Spin Sum Rules & Polarizabilities At Long Distance

10th International Workshop on Chiral Dynamics

Beijing, China
2021-11-17

Karl Slifer
University of New Hampshire

This Talk

Review of Inclusive Doubly Polarized e-Scattering

Structure Functions

Results from JLab

- g2p, sagdh & EG4 Experiments
- SF, Spin Polarizabilities & Moments

Thanks to these Collaborators

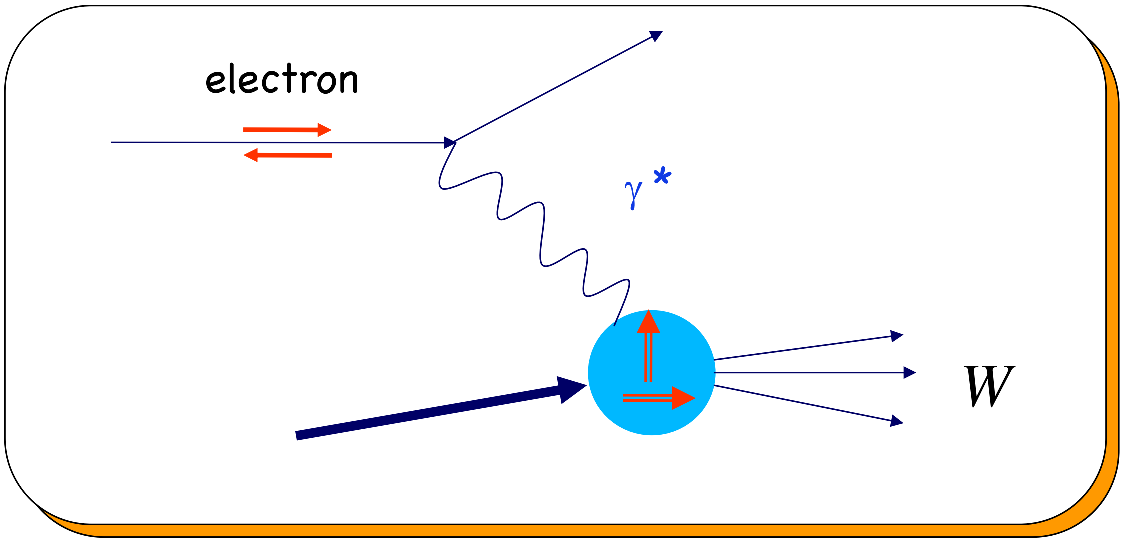
EG4 Proton : M. Ripani(Contact), M. Battaglieri, A. Deur, R. De Vita
H. Kang, X. Zheng, L. El Fassi, J. Zhang

EG4 Deuteron : A. Deur(Contact), G. Dodge, K. Slifer
K. Adhikari

g2p Proton: K. Slifer (Contact), JP Chen, D. Crabb, A. Camsonne
D. Ruth, R. Zielinski, T. Badman, Chao Gu,

sagdh Neutron : JP Chen(contact), A. Deur, F. Garibaldi
V. Sulkosky, Chao Peng, Nguyen Ton

Inclusive Scattering



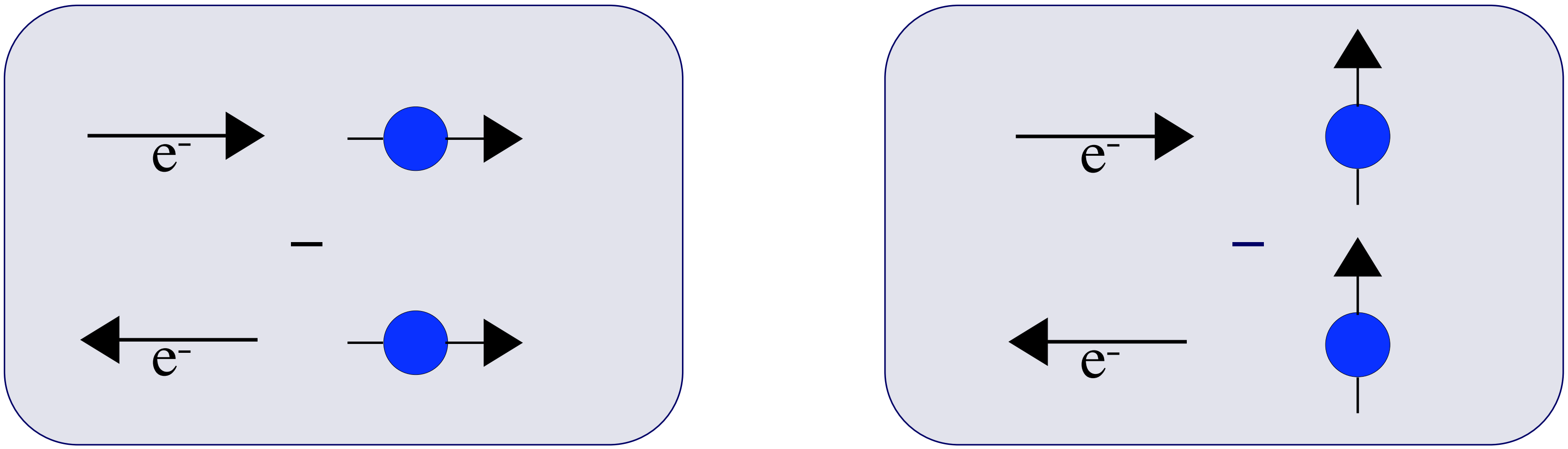
When we add spin degrees of freedom to the target and beam, 2 Additonal SF needed.

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right.$$

$$\left. + \gamma\, g_1(x,Q^2) + \delta\, g_2(x,Q^2)\right]$$

Inclusive **Polarized** Cross Section

Cross Section Differences



$$\frac{d^2\sigma^{\uparrow\Uparrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\left[\left(E + E'\cos\theta\right) g_1 - 2Mx g_2\right]$$

$$\frac{d^2\sigma^{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2\sigma^{\downarrow\Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2}\frac{E'}{E}\sin\theta\left[g_1 + \frac{2ME}{\nu} g_2\right]$$

SSF Moments

Generalized GDH Sum

$$\mathit{\Gamma}_1(Q^2) = \int_0^{x_0} \mathrm{d}x \, g_1(x, Q^2)$$

Burkhardt Cottingham

$$\Gamma_2(Q^2) = \int_0^{x_0} dx \;\; g_2(x, Q^2)$$

Generalized Forward Spin polarizabilities

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[g_1 - \left(\frac{4M^2 x^2}{Q^2} \right) g_2 \right] dx$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[g_1(x, Q^2) + g_2(x, Q^2) \right] dx$$

Spin Experiments at Low Q^2

Jefferson Lab

g2p

NH_3 Target (proton)
$0.02 < Q^2 < 0.4$
$W < 1.8$ GeV
θ = **6 deg**

Spin Experiments at Low Q^2

Jefferson Lab

g2p

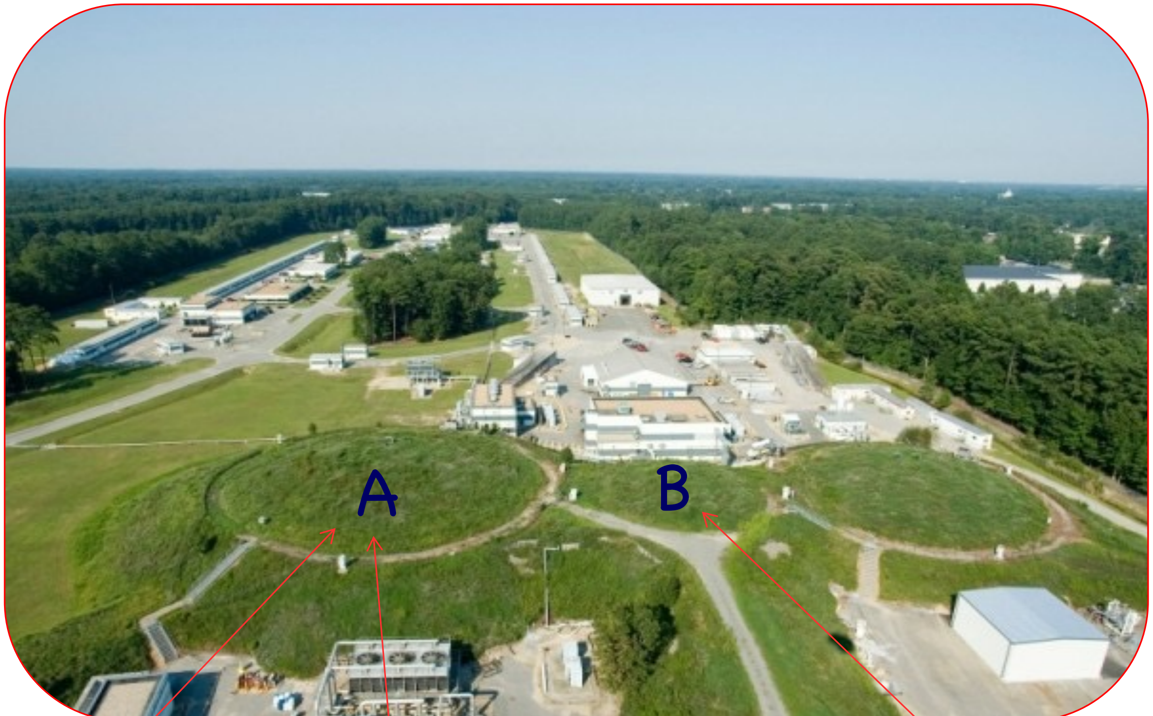
NH_3 Target (proton)
$0.02 < Q^2 < 0.4$
W< 1.8 GeV
θ = 6 deg

sagdh

^3He Target (neutron)
$0.035 < Q^2 < 0.24$
W< 1.8 GeV
θ = 6&9 degs

Spin Experiments at Low Q^2

Jefferson Lab



g2p

NH_3 Target (proton)
$0.02 < Q^2 < 0.4$
W< 1.8 GeV
θ = 6 deg

sagdh

^3He Target (neutron)
$0.035 < Q^2 < 0.24$
W< 1.8 GeV
θ = 6&9 degs

Eg4

NH_3 (proton) and ND_3(neutron)
$0.02 < Q^2 < 0.5$
W< 2.5 GeV
θ < 15 deg

E08-027 : The Proton g2p Experiment

A.Camsonne, D. Crabb,

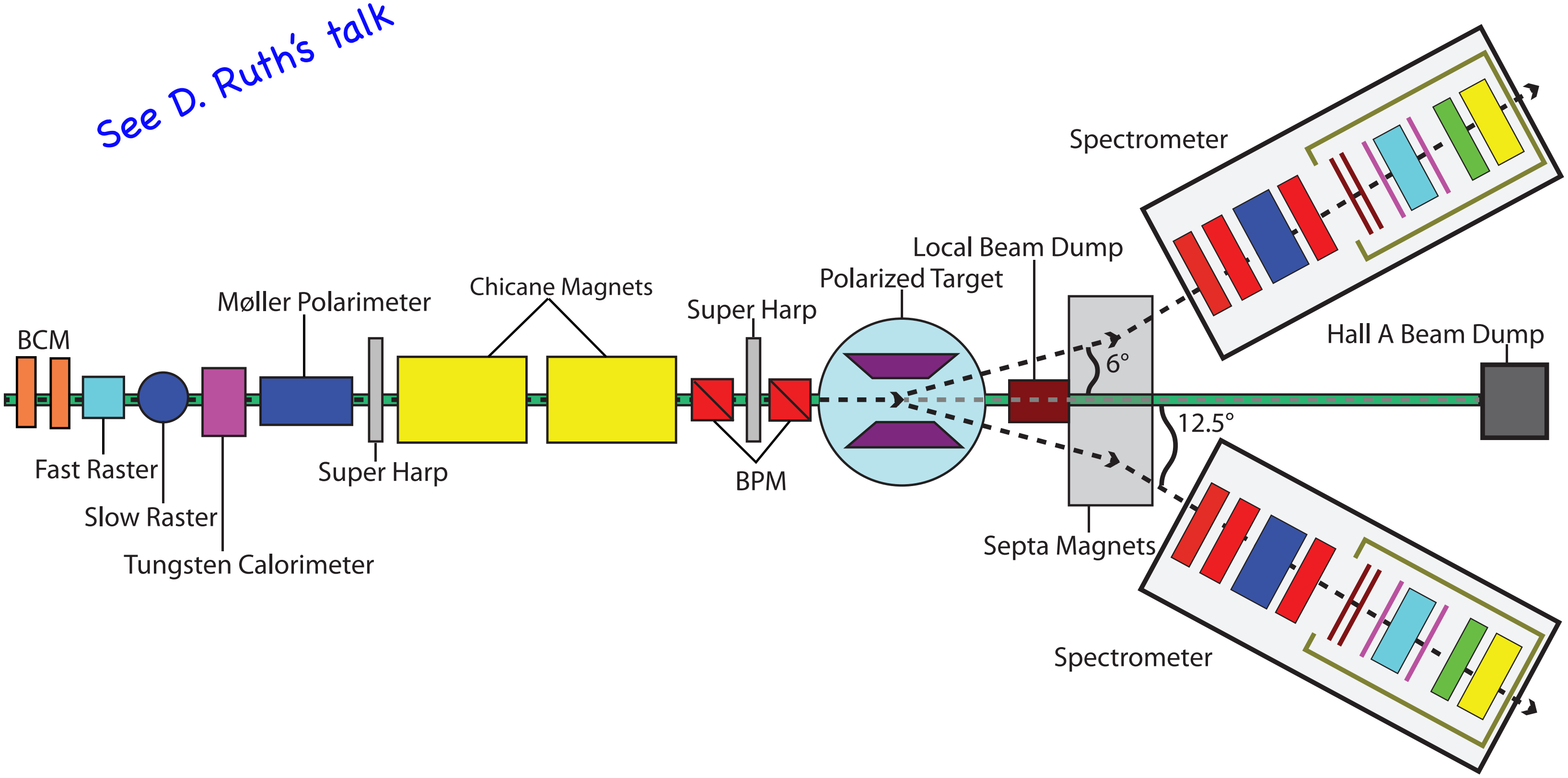
J. P. Chen, K. Slifer

BC Sum Rule : violation suggested for proton at large Q^2, but found satisfied for the neutron & ^3He.

Spin Polarizability : Major failure (>8σ) of χPT for neutron δ_{LT}

Hydrogen HFS: Structure dependent corrections

See D. Ruth's talk



Polarized Target Installation in Hall A

Polarized proton target

upstream chicane

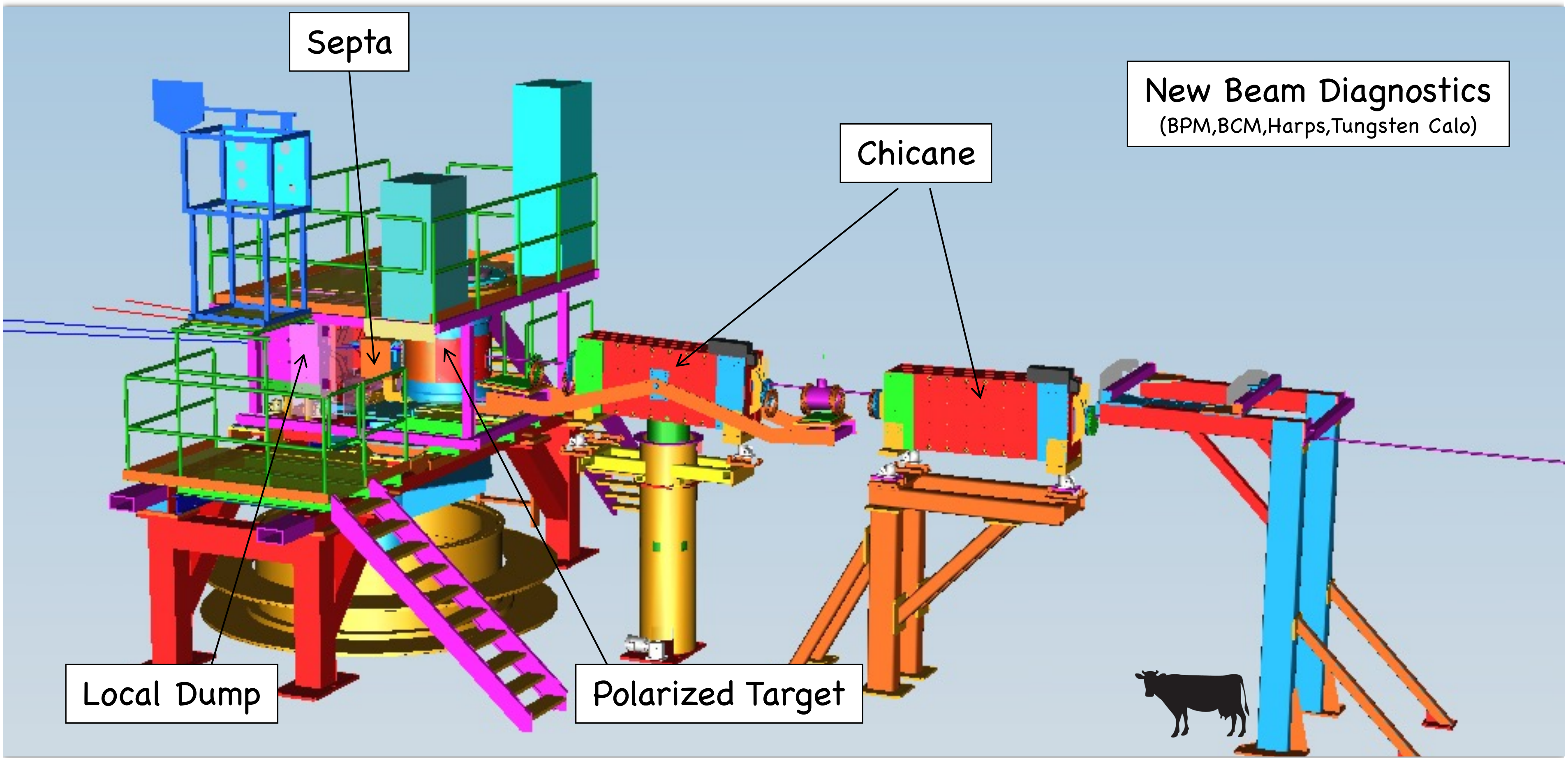
downstream local dump

Low current polarized beam

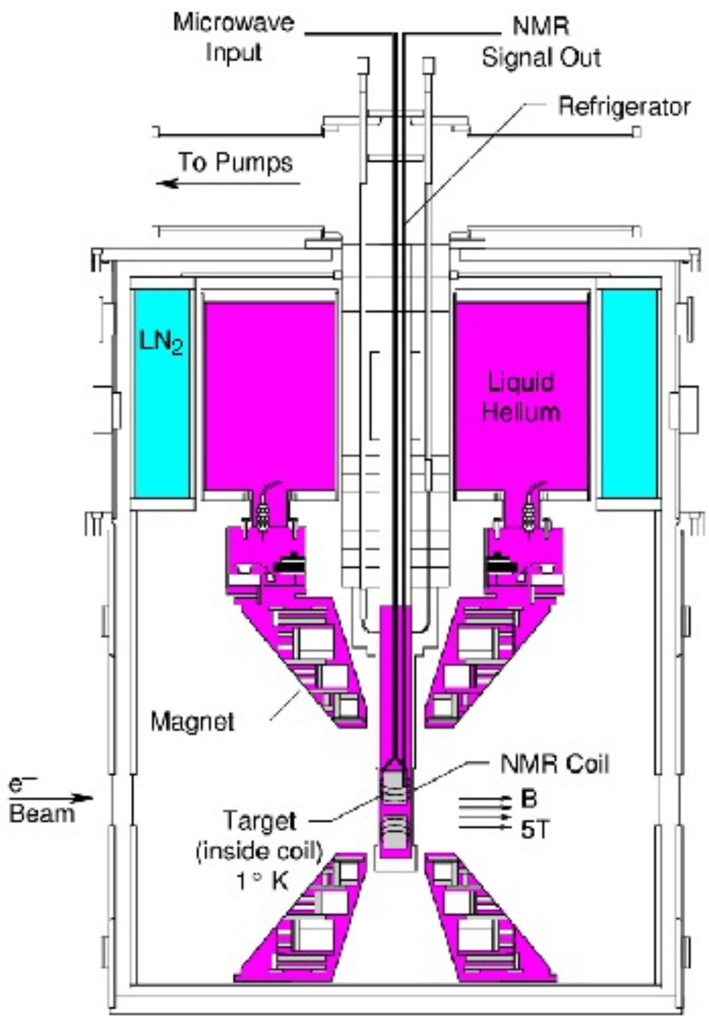
Upgrades to existing Beam Diagnostics to work at 85 nA

Lowest possible Q^2 in the resonance region

Septa Magnets to detect forward scattering



Polarized Ammonia Target



Dynamic Nuclear Polarization of NH_3

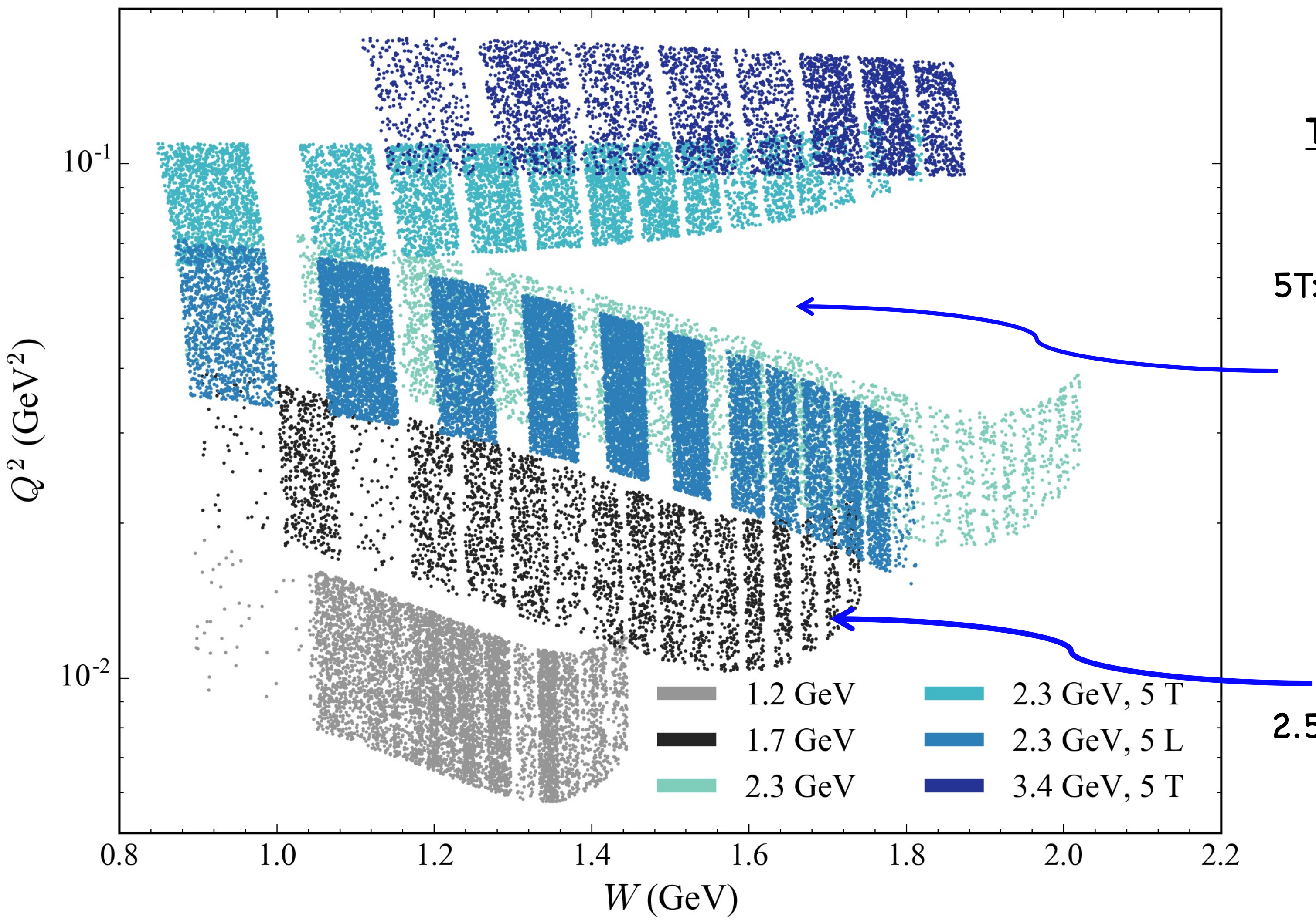
5 T field
Helmholtz superconduct magnet

1K ^{4}He evaporation refrigerator
Cooling power: about 1 W

Microwave Power
1W at 140 GHz to pump electrons

Insulated cryostat
85 L Liquid He resevoir
57 L Liquid N shield (300K BB shield)

E08-027 : Kinematics

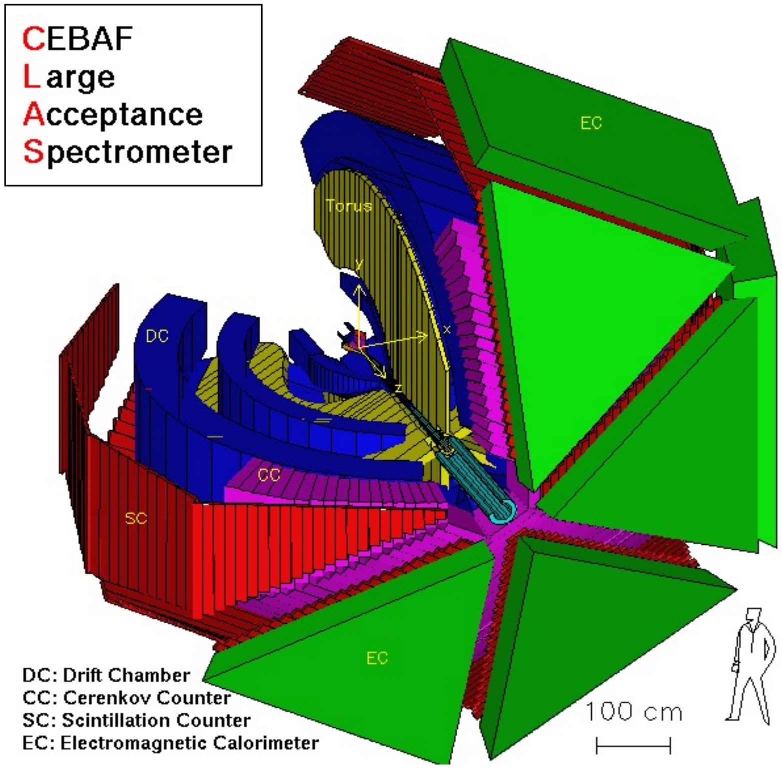


Target Field

5T: Large Q^2 data

2.5T: Low Q^2 data

EG4 experiment



Measurement of g_1 at low Q^2

Test of ChPT as $Q^2 \longrightarrow 0$

Measured Absolute XS differences

Goal : Extended GDH Sum Rule
Proton
Deuteron

See A. Deur's talk

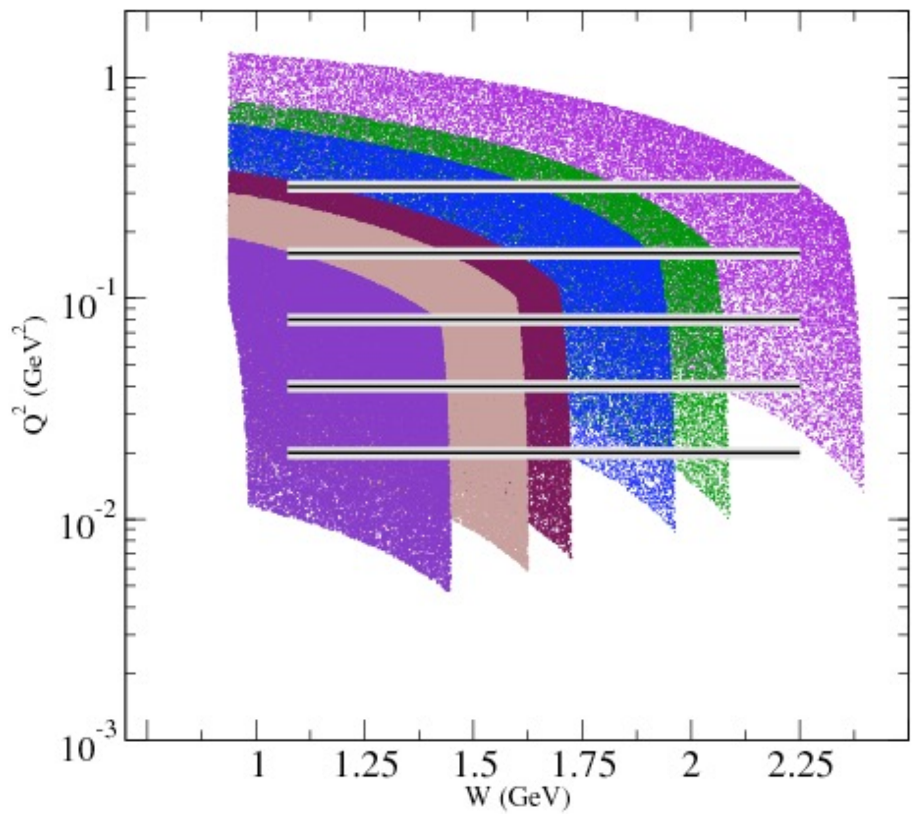
Spokespersons

NH_3: M. Battaglieri, A. Deur, R. De Vita, M. Ripani (Contact)
ND_3: A. Deur(Contact), G. Dodge, K. Slifer

PhD. Students
K. Adhikari, H. Kang, K. Kovacs

EG4 Kinematic Coverage

EG4: g1 (Proton and Deuteron)



$0.02 < Q^2 < 0.5$ GeV^2
Resonance Region

Incident Energies

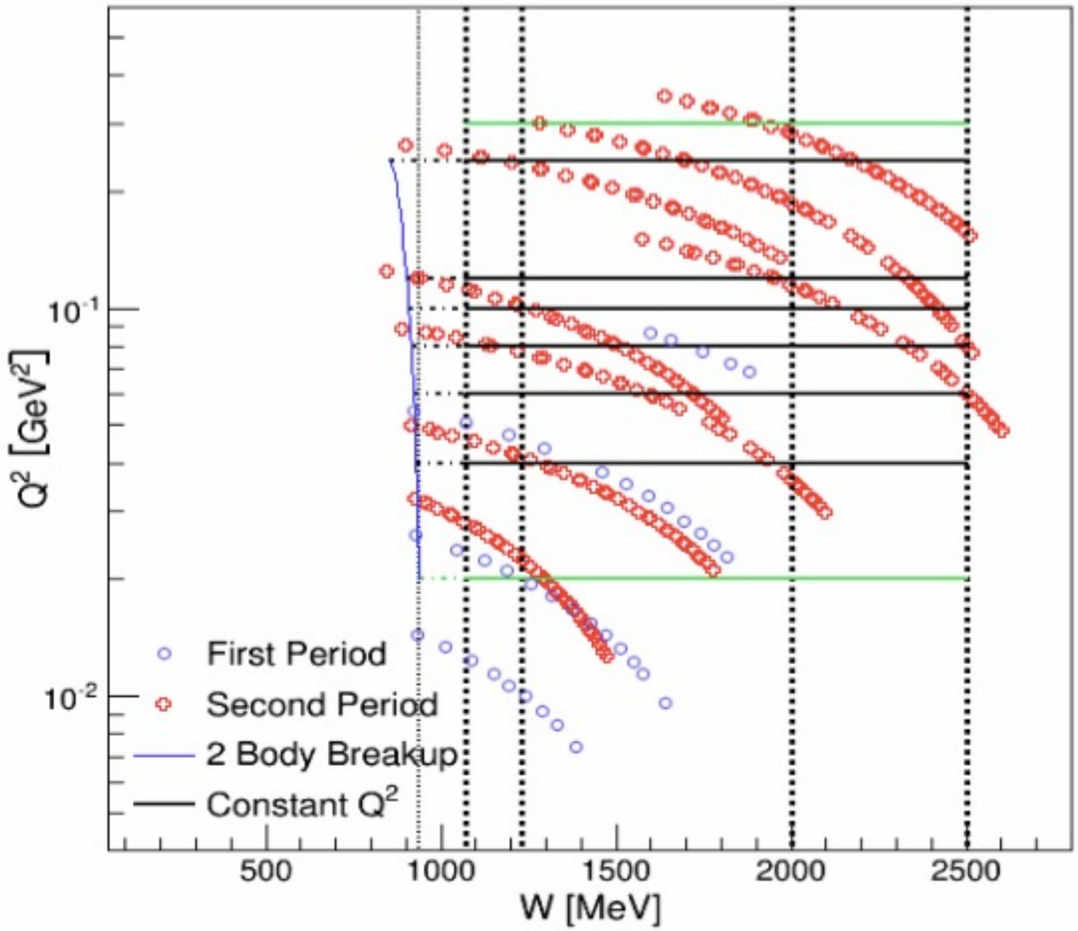
Proton Target
E=1.0, 1.3, 2.0, 2.3 and 3.0 GeV.

Deuteron Target
E=1.3 and 2.0 GeV

sagdh experiment

Spokesmen: J.-P. Chen, A. Deur, F. Garibaldi

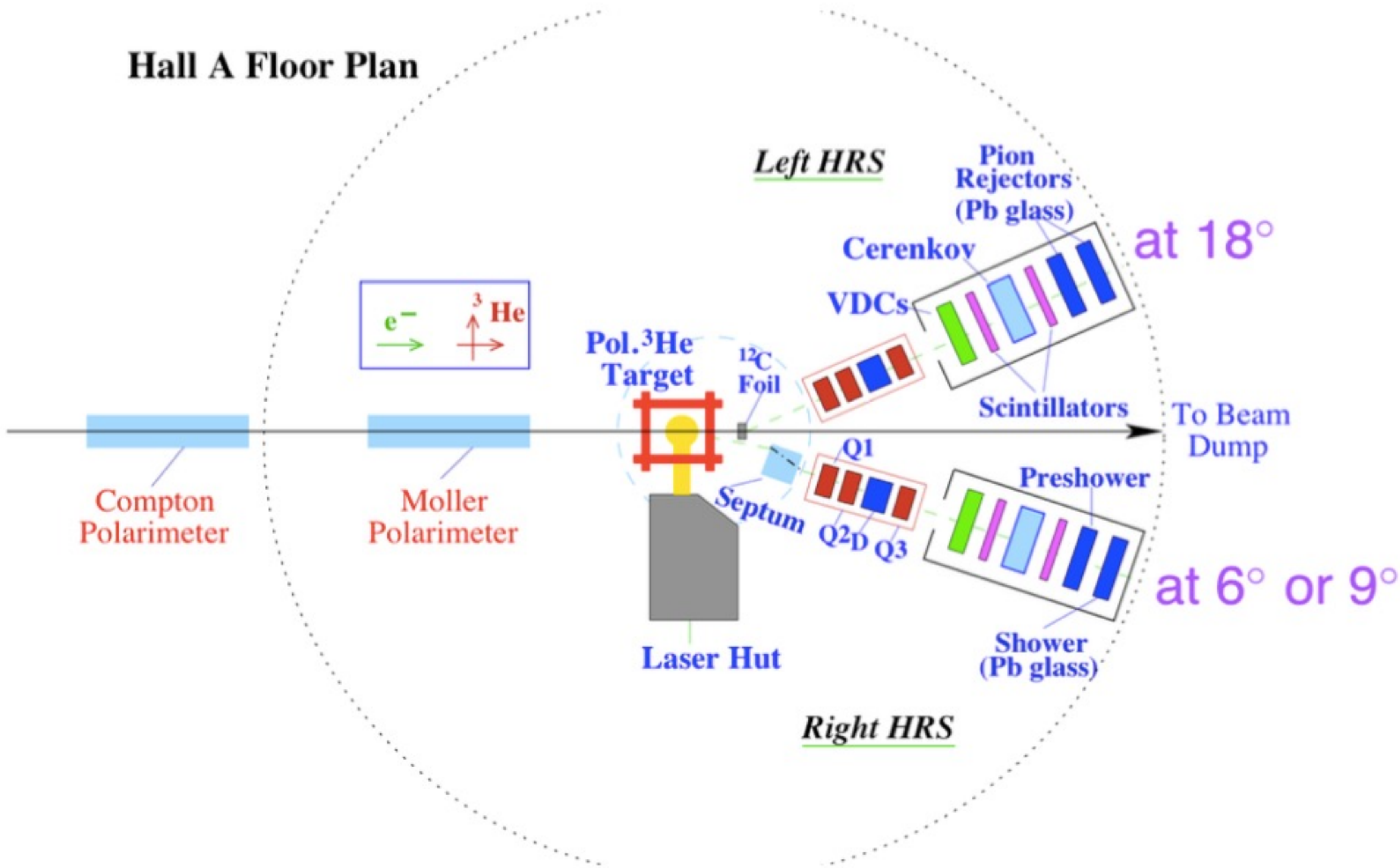
PhD Students: V. Solkosky, J. Singh, J. Yuan, C. Peng, N. Ton



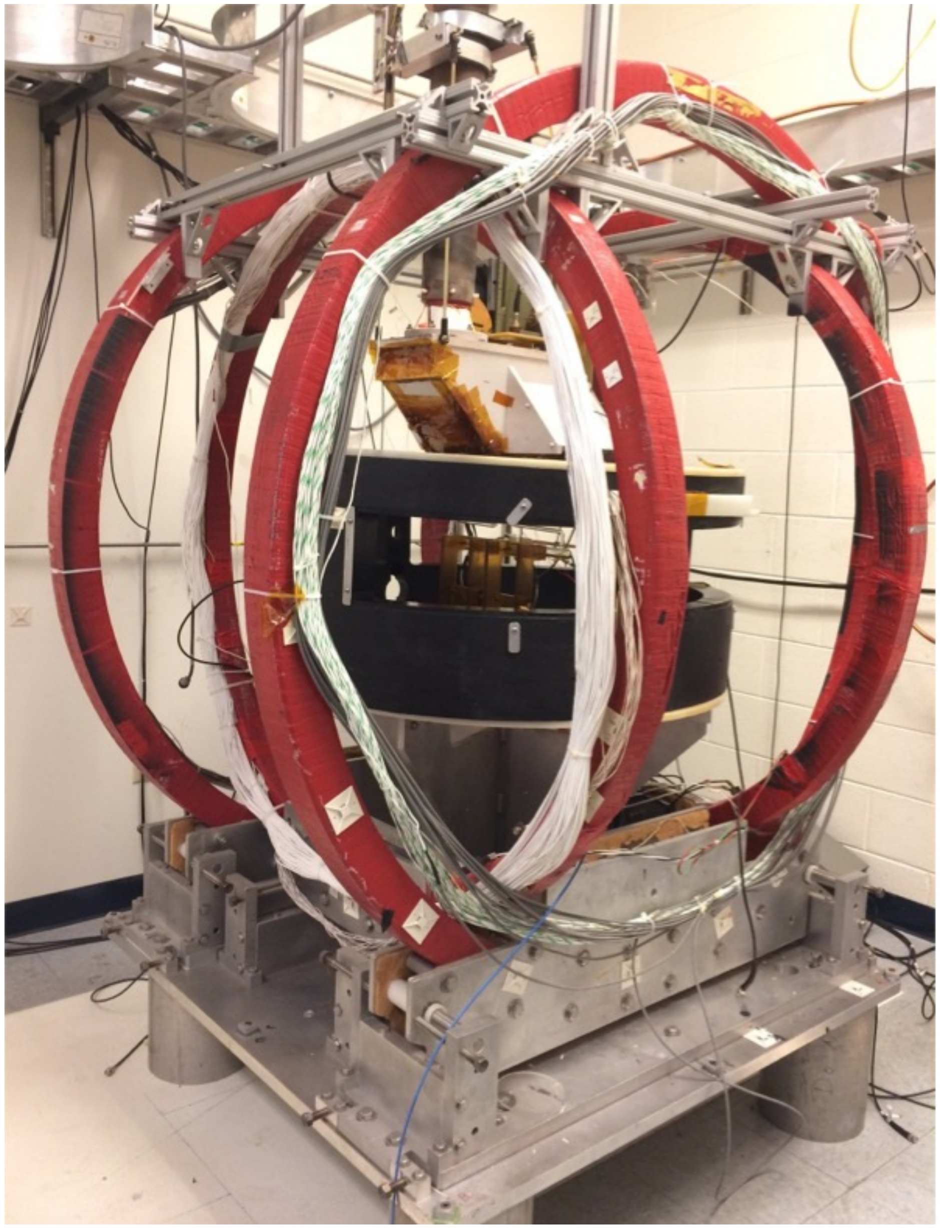
Target Cell	Angle	Beam Energy (MeV)
Penelope	6.10°	2134.2
Priapus	6.10°	2134.9
Priapus	6.10°	2844.8
Priapus	6.10°	4208.8
Priapus	9.03°	1147.3
Priapus	9.03°	2233.9
Priapus	9.03°	3318.8
Priapus	9.03°	3775.4
Priapus	9.03°	4404.2

See Chao Peng's talk

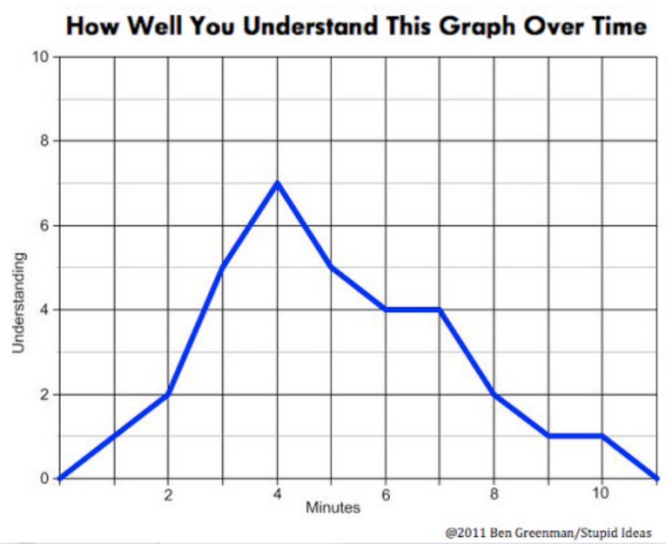
sagdh Floor Plan

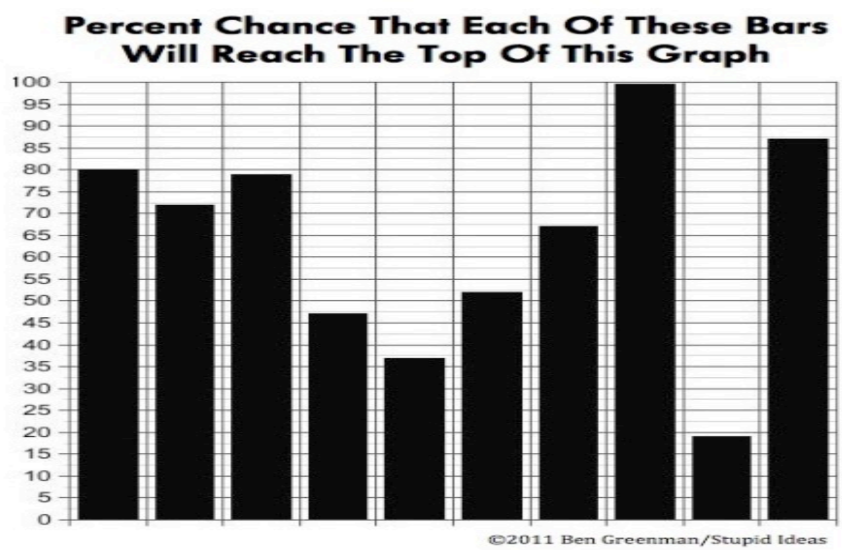


sagh experiment

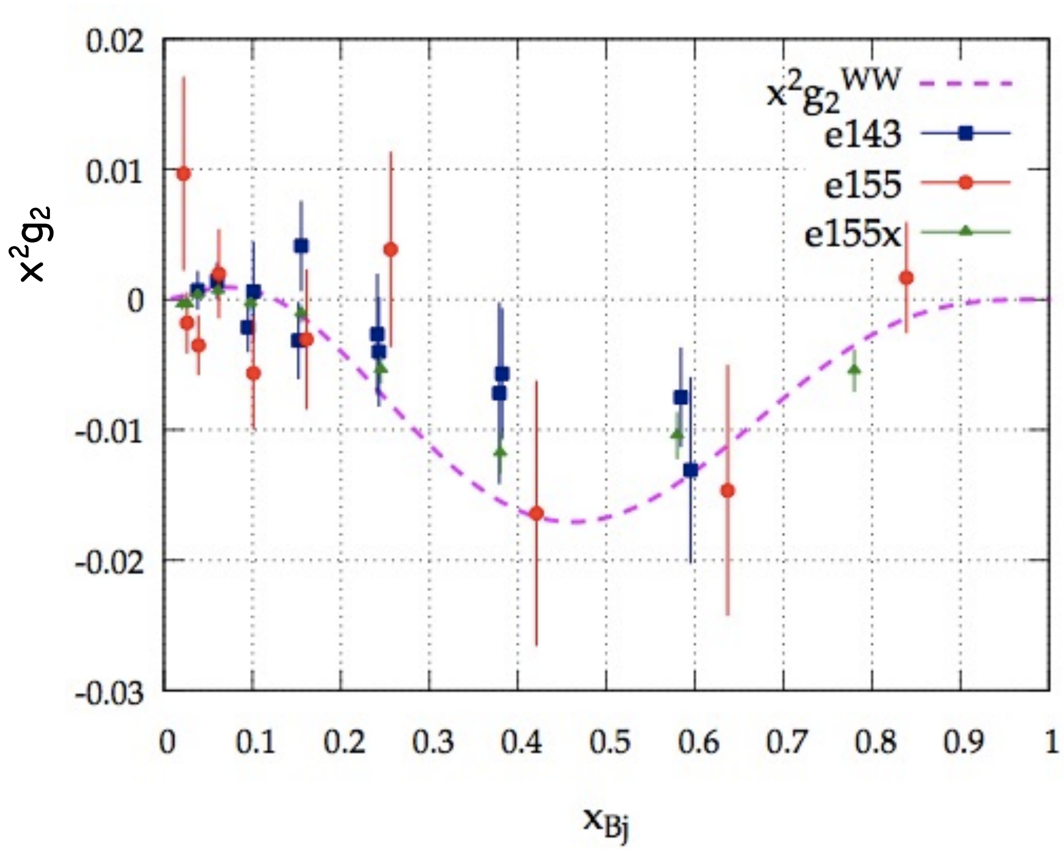


Experimental results





Proton g_2 data from SLAC

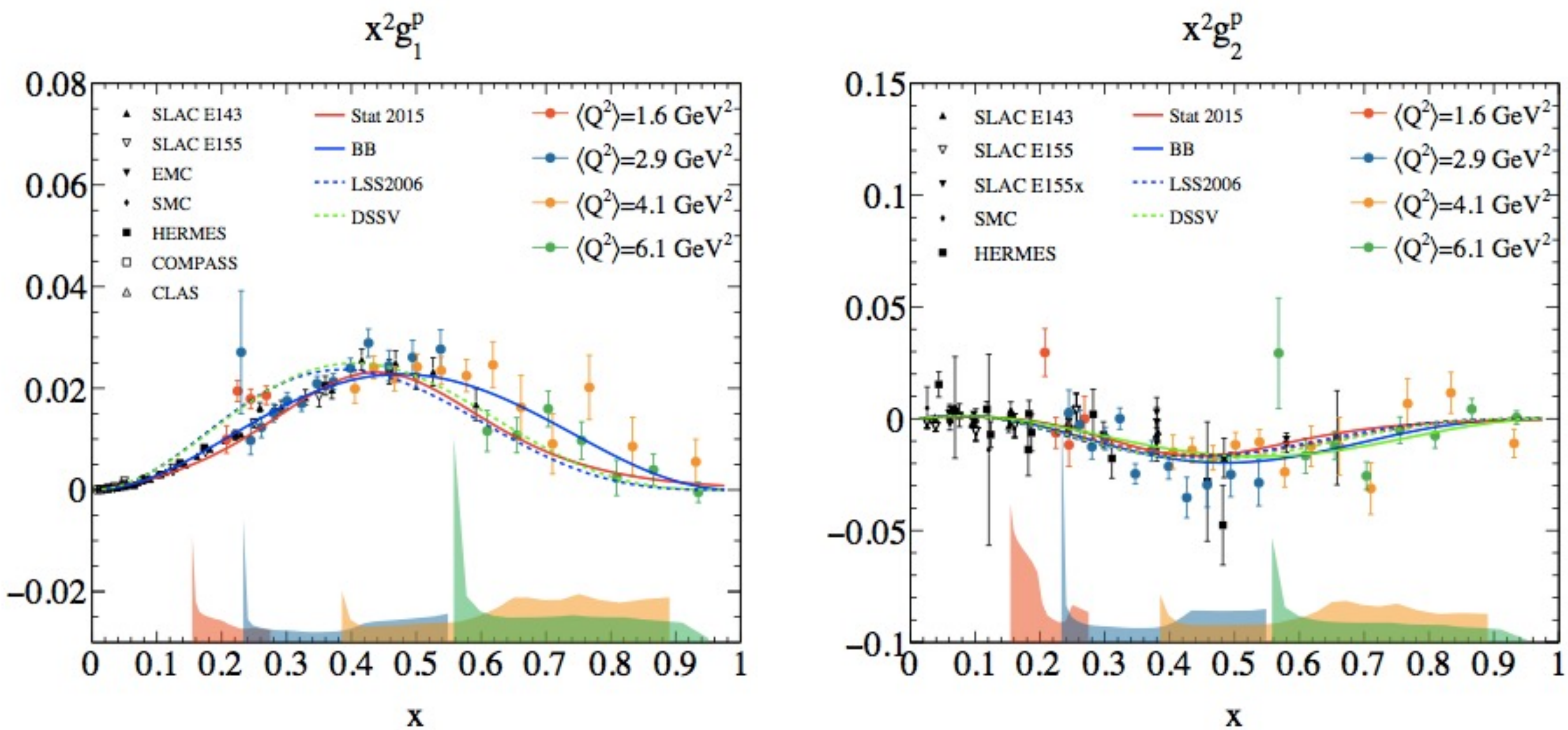


$Q^2 \approx 5\ GeV^2$

Leading Twist

Precision does not allow unambiguous HT extraction

SANE Proton g_1 and g_2 ($Q^2 \approx$ 2-6 GeV^2)

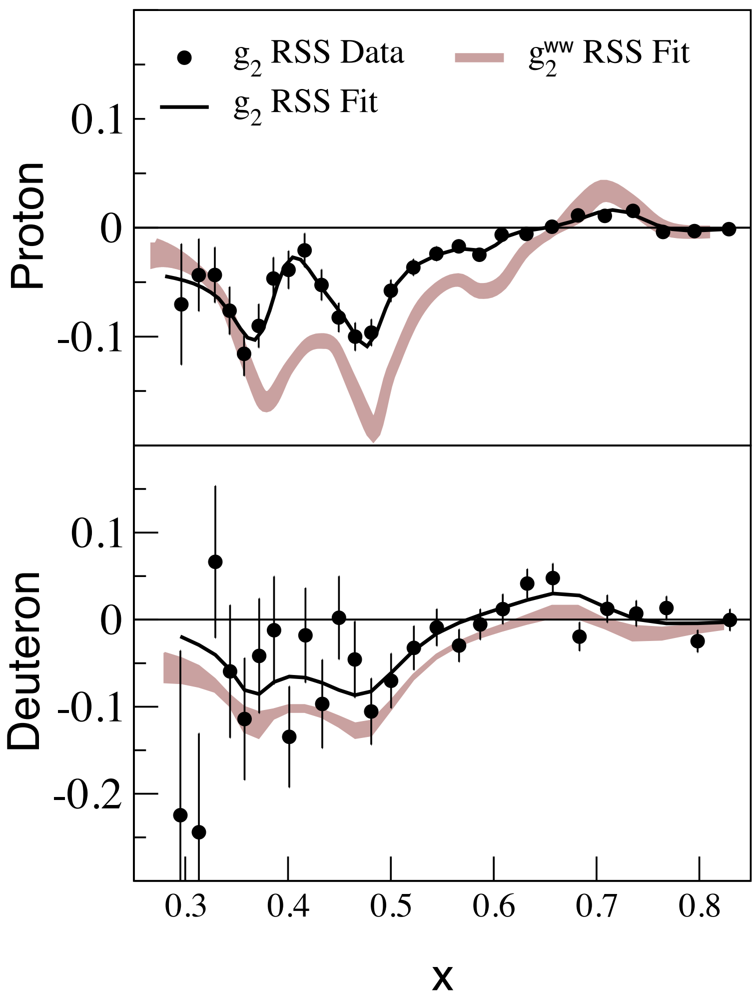


Models are showing g_2^{WW}.

W. Armstrong et al. arXiv:1805.08835

RSS Experiment

$Q^2 = 1.3\ \mathrm{GeV}^2$



$$\overline{\Delta}\Gamma_2 = -0.0006 \pm 0.0021$$ (proton)

consistent with zero
=> low x HT are small in proton.

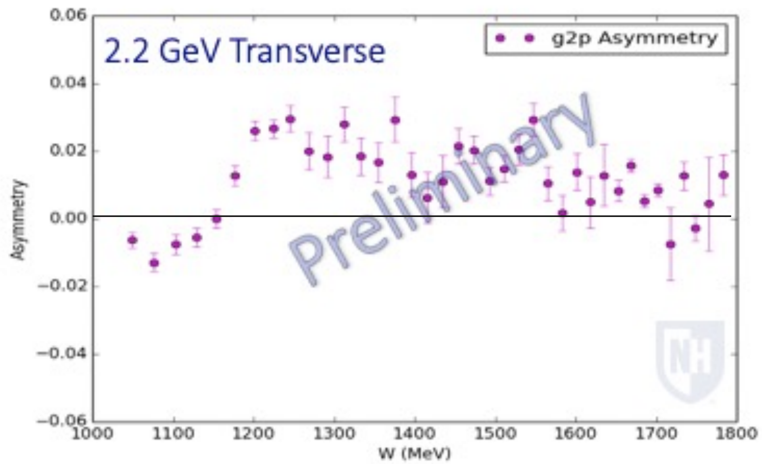
$$\overline{\Delta}\Gamma_2 = -0.0092 \pm 0.0035$$ (neutron)

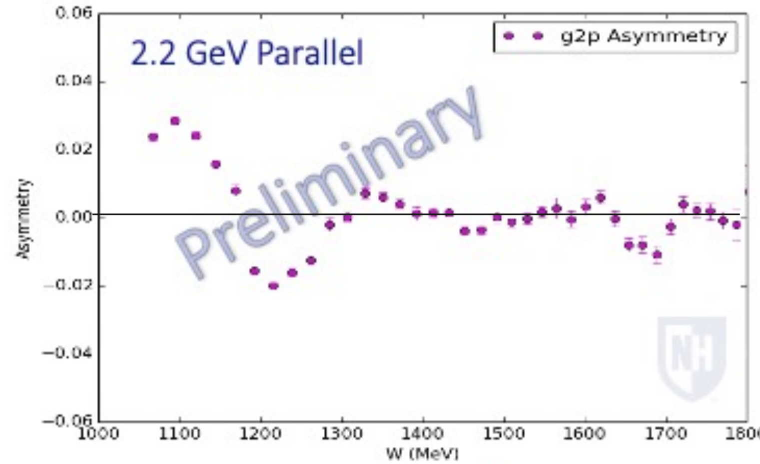
non-zero by 2.6σ

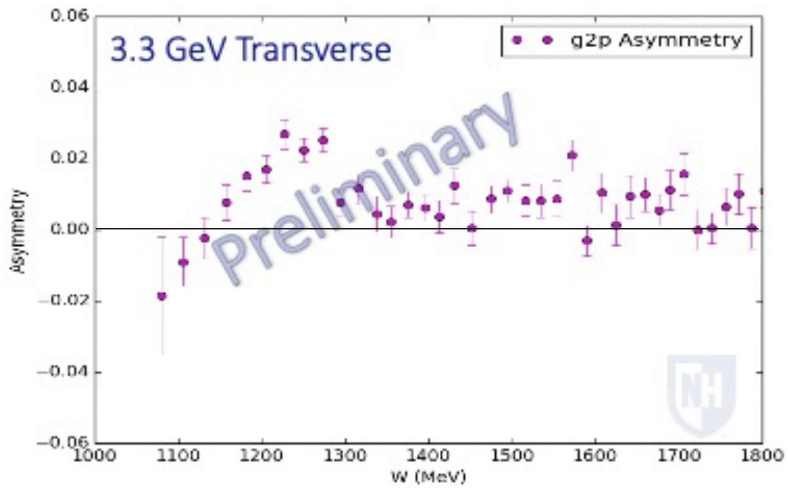
=>Significant HT at low x
needed to satisfy Neutron
BC sum rule.

K.Slifer., O. Rondon *et al.*
PRL 105, 101601 (2010)

E08-027 Asymmetries (5T)







Raw Counts:

$$Y_{\pm} = \frac{N_{\pm}}{LT_{\pm}Q_{\pm}}$$

Measured Asymmetries:

$$A^{\text{raw}} = \frac{Y_+ - Y_-}{Y_+ + Y_-},$$

$$A^{\text{exp}} = \frac{1}{f \cdot P_t \cdot P_b} A^{\text{raw}}$$

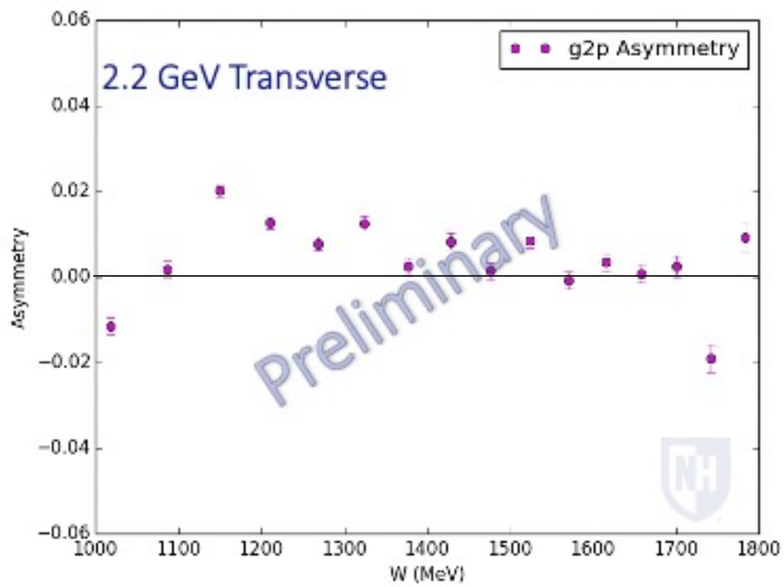
dilution factor

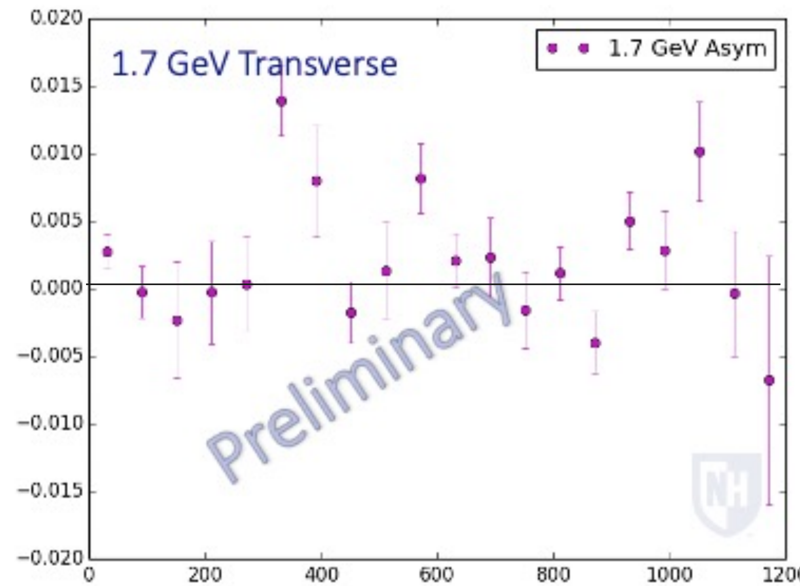
beam/target pol

Combine both HRS for best statistics!

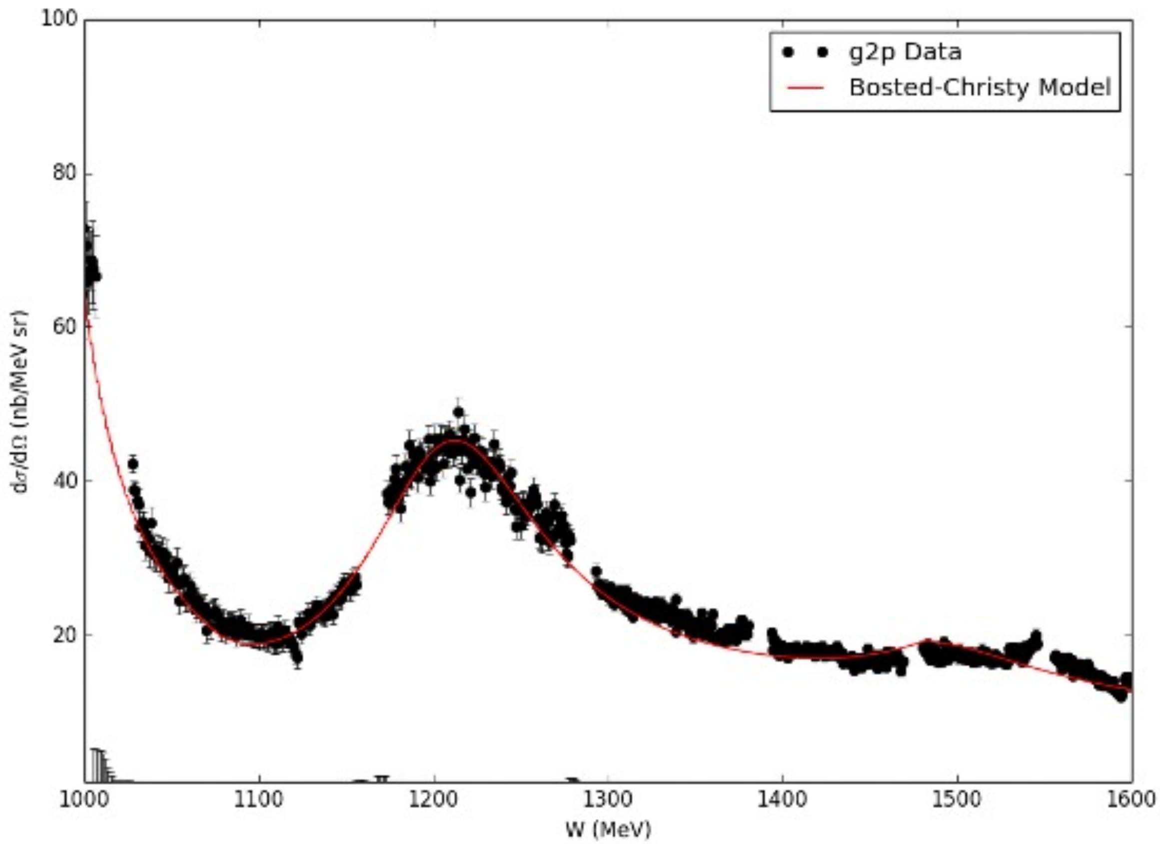
University of New Hampshire

E08-027 Asymmetries (2.5T)



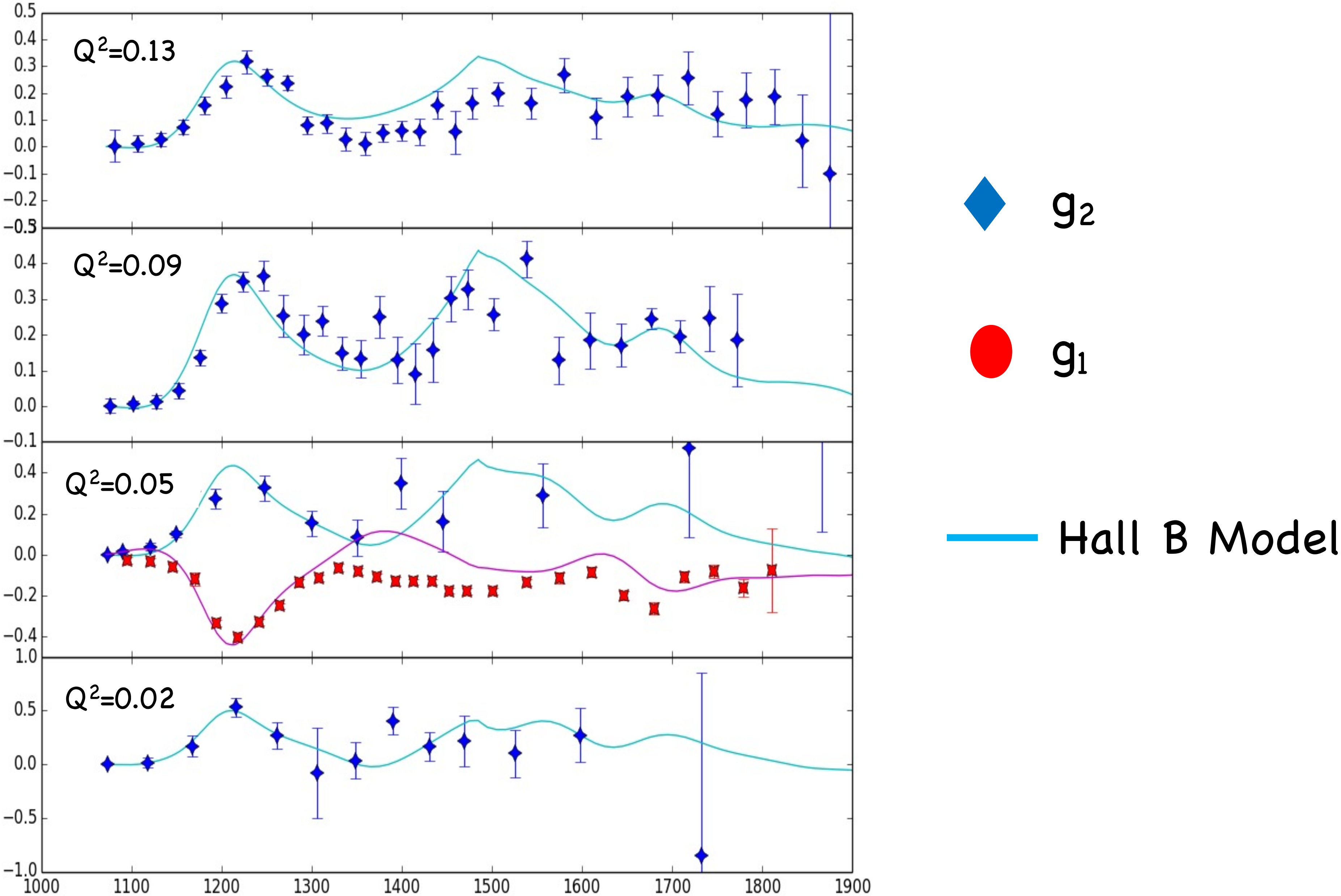


E08-027 Model Cross Section



- Direct comparison to g2p Longitudinal cross section yields very similar comparison results
- Necessary to scale model by 1.15 to obtain good agreement for both SLAC and g2p
- Systematic impact on the moments is very small

E08-027 Structure Functions



BC Sum Rule

$$\int_0^1 g_2(x,Q^2)dx = 0$$

H.Burkhardt and W.N. Cottingham
Annals Phys. **56** (1970) 453.

Assumptions:

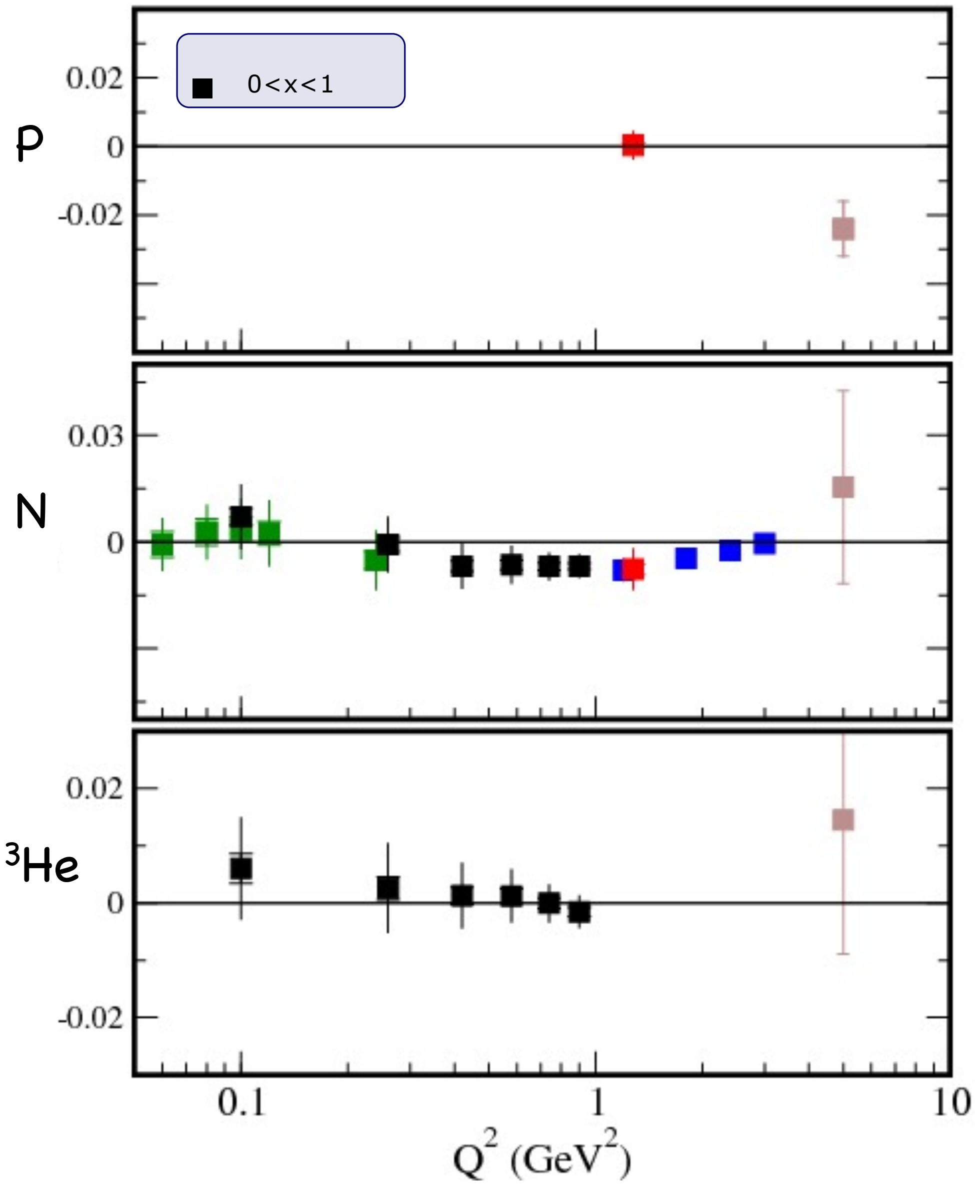
the virtual Compton scattering amplitude S_2 falls to zero faster than 1/x

g_2 does not behave as $\delta(x)$ at x=0.

Discussion of possible causes of violations

R.L. Jaffe Comm. Nucl. Part. Phys. 19, 239 (1990)
"If it holds for one Q^2 it holds for all"

BC Sum Rule (old data)

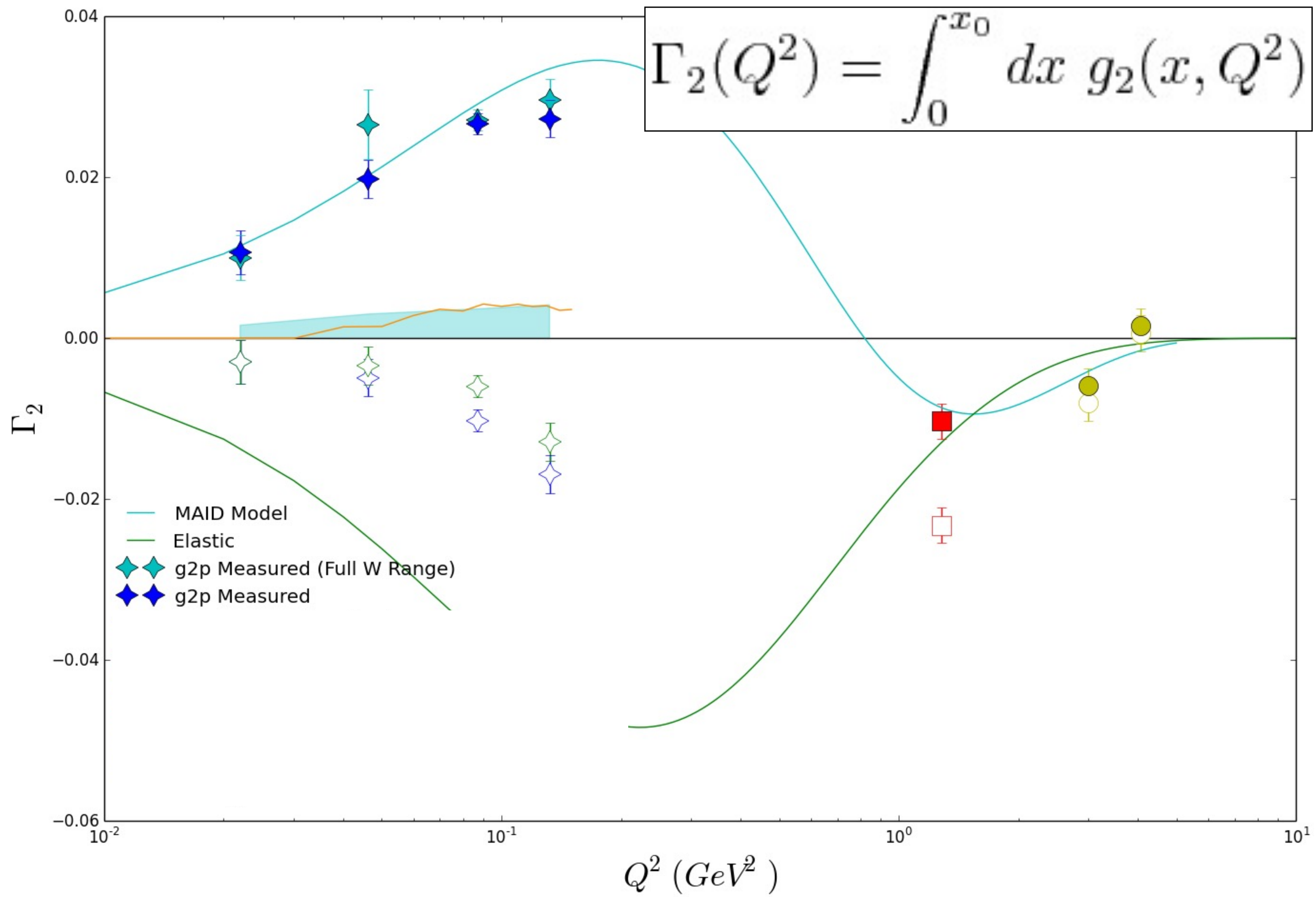


BC satisfied w/in errors for JLab Proton
2.8σ violation seen in SLAC data

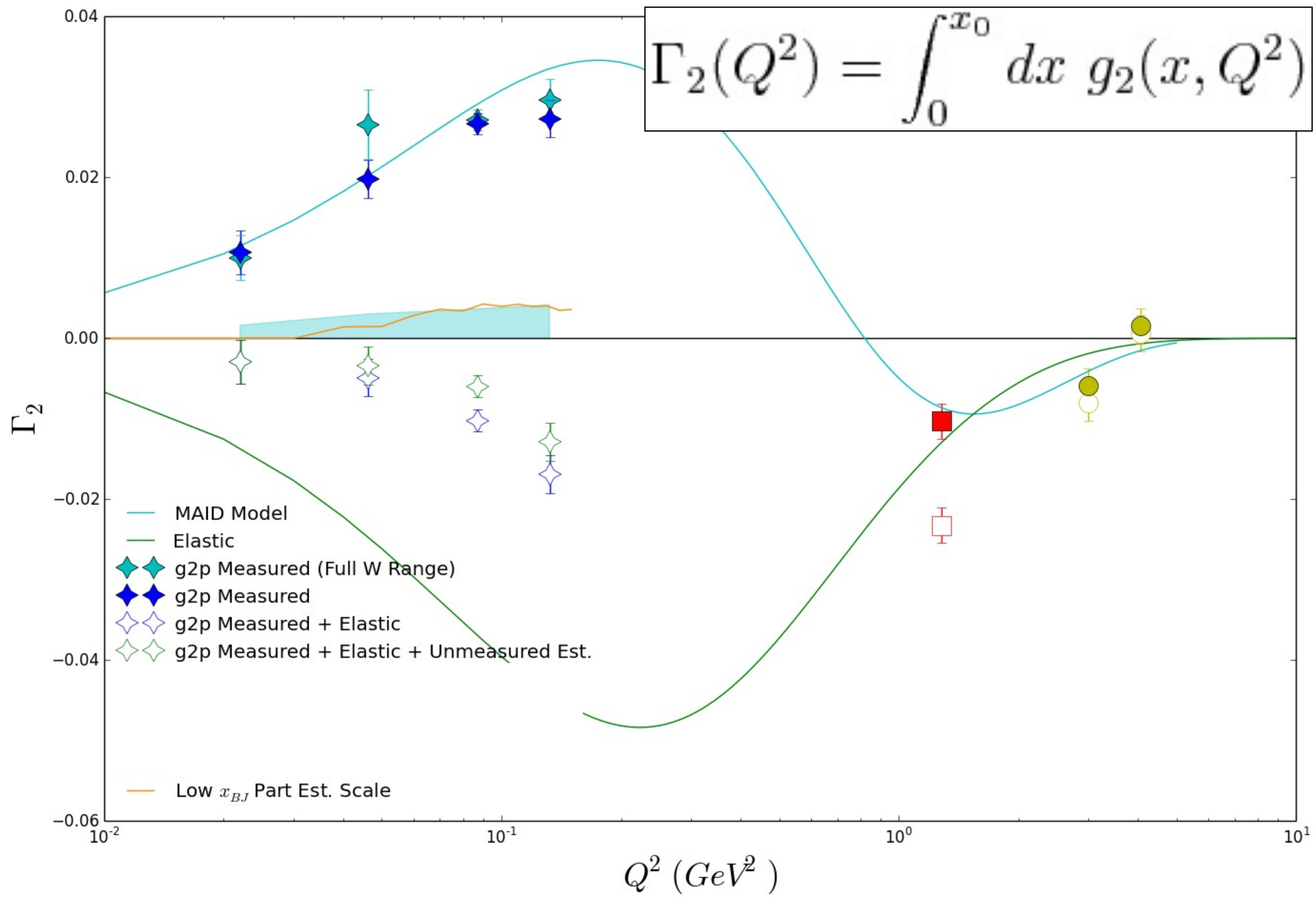
$$\Gamma_2(Q^2) = \int_0^{x_0} dx\ g_2(x, Q^2)$$

← Nuclear Sum Rule

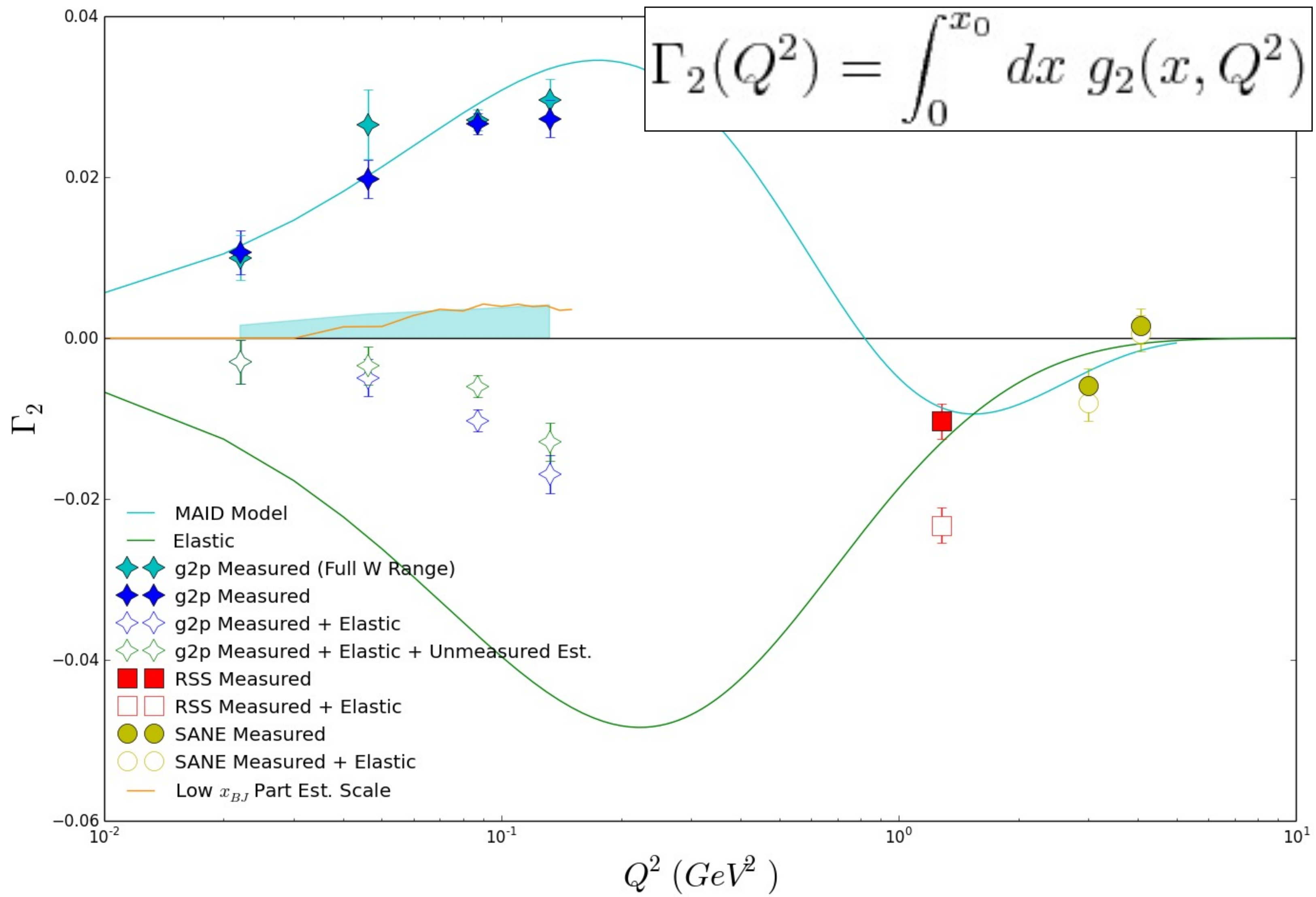
E08-027 Proton BC Integral



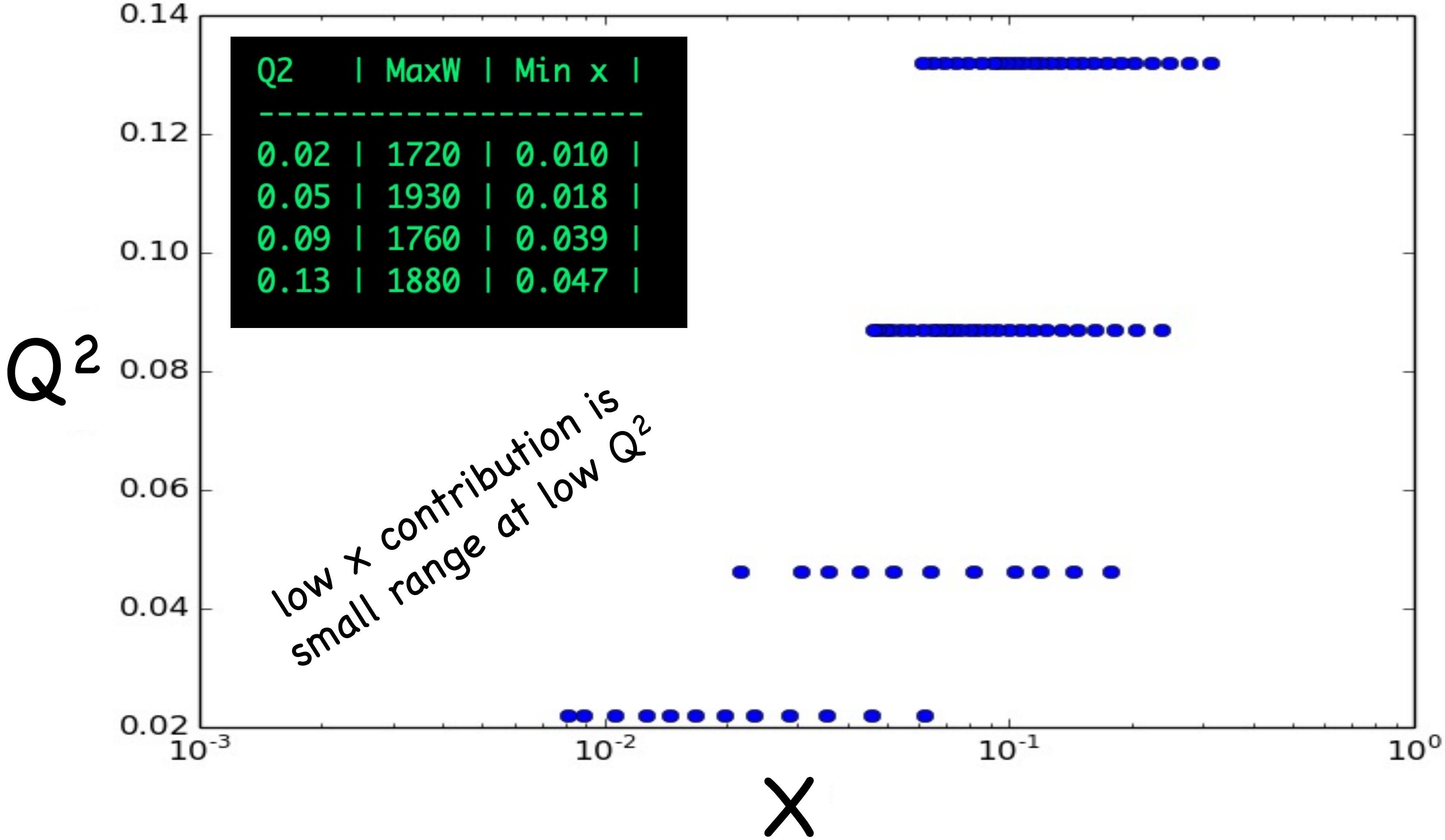
E08-027 Proton BC Integral



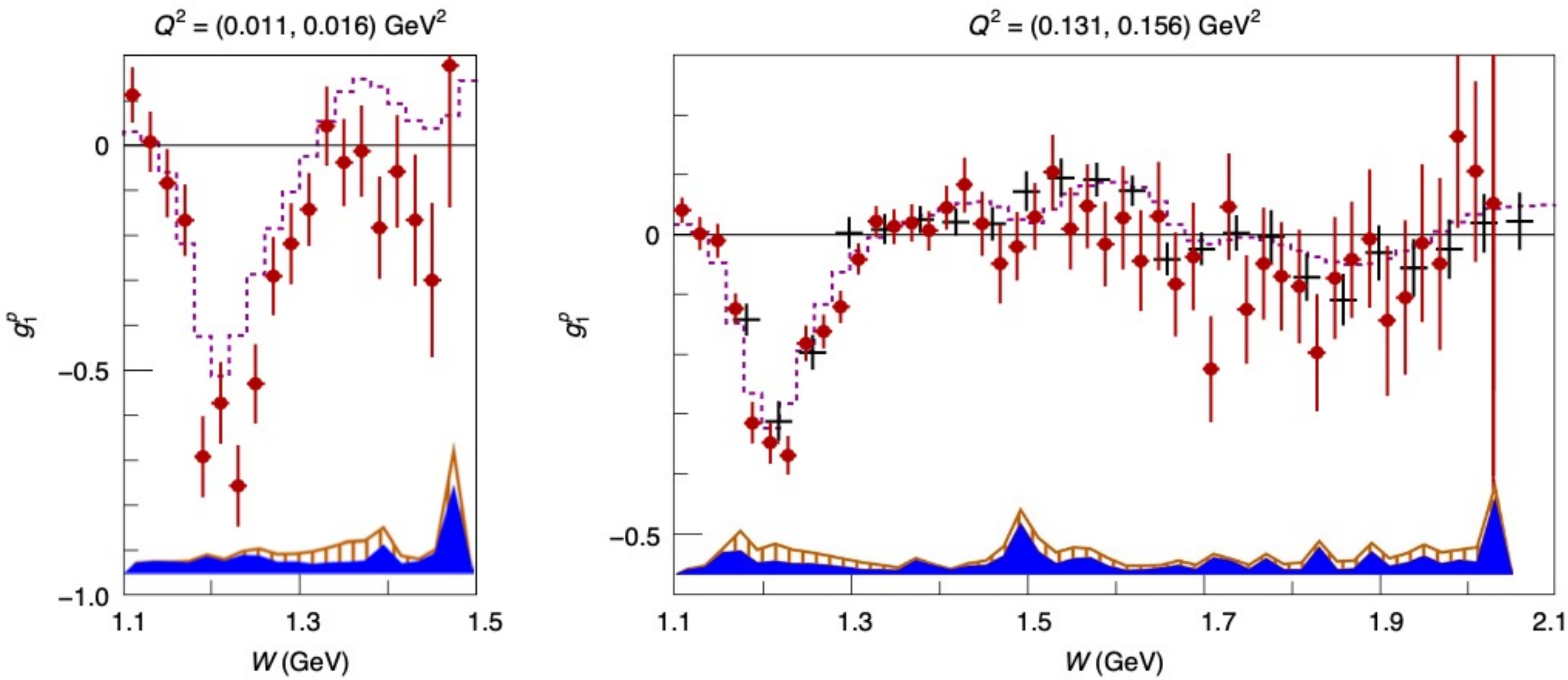
E08-027 Proton BC Integral



E08-027 Kinematic Coverage



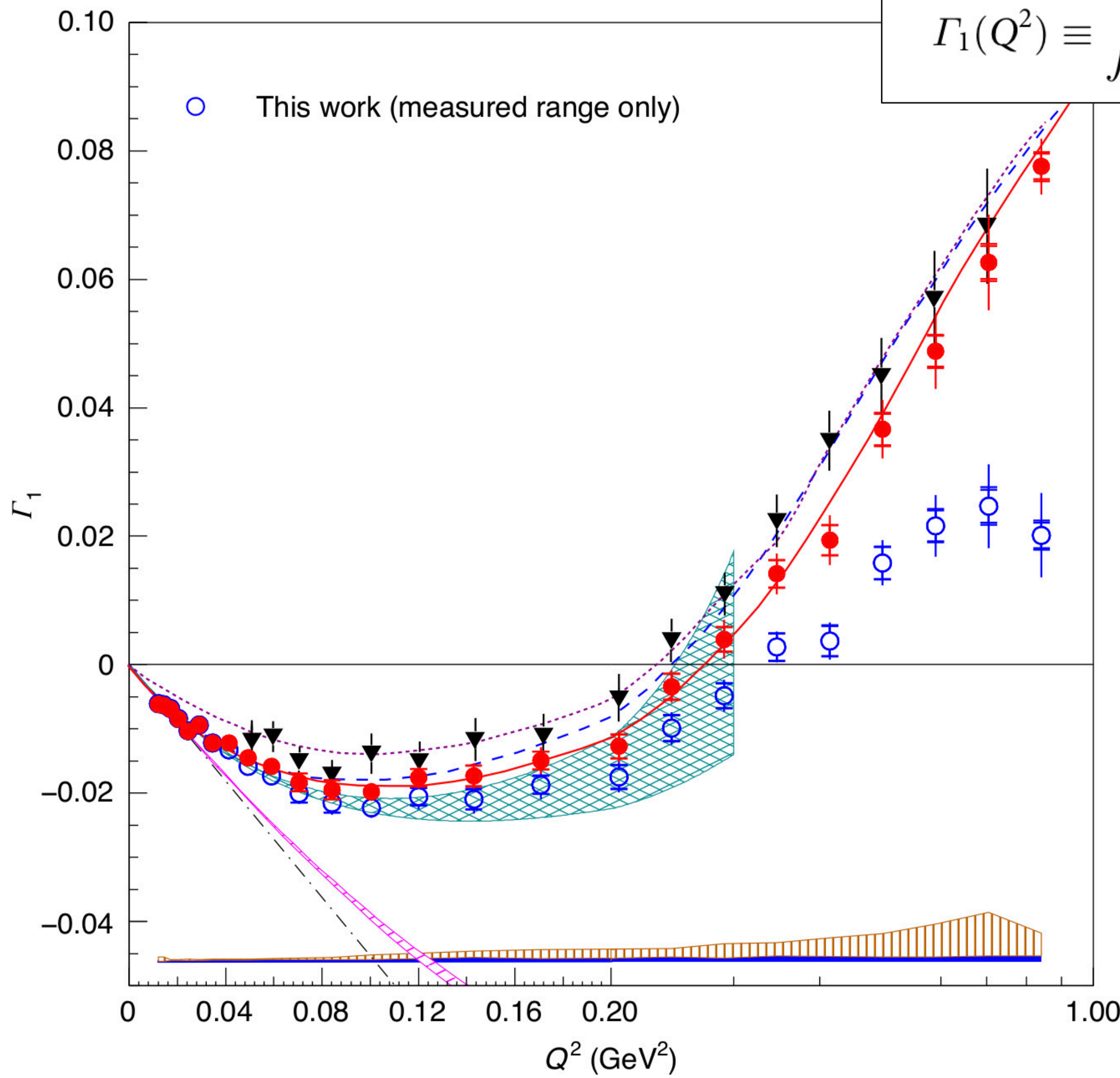
Sample of EG4 Proton g_1 data



Large Data Set covering $0.12 < Q^2 < 1.0$ GeV^2

EG4 Proton 1st Moment

$$\Gamma_1(Q^2) \equiv \int_0^{x_0} g_1(x, Q^2)\,\mathrm{d}x = \frac{Q^2}{2M^2} I_1(Q^2)$$



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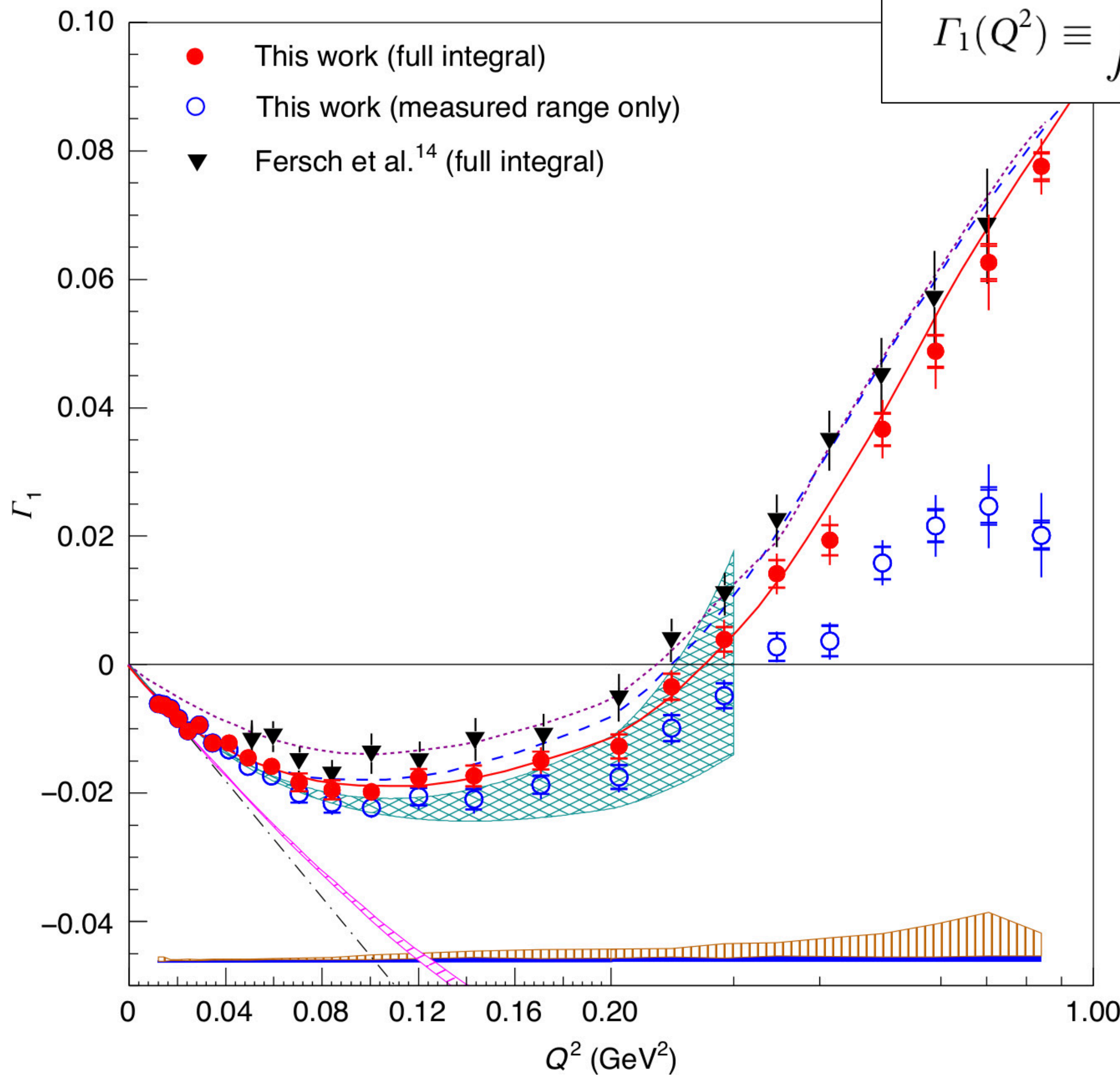

EG4 Proton 1st Moment

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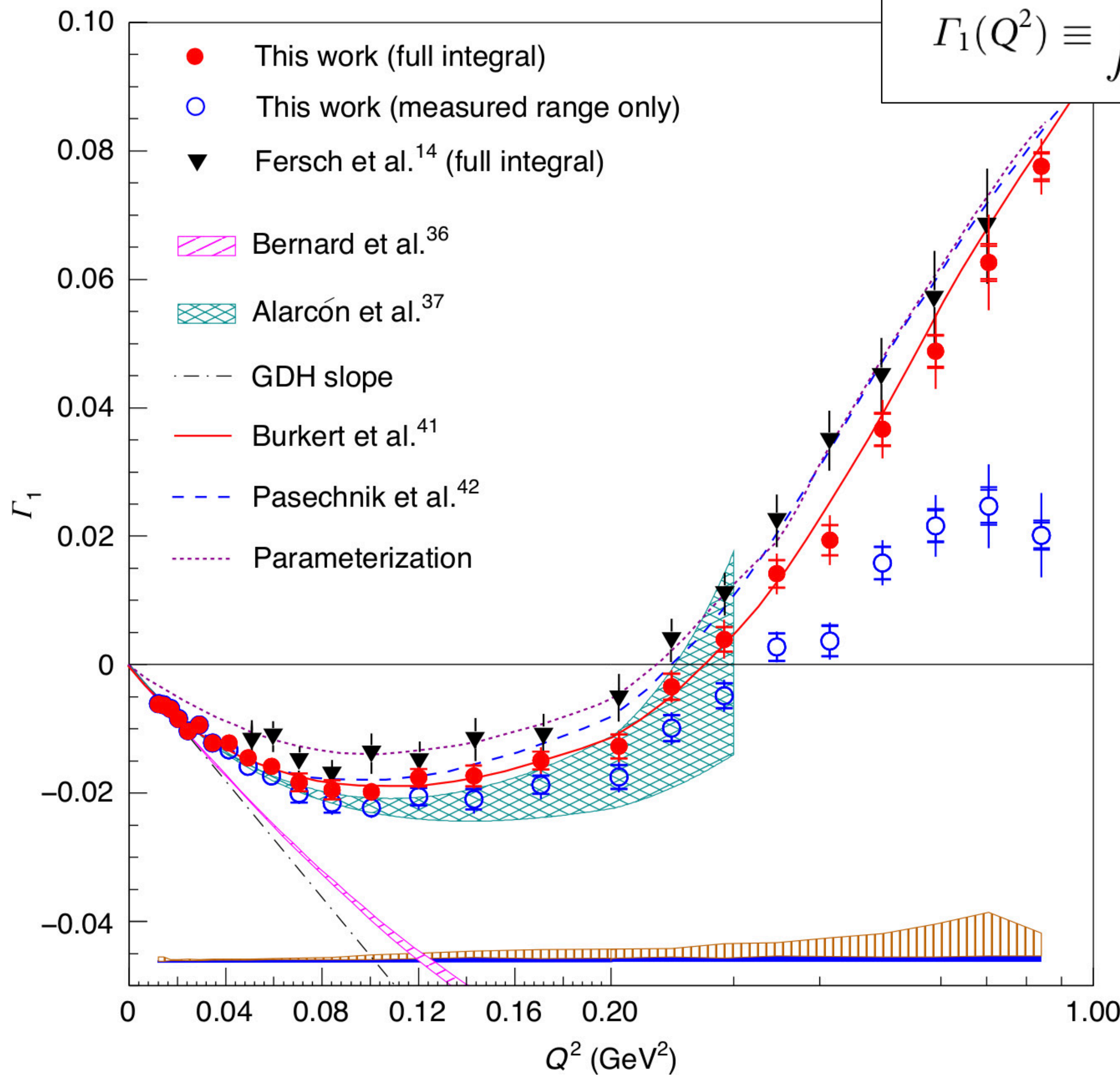


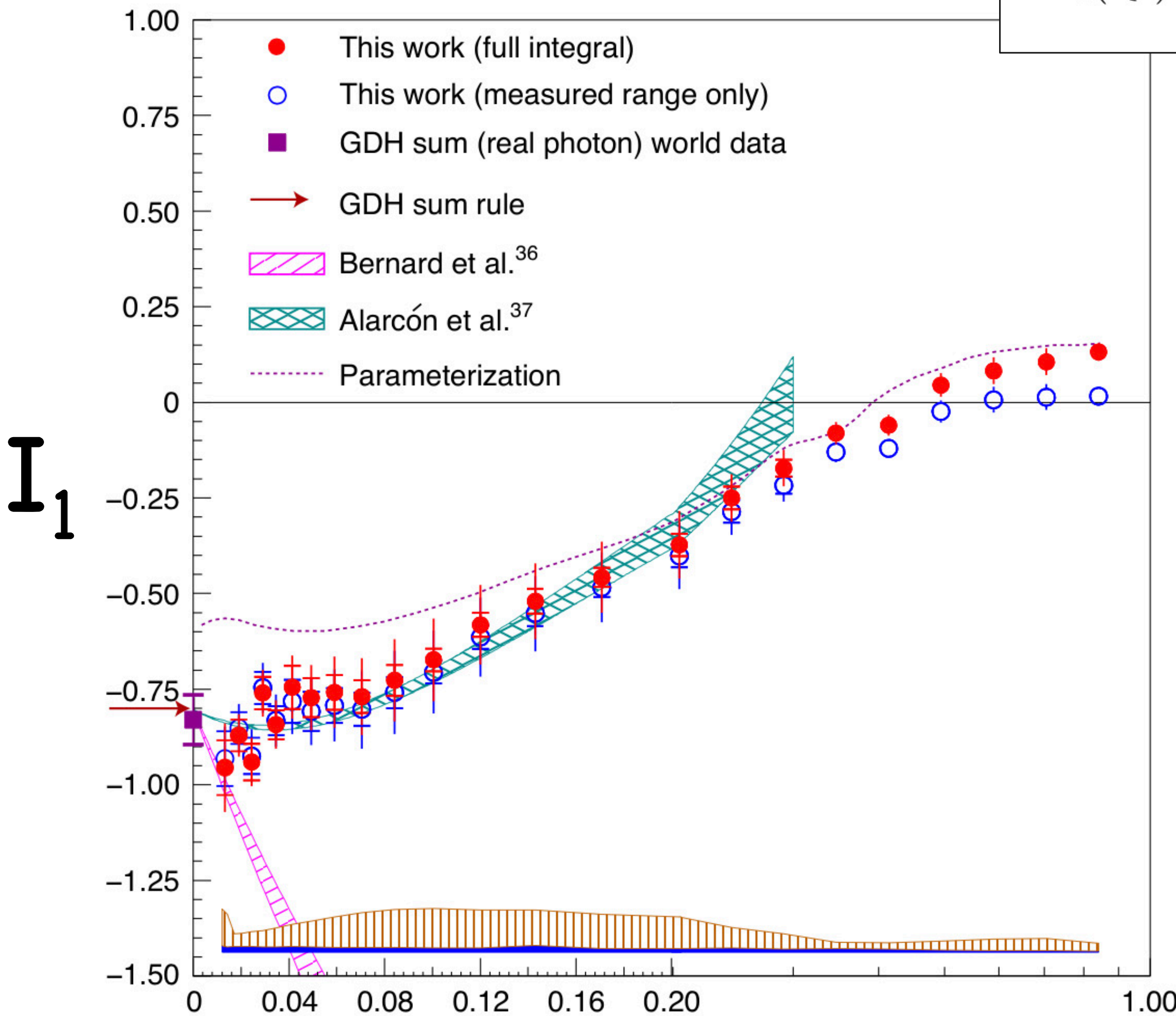
Agreement with chpt

Bernard et al $Q^2 < 0.024$

Alarcon et al $Q^2 < 0.3$

EG4 Proton I_1 Moment

$$\Gamma_1(Q^2) \equiv \int_0^{x_0} g_1(x, Q^2)\,dx = \frac{Q^2}{2M^2} I_1(Q^2)$$

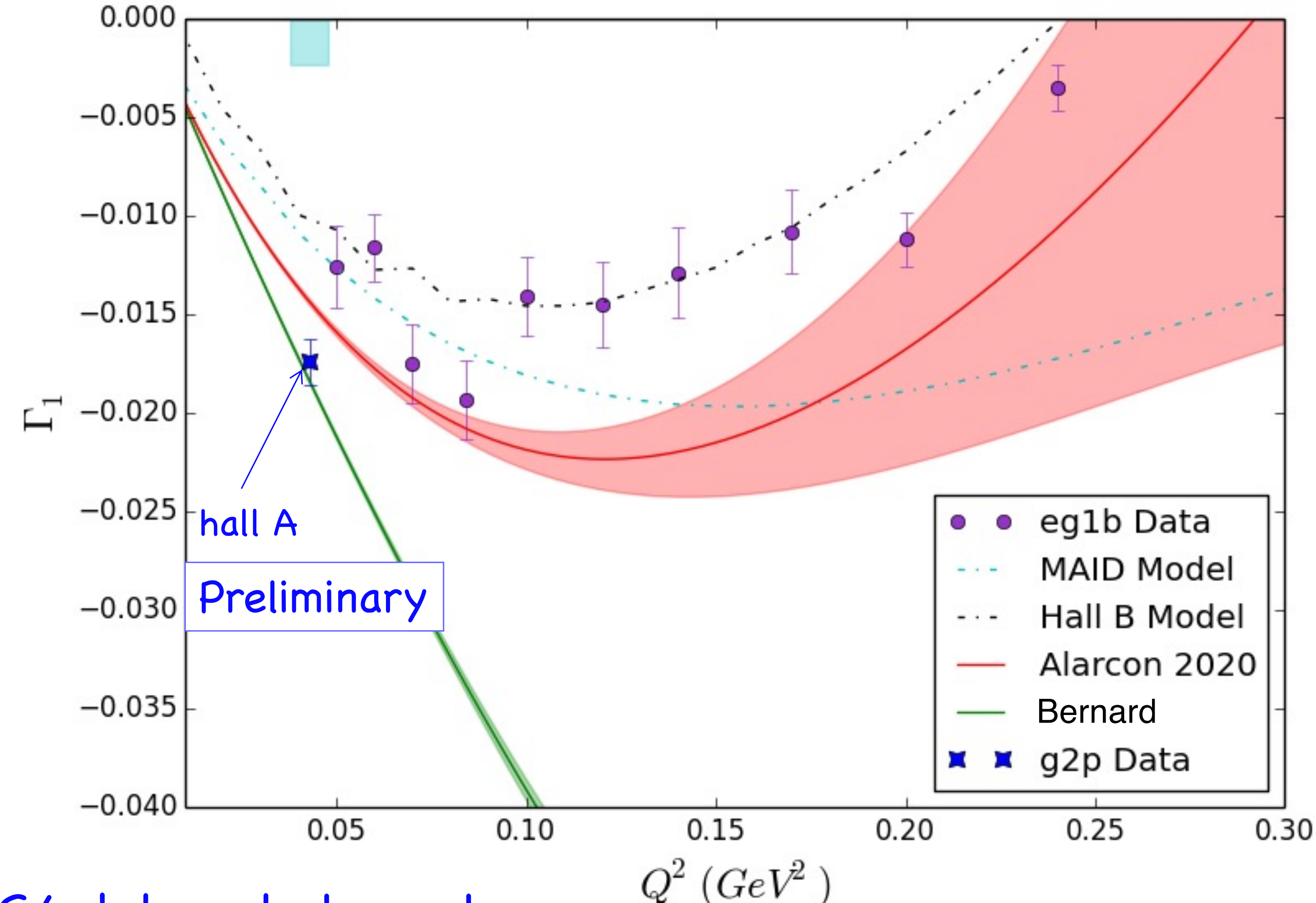


Agreement with chpt

Bernard et al $Q^2 < 0.014$

Alarcon et al $Q^2 < 0.3$

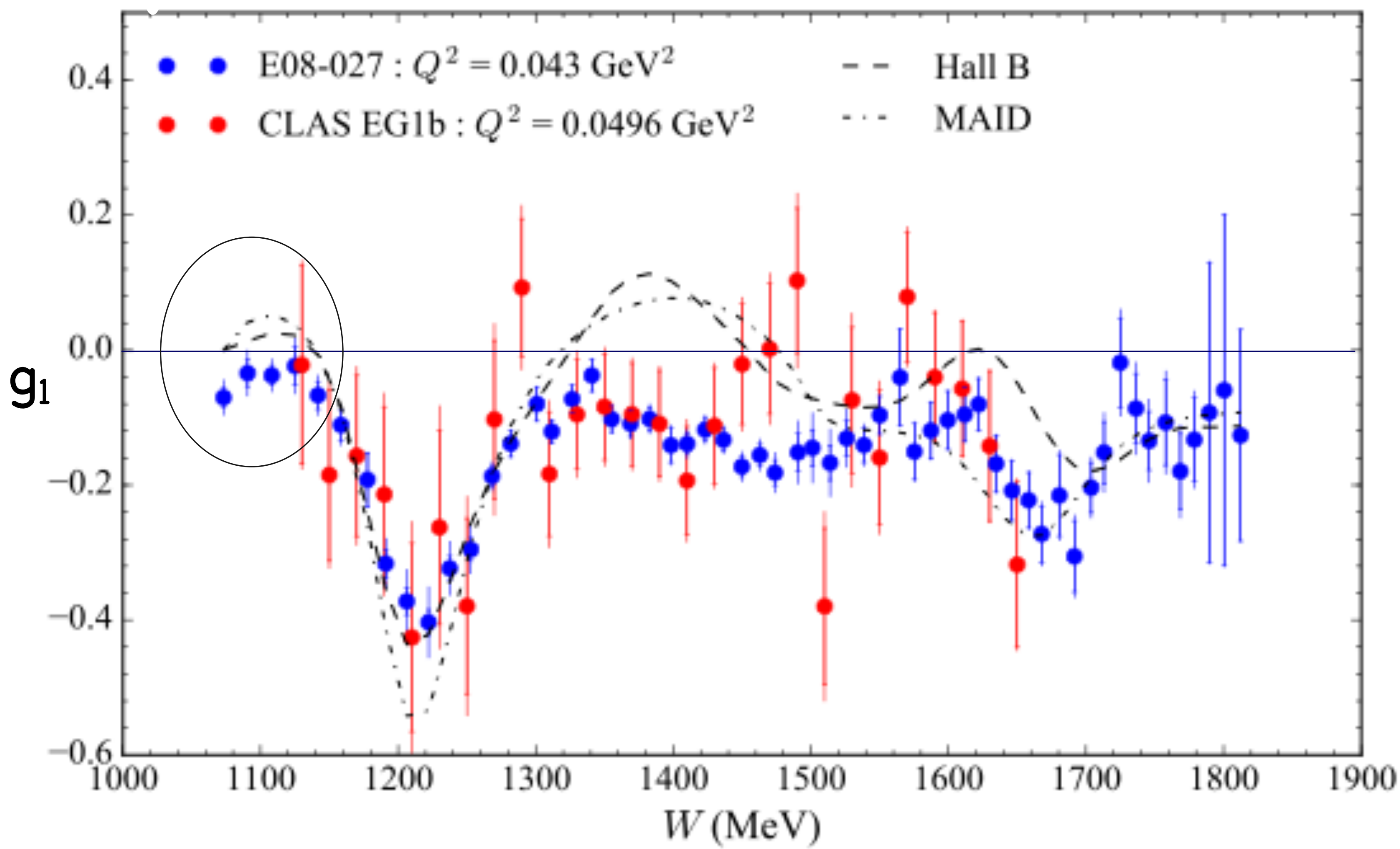
E08-027 Proton 1st Moment



EG4 data not shown here

courtesy D. Ruth, UNH

E08-027 compared to CLAS DATA



Some small disagreement between Hall A E08-027 and Hall B EG1B data at low nu

Spin Polarizabilities

$$\gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 g_{TT}(x, Q^2),$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 \Big[g_1(x, Q^2) + g_2(x, Q^2)\Big]$$

$$g_{TT} = g_1 - (4M_N^2 x^2/Q^2) g_2$$

Good Test of ChPT.

Chpt respects all symmetries of QCD but its Lagrangian is constructed from hadron degrees of freedom

Heavy Baryon χPT : Treats the Baryon as a heavy static particle
Kao, Vanderhaeghen, et al

Relativistic Baryon : large momentum effects are absorbed in the low energy consts
Δ(1232) included explicitly. Other Resonances are included
systematically through additional low energy constants

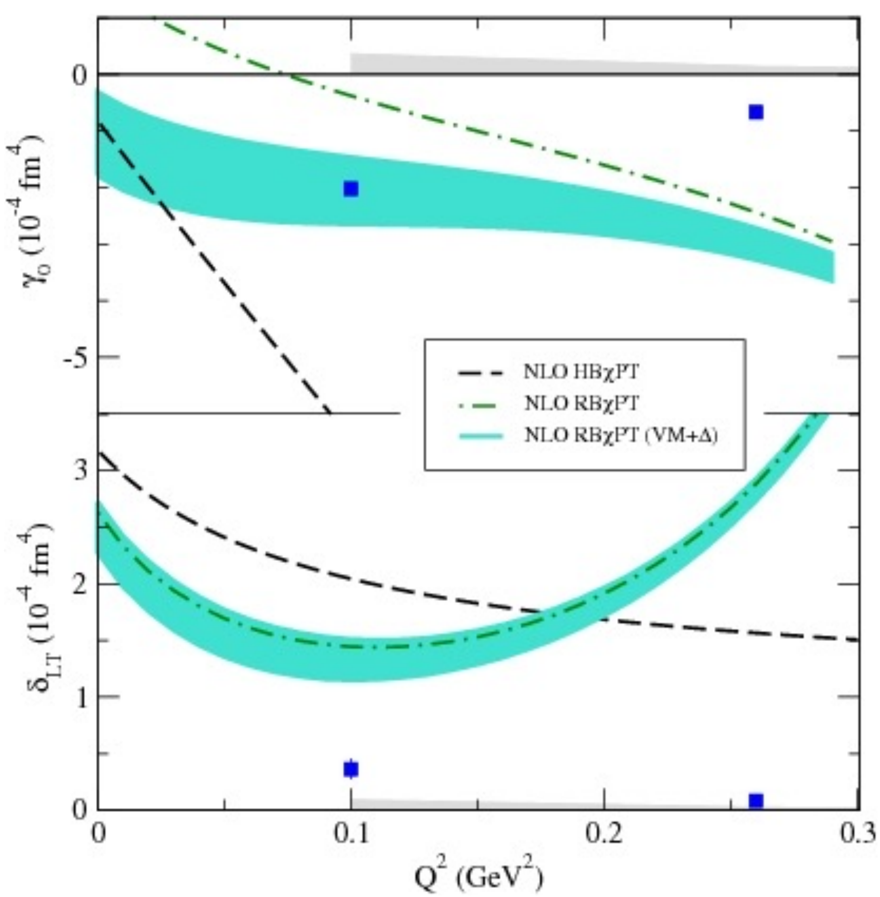
1: Meissner, Bernard, Krebs, Epelbaum
2: Lensky, Alarcon, Pascalutsa

Old calcs

δ_{LT} E94010 (old data)

Neutron

PRL 93: 152301 (2004)



$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 \left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

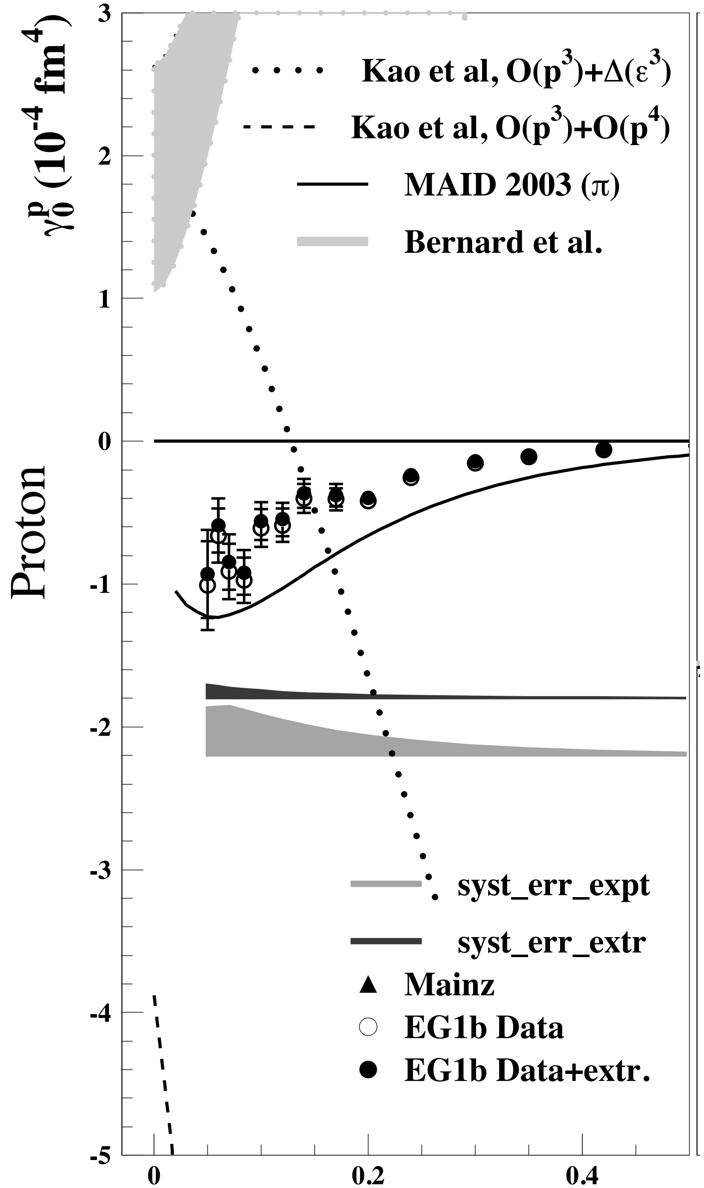
$$\delta_{LT} = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 \left[g_1 + g_2\right]$$

Dramatic Discrepency
with χPT

Heavy Baryon ChPT Calculation
Kao, Spitzenberg, Vanderhaeghen
PRD 67:016001(2003)

Infrared Relativistic Baryon ChPT
Bernard, Hemmert, Meissner
PRD 67:076008(2003)

Proton γ_0 (old data)



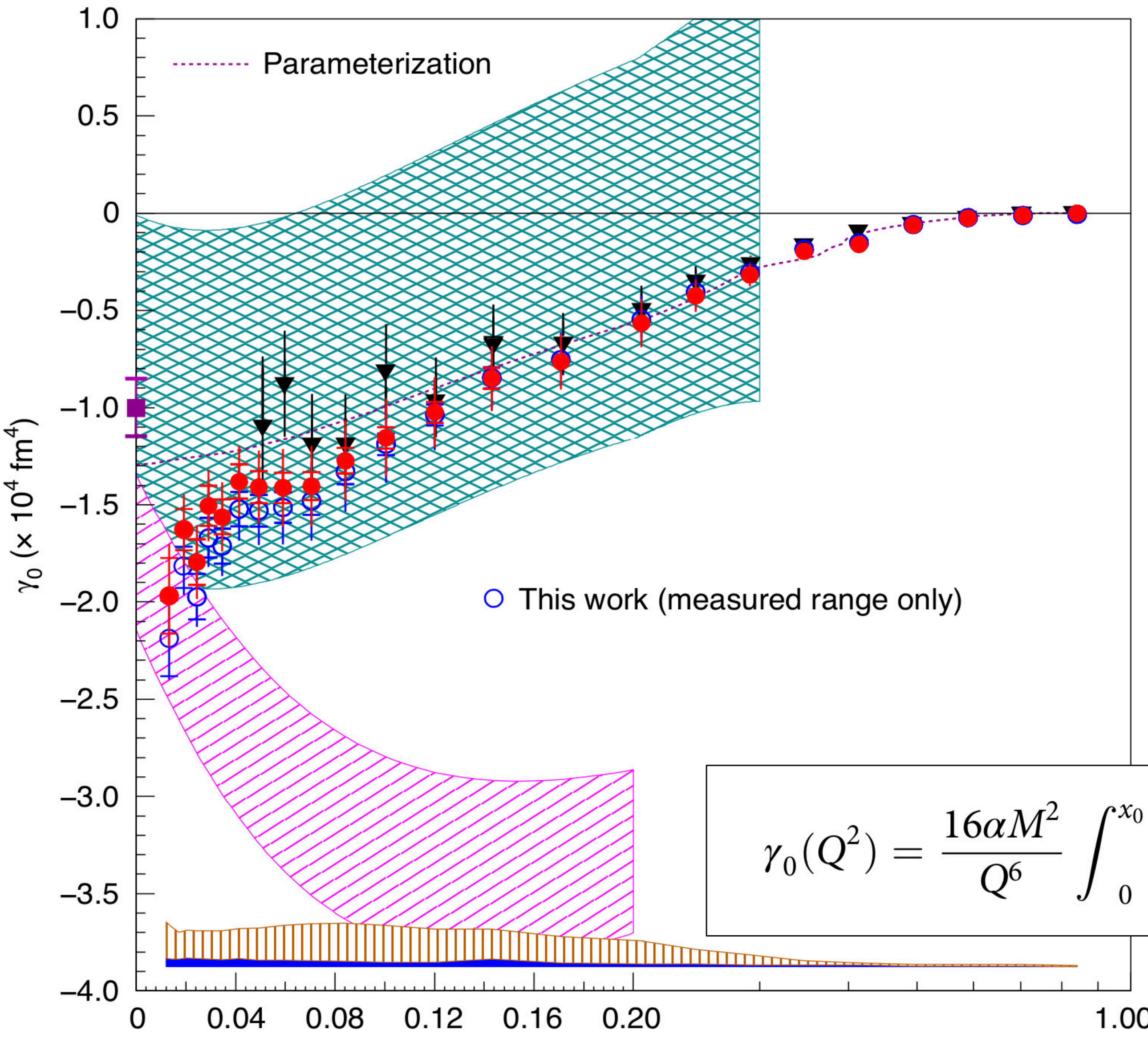
$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 \left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

Older Calcs also failed for proton γ_0

PLB 672 12, 2009

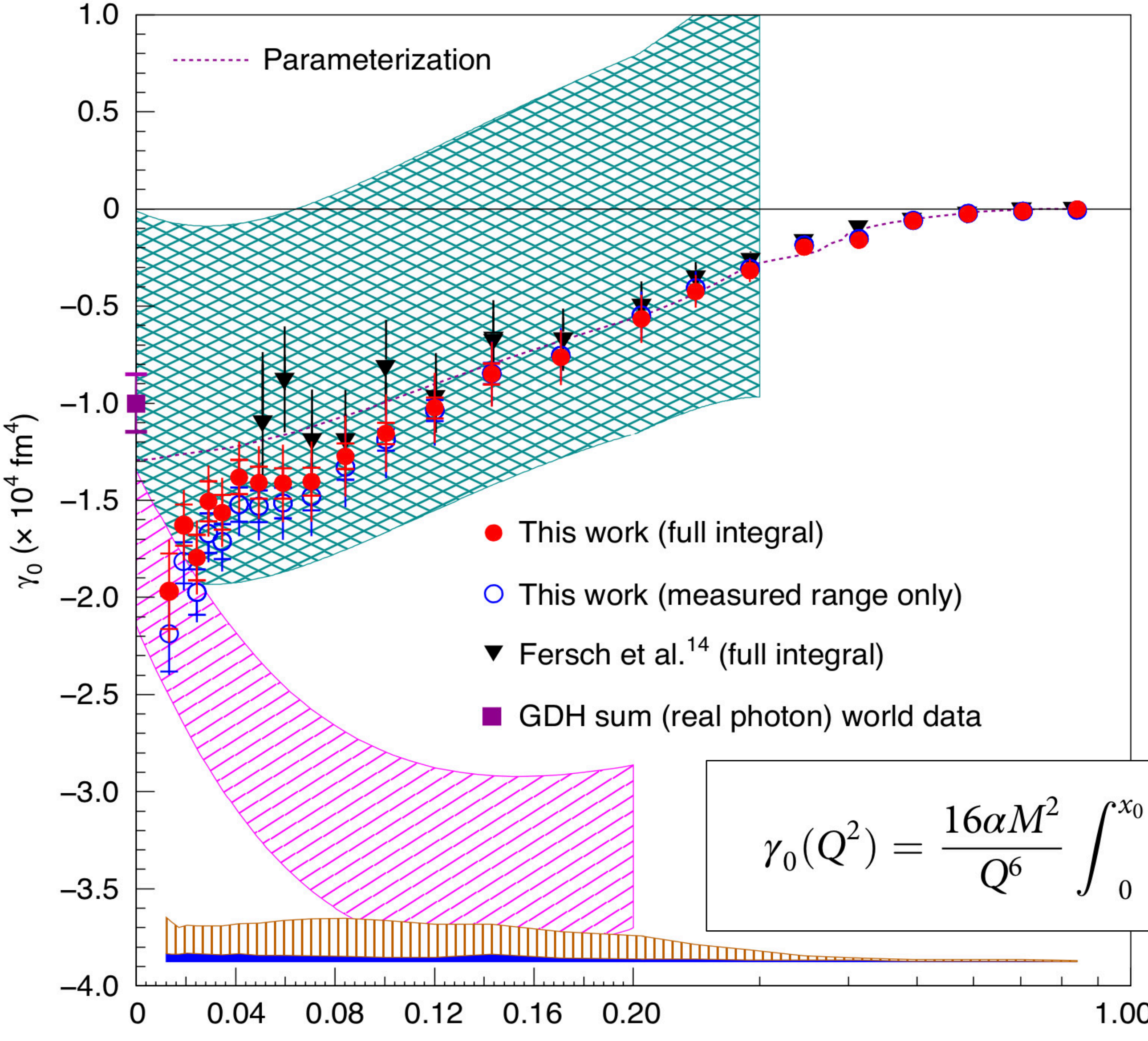
published data goes down to about 0.06 GeV^2

EG4 Proton



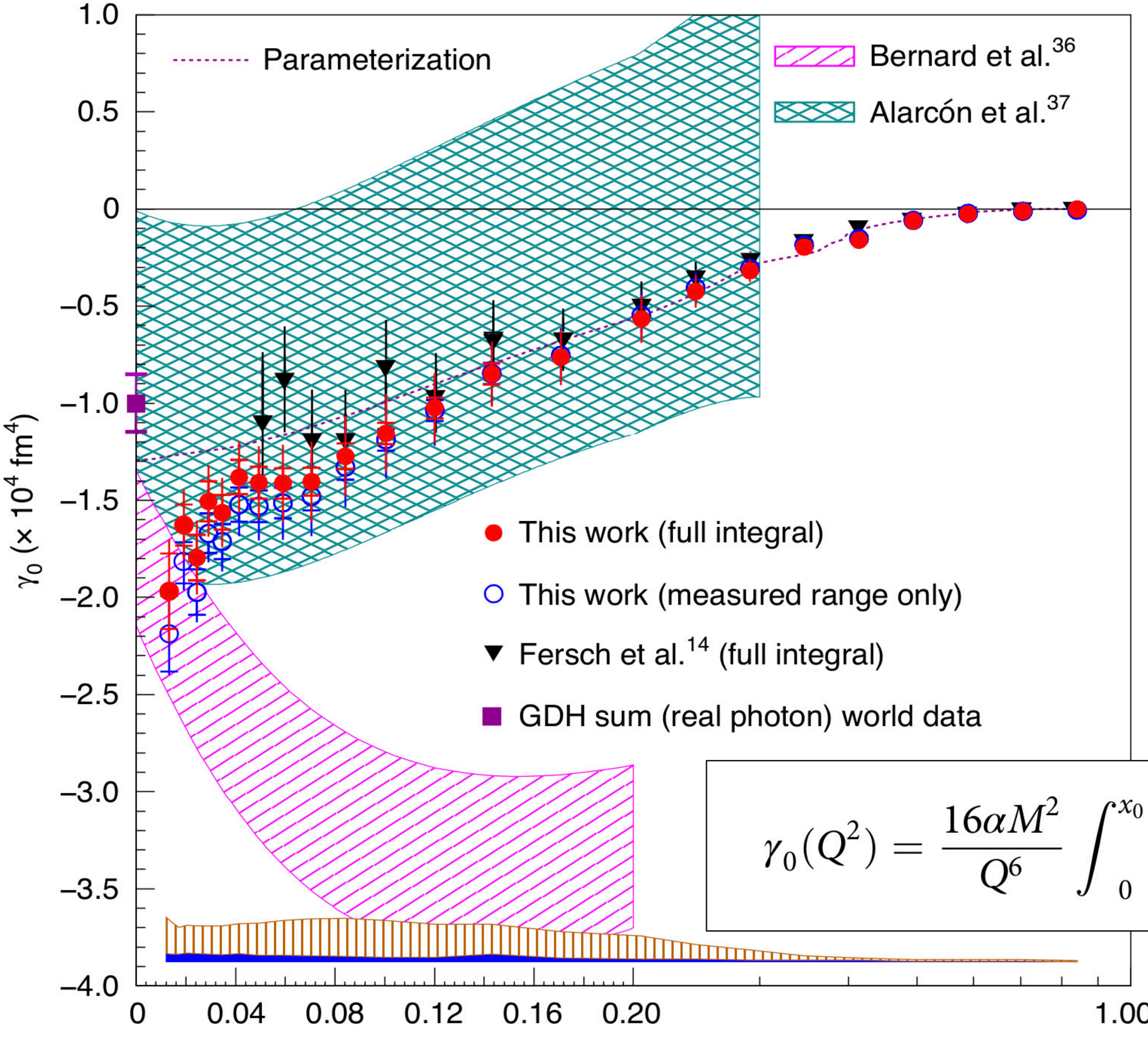
$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 A_1(x, Q^2) F_1(x, Q^2)\,\mathrm{d}x.$$

EG4 Proton



$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 A_1(x, Q^2) F_1(x, Q^2)\,\mathrm{d}x.$$

EG4 Proton



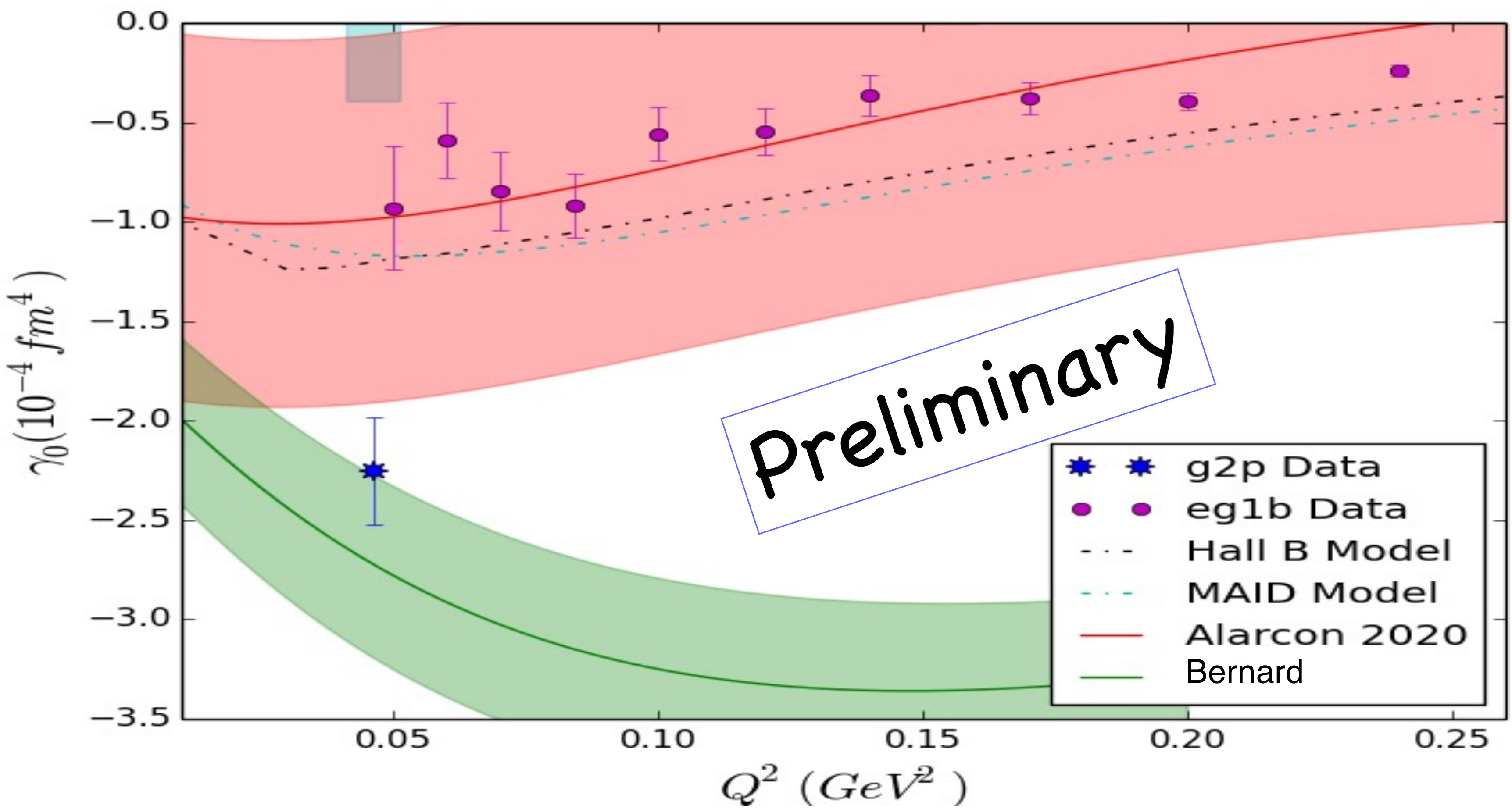
$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2 A_1(x, Q^2) F_1(x, Q^2) \mathrm{d}x.$$

Agreement with chpt

Bernard et al $Q^2 < 0.014$

Alarcon et al larger Q^2

E08-027 Proton γ_0 (latest Hall A data)

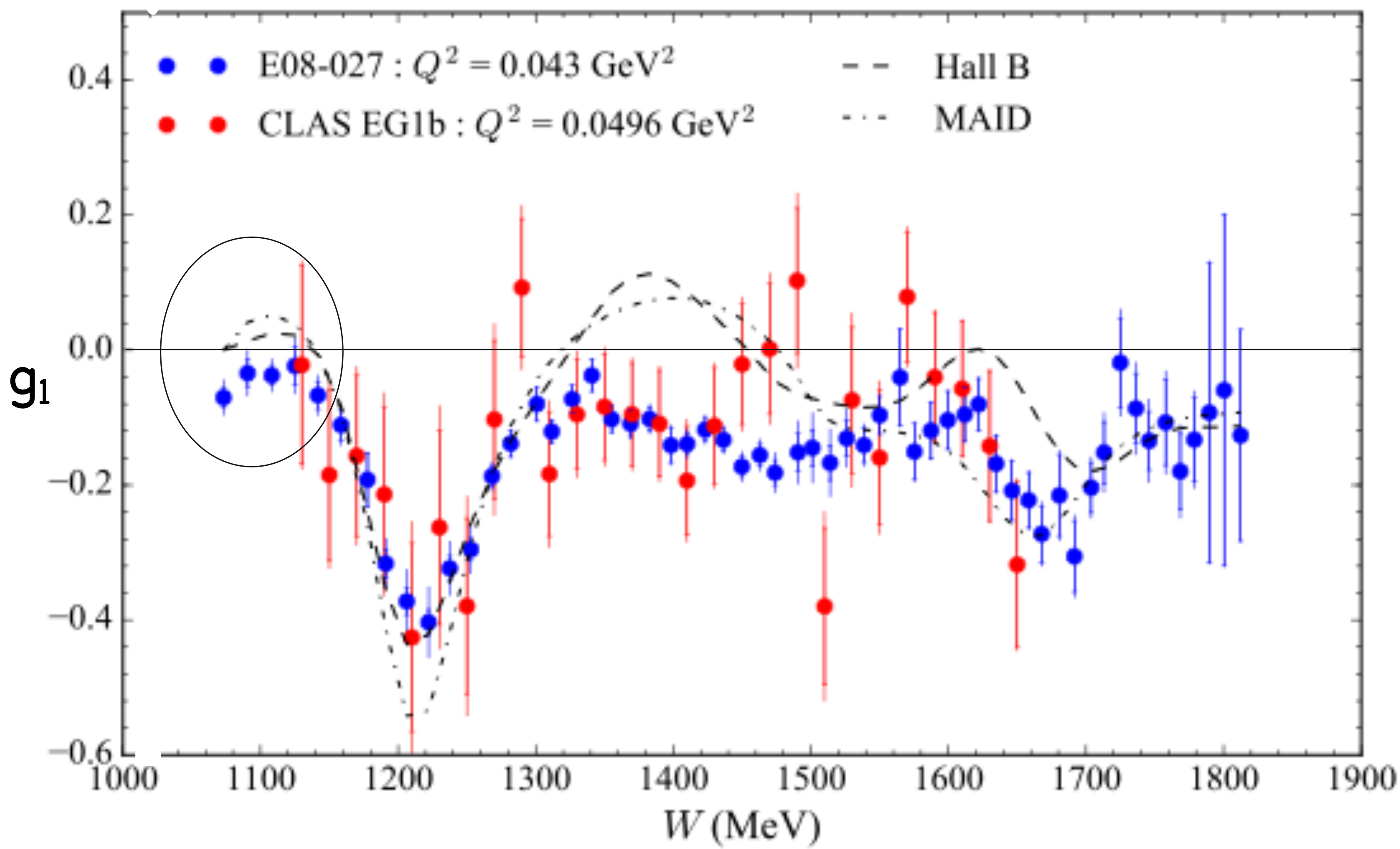


$$\gamma_0 = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1 - \frac{4M^2}{Q^2}x^2 g_2\right]$$

disagreement with EG1B data
HallA data includes measured g_2
And goes closer to threshold

Zheng et al. (2021) not shown here.

E08-027 compared to CLAS DATA



Some small disagreement between Hall A E08-027 and Hall B EG1B data at low nu

σ_{TT} and σ_{LT}

$$\sigma'_{LT} = -\frac{4\pi^2\alpha}{mK}\gamma(g_1 + g_2)$$

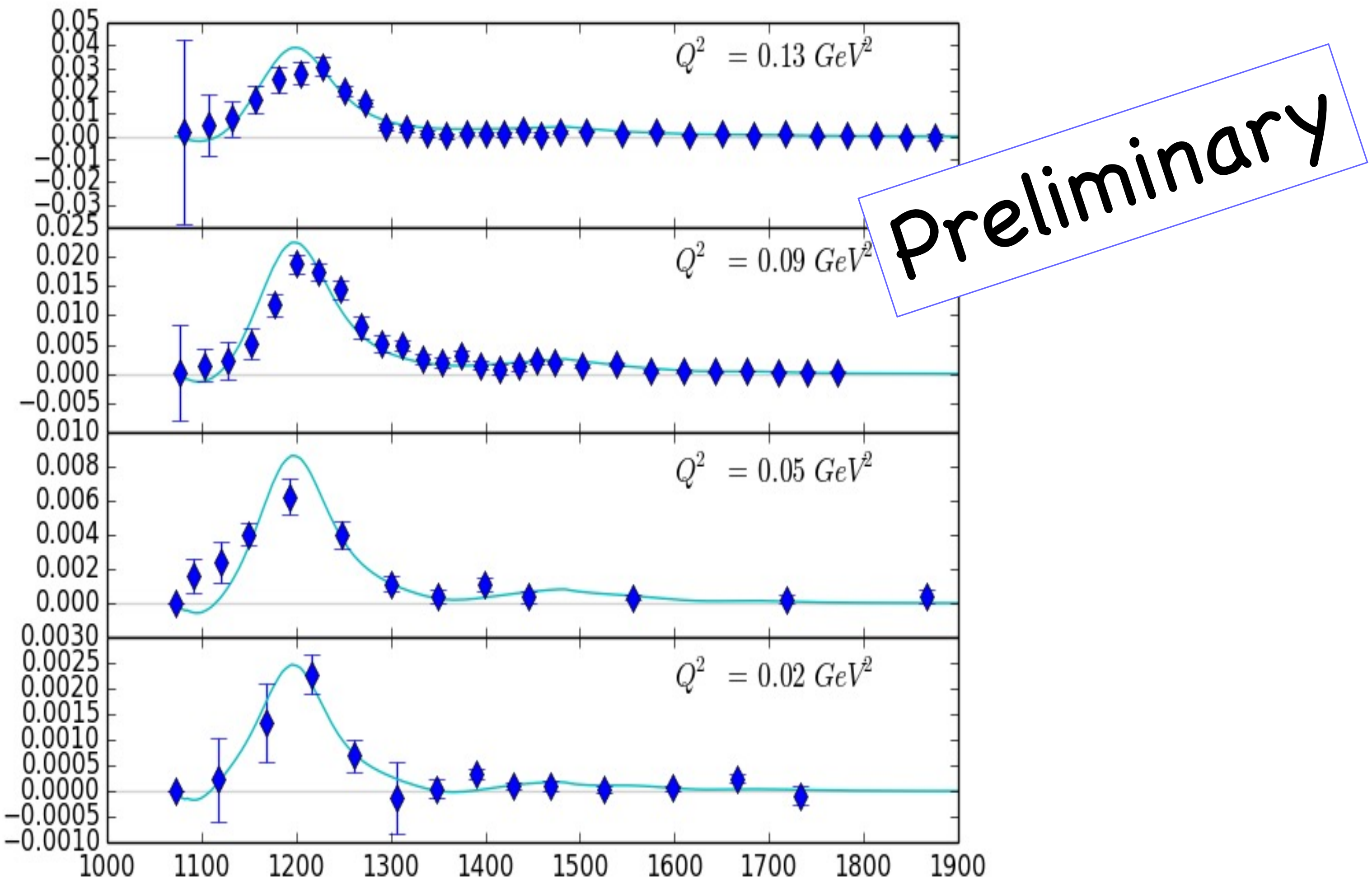
$$\sigma'_{TT} = -\frac{4\pi^2\alpha}{mK}(g_1 - \gamma^2 g_2)$$

σ_{TT} and σ_{LT}

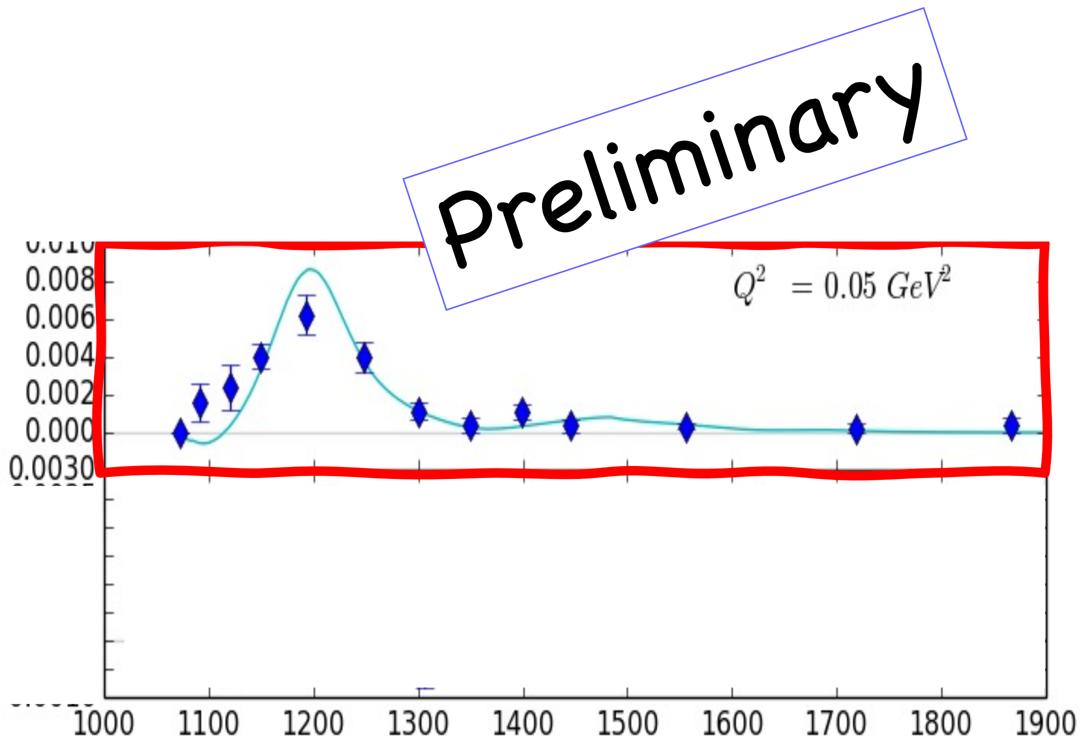
$$
\begin{aligned}
\sigma'_{LT} &= -\frac{4\pi^2\alpha}{mK}\gamma(g_1 + g_2) \\
\sigma'_{TT} &= -\frac{4\pi^2\alpha}{mK}(g_1 - \gamma^2 g_2)
\end{aligned}
$$

$$
\begin{aligned}
K_{\mathsf{A}} &= \nu \\
K_{\mathsf{G}} &= \sqrt{\nu^2 + Q^2} \\
K_{\mathsf{H}} &= \frac{W^2 - M^2}{2M} = \nu - \frac{Q^2}{2M}
\end{aligned}
$$

σ_{TT} Proton (E08-027)

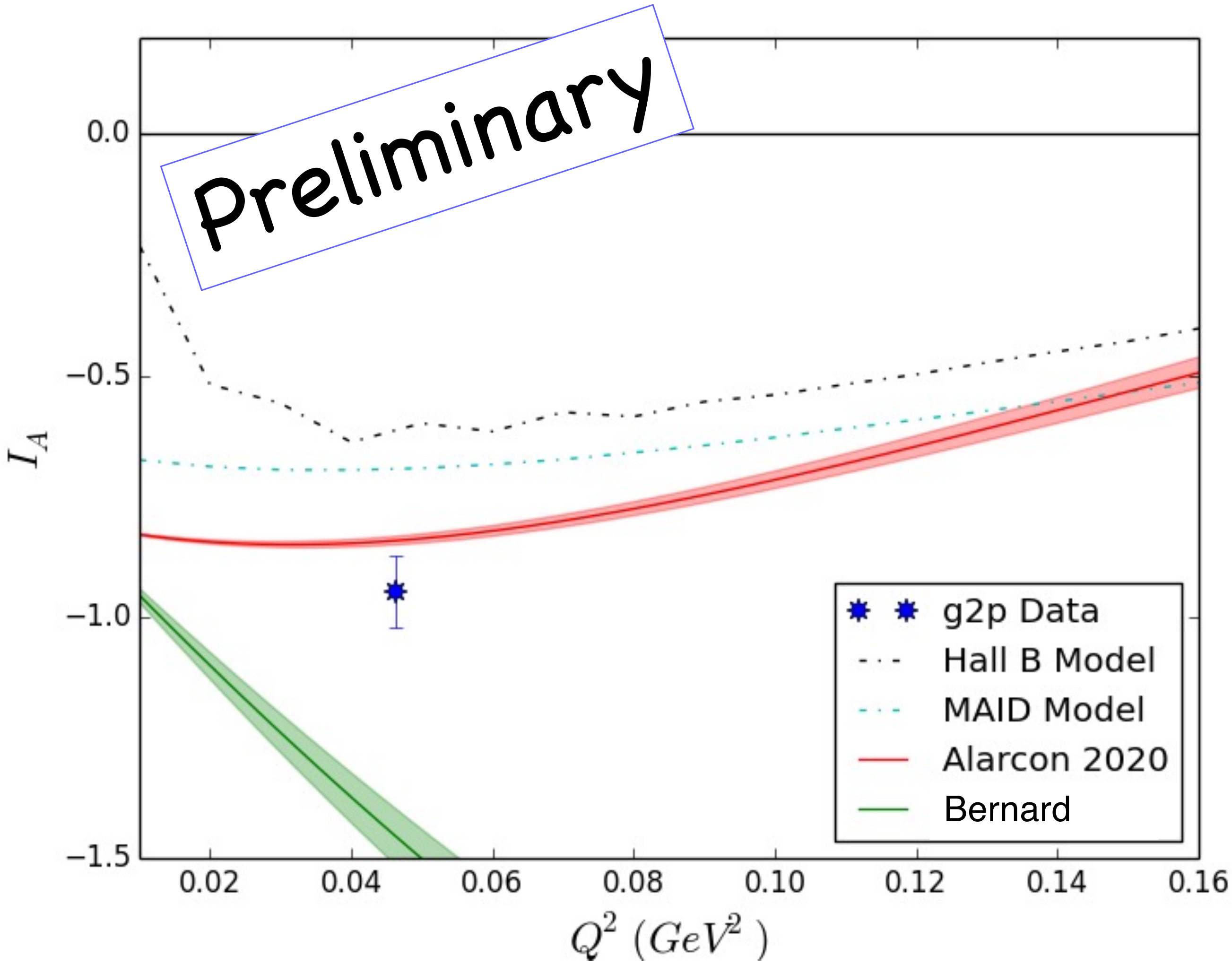


σ_{TT} Proton (E08-027)



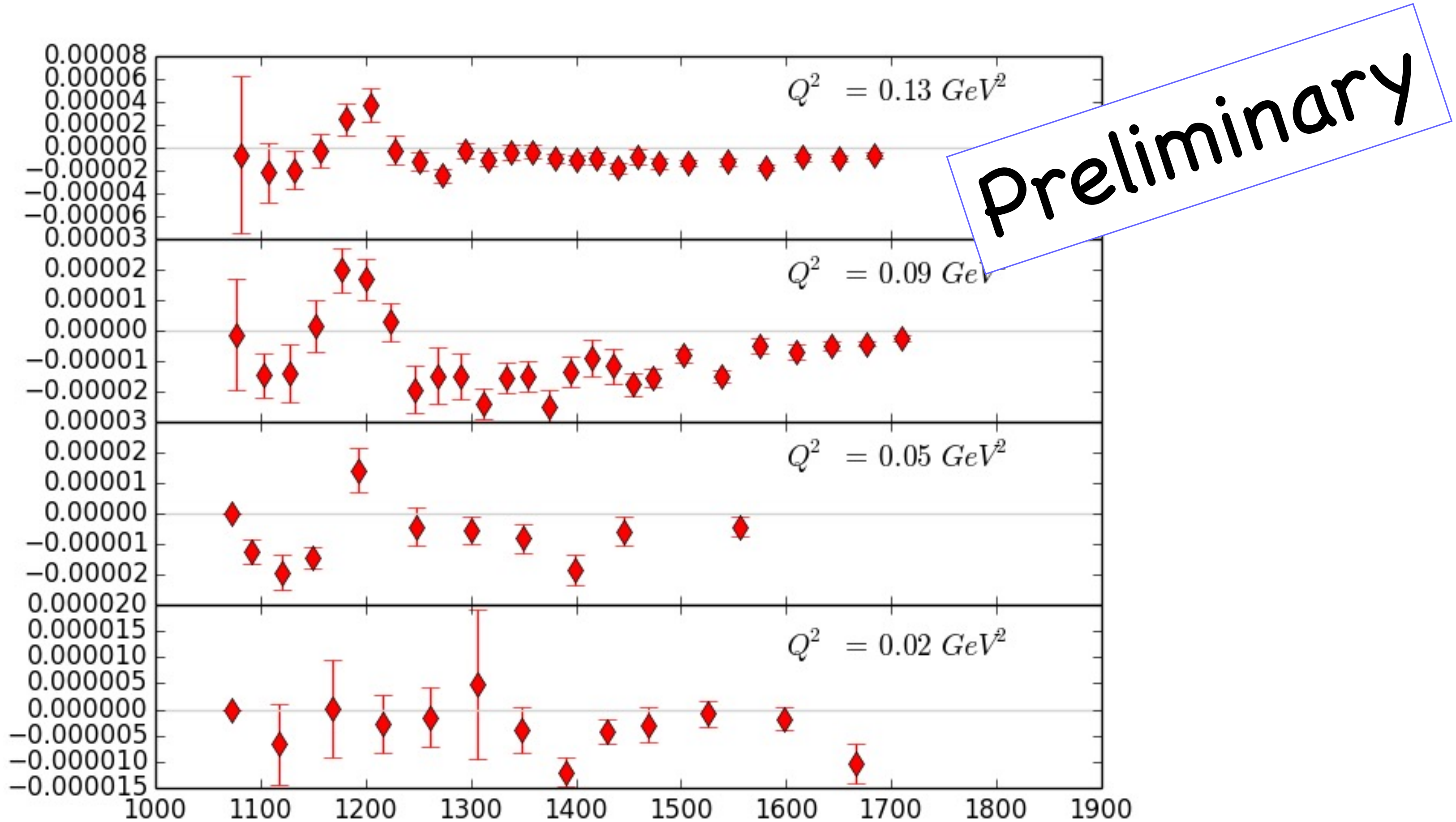
From longitudinal Data
So, the least
Model dependence

Proton GDH Integral (E08-027)

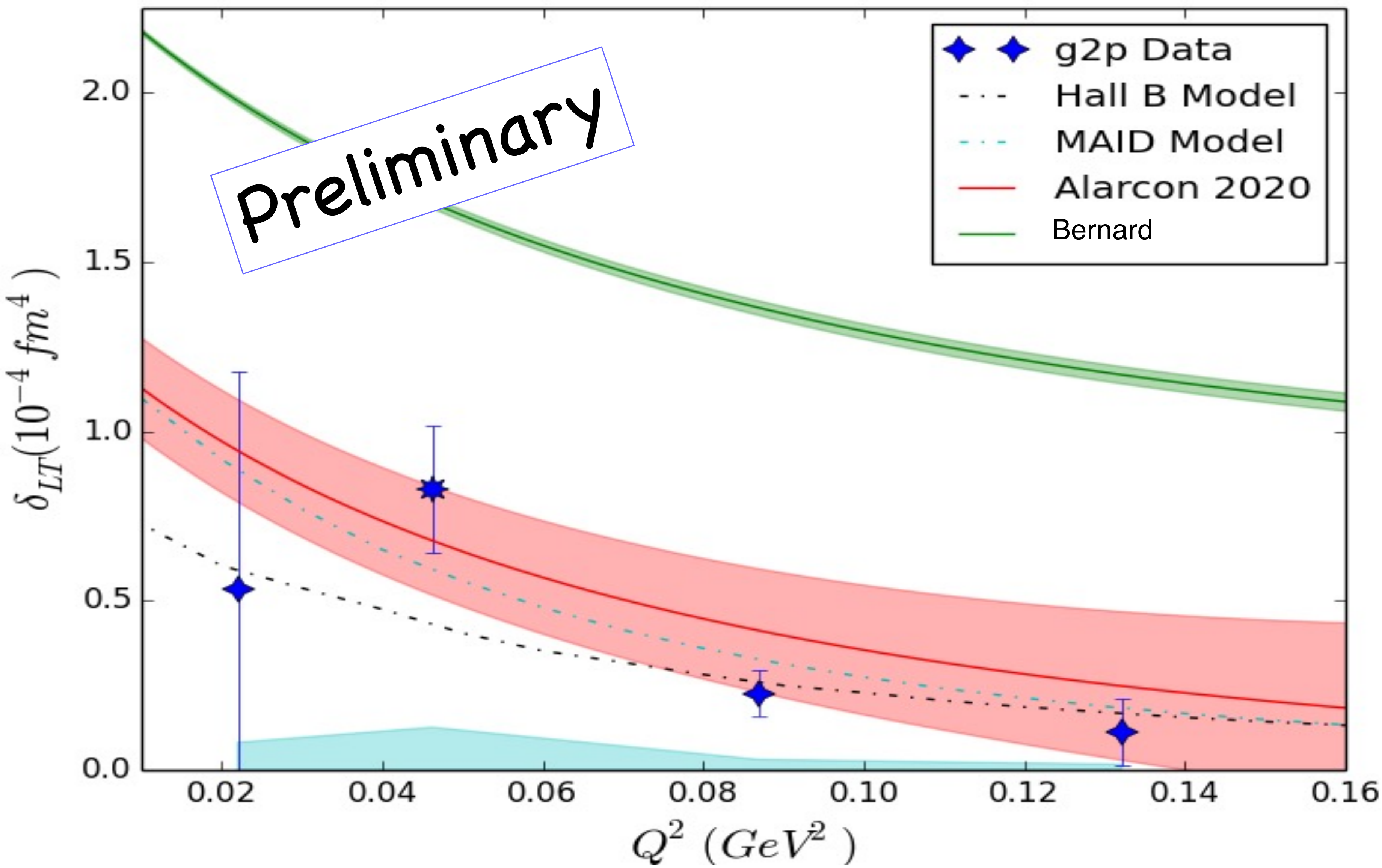


$$I_A = \frac{2M^2}{Q^2}\int_0^{x_{th}} g_1(x,Q^2) - \frac{4M^2}{Q^2}x^2 g_2(x,Q^2)dx$$

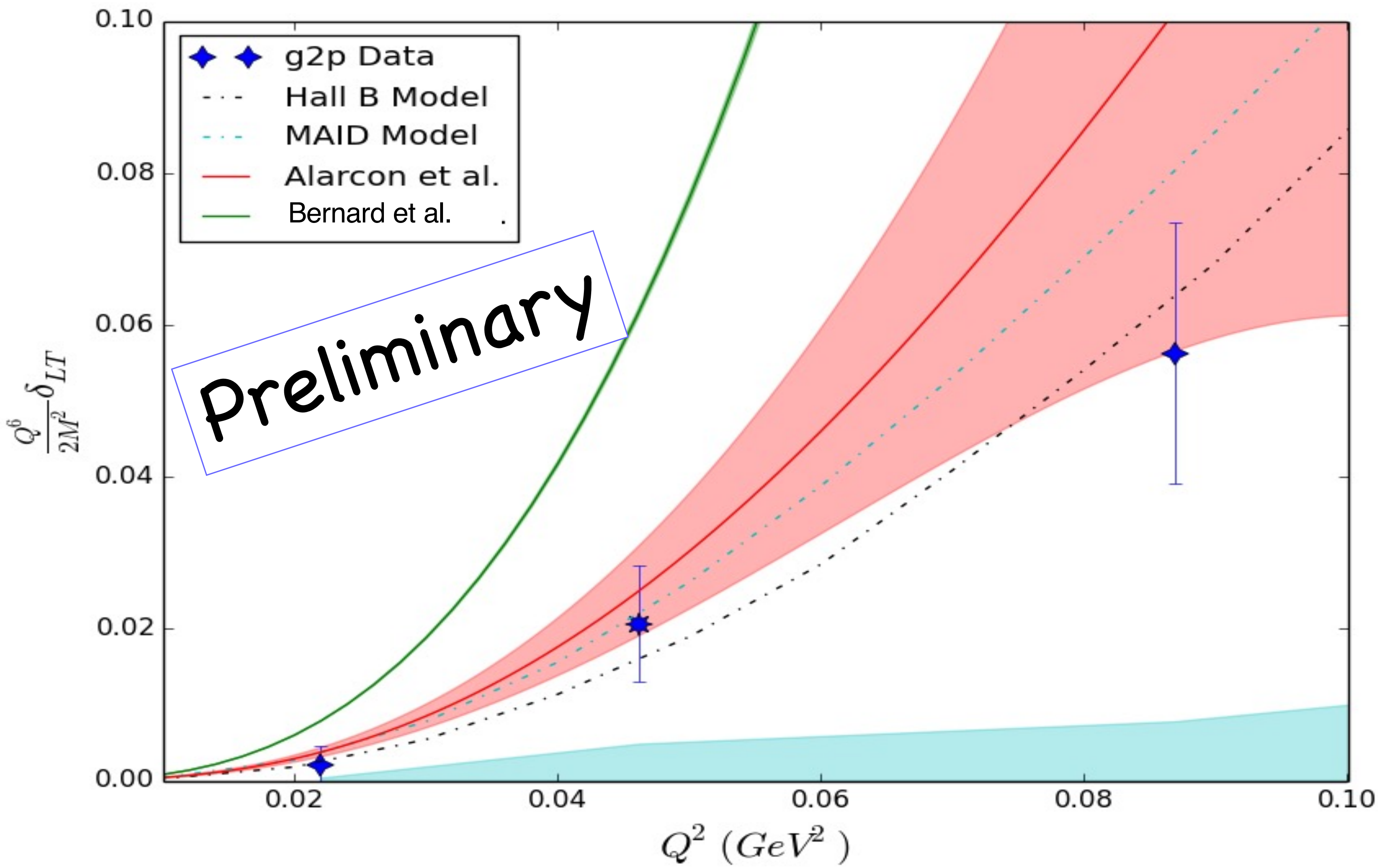
σ_{LT} Proton (E08-027)



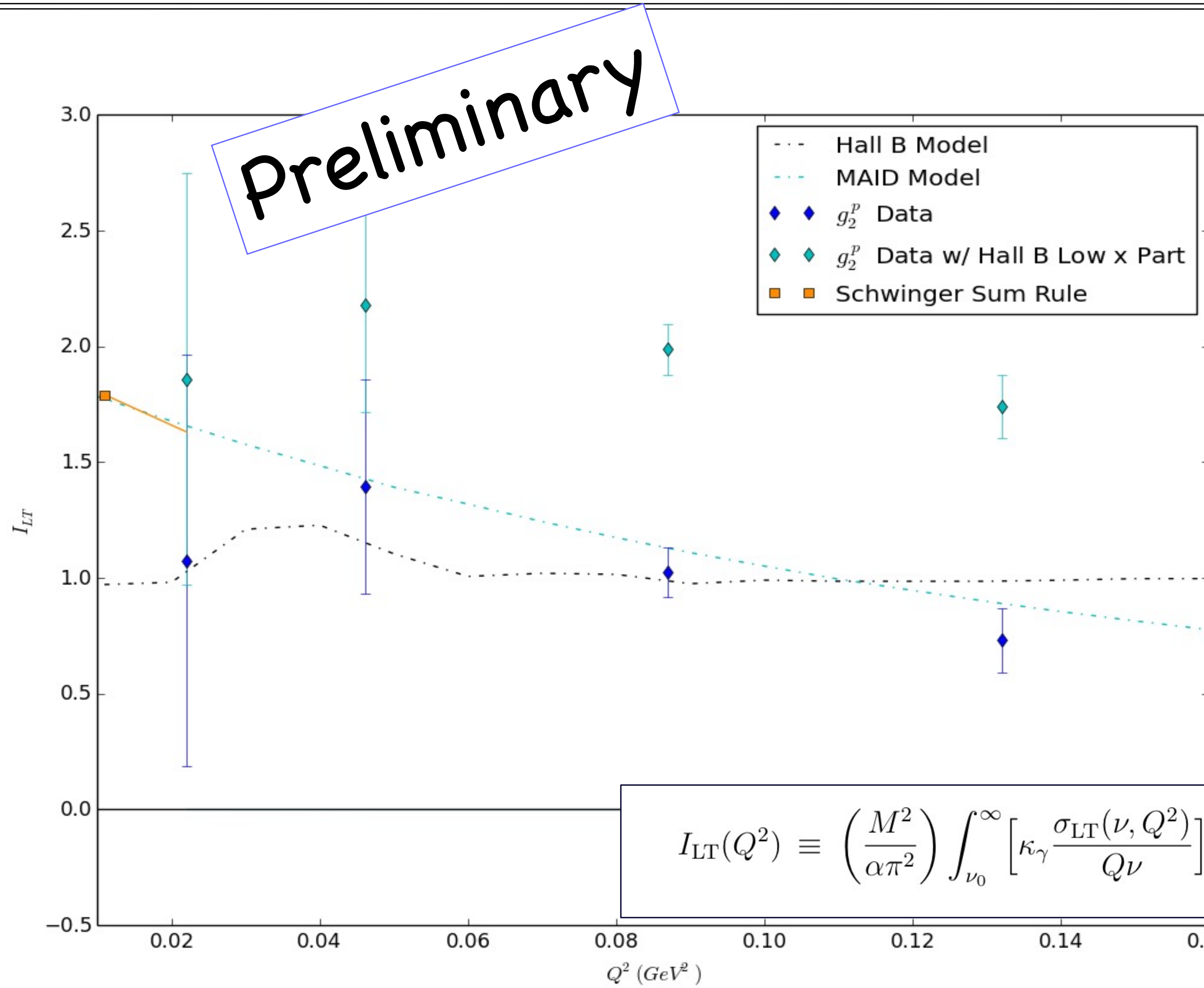
δ_{LT} Proton (E08-027)



$Q^6\delta_{LT}$ Proton (E08-027)



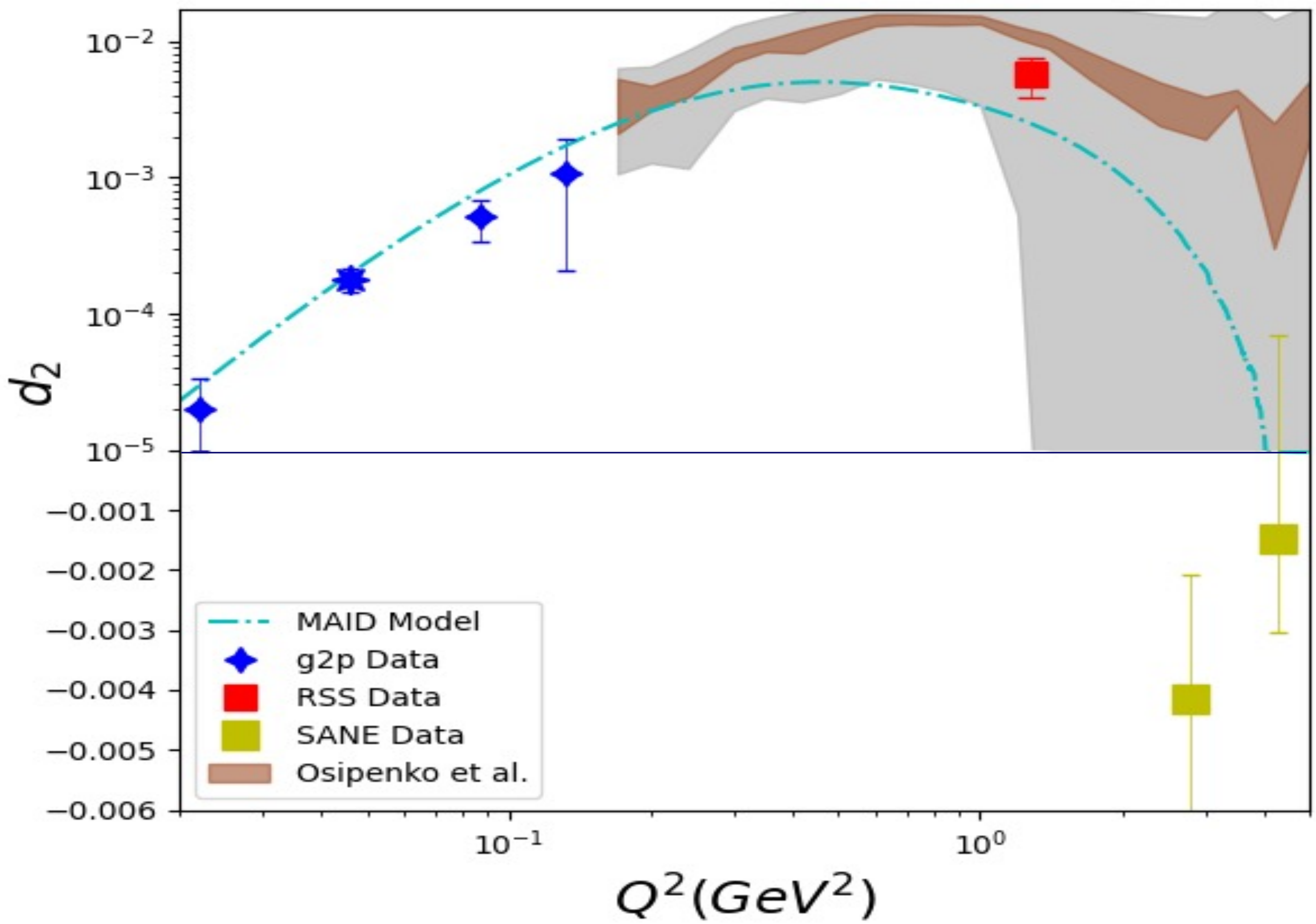
I_{LT} Proton (E08-027)



$$I_{\rm LT}(Q^2) \equiv \left(\frac{M^2}{\alpha\pi^2}\right)\int_{\nu_0}^{\infty}\left[\kappa_\gamma \frac{\sigma_{\rm LT}(\nu,Q^2)}{Q\nu}\right]_{Q=0} d\nu$$

E08–027 d_2

$$\overline{d}_2(Q^2) = \int_0^{x_0} x^2\left(2g_1(x,Q^2) + 3g_2(x,Q^2)\right)dx$$



Summary

g2p EG4 and sagdh

New Data at Low Q^2

See talks of A. Deur, Chao Peng, D. Ruth for more details

Some good success in reproducing data with ChPT

Proton δ_{lt} data favors Alarcon et. al

data consistent with Schwinger sum rule expectations

Some challenges remain to resolve differences between data and theory