Review of light quark mass determination via $\eta \rightarrow 3\pi$

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Outline

1. Introduction and Motivation
2. Light quark masses from $\eta \rightarrow \pi^+ \pi^- \pi^0$
3. Light quark masses from $\eta \rightarrow \pi^0 \pi^0 \pi^0$
4. Comparison with Lattice QCD
5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Why is it interesting to study η and η’ physics?

- In the study of η and η’ physics, large amount of data have been collected:

  ⇨ *CBall, WASA, KLOE & KLOEII, BESIII, A2@MAMI, CLAS, GlueX*

  More to come: *JEF, REDTOP* ⇨ see talk by *A. Somov*

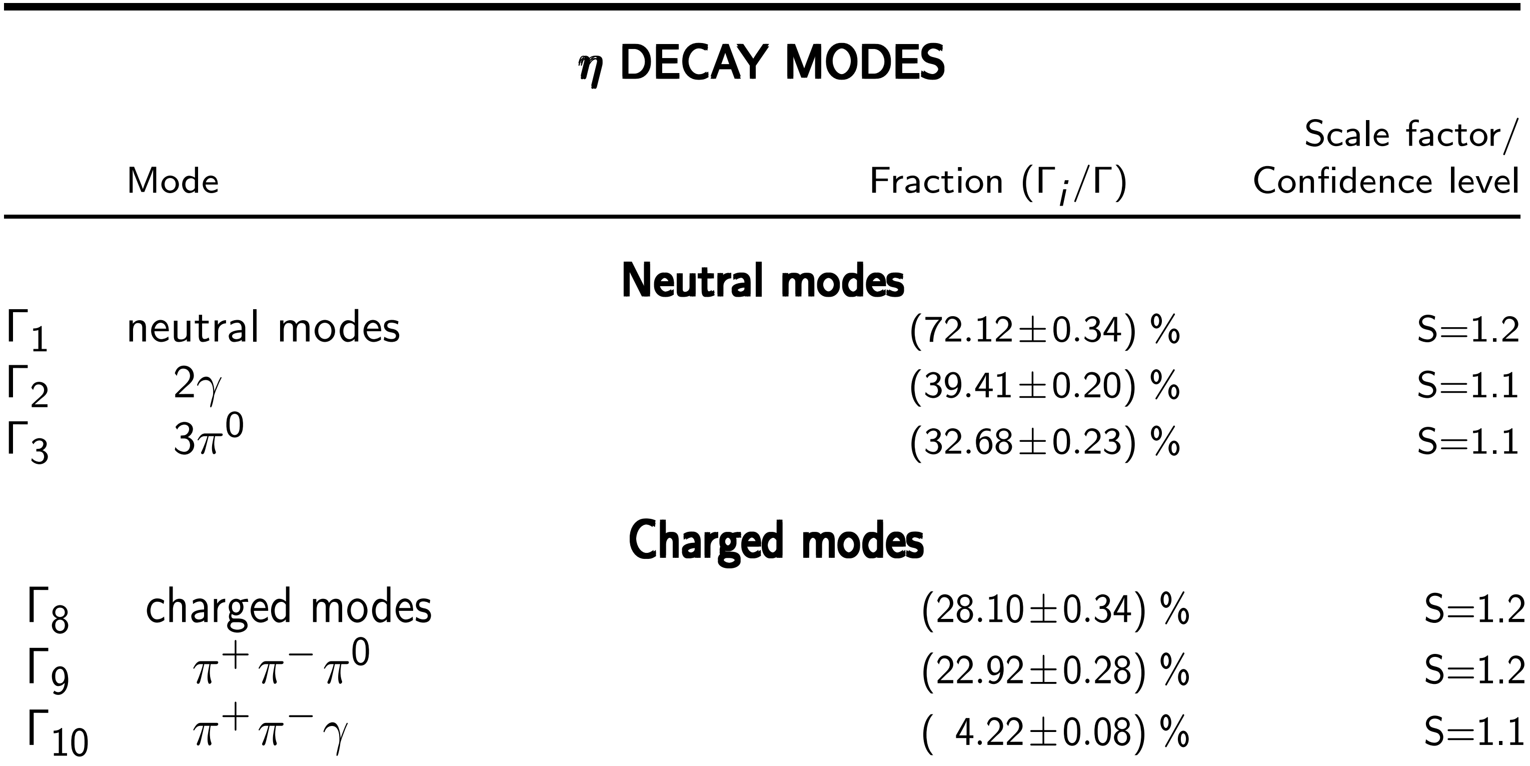
- Unique opportunity:
  - Test chiral dynamics at low energy
  - Extract fundamental parameters of the Standard Model: ex: light quark masses
  - Study of fundamental symmetries: C, P & T violation
  - Looking for beyond Standard Model Physics

⇨ See talk by *B. Kubis*

1.2 Decays of η

M_η = 547.862(17) MeV

- η decay from PDG:



η DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
	Neutral modes		
Γ_1	neutral modes	(72.12±0.34) %	S=1.2
Γ_2	2γ	(39.41±0.20) %	S=1.1
Γ_3	$3\pi^0$	(32.68±0.23) %	S=1.1
	Charged modes		
Γ_8	charged modes	(28.10±0.34) %	S=1.2
Γ_9	$\pi^+\pi^-\pi^0$	(22.92±0.28) %	S=1.2
Γ_{10}	$\pi^+\pi^-\gamma$	(4.22±0.08) %	S=1.1

1.3 Why is it interesting to study $\eta \to 3\pi$?

- Decay forbidden by isospin symmetry

⇨ $$\boxed{A = (m_u - m_d) A_1 + \alpha_{em} A_2}$$

- α_{em} effects are small

Sutherland'66, Bell & Sutherland'68
Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

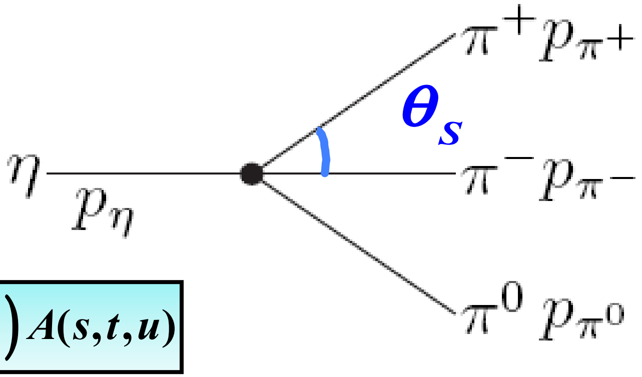
- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \to \boxed{L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)}$$

⇨ Unique access to ($m_u - m_d$)

2. Light quark masses from $\eta \rightarrow \pi^+ \pi^- \pi^0$

2.1 Definitions



- η decay: $\eta \to \pi^+ \pi^- \pi^0$

$$\boxed{\langle \pi^+\pi^-\pi^0{}_{out}|\eta\rangle = i(2\pi)^4\delta^4\left(p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0}\right)A(s,t,u)}$$

- Mandelstam variables $s=\left(p_{\pi^+}+p_{\pi^-}\right)^2,\ t=\left(p_{\pi^-}+p_{\pi^0}\right)^2,\ u=\left(p_{\pi^0}+p_{\pi^+}\right)^2$

⇨ only two independent variables

$$s+t+u = M_\eta^2 + M_{\pi^0}^2 + 2M_{\pi^+}^2 \equiv 3s_0$$

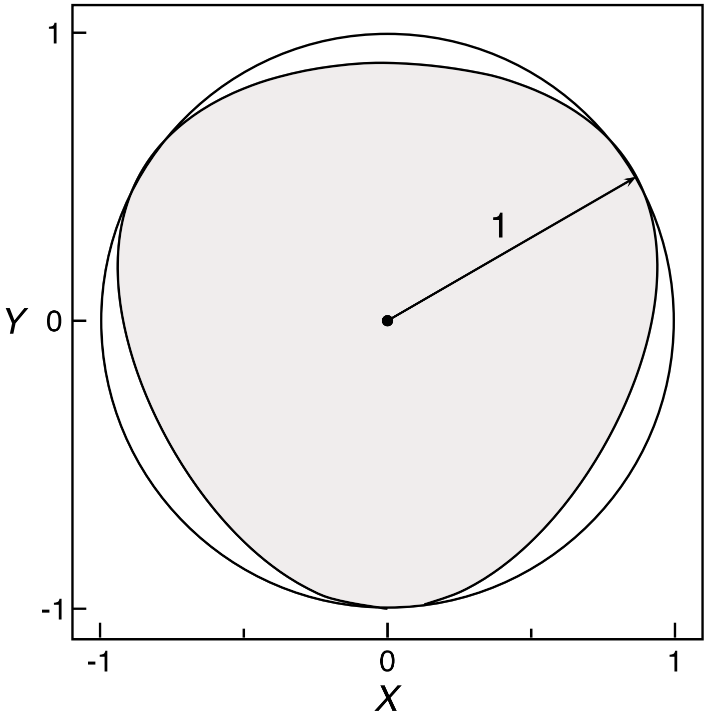
- 3 body decay ⇨ Dalitz plot

$$\boxed{\left|A(s,t,u)\right|^2 = N\left(1+aY+bY^2+dX^2+fY^3+\ldots\right)}$$

Expansion around X=Y=0

$$X = \sqrt{3}\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}\left(u-t\right)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

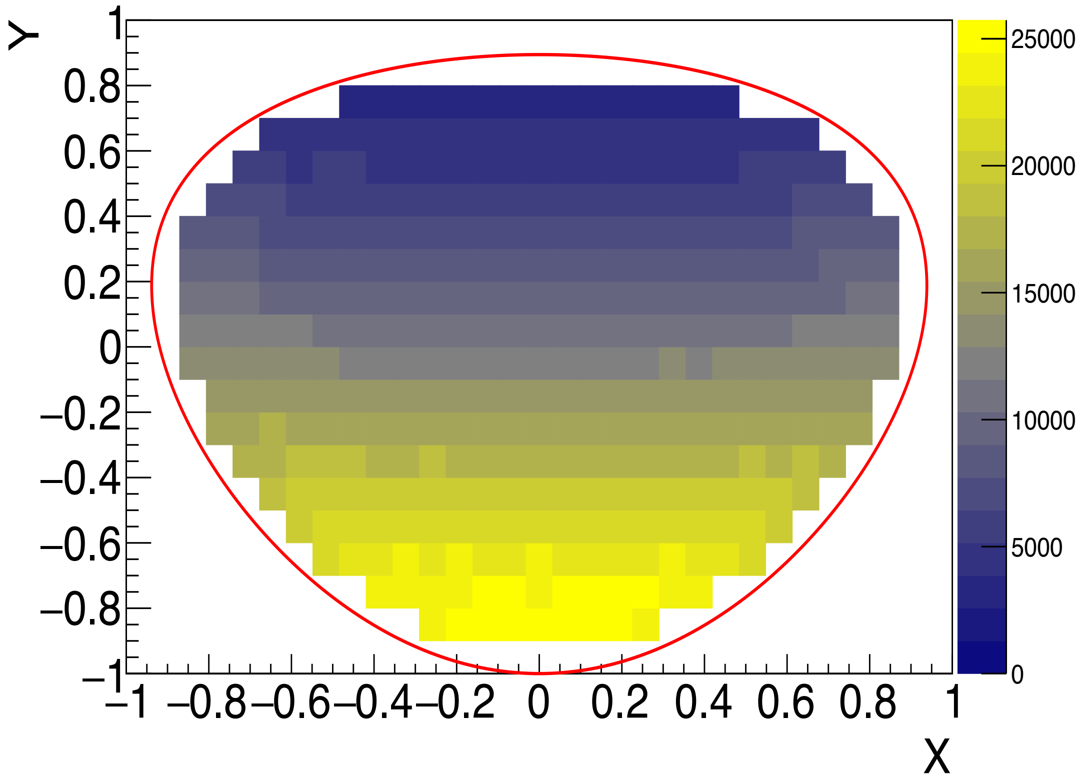


$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

2.2 $\eta \to 3\pi$ Dalitz plot measurements

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

2.3 Quark mass ratio

- In the following, extraction of Q from $\eta \to \pi^+ \pi^- \pi^0$

$$\boxed{\Gamma_{\eta\to\pi^+\pi^-\pi^0} = \frac{1}{Q^4}\frac{M_K^4}{M_\pi^4}\frac{\left(M_K^2 - M_\pi^2\right)^2}{6912\pi^3 F_\pi^4 M_\eta^3}\int_{s_{\min}}^{s_{\max}} ds \int_{u_-(s)}^{u_+(s)} du \, \left|M(s,t,u)\right|^2}$$

Determined from experiment

Determined from:
- Dispersive calculation
- ChPT

Fit to Dalitz distr.

$$\boxed{\left[Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} \right]}$$

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$$

- Aim: Compute M(s,t,u) with the *best accuracy*

2.4 Computation of the amplitude

- What do we know?
- Compute the amplitude using ChPT :

LO: *Osborn, Wallace'70*

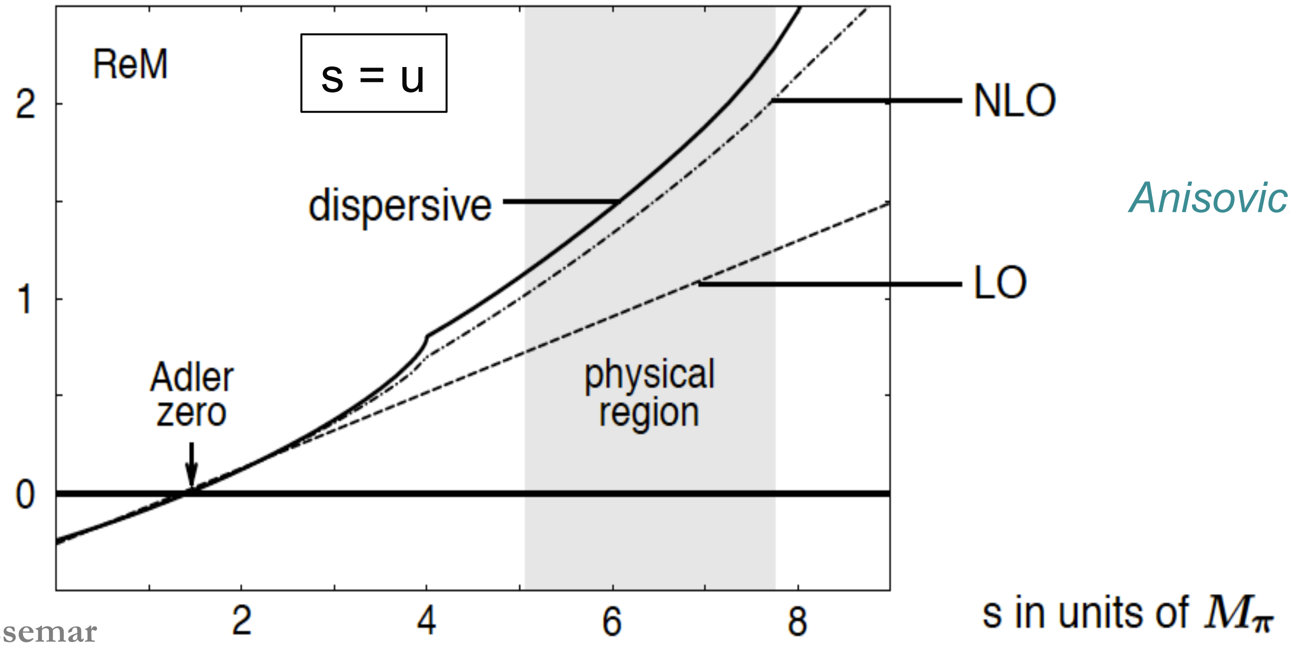
NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

$$\boxed{\Gamma_{\eta\to3\pi} = \left(\underset{\underset{\text{LO}}{\uparrow}}{66} + \underset{\underset{\text{NLO}}{\uparrow}}{94} + \underset{\underset{\text{NNLO}}{\nwarrow}}{\ldots} + \ldots\right)\text{eV} = \left(300 \pm 12\right)\text{eV}}$$

PDG'19

The Chiral series has convergence problems

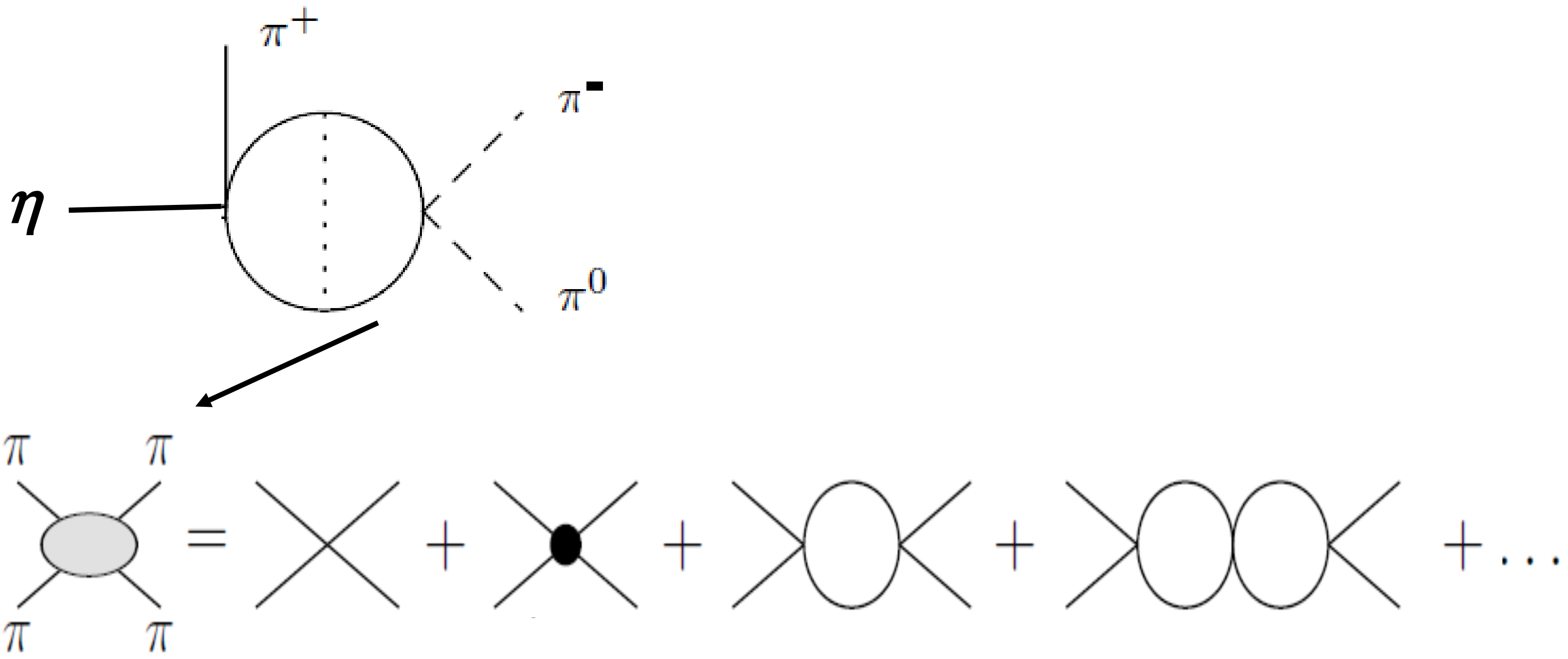


Anisovich & Leutwyler'96

2.5 Dispersive treatment

- The Chiral series has convergence problems

⇨ Large *ππ final state interactions* *Roiesnel & Truong'81*



- *Dispersive treatment :*
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

2.5 Dispersive treatment

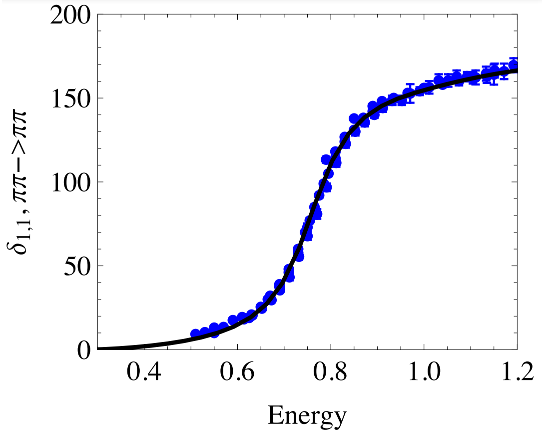
See talk by *T. Isken*

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)}$$

Roy analysis
Colangelo et al.'01

input

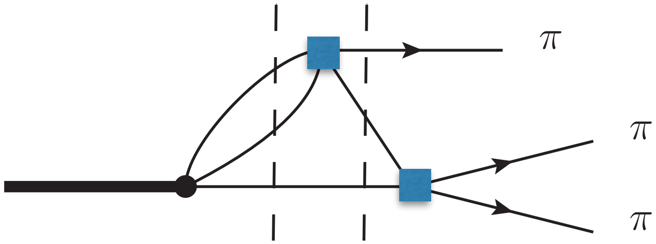


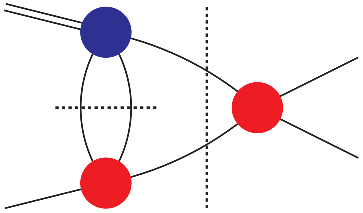
- Unitarity relation:

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)\overbrace{t_\ell^*(s)}^{\pi\pi\to\pi\pi}\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

right-hand cut

left-hand cut





2.5 Dispersive treatment

See talk by *T. Isken*

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)}$$

- Unitarity relation:

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$\boxed{M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)}$$

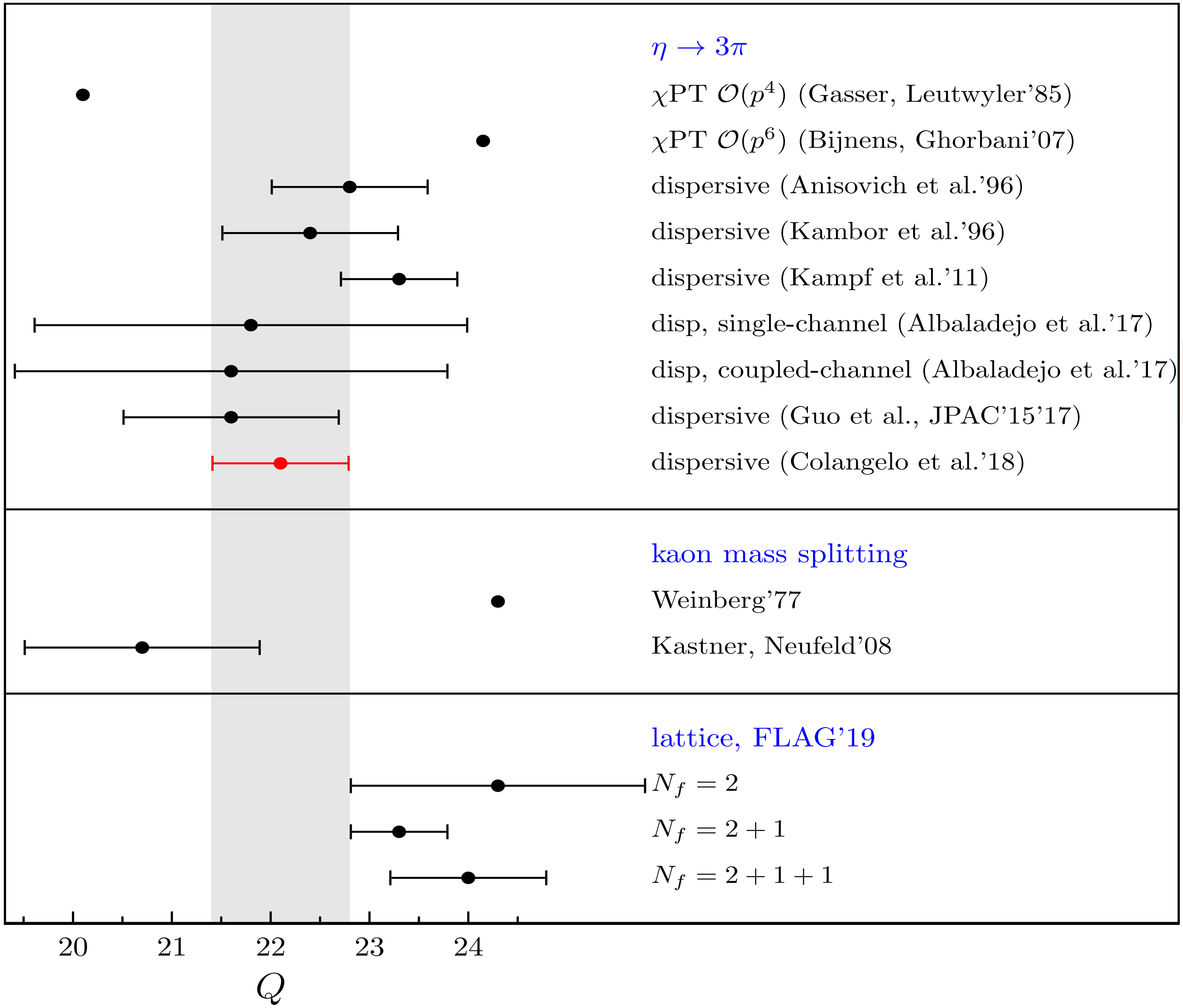
$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.6 Quark mass ratio



$\boldsymbol{Q = 22.04 \pm 0.72}$

- Experimental systematics needs to be taken into account

3. Light quark masses from $\eta \to 3\pi^0$

3.1 Neutral channel : $\eta \rightarrow \pi^0 \pi^0 \pi^0$

- What do we know?

- We can relate charged and neutral channels

$$\boxed{\overline{A}(s,t,u) = A(s,t,u) + A(t,u,s) + A(u,s,t)}$$

⇨ *Correct formalism should be able to reproduce both charged and neutral channels*

- Ratio of decay width precisely measured

$$\boxed{r = \frac{\Gamma\left(\eta \to \pi^0\pi^0\pi^0\right)}{\Gamma\left(\eta \to \pi^+\pi^-\pi^0\right)}} = 1.426 \pm 0.026$$

PDG'19

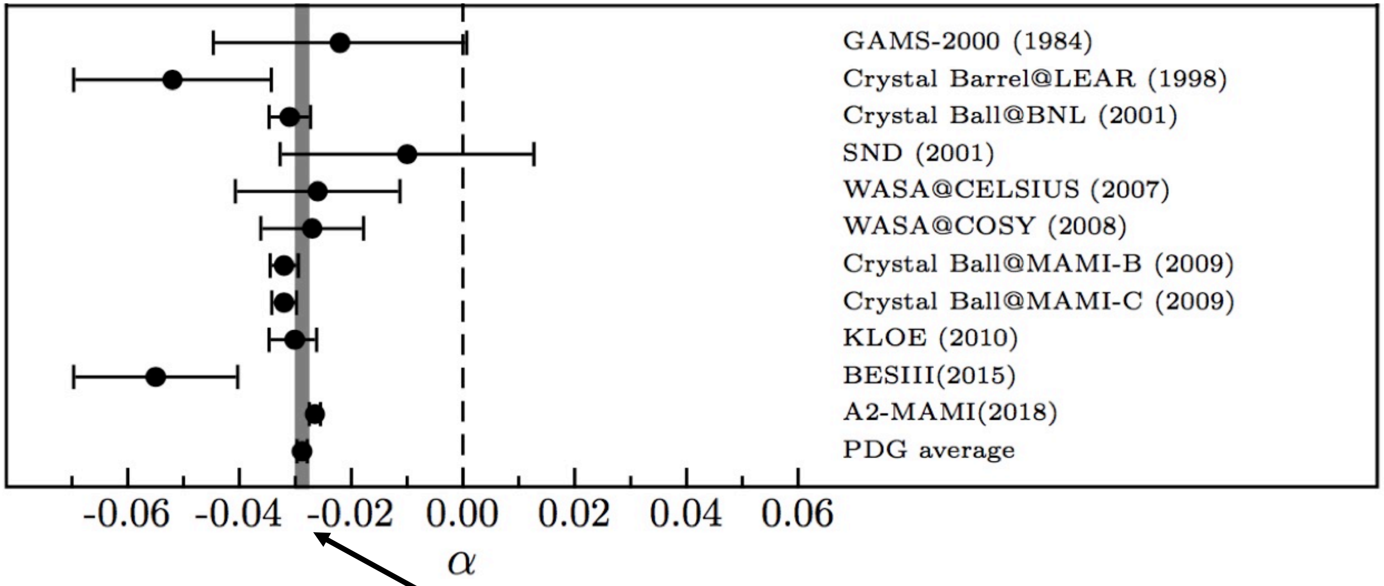
3.1 Neutral Channel : $\eta \rightarrow \pi^0 \pi^0 \pi^0$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

- Decay amplitude $\boxed{\Gamma_{\eta\to 3\pi} \propto |\bar{A}|^2 \propto 1 + 2\alpha Z}$ with $Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$

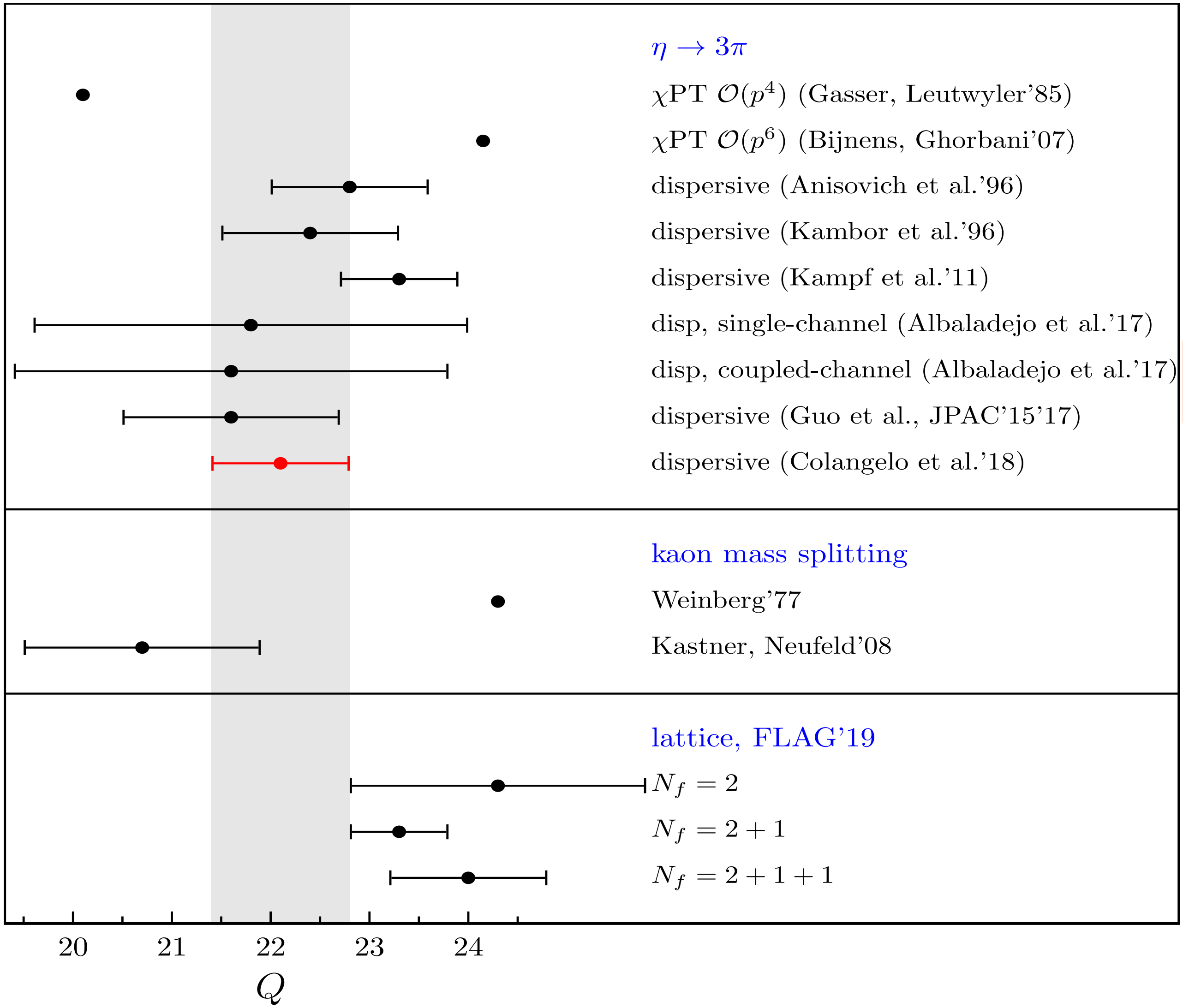
- α has been precisely measured for a long time

➡ recently very high-statistics from *A2@MAMI'2018*



$$\boxed{\alpha = -0.0288 \pm 0.0012}$$

3.2 Quark mass ratio

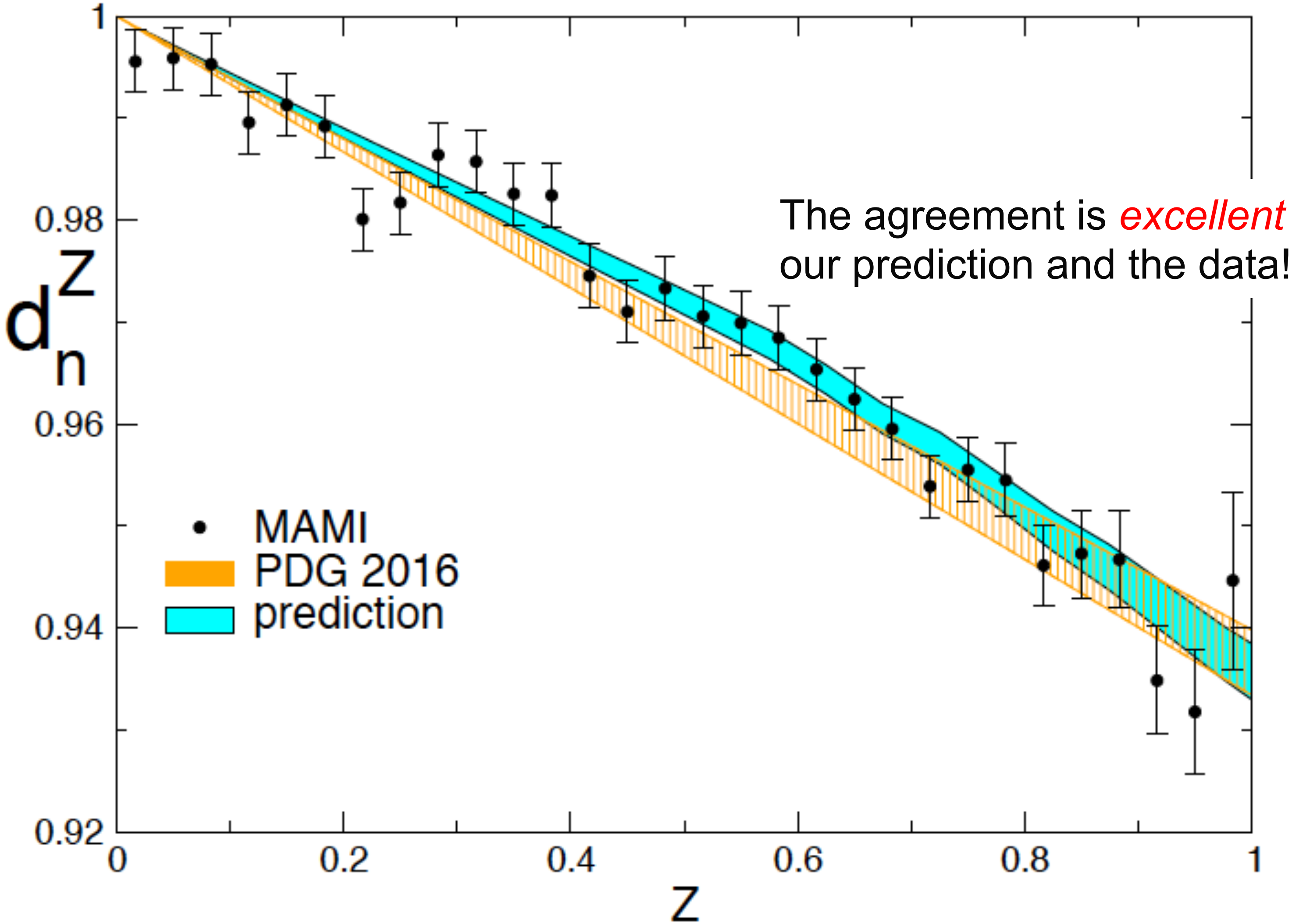


$$\boxed{Q = 22.08 \pm 0.66}$$

- Experimental systematics needs to be taken into account

3.3 Z distribution for $\eta \rightarrow \pi^0 \pi^0 \pi^0$ decays

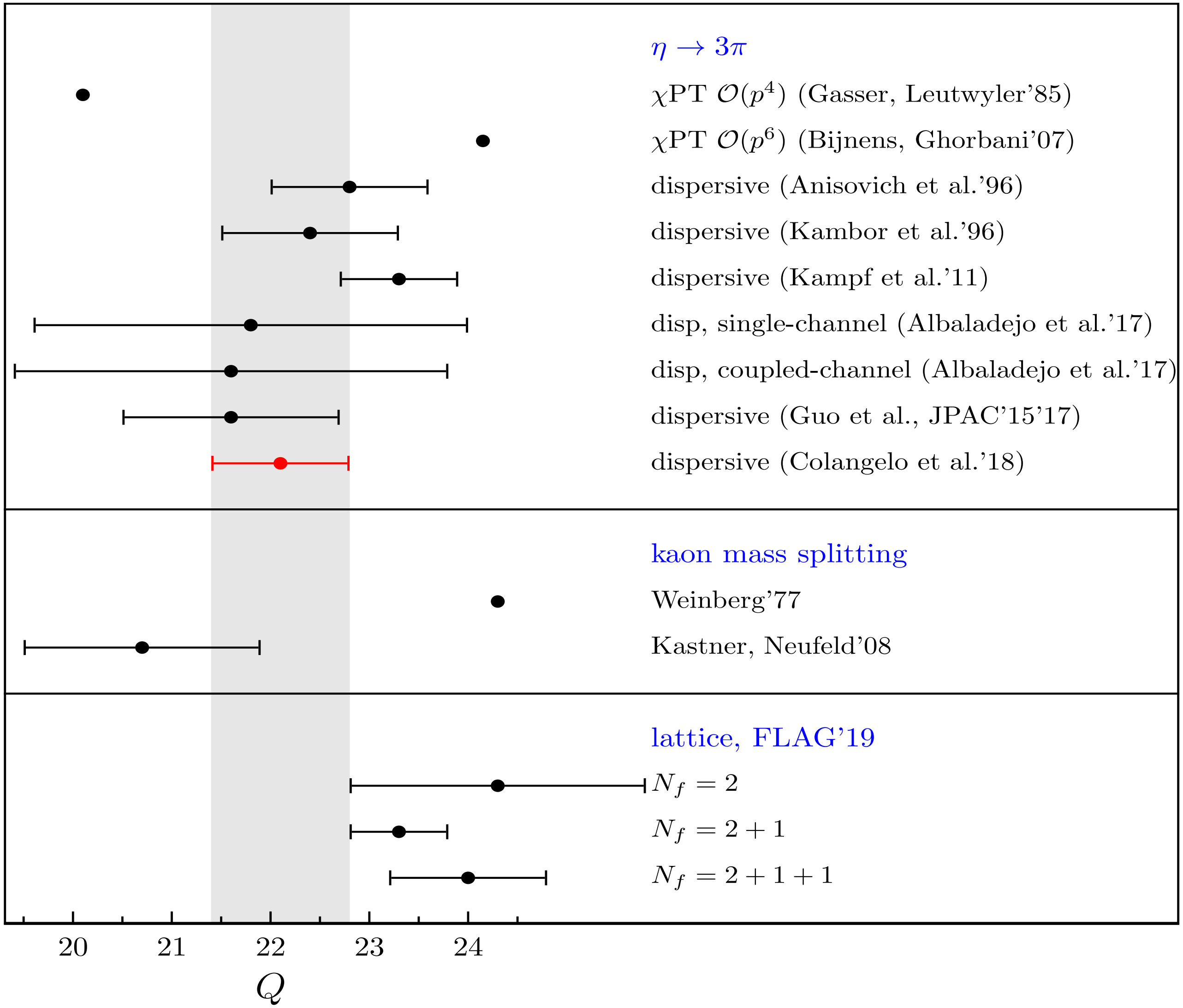
- The amplitude squared in the neutral channel is



The agreement is *excellent* between our prediction and the data!

4. Comparison with Lattice QCD and uncertainties

4.1 Quark mass ratio



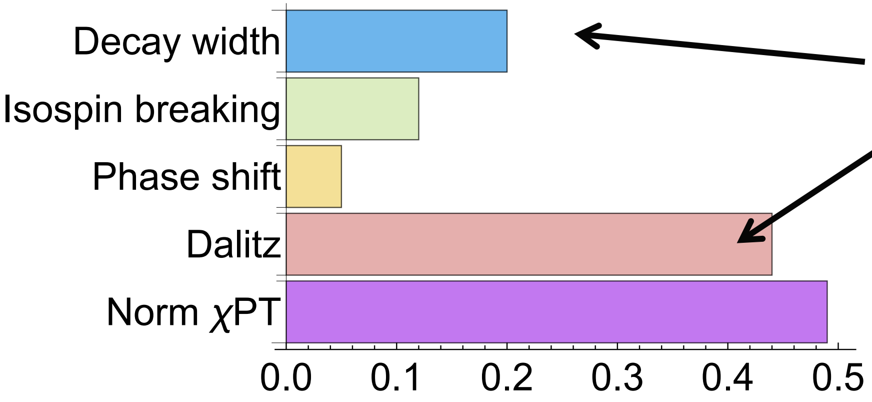
$\boxed{Q = 22.1 \pm 0.7}$

- Experimental systematics needs to be taken into account

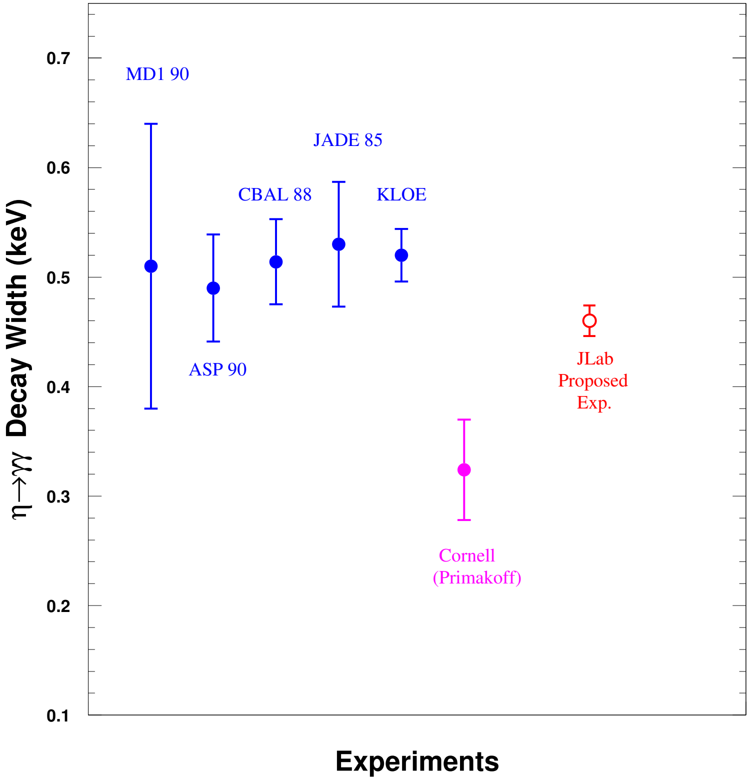
4.2 Uncertainties and Prospects

Colangelo et al.'18
Gan, Kubis, E. P., Tulin'20

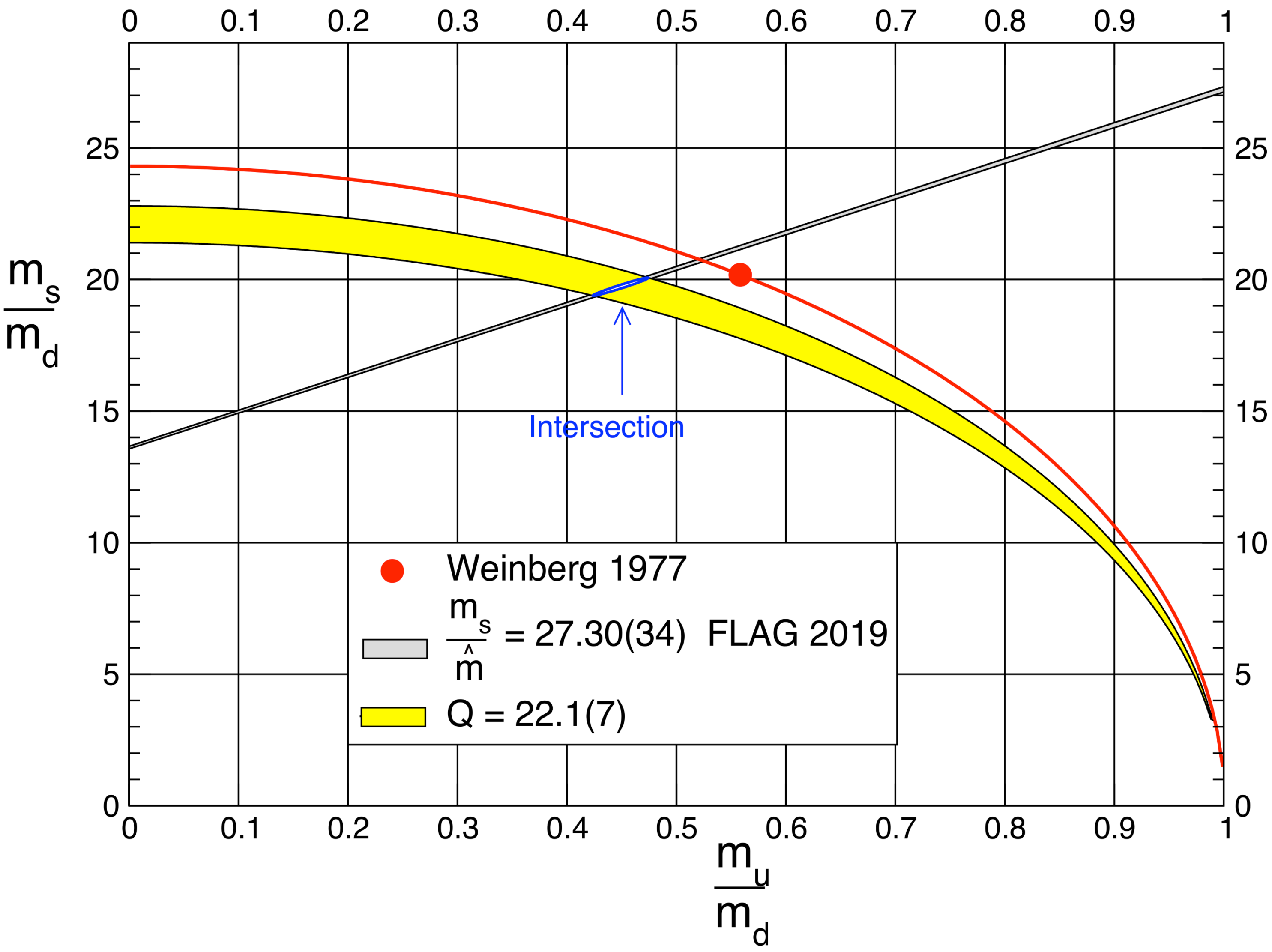
- Uncertainties in the quark mass ratio



Can be investigated and reduced at *future facilities*



4.2 Light quark masses

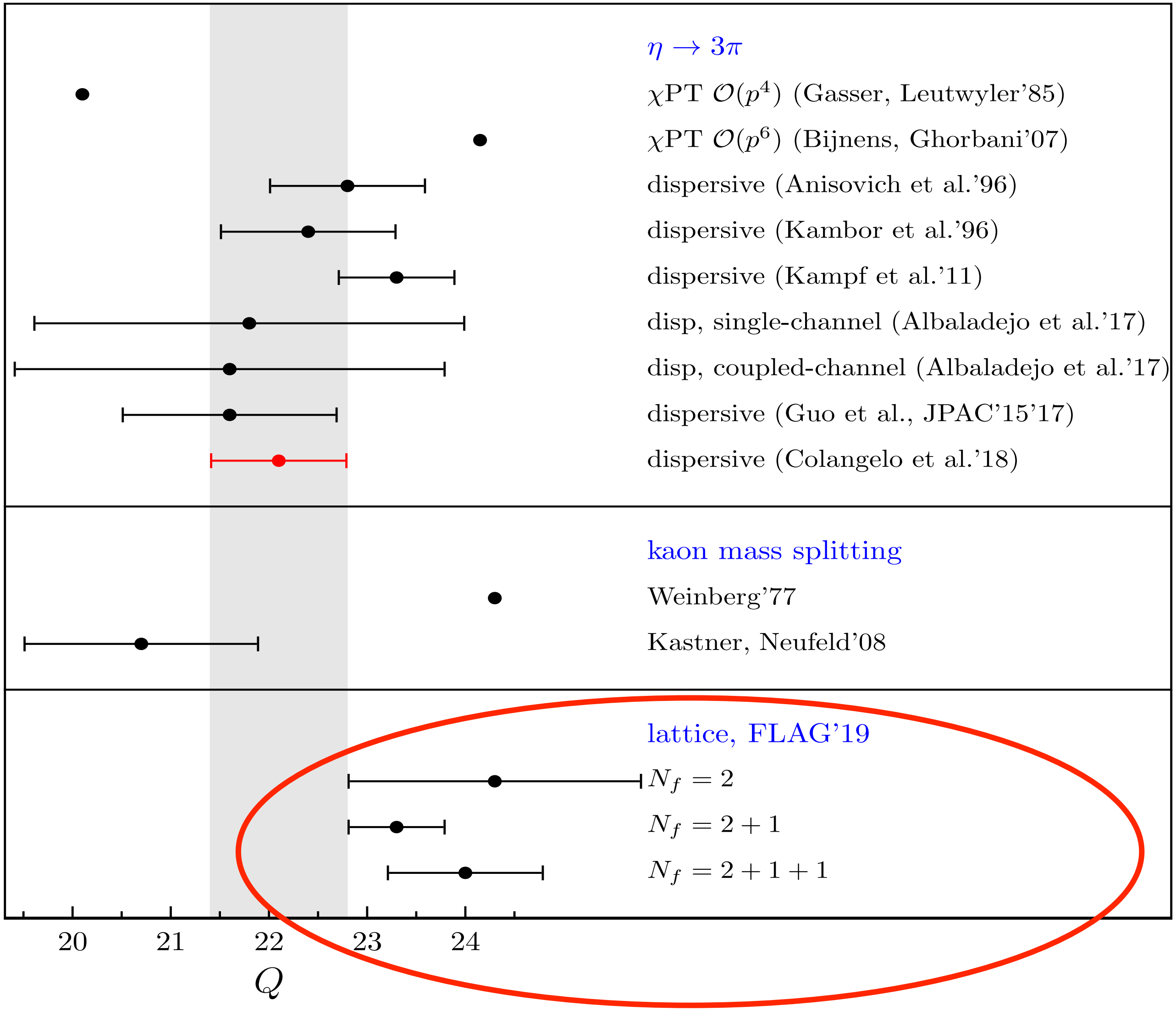


$$\boxed{Q = 22.1 \pm 0.7}$$

$$\boxed{\frac{m_u}{m_d} = 0.44 \pm 0.03}$$

- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

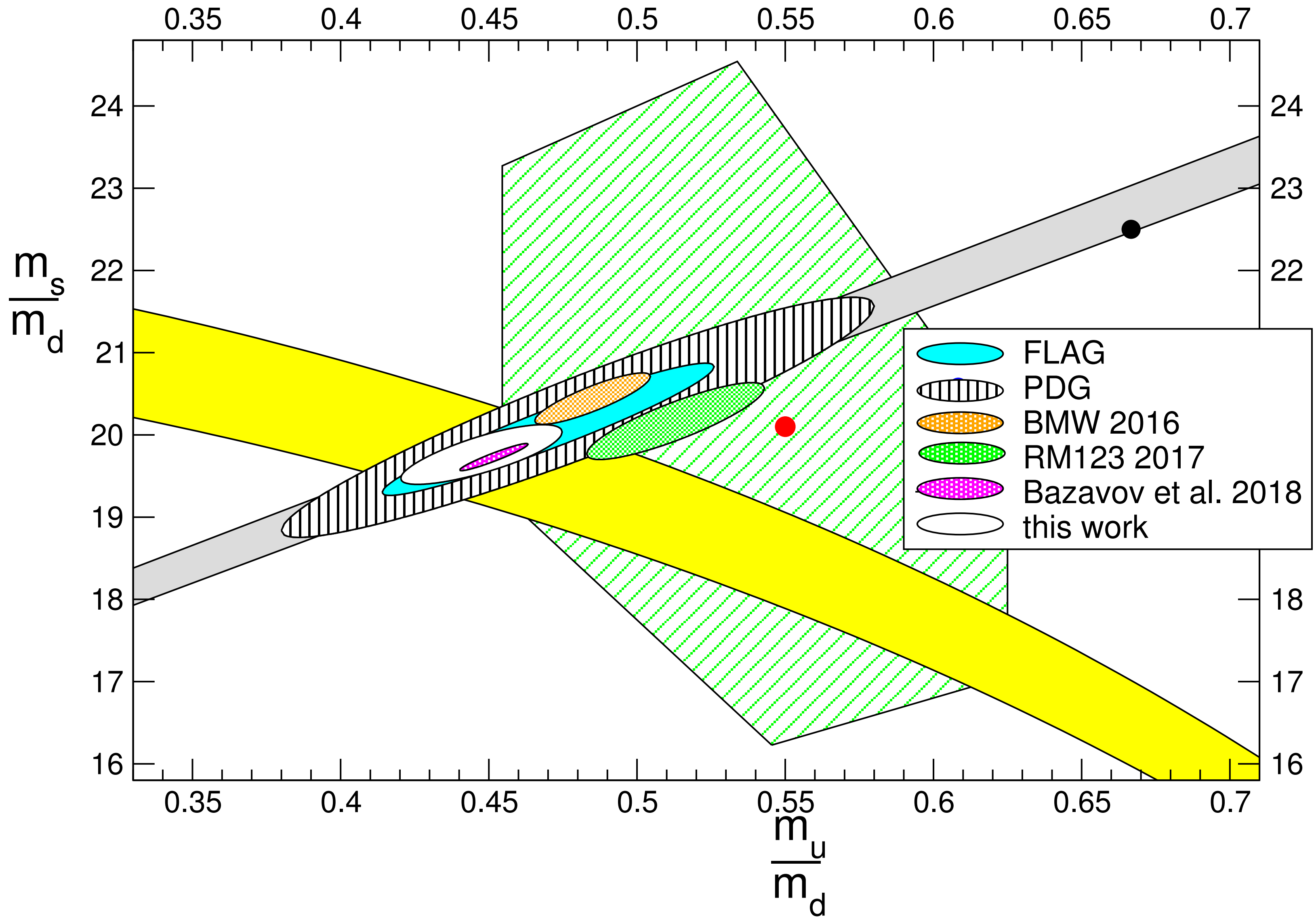
4.3 Comparison with Lattice



$Q = 22.1 \pm 0.7$

- Tension with lattice results

4.3 Comparison with Lattice



4.3 Comparison with Lattice

- Low energy theorem:

  Gell-Mann-Oakes-Renner relations:

  (meson mass)^2 = (spontaneous ChSB) x (explicit ChSB)

  $\langle \bar{q} q \rangle$ ↗ ↖ m_q

- From LO ChPT without e.m effects:

$$
\begin{aligned}
M^2_{\pi^+} &= (m_u + m_d)\, B_0 + O(m^2)\\
M^2_{K^+} &= (m_u + m_s)\, B_0 + O(m^2)\\
M^2_{K^0} &= (m_d + m_s)\, B_0 + O(m^2)
\end{aligned}
$$

- Electromagnetic effects: *Dashen's theorem*

$$
\boxed{\left(M^2_{K^+} - M^2_{K^0}\right)_{em} - \left(M^2_{\pi^+} - M^2_{\pi^0}\right)_{em} = O\left(e^2 m\right)}
$$

Dashen'69

$$
\left.\right\} \boxed{
\begin{aligned}
M^2_{\pi^0} &= B_0\left(m_u + m_d\right)\\
M^2_{\pi^+} &= B_0\left(m_u + m_d\right) + \Delta_{em}\\
M^2_{K^0} &= B_0\left(m_d + m_s\right)\\
M^2_{K^+} &= B_0\left(m_u + m_s\right) + \Delta_{em}
\end{aligned}
}
$$

2 unknowns B_0 and Δ_{em}

Meson masses from ChPT

- Quark mass ratios

Weinberg'77

$$\frac{m_u}{m_d} \overset{\text{LO}}{=} \frac{M_{K^+}^2 - M_{K^0}^2 + 2M_{\pi^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} = 0.56\,,$$

$$\frac{m_s}{m_d} \overset{\text{LO}}{=} \frac{M_{K^+}^2 + M_{K^0}^2 - M_{\pi^+}^2}{M_{K^0}^2 - M_{K^+}^2 + M_{\pi^+}^2} = 20.2$$

Meson masses from ChPT

- Mass formulae to second chiral order

Gasser & Leutwyler'85

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}}\left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}}\left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

with $\Delta_M = \frac{8(M_K^2 - M_\pi^2)}{F_\pi^2}(2L_8 - L_5) + \chi\text{-logs}$

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

- The same O(m) correction appears in both ratios

⇨ Take the double ratio

$$\boxed{Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}} = \frac{M_K^2}{M_\pi^2}\frac{M_K^2 - M_\pi^2}{\left(M_{K^0}^2 - M_{K^+}^2\right)_{QCD}}\left[1 + O(m_q^2, e^2)\right]$$

Very Interesting quantity to determine since Q^2 does not receive any correction at NLO!

Meson masses from ChPT

- Mass formulae to second chiral order

Gasser & Leutwyler'85

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}}\left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}}\left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

with $\Delta_M = \frac{8(M_K^2 - M_\pi^2)}{F_\pi^2}(2L_8 - L_5) + \chi$-logs

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

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⇨ Take the double ratio

$$\boxed{Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}} = \frac{M_K^2}{M_\pi^2}\frac{M_K^2 - M_\pi^2}{\left(M_{K^0}^2 - M_{K^+}^2\right)_{QCD}}\left[1 + O(m_q^2, e^2)\right]$$

$$\left(1 + \Delta Q\right)$$

- In our calculation we take $\Delta Q = 0$

4.3 Comparison with Lattice

- Mass formulae to second chiral order

Gasser & Leutwyler'85

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{2\hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

$$\Rightarrow \frac{2M_K^2}{M_\pi^2} = (S+1)(1+\Delta_S)$$ with $$\boxed{S = \frac{m_s}{\hat{m}}}$$

$$\frac{M_{K^0}^2 - M_{K^+}^2}{M_K^2 - M_\pi^2} = \frac{m_d - m_u}{m_s - \hat{m}} \left[1 + \Delta_M + \mathcal{O}(m^2)\right]$$

$$\Rightarrow \frac{M_K^2 - M_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2} = R(1+\Delta_R)$$ with $$\boxed{R = \frac{m_s - \hat{m}}{m_d - m_u}}$$

$$\Rightarrow \boxed{2Q^2 \equiv R(S+1)} \Rightarrow \boxed{(1+\Delta_Q) = (1+\Delta_S)(1+\Delta_R)}$$

4.3 Comparison with Lattice

$$\frac{2M_K^2}{M_\pi^2} = (S+1)(1+\Delta_S) \quad \text{with} \quad \boxed{S = \frac{m_s}{\hat{m}}}$$

$$\left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

$$\frac{M_K^2 - M_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2} = R(1+\Delta_R) \quad \text{with} \quad \boxed{R = \frac{m_s - \hat{m}}{m_d - m_u}}$$

$$\boxed{2Q^2 \equiv R(S+1)} \quad \Rightarrow \quad \boxed{(1+\Delta_Q) = (1+\Delta_S)(1+\Delta_R)}$$

	Q	Δ_S	Δ_R	Δ_Q
BMW [92]	23.4(6)	−0.063	−0.028	−0.089
RM123 [93]	23.8(1.1)	−0.042	−0.060	−0.099
this work	22.1(7)	−0.051(12)	+0.053(14)	0

Important corrections for Δ_Q from lattice QCD in contradiction with convergence of chiral series!

5. Conclusion and Outlook

Conclusion and Outlook

- $\eta \rightarrow 3\pi$ gives a unique opportunity to access the light quark mass double ratio Q experimentally

- To do so we need a parametrization of the amplitude + fix the normalization

- To extract Q with the best precision: Development of amplitude analysis techniques consistent with analyticity, unitarity, crossing symmetry dispersion relations allow to take into account *all rescattering effects* being as model independent as possible combined with ChPT → Provide very precise and robust parametrization for experimental studies especially to extract Q → systematic uncertainties to be extracted

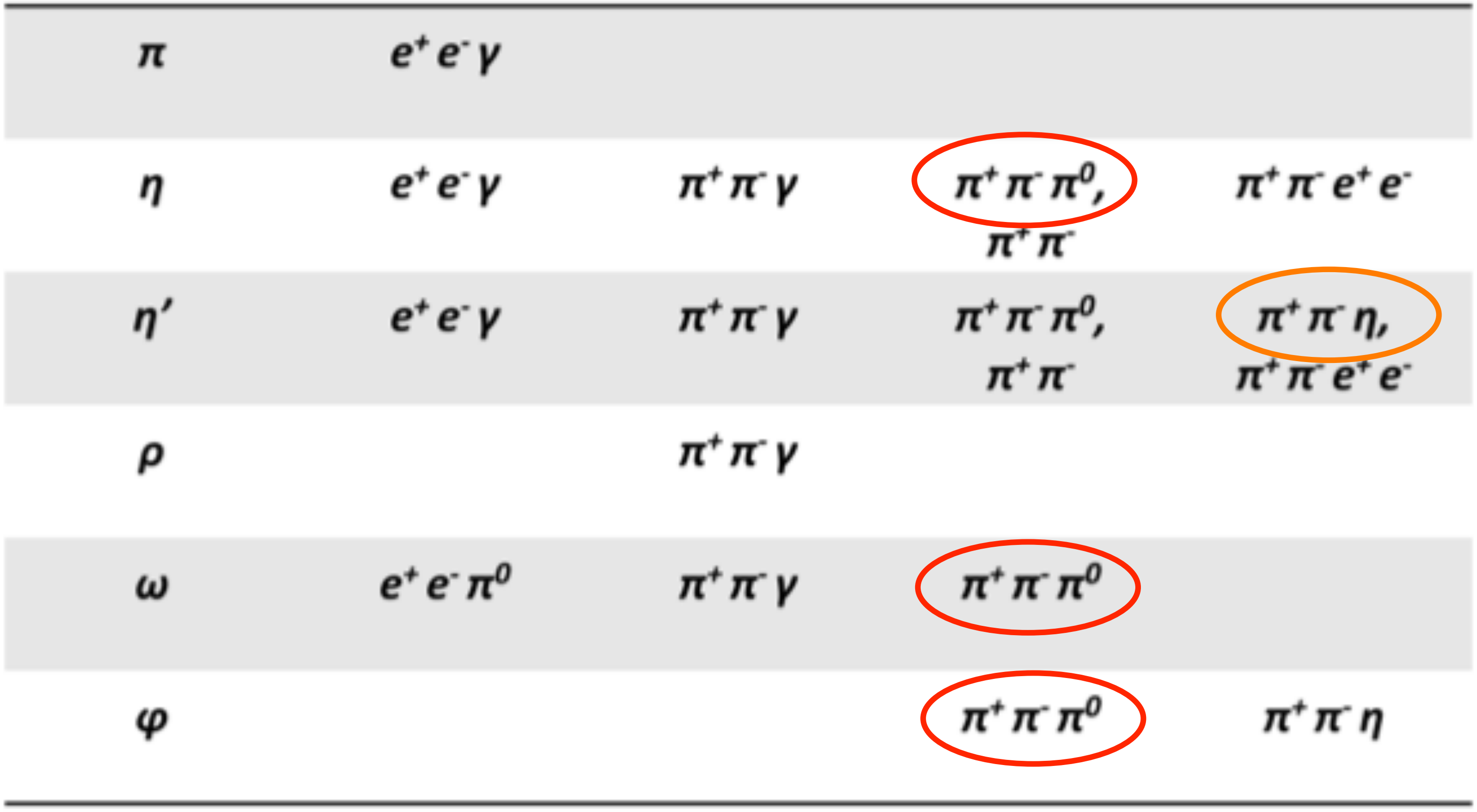
- Charged channel and neutral channels give results consistent → good check

- Tensions with some lattice results exist → need to be understood.

6. Back-up

Experimental Facilities and Role of JLab 12

M. J. Amaryan et al.
CLAS Analysis Proposal, (2014)



π	$e^+e^-\gamma$			
η	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-e^+e^-$
η'	$e^+e^-\gamma$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$, $\pi^+\pi^-$	$\pi^+\pi^-\eta$, $\pi^+\pi^-e^+e^-$
ρ		$\pi^+\pi^-\gamma$		
ω	$e^+e^-\pi^0$	$\pi^+\pi^-\gamma$	$\pi^+\pi^-\pi^0$	
φ			$\pi^+\pi^-\pi^0$	$\pi^+\pi^-\eta$

2.3 Computation of the amplitude

- What do we know?

- Compute the amplitude using ChPT : the effective theory that describe dynamics of the Goldstone bosons (kaons, pions, eta) at low energy

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$ Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d\geq 2} \mathcal{L}_d \ , \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}}$$

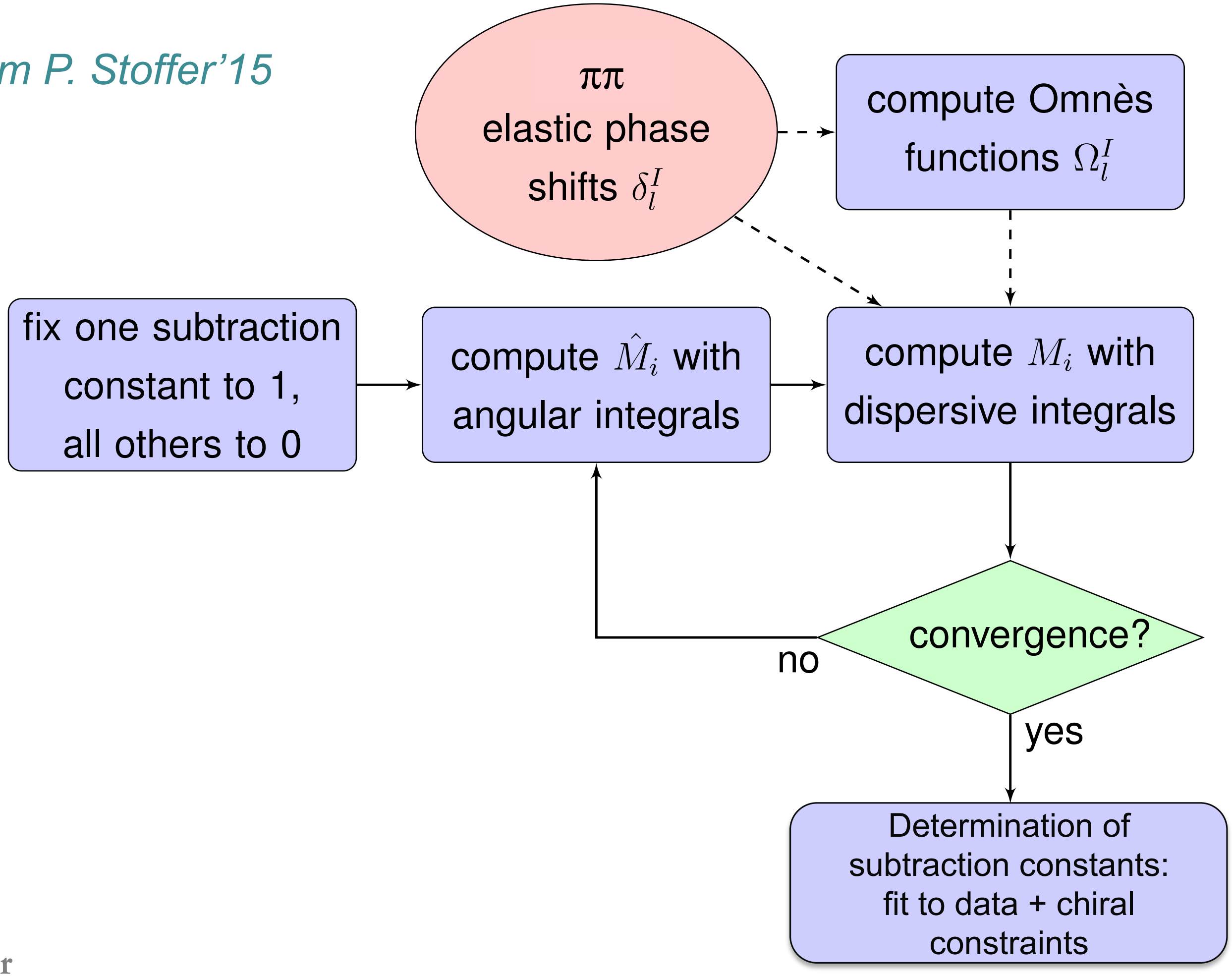
$$\boxed{\mathbf{p} << \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}}$$

2.5 Iterative Procedure

- Solution *linear* in the *subtraction constants* *Anisovich & Leutwyler'96*

$$M(s,t,u) = \alpha_0 M_{\alpha_0}(s,t,u) + \beta_0 M_{\beta_0}(s,t,u) + \dots$$

⇨ makes the fit much easier



Adapted from P. Stoffer'15

ππ elastic phase shifts δ_l^I

compute Omnès functions Ω_l^I

fix one subtraction constant to 1, all others to 0

compute $\hat{M}_i$ with angular integrals

compute M_i with dispersive integrals

convergence?

no

yes

Determination of subtraction constants: fit to data + chiral constraints

2.6 Subtraction constants

- Extension of the numbers of parameters compared to *Anisovich & Leutwyler'96*

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$

$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$

$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2$$

- In the work of *Anisovich & Leutwyler'96* matching to one loop ChPT
  Use of the SU(2) x SU(2) chiral theorem
  ⇨ The amplitude has an *Adler zero* along the line s=u

- Now data on the Dalitz plot exist from KLOE, WASA, MAMI and BES III

  ⇨ Use the data to directly fit the subtraction constants

- However normalization to be fixed to ChPT!

2.7 Subtraction constants

- The subtraction constants are

$$P_0(s) = \alpha_0 + \beta_0 s + \gamma_0 s^2 + \delta_0 s^3$$

$$P_1(s) = \alpha_1 + \beta_1 s + \gamma_1 s^2$$

$$P_2(s) = \alpha_2 + \beta_2 s + \gamma_2 s^2 + \delta_0 s^3$$

Only *6 coefficients* are of physical relevance

- They are determined from combining ChPT with a fit to KLOE Dalitz plot

- Taylor expand the dispersive M_I
  Subtraction constants ⇔ Taylor coefficients

$$M_0(s) = A_0 + B_0 s + C_0 s^2 + D_0 s^3 + \ldots$$

$$M_1(s) = A_1 + B_1 s + C_1 s^2 + \ldots$$

$$M_2(s) = A_2 + B_2 s + C_2 s^2 + D_2 s^3 +$$

- Gauge freedom in the decomposition of M(s,t,u)

2.7 Subtraction constants

- Build some gauge independent combinations of Taylor coefficients

$$H_0 = A_0 + \frac{4}{3}A_2 + s_0\left(B_0 + \frac{4}{3}B_2\right)$$

$$H_1 = A_1 + \frac{1}{9}\left(3B_0 - 5B_2\right) - 3C_2 s_0$$

$$H_2 = C_0 + \frac{4}{3}C_2, \qquad H_3 = B_1 + C_2$$

$$H_4 = D_0 + \frac{4}{3}D_2, \qquad H_5 = C_1 - 3D_2$$

$\Rightarrow$

$$H_0^{ChPT} = 1 + 0.176 + O\left(p^4\right)$$

$$h_1^{ChPT} = \frac{1}{\Delta_{\eta\pi}}\left(1 - 0.21 + O\left(p^4\right)\right)$$

$$h_2^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(4.9 + O\left(p^4\right)\right)$$

$$h_3^{ChPT} = \frac{1}{\Delta_{\eta\pi}^2}\left(1.3 + O\left(p^4\right)\right)$$

$$\left[h_i \equiv \frac{H_i}{H_0}\right]$$

$\Rightarrow$

$$\boxed{\chi^2_{theo} = \sum_{i=1}^{3}\left(\frac{h_i - h_i^{ChPT}}{\sigma_{h_i^{ChPT}}}\right)^2}$$

$$\boxed{\sigma_{h_i^{ChPT}} = 0.3\left|h_i^{NLO} - h_i^{LO}\right|}$$

Isospin breaking corrections

- Dispersive calculations in the isospin limit ⇨ to fit to data one has to include isospin breaking corrections
- $$\boxed{M_{c/n}(s,t,u) = M_{disp}(s,t,u)\frac{M_{DKM}(s,t,u)}{\tilde{M}_{GL}(s,t,u)}}$$

with M_{DKM} : amplitude at one loop with $\mathcal{O}(e^2 m)$ effects

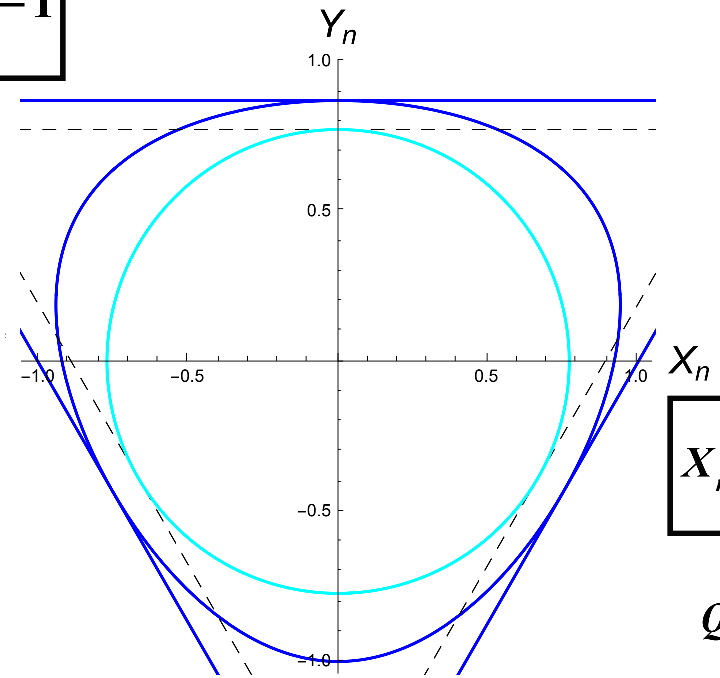
Ditsche, Kubis, Meissner'09

M_{GL}: amplitude at one loop in the isospin limit

Gasser & Leutwyler' 85

$$\boxed{Y_n = \frac{3T_3}{Q_n} - 1}$$

Neutral channel



$$\boxed{X_n = \sqrt{3}\frac{T_2 - T_1}{Q_n}}$$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

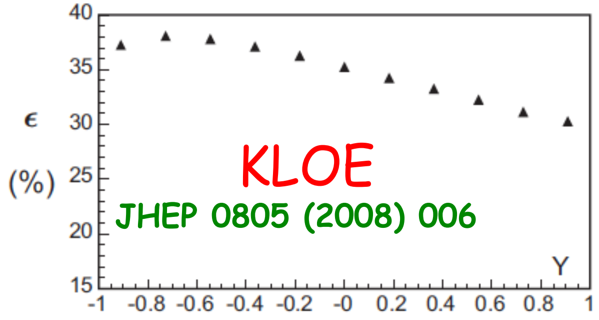
Kinematic map:
isospin symmetric boundaries

⇨ physical boundaries

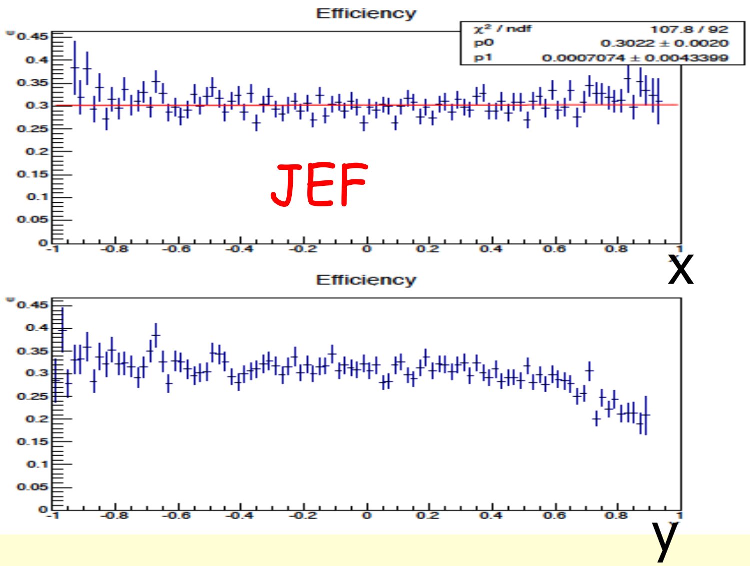
$$M_{GL} \rightarrow \tilde{M}_{GL}$$

2.15 Prospects

Exp.	$3\pi^0$ Events (10^6)	$\pi^+ \pi^- \pi^0$ Events (10^6)
Total world data (include prel. WASA and prel. KLOE)	6.5	6.0
GlueX+PrimEx-η +JEF	20	19.6



- Existing data from the low energy facilities are sensitive to the detection threshold effects
- JEF at high energy has uniform detection efficiency over Dalitz phase space
- JEF will offer large statistics and different systematics



2.3 Computation of the amplitude

- What do we know?
- The amplitude has an Adler zero: soft pion theorem *Adler'85*

⇒ Amplitude has a zero for :

$$p_{\pi^+} \to 0 \Rightarrow s = u = 0,\ t = M_\eta^2$$

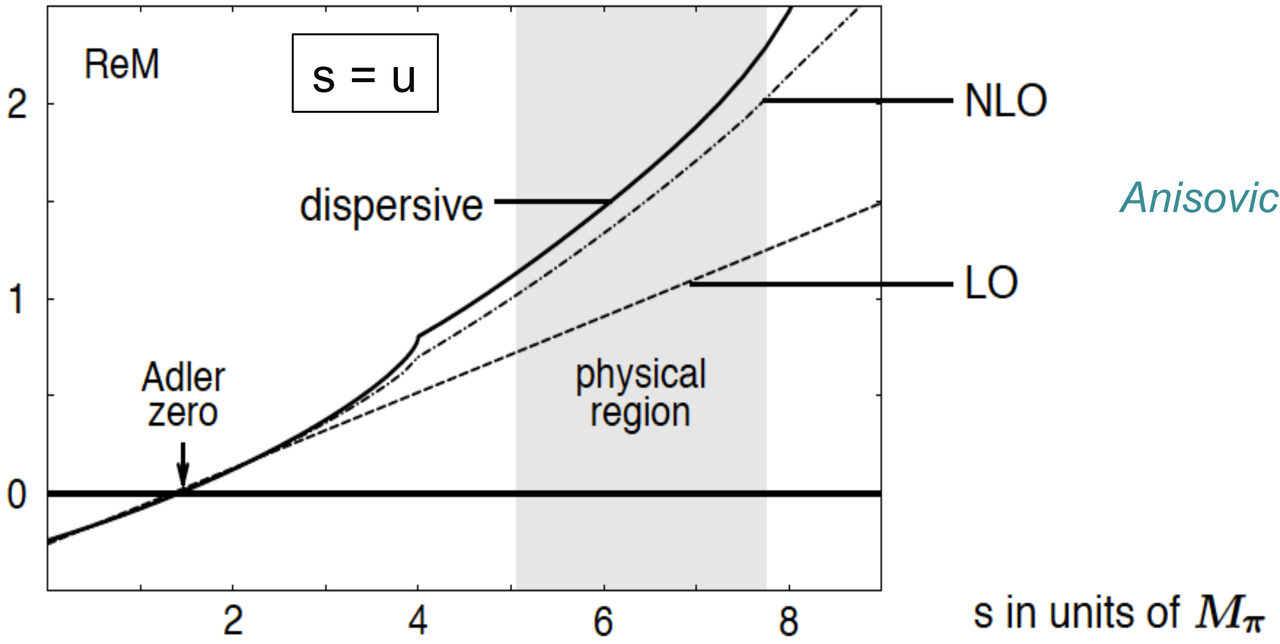
$$p_{\pi^-} \to 0 \Rightarrow s = t = 0,\ u = M_\eta^2$$

$$M_\pi \neq 0 \Rightarrow$$

$$s = u = \frac{4}{3} M_\pi^2,\ t = M_\eta^2 + \frac{M_\pi^2}{3}$$

$$s = t = \frac{4}{3} M_\pi^2,\ u = M_\eta^2 + \frac{M_\pi^2}{3}$$

SU(2) corrections



Anisovich & Leutwyler'96

2.4 Neutral channel : $\eta \to \pi^0 \pi^0 \pi^0$

- What do we know?

- We can relate charged and neutral channels

$$\boxed{\overline{A}(s,t,u) = A(s,t,u) + A(t,u,s) + A(u,s,t)}$$

⇨ *Correct formalism should be able to reproduce both charged and neutral channels*

- Ratio of decay width precisely measured

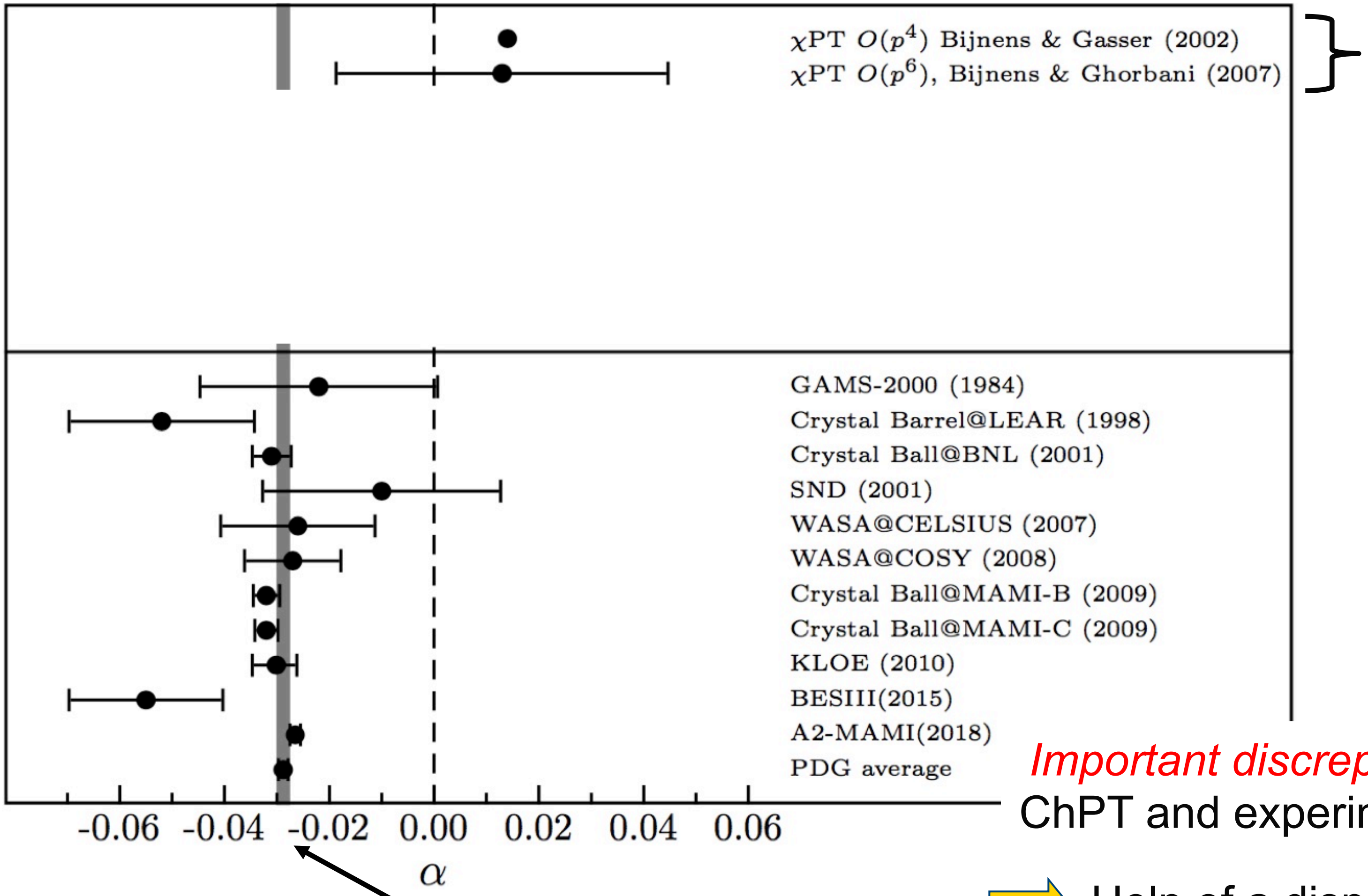
$$\boxed{r = \frac{\Gamma\left(\eta \to \pi^0\pi^0\pi^0\right)}{\Gamma\left(\eta \to \pi^+\pi^-\pi^0\right)}} = 1.426 \pm 0.026$$

PDG'19

2.4 Neutral Channel : $\eta \to \pi^0 \pi^0 \pi^0$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$

- Decay amplitude $\boxed{\Gamma_{\eta\to 3\pi} \propto \left|\bar{A}\right|^2 \propto 1+2\alpha Z}$ with $Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n}-1\right)^2$



$\boxed{\alpha \sim 0.015}$

$\boxed{\alpha = -0.0288 \pm 0.0012}$

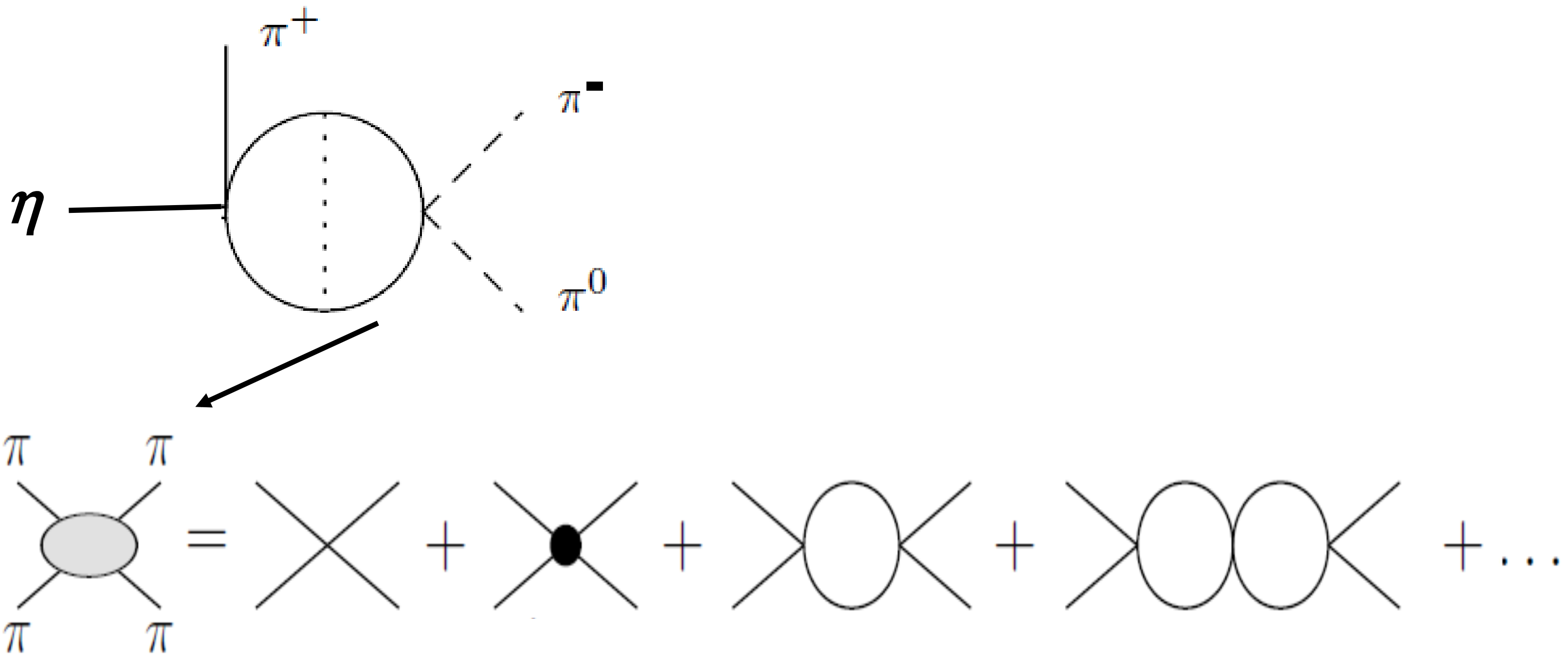
Important discrepancy between ChPT and experiment!

⇨ Help of a dispersive treatment?

2.5 Dispersive treatment

- The Chiral series has convergence problems

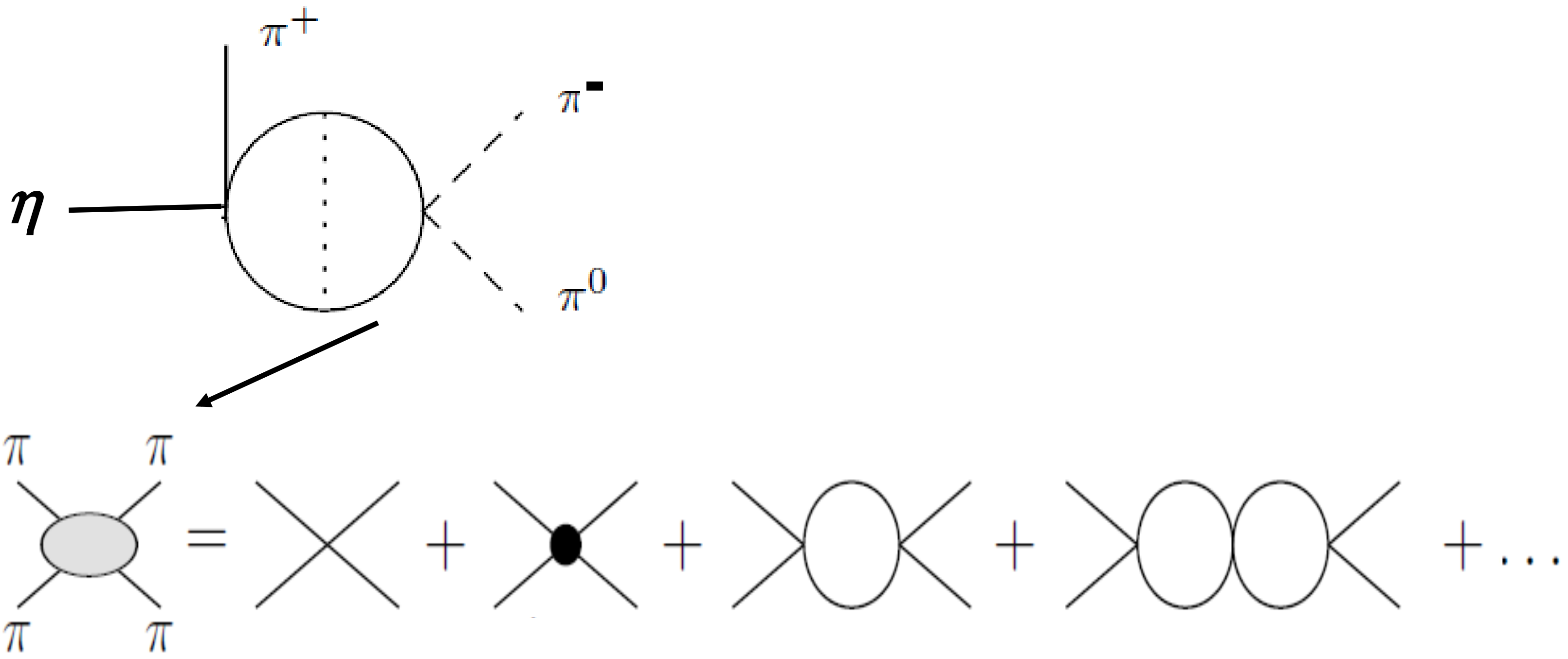
Large *$\pi\pi$ final state interactions* *Roiesnel & Truong’81*



2.5 Dispersive treatment

- The Chiral series has convergence problems

⇒ Large *$\pi\pi$ final state interactions* *Roiesnel & Truong'81*



- *Dispersive treatment :*
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction $\pi\pi$ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01
    Descotes-Genon et al'01
    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)
    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)
    BES III (Beijing)

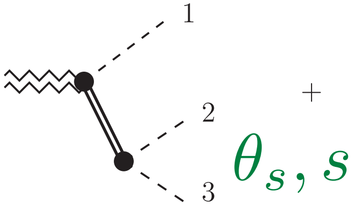
  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects *Gullstrom, Kupsc, Rusetsky'09, Schneider, Kubis, Ditsche'11*
    - Inelasticities *Albaladejo & Moussallam'15*

3. Dispersive analysis of $\eta \rightarrow 3\pi$

3.1 Method

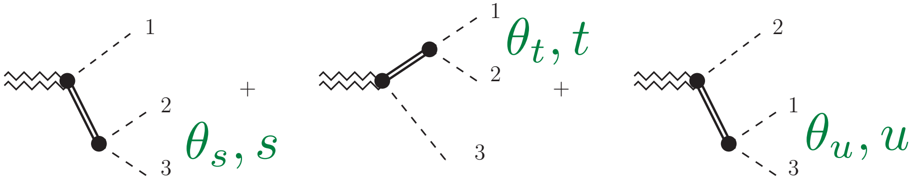
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^\infty (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ⇨ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$

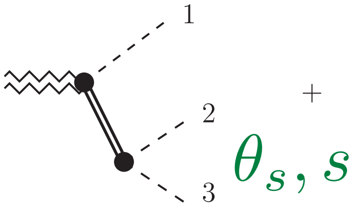


3 BWs (ρ^+, ρ^-, ρ^0) + background term

⇨ Improve to include final states interactions

3.1 Method

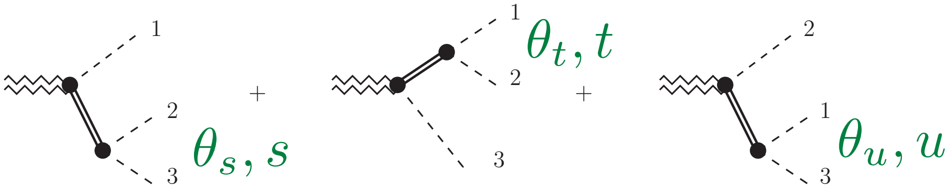
- S-channel partial wave decomposition

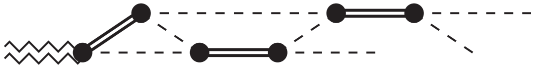


$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$

- One truncates the partial wave expansion : ⇨ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$





- Use a Khuri-Treiman approach or dispersive approach

⇨ Restore 3 body unitarity and take into account the final state interactions in a systematic way

3.2 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)}$$

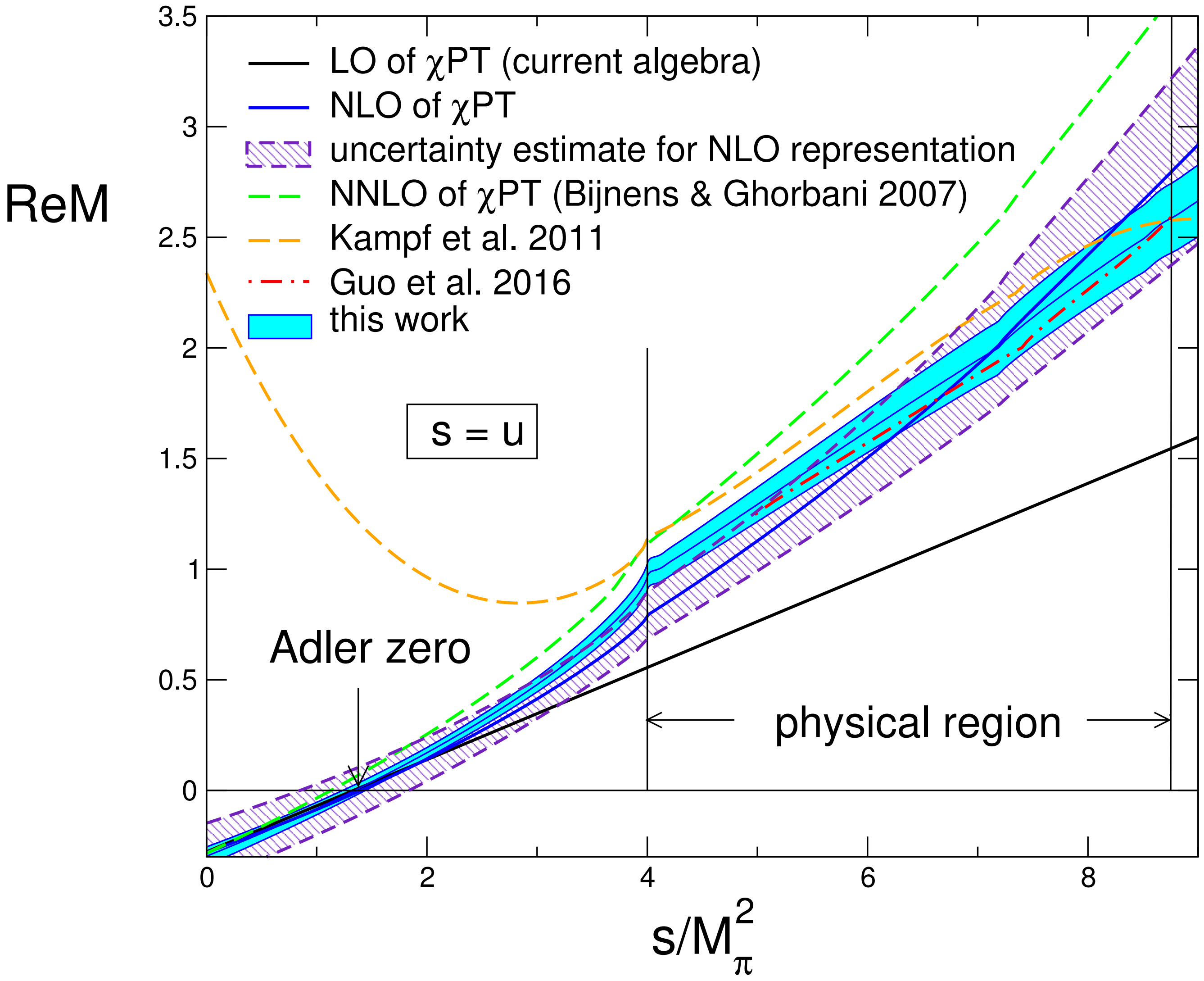
Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

- ➢ M_I isospin *I* rescattering in two particles
- ➢ Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
- ➢ Main two body rescattering corrections inside M_I

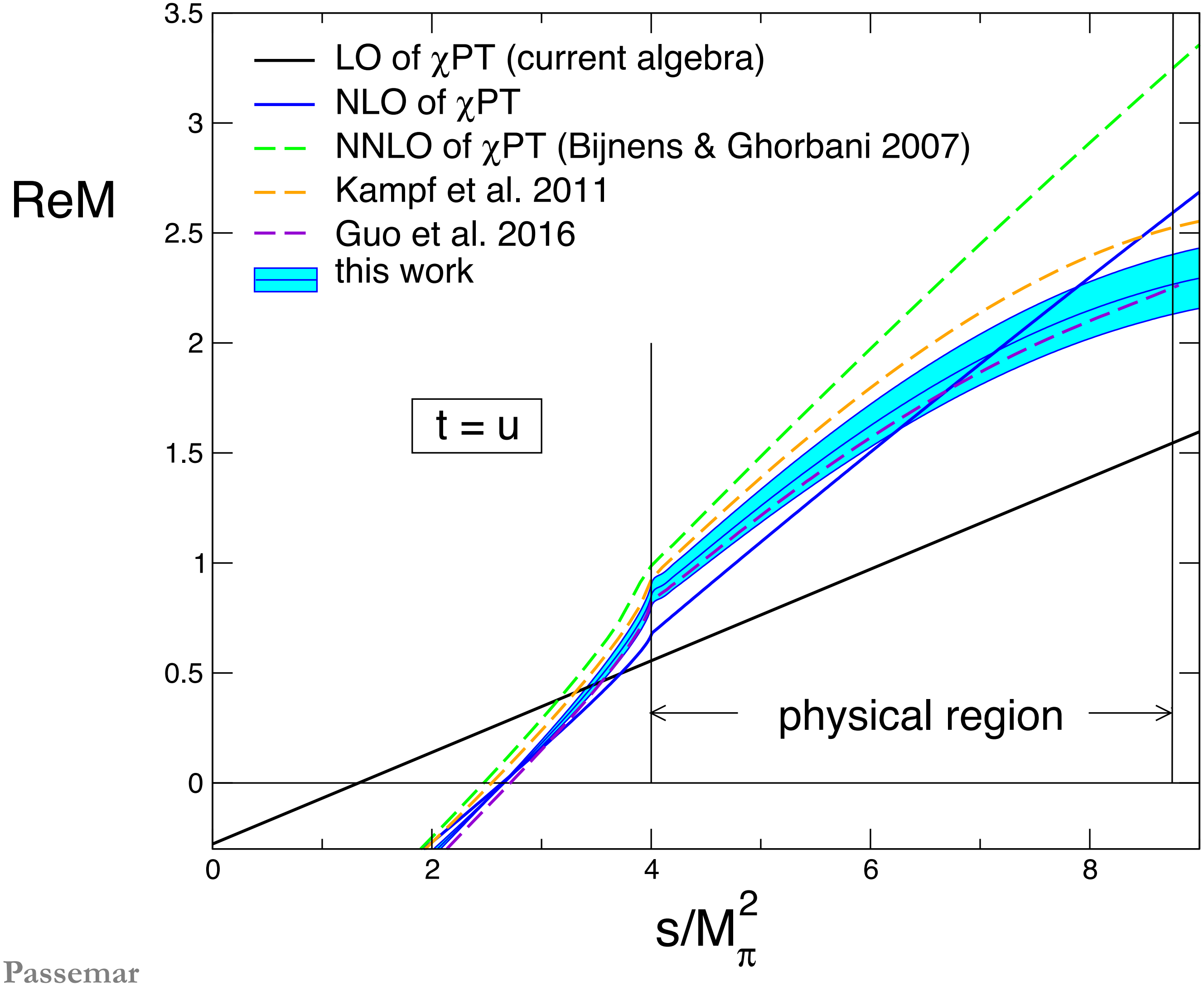
3.4 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :

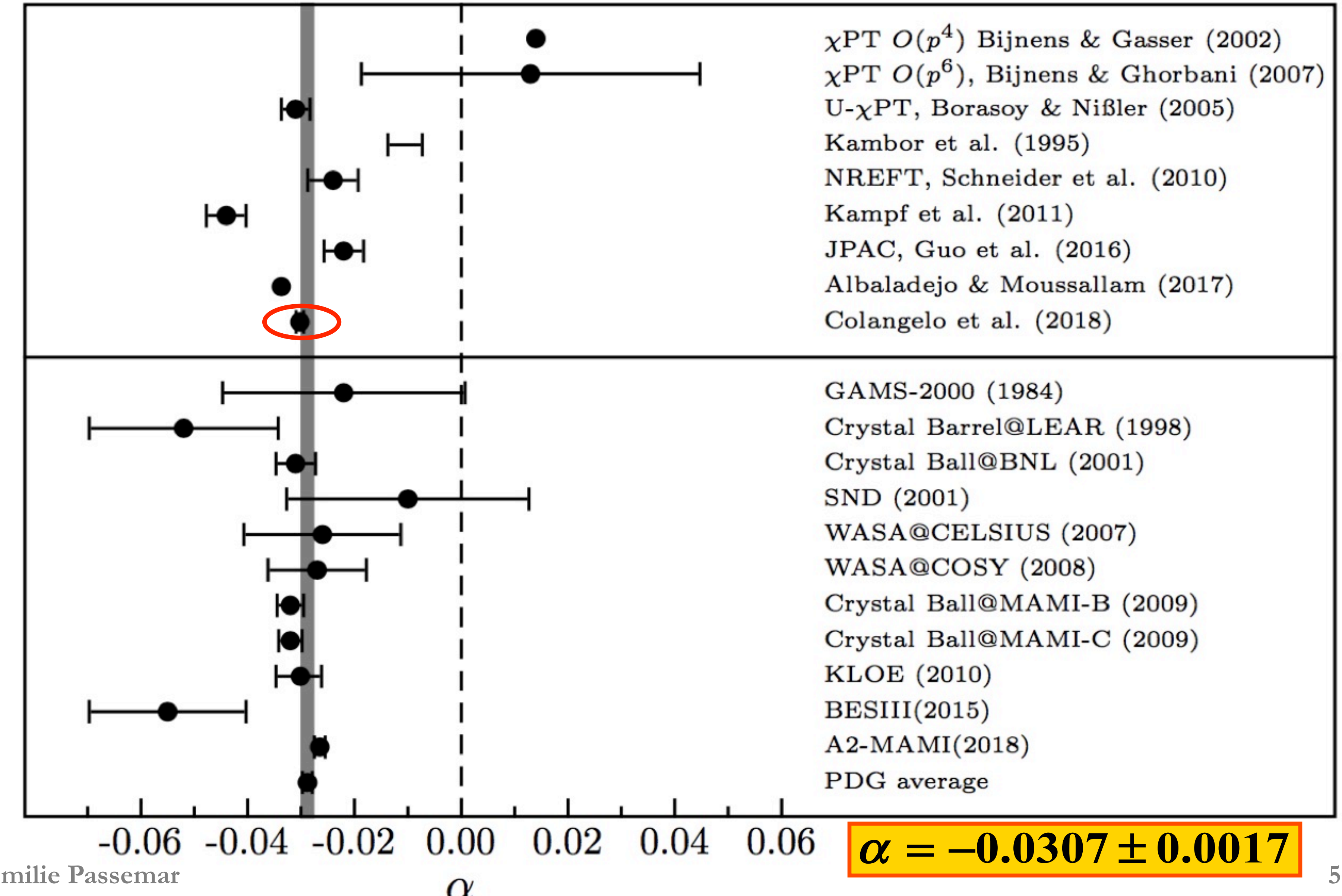


3.4 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



2.12 Comparison of results for α



$$\alpha = -0.0307 \pm 0.0017$$