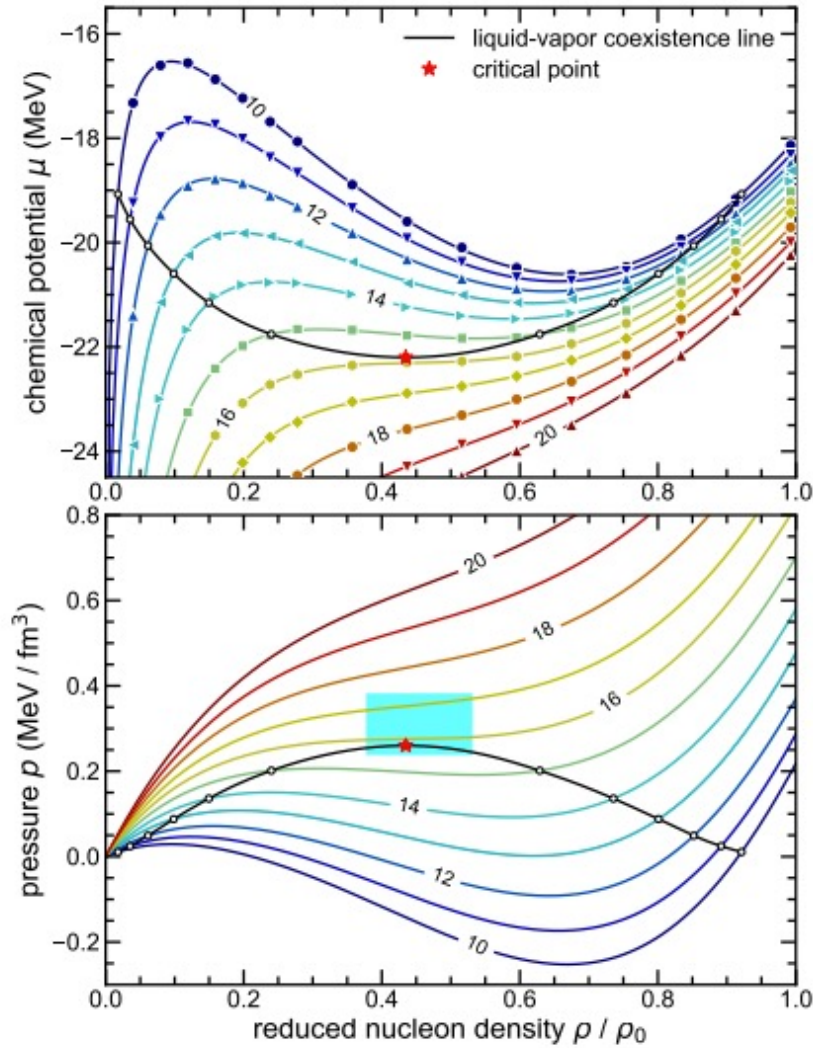
This can be used to calculate the partition function in the canonical ensemble.

Metropolis updates of pinholes







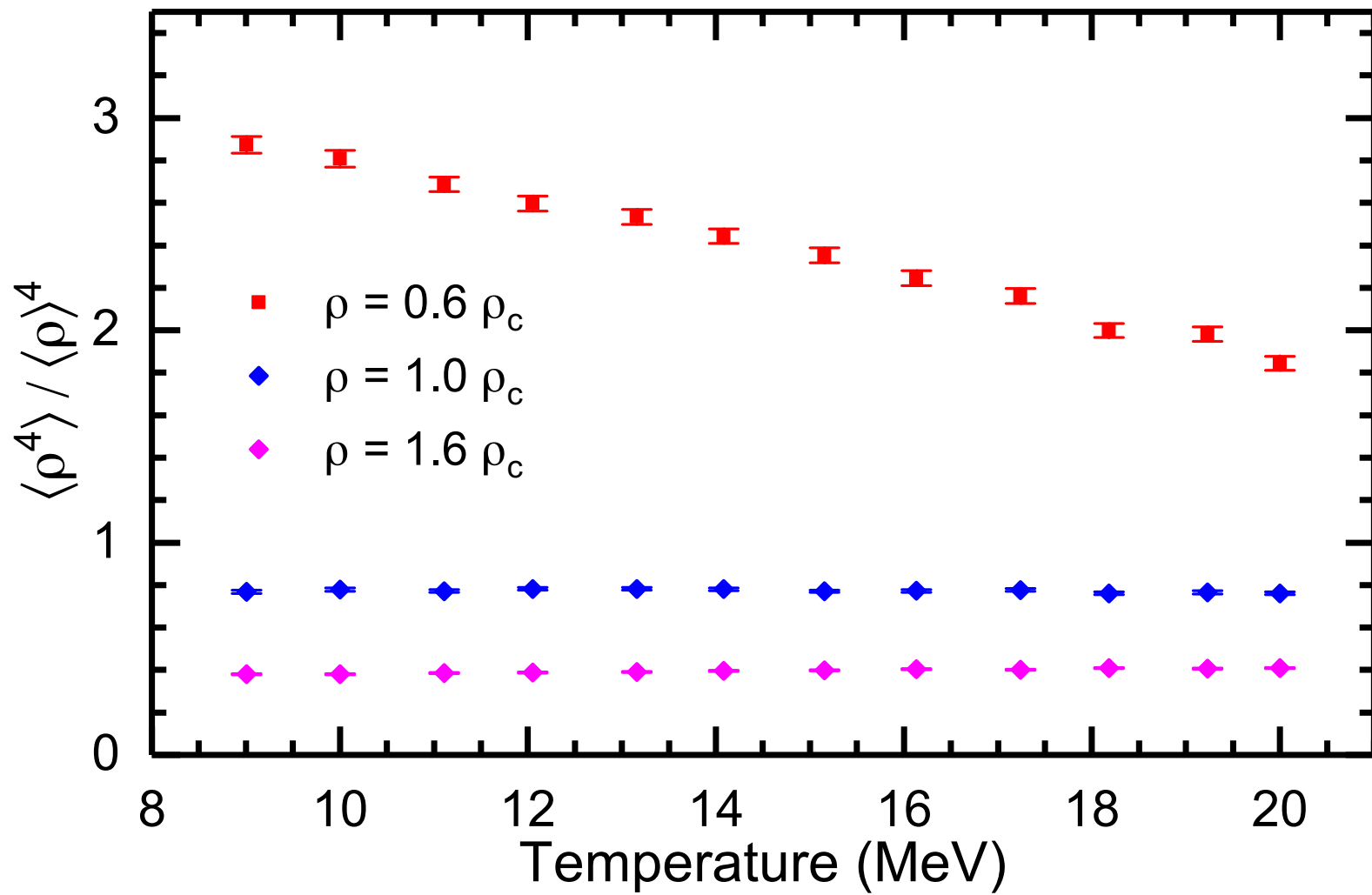


$$T_c = 15.80(0.32)(1.60) \text{ MeV}$$

$$\rho_c = 0.089(04)(18) \text{ fm}^{-3}$$

$$\mu_c = -22.20(0.44)(2.20) \text{ MeV}$$

$$P_c = 0.260(05)(30) \text{ MeV fm}^{-3}$$



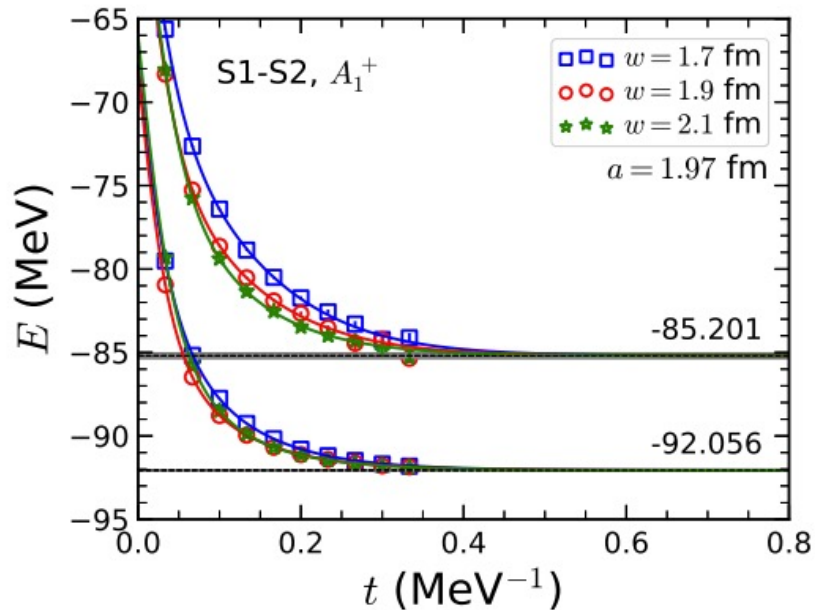
Structure and clustering in carbon-12

Prior work:

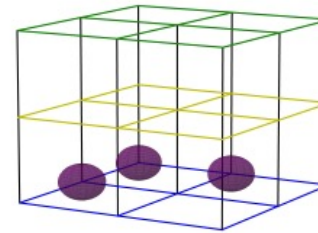
Epelbaum, Krebs, D.L., Meißner, PRL 106, 192501 (2011)

Epelbaum, Krebs, Lähde, D.L., Meißner, PRL 109, 252501 (2012)

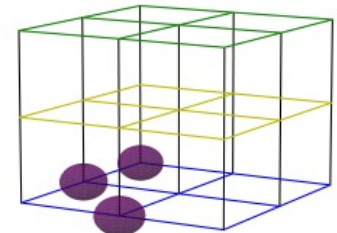
$$0_1^+ - 0_2^+$$



S1

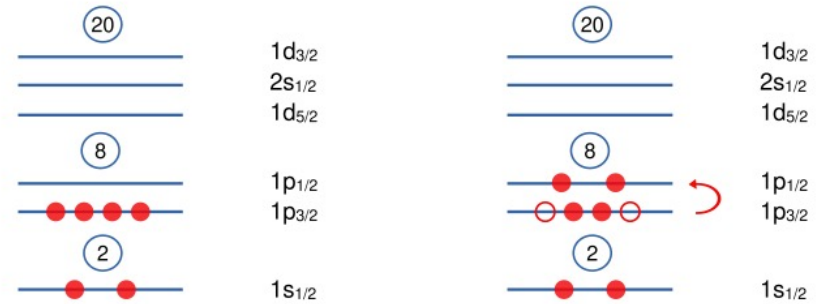
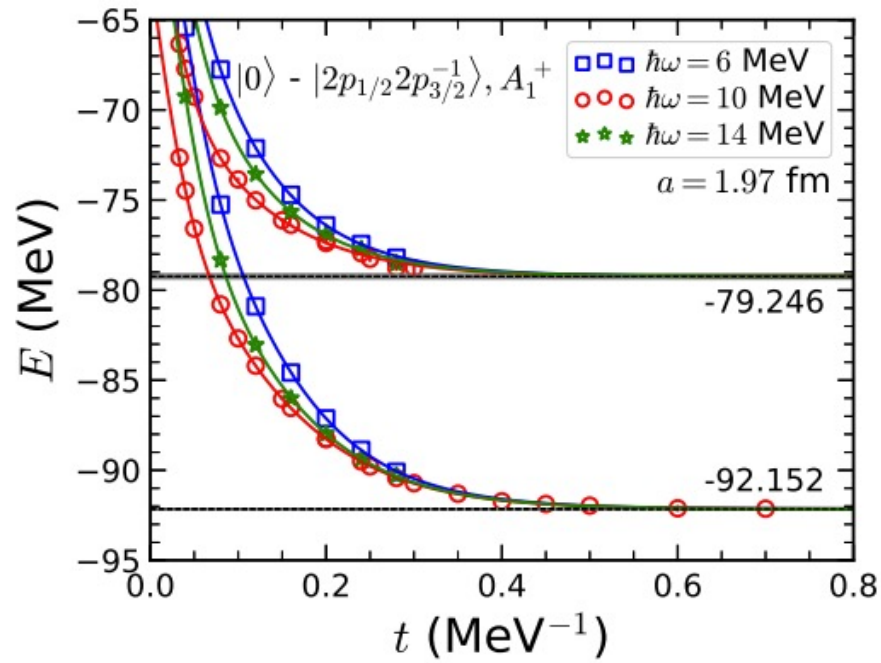


S2



Shen, Lähde, D.L., Meißner, EPJA 57, 276 (2021)

$$0_1^+ - 0_3^+$$



State	$a = 1.97$ fm	$a = 1.64$ fm	Experiment
0_1^+	$-92.15(3)$	$-92.12(4)$	-92.162
2_1^+	$-88.87(4)$	$-88.19(17)$	-87.722
0_2^+	$-85.20(15)$	$-85.23(22)$	-84.508
3_1^-	$-84.9(2)$	$-83.3(5)$	$-82.521(5)$
2_2^+	$-83.5(2)$	$-83.1(5)$	$-82.29(6)$
0_3^+	$-80.0(3)$	$-79.2(6)$	$-81.9(3)$
1_1^-	$-81.5(4)$	$-79.7(4)$	$-81.315(4)$
2_1^-	$-78.6(2)$	$-76.1(2)$	$-80.326(4)$
1_1^+	$-79.67(11)$	$-78.14(24)$	$-79.452(6)$
4_1^-	$-78.1(2)$	$-75.5(5)$	$-78.846(20)$
4_1^+	$-80.99(11)$	$-79.1(6)$	$-78.083(5)$
2_3^+	$-79.9(4)$	$-77.9(2)$	-76.056
0_4^+	$-79.25(11)$	$-76.94(18)$	-74.402

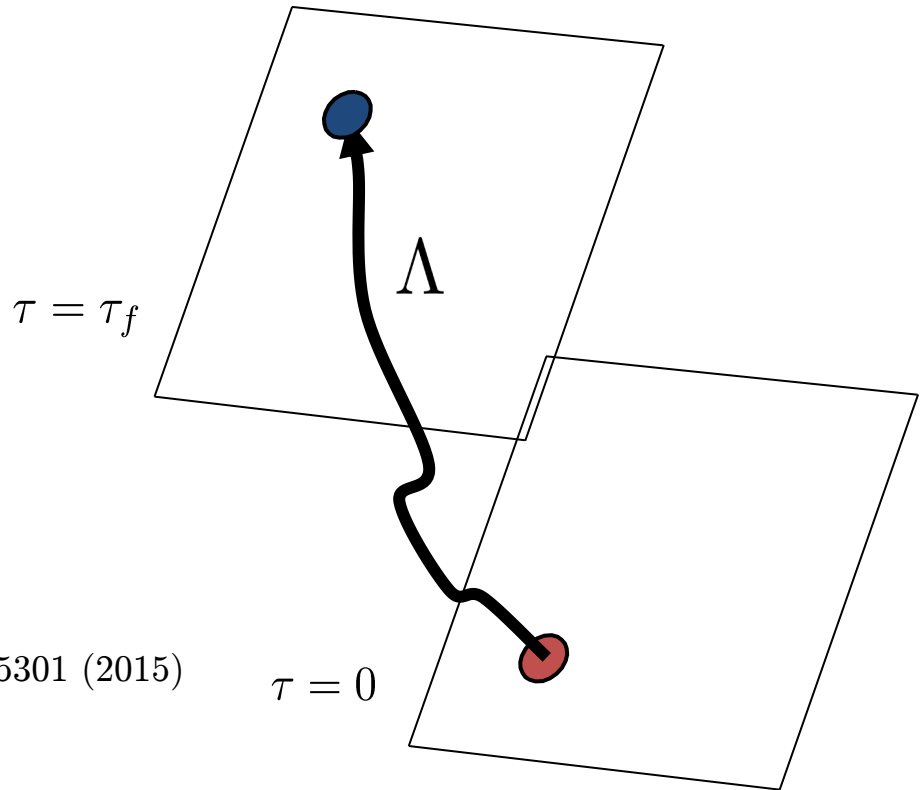
Hypernuclei with impurity lattice Monte Carlo

Auxiliary-field lattice Monte Carlo simulations that treat hyperons and nucleons in the same manner would cause large sign cancellations. Instead, we use impurity lattice Monte Carlo, where hyperon worldlines are sampled explicitly.

Prior work:

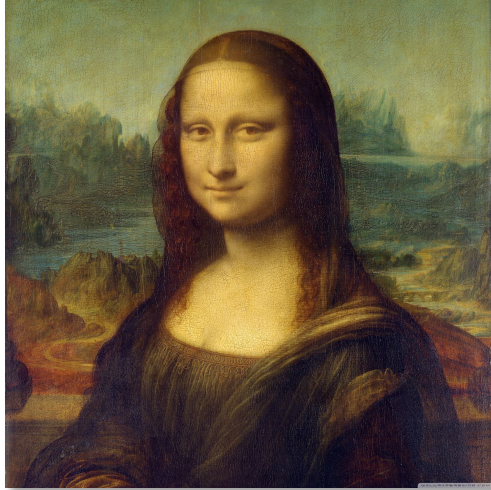
Elhatisari, D.L., PRC 90, 064001 (2014)

Bour, D.L., H.-W. Hammer, Meißner, PRL 115, 185301 (2015)



Nucleus	L (fm)	E (MeV)	B_Λ (MeV)	B_Λ^{exp} (MeV)
${}^2\text{H}$	29.6	-1.100	—	—
${}^3_\Lambda\text{H}$	29.6	-1.319	0.220	0.13(5) [29, 30, 31]
${}^3\text{H}/\text{He}$	15.8	-8.725	—	—
${}^4_\Lambda\text{H}/\text{He}$	15.8	$-9.19(5)$	0.46(5)	1.39(4) [29, 30, 31, 32, 33]
${}^4\text{He}$	9.9	$-25.698(9)$	—	—
${}^5_\Lambda\text{He}$	9.9	$-29.66(6)$	3.96(6)	3.12(2) [29, 30, 31]

Wave function matching



Work in progress: Elhatisari, Bovermann, et al.

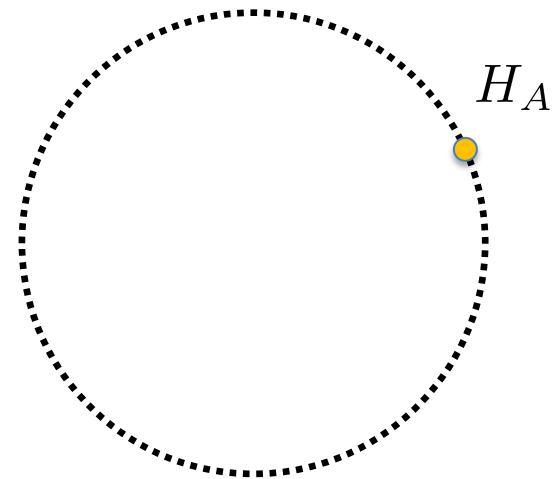
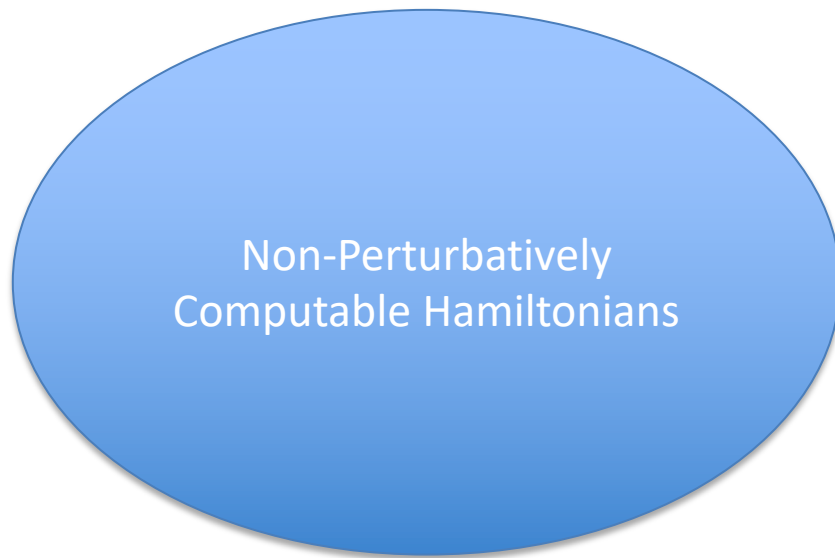
Lattice Monte Carlo simulations can compute highly nontrivial correlations in nuclear many-body systems. Unfortunately, sign oscillations prevent direct simulations using a high-fidelity Hamiltonian based on chiral effective field theory due to short-range repulsion.

Wave function matching solves this problem by means of unitary transformations and perturbation theory. By using unitary transformations, we construct a high-fidelity Hamiltonian that can be reached by perturbation theory, starting from a Hamiltonian without a sign problem.

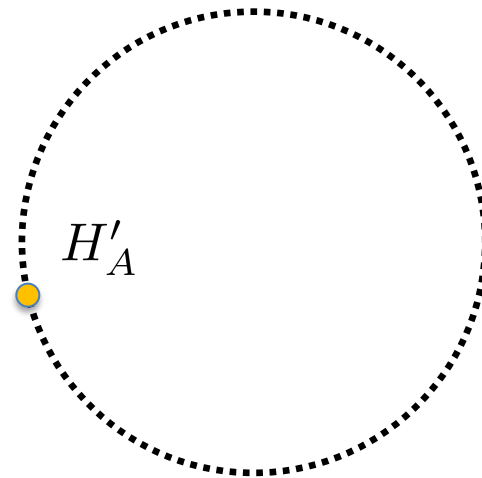
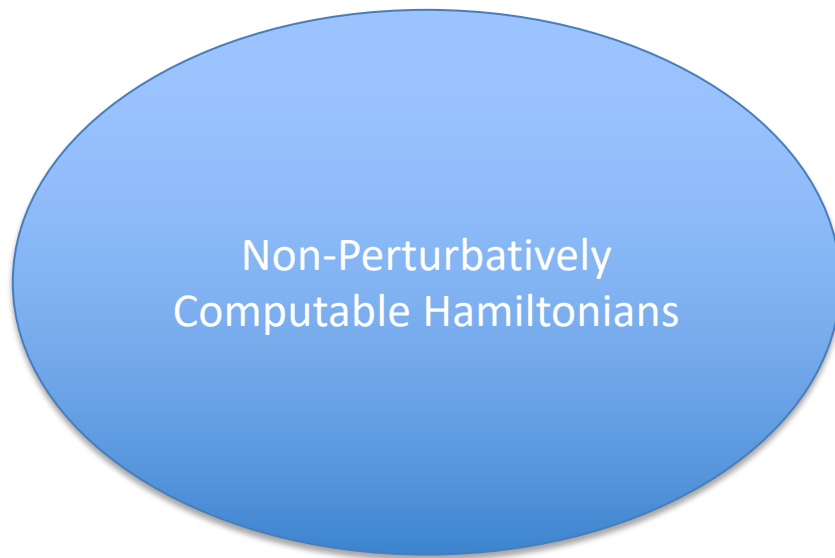
Non-Perturbatively
Computable Hamiltonians

H_A





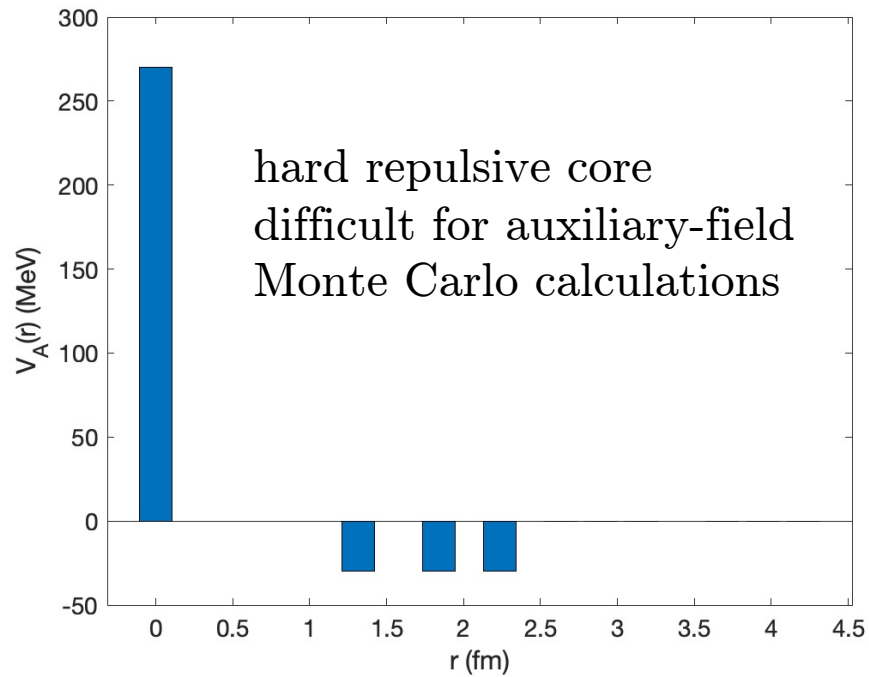
unitarily equivalent
Hamiltonians



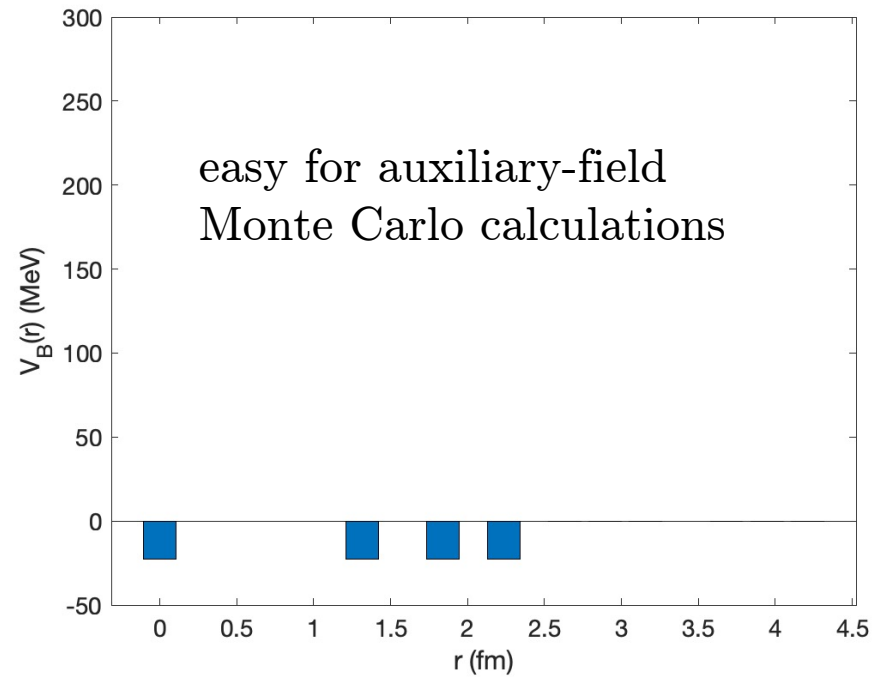
unitarily equivalent
Hamiltonians

Wave function matching

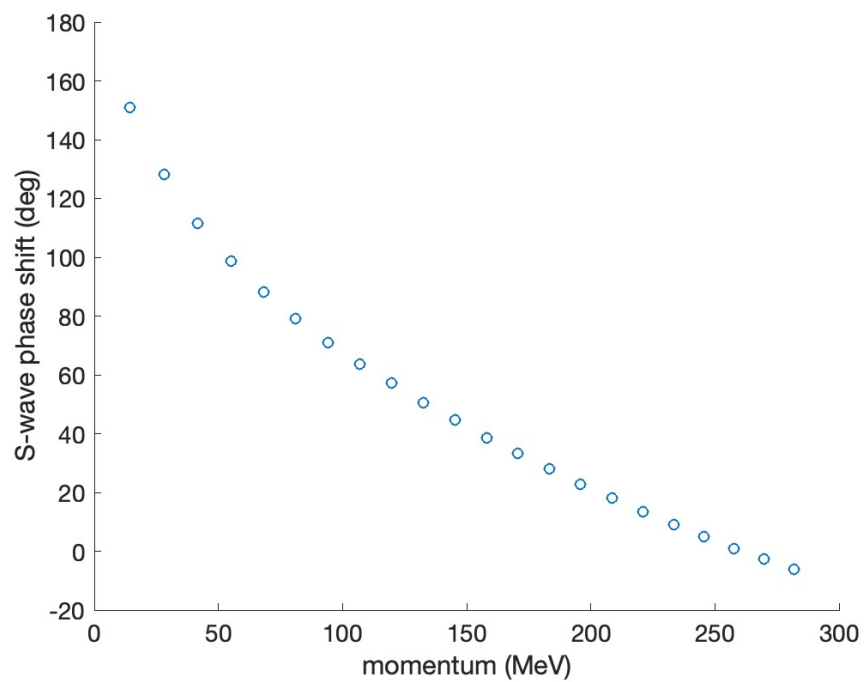
$$V_A(r)$$



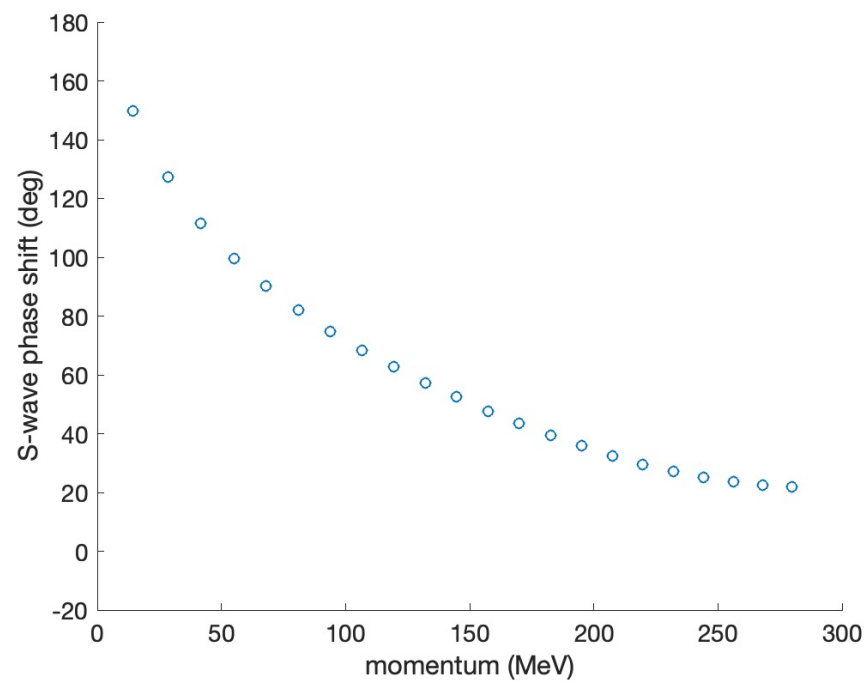
$$V_B(r)$$



$$V_A(r)$$



$$V_B(r)$$



Let us write the eigenenergies and eigenfunctions for the two interactions as

$$H_A |\psi_{A,n}\rangle = (K + V_A) |\psi_{A,n}\rangle = E_{A,n} |\psi_{A,n}\rangle$$

$$H_B |\psi_{B,n}\rangle = (K + V_B) |\psi_{B,n}\rangle = E_{B,n} |\psi_{B,n}\rangle$$

We would like to compute the eigenenergies of H_A starting from the eigenfunctions of H_B and using first-order perturbation theory.

Not surprisingly, this does not work very well. The interactions V_A and V_B are quite different.

$E_{A,n}$ (MeV)	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle$ (MeV)
-1.2186	3.0088
0.2196	0.3289
0.8523	1.1275
1.8610	2.2528
3.2279	3.6991
4.9454	5.4786
7.0104	7.5996
9.4208	10.0674
12.1721	12.8799
15.2669	16.0458

Let P be a projection operator that is nonzero only for separation distances r less than R . We define a short-distance unitary operator U such that

$$U : P |\psi_A^0\rangle / \|P |\psi_A^0\rangle\| \rightarrow P |\psi_B^0\rangle / \|P |\psi_B^0\rangle\|$$

There are many possible choices for U . The corresponding action of U on the Hamiltonian is

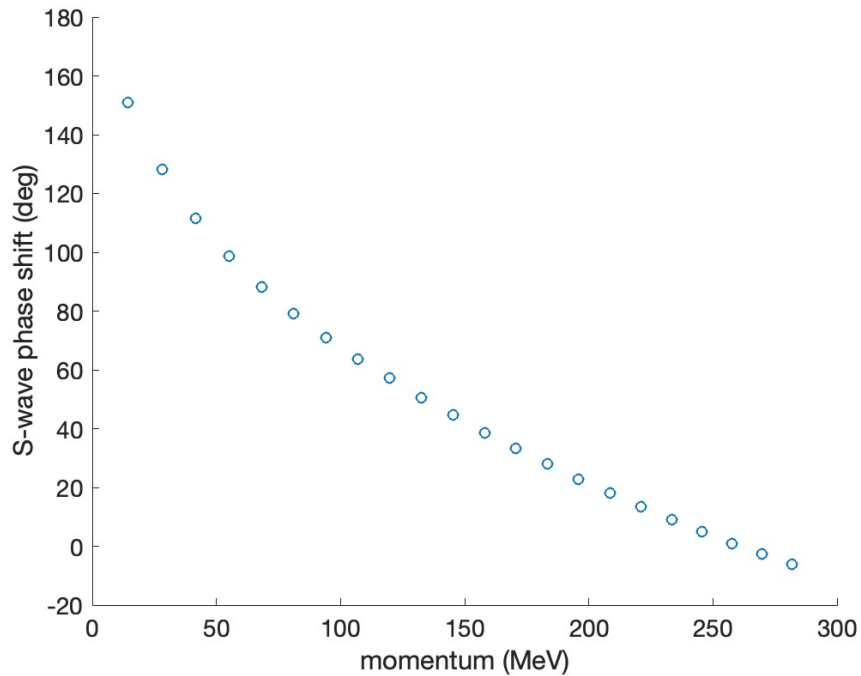
$$U : H_A \rightarrow H'_A = U^\dagger H_A U$$

and the resulting nonlocal interaction is

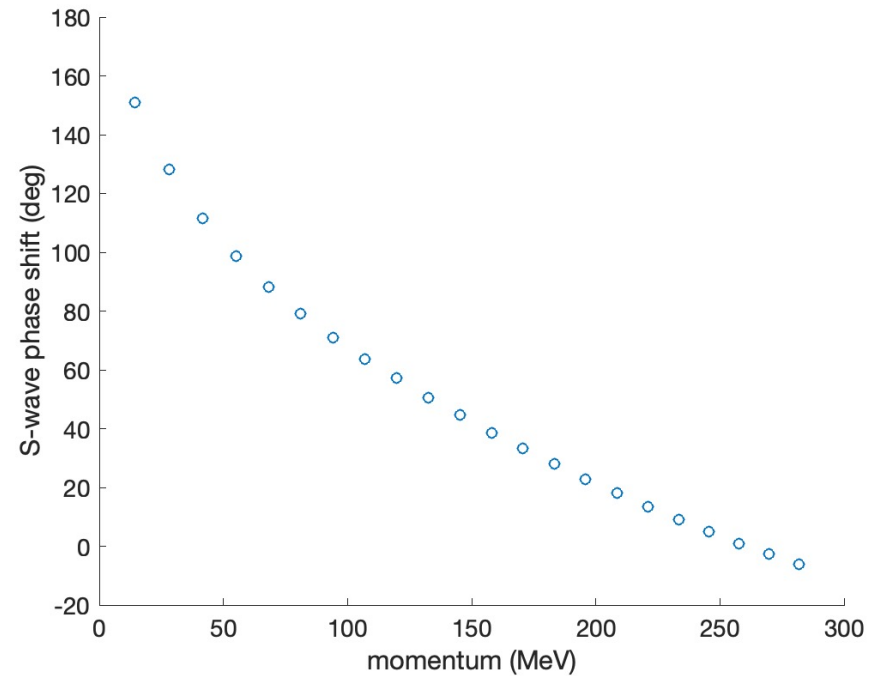
$$V'_A = H'_A - K = U^\dagger H_A U - K$$

Since they are unitarily equivalent, the phase shifts are exactly the same.

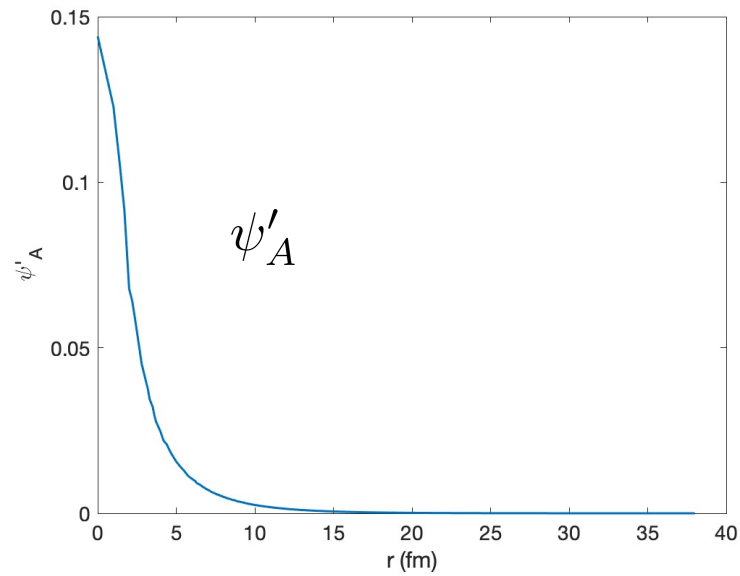
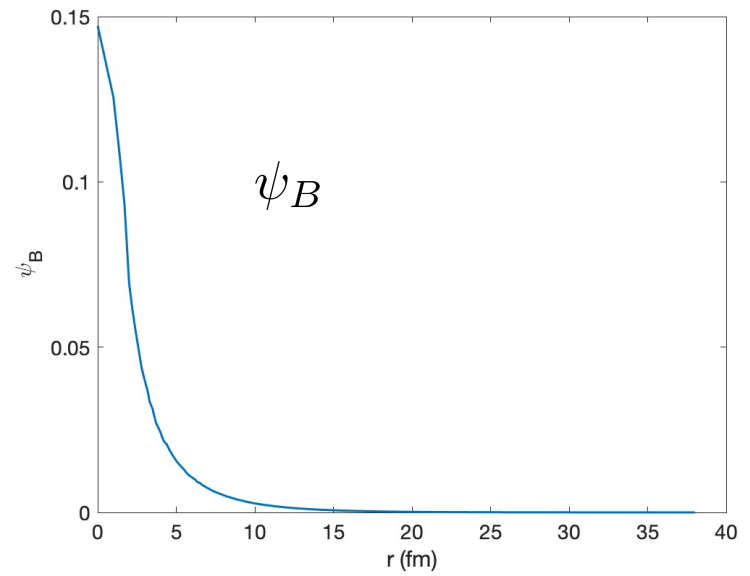
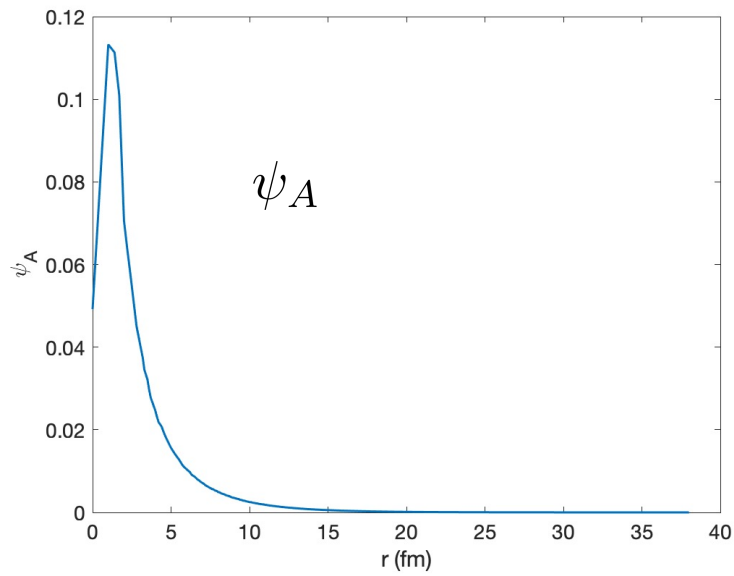
$$V_A(r)$$



$$V'_A(r, r')$$



Ground state wave functions



With wave function matching, we can now compute the eigenenergies starting from the eigenfunctions of H_B and using first-order perturbation theory.

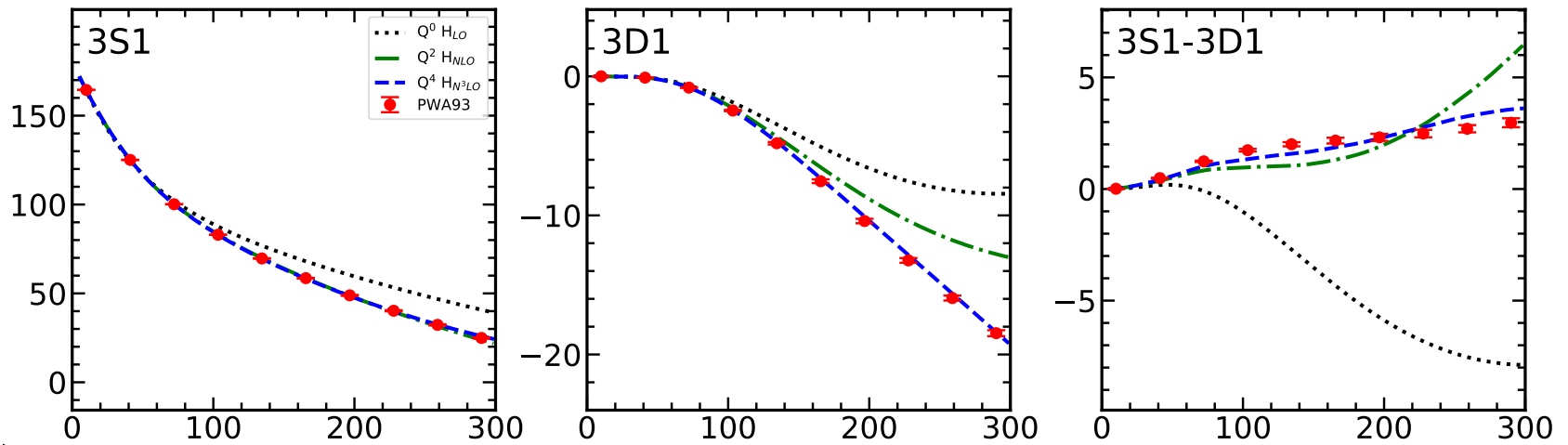
$$R = 1.3 \text{ fm}$$

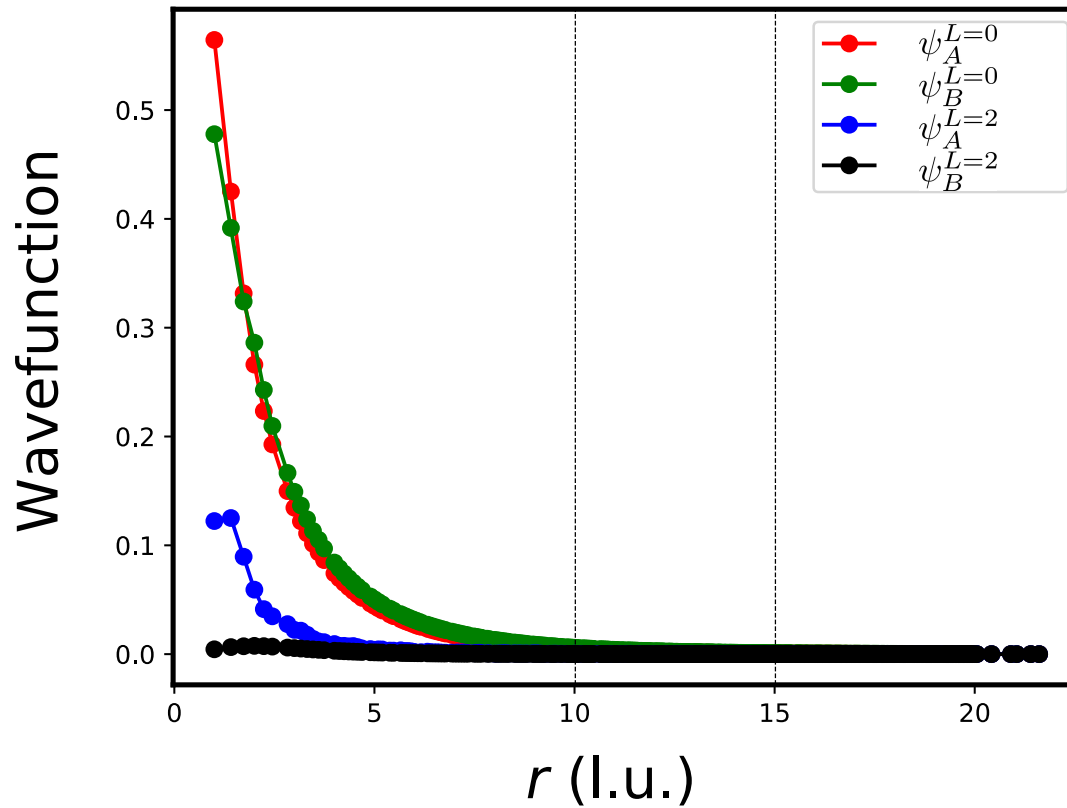
$E_{A,n} = E'_{A,n} \text{ (MeV)}$	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle \text{ (MeV)}$	$\langle \psi_{B,n} H'_A \psi_{B,n} \rangle \text{ (MeV)}$
-1.2186	3.0088	-0.5134
0.2196	0.3289	0.2377
0.8523	1.1275	0.8982
1.8610	2.2528	1.9270
3.2279	3.6991	3.3083
4.9454	5.4786	5.0378
7.0104	7.5996	7.1146
9.4208	10.0674	9.5379
12.1721	12.8799	12.3039
15.2669	16.0458	15.4170

$$R = 2.6 \text{ fm}$$

$E_{A,n} = E'_{A,n} \text{ (MeV)}$	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle \text{ (MeV)}$	$\langle \psi_{B,n} H'_A \psi_{B,n} \rangle \text{ (MeV)}$
-1.2186	3.0088	-1.1597
0.2196	0.3289	0.2212
0.8523	1.1275	0.8577
1.8610	2.2528	1.8719
3.2279	3.6991	3.2477
4.9454	5.4786	4.9798
7.0104	7.5996	7.0680
9.4208	10.0674	9.5137
12.1721	12.8799	12.3163
15.2669	16.0458	15.4840

Application of wave function matching to the 3S1-3D1 chiral interaction at N3LO.





$E_{A,n} = E'_{A,n}$ (MeV)	$\langle \psi_{B,n} H_A \psi_{B,n} \rangle$ (MeV)	$\langle \psi_{B,n} H'_A \psi_{B,n} \rangle$ (MeV)
-2.2234	0.9104	-2.1380
0.7188	1.3362	0.8344

Work in progress: Elhatisari, Bovermann, et al.

Summary

We began with an introduction to lattice simulations using chiral effective field theory. We then discussed a hidden spin-isospin symmetry of the nucleonic interactions that can be derived in the large- N_c limit.

We constructed a minimal nuclear interaction that can reproduce the ground state properties of light nuclei, medium-mass nuclei, and neutron matter.

We presented first principles calculations of nuclear thermodynamics using the pinhole trace algorithm. We probed the nuclear liquid-vapor phase diagram, the location of the critical point, and alpha clustering as a function of density and temperature.

We discussed calculations of the structure of the nuclear states in carbon-12. In particular, we confirm the prolate alpha-cluster structure of the Hoyle state and the second 0^+ state. We then showed how hypernuclei can be computed using impurity lattice Monte Carlo simulations.

We concluded with a discussion of a new method called wave function matching. Using unitary transformations, we construct a high-fidelity Hamiltonian that can be reached via perturbation theory from a Hamiltonian that doesn't produce a Monte Carlo sign problem.