Theoretical analysis of the doubly radiative decays $\eta^{(\prime)} \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$

10th International Workshop on Chiral Dynamics, Beijing (China), 15-19 November, 2021

Sergi Gonzàlez-Solís

Email: sergig@lanl.gov

Theoretical Division, Los Alamos National Laboratory

Based on:
- R. Escribano, S. Gonzàlez-Solís, R. Jora and E. Royo, Phys.Rev.D 102 (2020) 3, 034026

Los Alamos NATIONAL LABORATORY

- η and η' decays offer **fantastic opportunities** to:
  - test low-energy QCD
  - search for New Physics beyond SM
- **High priority** $\eta^{(\prime)}$ decays for experiment and theory

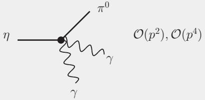
  (L. Gan, B. Kubis, E. Passemar and S. Tulin, 2007.00664)

Decay channel	Standard Model	Discrete symmetries	BSM particles
$\eta^{(\prime)} \to \pi^+\pi^-\pi^0$	light quark masses	C/CP violation	scalar bosons
$\eta^{(\prime)} \to \gamma\gamma$	η-η' mixing, width	—	—
$\eta^{(\prime)} \to \ell^+\ell^-\gamma$	$(g-2)_\mu$	—	Z', dark photon
$\eta^{(\prime)} \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$	higher-order χPT, scalar dynamics	—	$U(1)_B$ boson, scalar boson
$\eta^{(\prime)} \to \mu^+\mu^-$	$(g-2)_\mu$, precision tests	CP violation	—
$\eta^{(\prime)} \to \pi^0\ell^+\ell^-$	—	C violation	scalar bosons
$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$	$(g-2)_\mu$	—	ALP, dark photon
$\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$	—	C violation	ALP

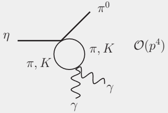
- **Important** experimental activities: A2, Belle-II, BESIII, GlueX, LHCb, KLOE-II, WASA-at-COSY
- **Forthcoming** experiments: JLab Eta Factory (JEF) and REDTOP

$\eta \to \pi^0\gamma\gamma$ decays: Theoretical motivation

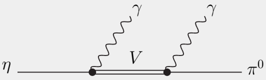
- **SM motivation**:



Reference	$\Gamma(\eta \to \pi^0\gamma\gamma)$ [eV]
$\mathcal{O}(p^2)$, $\mathcal{O}(p^4)$ tree-level χPT	0
$\pi + K$ loops at $\mathcal{O}(p^4)$	1.87×10^{-3}
pdg	0.34(3)



  - 1^{st} sizable contribution comes at $\mathcal{O}(p^6)$, but LEC's are not well known
  - To **test ChPT** and a wide range of chiral models, *e. g.* VMD and LσM

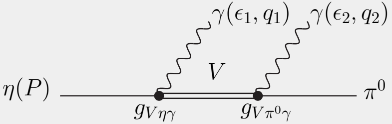


- **BSM motivation**: search for a *B* boson via $\eta \to B\gamma \to \pi^0\gamma\gamma$

(see talks by: Gatto, Somov, Kang)

$\eta \to \pi^0\gamma\gamma$ DECAYS: VMD CALCULATION

- Six diagrams corresponding to the exchange of $V = \rho^0, \omega, \phi$



$$\mathcal{A}^{\text{VMD}}_{\eta\to\pi^0\gamma\gamma} = \sum_{V=\rho^0,\omega,\phi} g_{V\eta\gamma} g_{V\pi^0\gamma} \left[\frac{(P\cdot q_2 - m_\eta^2)\{a\} - \{b\}}{D_V(t)} + \left\{ \begin{array}{c} q_2 \leftrightarrow q_1 \\ t \leftrightarrow u \end{array} \right\} \right] ,$$

- Mandelstam variables and Lorentz structures given by:

$$\begin{aligned} t, u &= (P - q_{2,1})^2 = m_\eta^2 - 2P\cdot q_{2,1} , \\ \{a\} &= (\epsilon_1\cdot\epsilon_2)(q_1\cdot q_2) - (\epsilon_1\cdot q_2)(\epsilon_2\cdot q_1) , \\ \{b\} &= (\epsilon_1\cdot q_2)(\epsilon_2\cdot P)(P\cdot q_1) + (\epsilon_2\cdot q_1)(\epsilon_1\cdot P)(P\cdot q_2) \\ &\quad - (\epsilon_1\cdot\epsilon_2)(P\cdot q_1)(P\cdot q_2) - (\epsilon_1\cdot P)(\epsilon_2\cdot P)(q_1\cdot q_2) , \end{aligned}$$

- The decays $\eta' \to \{\pi^0, \eta\}\gamma\gamma$ are formally identical, with:

$$g_{V\eta\gamma} g_{V\pi^0\gamma} \to g_{V\eta'\gamma} g_{V\{\pi^0,\eta\}\gamma}$$

Input for the $g_{VP\gamma}$ couplings

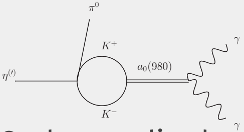
- $g_{VP\gamma}$ couplings fixed from the measured widths ($P = \pi^0, \eta, \eta'$)

$$\Gamma^{\rm exp}_{V\to P\gamma} = \frac{1}{3}\frac{g^2_{VP\gamma}}{32\pi}\left(\frac{m_V^2 - m_P^2}{m_V}\right)^3 , \quad \Gamma^{\rm exp}_{P\to V\gamma} = \frac{g^2_{VP\gamma}}{32\pi}\left(\frac{m_P^2 - m_V^2}{m_P}\right)^3 ,$$

Decay	**Branching ratio** (pdg)	$\lvert g_{VP\gamma}\rvert$ GeV^{-1}
$\rho^0 \to \pi^0\gamma$	$(4.7 \pm 0.6) \times 10^{-4}$	0.22(1)
$\rho^0 \to \eta\gamma$	$(3.00 \pm 0.21) \times 10^{-4}$	0.48(2)
$\eta' \to \rho^0\gamma$	$(28.9 \pm 0.5)\%$	0.40(1)
$\omega \to \pi^0\gamma$	$(8.40 \pm 0.22)\%$	0.70(1)
$\omega \to \eta\gamma$	$(4.5 \pm 0.4) \times 10^{-4}$	0.135(6)
$\eta' \to \omega\gamma$	$(2.62 \pm 0.13)\%$	0.127(4)
$\phi \to \pi^0\gamma$	$(1.30 \pm 0.05) \times 10^{-3}$	0.041(1)
$\phi \to \eta\gamma$	$(1.303 \pm 0.025)\%$	0.2093(20)
$\phi \to \eta'\gamma$	$(6.22 \pm 0.21) \times 10^{-5}$	0.216(4)

LσM FOR THE SCALAR RESONANCE CONTRIBUTIONS

- χPT loops complemented by the exchange of scalar resonances, $a_0(980), \kappa, \sigma, f_0(980)$, e.g.:



$$\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{\eta^{(\prime)}\to\pi^0\gamma\gamma} = \frac{2\alpha}{\pi}\frac{1}{m^2_{K^+}}L(s_K)\{a\} \times \mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{K^+K^-\to\pi^0\eta^{(\prime)}} \, ,$$

- Scalar amplitudes:

$$\mathcal{A}^{\mathrm{L}\sigma\mathrm{M}}_{K^+K^-\to\pi^0\eta^{(\prime)}} = \frac{1}{2f_\pi f_K}\Big\{(s-m^2_{\eta^{(\prime)}})\frac{m_K^2-m_{a_0}^2}{D_{a_0}(s)}\cos\varphi_P + \frac{1}{6}\Big[(5m^2_{\eta^{(\prime)}}+m_\pi^2-3s)\cos\varphi_P - \sqrt{2}(m^2_{\eta^{(\prime)}}+4m_K^2+m_\pi^2-3s)\sin\varphi_P\Big]\Big\} \, ,$$

- Complete one-loop propagator for the scalar resonances:

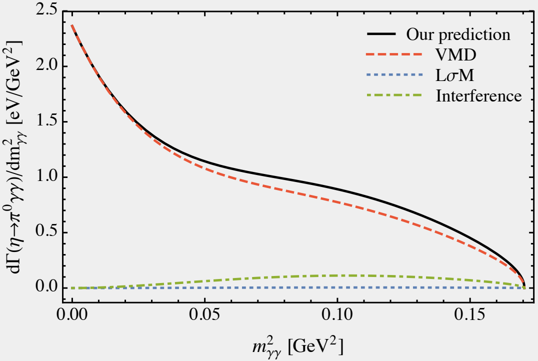
$$D_R(s) = s - m_R^2 + \mathrm{Re}\Pi(s) - \mathrm{Re}\Pi(m_R^2) + i\,\mathrm{Im}\Pi(s) \, ,$$

$\eta \to \pi^0\gamma\gamma$ PREDICTIONS

- Our theoretical prediction $BR = 1.35(8) \times 10^{-4}$

  (R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))

  - VMD dominates:
  - ρ: 27% of the signal
  - ω: 21% of the signal
  - ϕ: 0% of the signal
  - interference between ρ-ω-ϕ: 52%
  - interference between scalar and vector mesons: 7%

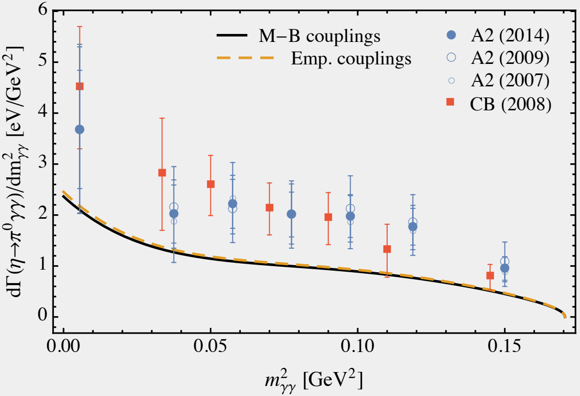


$\eta \to \pi^0\gamma\gamma$ PREDICTIONS VS DATA (SPECTRA)

- VMD comparison with A2 and CB data

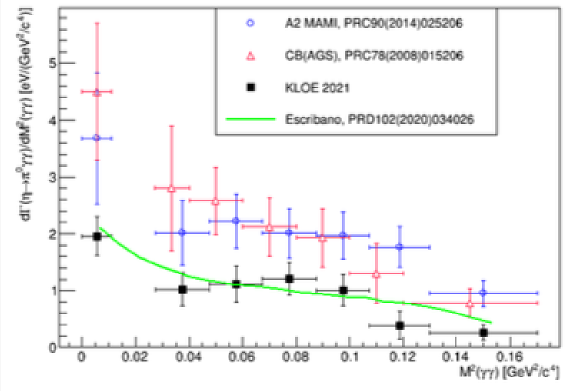
  (R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))

  - Shape of the spectra is captured well
  - Normalization offset



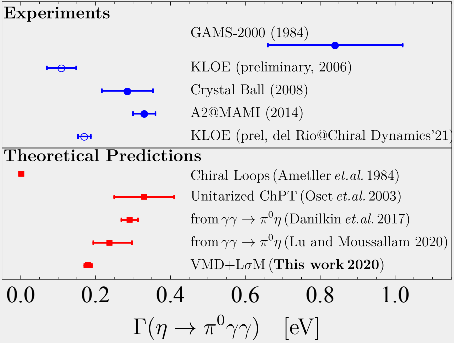
$\eta \to \pi^0\gamma\gamma$ PREDICTIONS VS DATA (SPECTRA)

- Comparison with KLOE preliminary results (See talk by Elena P. del Rio, figure taken from her talk)
    - Good agreement



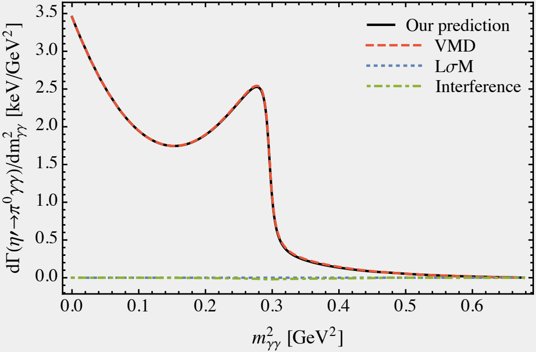
$\eta \to \pi^0\gamma\gamma$ PREDICTIONS VS DATA (BR)

- Our prediction, BR$= 1.35(8) \times 10^{-4}$, agrees with BR$= 1.30(13) \times 10^{-4}$ (KLOE prel, talk by del Rio)
- KLOE-II final measurement is forthcoming
- JEF experiment:
  (see talk by Somov)
  - BR and Dalitz distribution with $\sim 5\%$ precision
  - Improved understanding of the interplay of meson resonances
  - $\mathcal{O}(p^6)$ LEC's determination



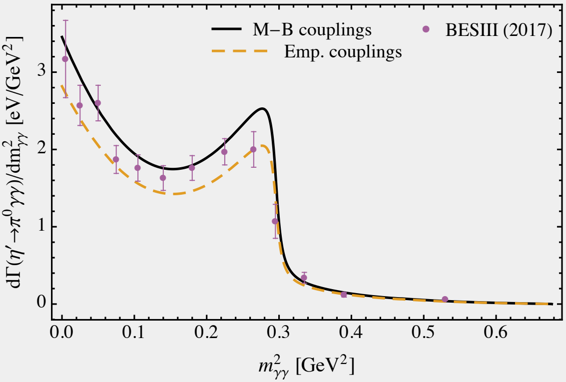
$\eta' \to \pi^0\gamma\gamma$ DECAYS

- Our theoretical predictions $BR = [2.91(21), 3.57(25)] \times 10^{-3}$
  (R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))
  - VMD completely dominates:
  - ω: 78% of the signal
  - ρ: 5% of the signal
  - ϕ: 0% of the signal
  - interference: 17%

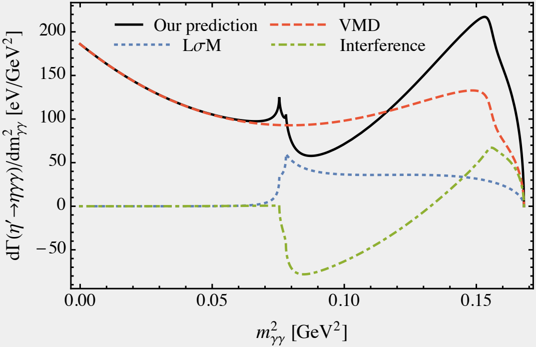


$\eta' \to \pi^0\gamma\gamma$ DECAYS

- Our theoretical predictions $BR = [2.91(21), 3.57(25)] \times 10^{-3}$ (R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))
- First time $m_{\gamma\gamma}$ invariant mass distribution by the BESIII coll.; $BR = 3.20(7)(23) \times 10^{-3}$ (Ablikim *et al.* Phys.Rev.D 96, 012005 (2017))



- 1[st] BR measurement by BESIII, $BR = 8.25(3.41)(0.72) \times 10^{-5}$ or $BR < 1.33 \times 10^{-4}$ at 90% C.L. (Ablikim *et al.* Phys.Rev.D 100, 052015 (2019))
- Our theoretical predictions $BR = [1.07(7), 1.17(8)] \times 10^{-4}$ (R. Escribano, S. G-S, R. Jora, E. Royo, Phys.Rev.D 102, 034026 (2020))
  - VMD predominates (91% of the signal)
  - Substantial scalar meson effects (16%)
  - Interference between scalar and vector mesons (7%)



- We look forward to the release of the $m_{\gamma\gamma}$ spectrum

Outlook

- Within the VMD and LσM frameworks **we have described**
  - $\eta \to \pi^0\gamma\gamma$: the situation is **not conclusive**

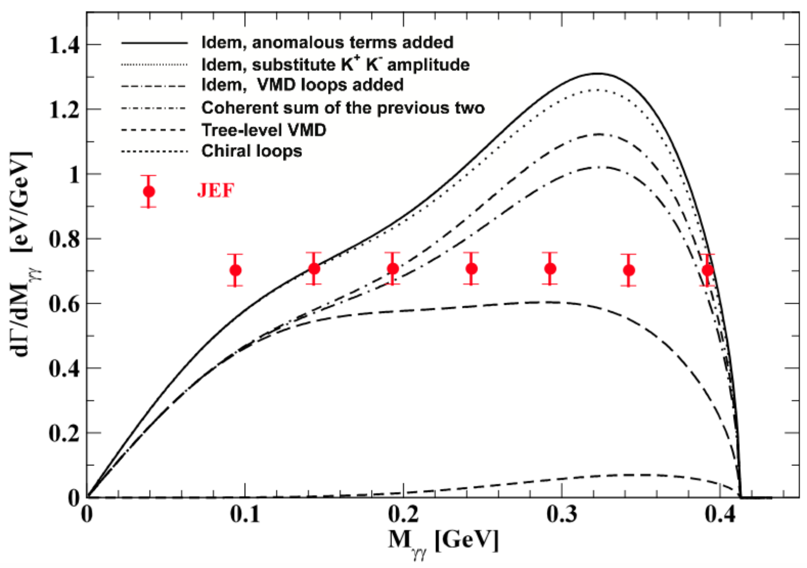
$$BR = 1.35(8)\times 10^{-4} \begin{cases} \sim 1/2 \text{ of } BR = 2.54(27)\times 10^{-4} & \text{(A2, 2014)} \\ \sim 1.6\sigma \text{ from } BR = 2.21(24)(47)\times 10^{-4} & \text{(CB, 2008)} \\ \text{agrees with } BR = 1.30(13)\times 10^{-4} & \text{(prel. KLOE, del Rio CD'21)} \end{cases}$$

  - $\eta' \to \pi^0\gamma\gamma$: **in fair agreement** with BESIII data
  - $\eta' \to \eta\gamma\gamma$: **in line** with BESIII data
- **Important experimental activity**: A2, Belle-II, BESIII, KLOE-II, GlueX, WASA. The contribution of **new experiments** (JEF, REDTOP), will be very welcome
- **Search for New Physics BSM**, e.g. $U(1)_B$ boson, requires: robust SM predictions and precise experiments
- A lot of **interesting physics** to be done in the η/η' sector

Phenomenological $VP\gamma$ couplings

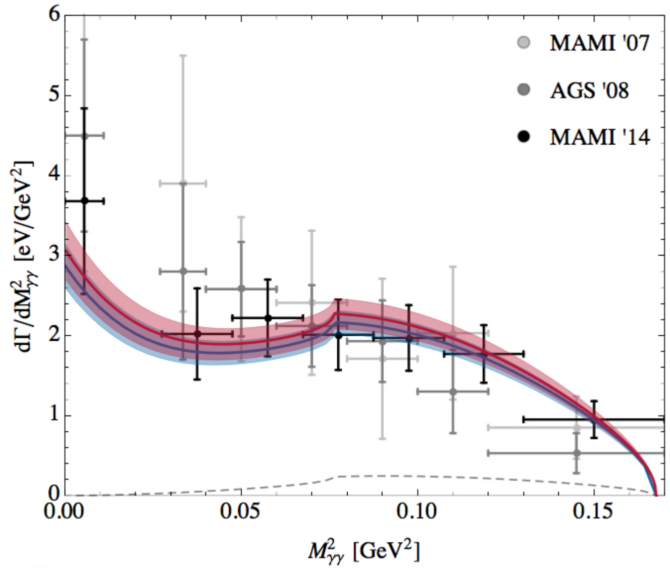
$$
\begin{aligned}
g_{\rho^0\pi^0\gamma} &= \frac{1}{3}g\,,\\
g_{\rho^0\eta\gamma} &= g z_{\mathrm{NS}}\cos\varphi_P\,,\\
g_{\rho^0\eta'\gamma} &= g z_{\mathrm{NS}}\sin\varphi_P\,,\\
g_{\omega\pi^0\gamma} &= g\cos\varphi_V\,,\\
g_{\omega\eta\gamma} &= \frac{1}{3}g\left(z_{\mathrm{NS}}\cos\varphi_P\cos\varphi_V - 2\frac{\overline{m}}{m_s}z_{\mathrm{S}}\sin\varphi_P\sin\varphi_V\right),\\
g_{\omega\eta'\gamma} &= \frac{1}{3}g\left(z_{\mathrm{NS}}\sin\varphi_P\cos\varphi_V + 2\frac{\overline{m}}{m_s}z_{\mathrm{S}}\cos\varphi_P\sin\varphi_V\right),\\
g_{\phi\pi^0\gamma} &= g\sin\varphi_V\,,\\
g_{\phi\eta\gamma} &= \frac{1}{3}g\left(z_{\mathrm{NS}}\cos\varphi_P\sin\varphi_V + 2\frac{\overline{m}}{m_s}z_{\mathrm{S}}\sin\varphi_P\cos\varphi_V\right),\\
g_{\phi\eta'\gamma} &= \frac{1}{3}g\left(z_{\mathrm{NS}}\sin\varphi_P\sin\varphi_V - 2\frac{\overline{m}}{m_s}z_{\mathrm{S}}\cos\varphi_P\cos\varphi_V\right),
\end{aligned}
$$

$\eta \to \pi^0\gamma\gamma$ DECAYS



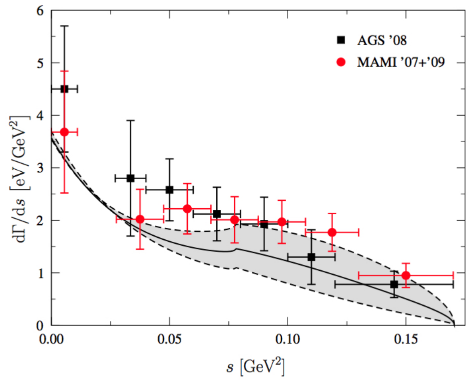
$\eta \to \pi^0\gamma\gamma$ DECAYS

Danilkin *et al.* Phys.Rev.D 96, 114018 (2017)



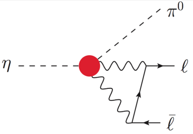
$\eta \to \pi^0\gamma\gamma$ DECAYS

- Dispersive comparison with A2 and CB data
  (Lu and Moussallam, Eur.Phys.J.C 80, 436 (2020))
  - Shape of the spectra is captured well
  - Normalization offset
  - New measurements from KLOE-II and JEF are welcome



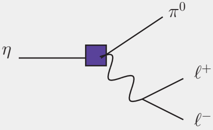
$\eta^{(\prime)} \to \{\pi^0, \eta\}\ell^+\ell^-$ DECAYS ($\ell = e, \mu$)

- In the SM:
  - $\eta \to \pi^0\gamma^* \to \pi^0\ell^+\ell^-$ forbidden by C and CP
  - $\eta \to \pi^0\ell^+\ell^-$ proceed via C-conserving two-photon intermediate state



Decay channel	BR_{th} (Escribano&Royo 2007.12467)	BR_{exp} (pdg)
$\eta \to \pi^0 e^+ e^-$	$2.1(1)(2) \times 10^{-9}$	$< 7.5 \times 10^{-6}$ (CL=90%)
$\eta \to \pi^0 \mu^+ \mu^-$	$1.2(1)(1) \times 10^{-9}$	$< 5 \times 10^{-6}$ (CL=90%)
$\eta' \to \pi^0 e^+ e^-$	$4.6(3)(7) \times 10^{-9}$	$< 1.4 \times 10^{-3}$ (CL=90%)
$\eta' \to \pi^0 \mu^+ \mu^-$	$1.8(1)(2) \times 10^{-9}$	$< 6.0 \times 10^{-5}$ (CL=90%)
$\eta' \to \eta e^+ e^-$	$3.9(3)(4) \times 10^{-10}$	$< 2.4 \times 10^{-3}$ (CL=90%)
$\eta' \to \eta \mu^+ \mu^-$	$1.6(1)(2) \times 10^{-10}$	$< 1.5 \times 10^{-5}$ (CL=90%)

- Background for BSM searches, *e.g.* C-violating virtual photon exchange or new scalar mediators
- REDTOP can improve the experimental state



Other interesting η and η' decays

- Standard Model decays:
  - $\eta \to 3\pi$: Dalitz plot measurements with improved precision (GlueX, REDTOP) $\Rightarrow$ more precise extraction of Q
  - $\eta' \to 3\pi$: theoretical advances $\Rightarrow$ extraction of Q also possible
  - $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$: detailed differential information $\Rightarrow$ access to the doubly-virtual transition form factors $\Rightarrow (g-2)_\mu$
- Discrete symmetry tests:
  - $\eta \to \mu^+\mu^-$: high-precision experimental test (REDTOP) can probe *CP* violation
  - $\eta^{(\prime)} \to \pi\pi$: improved experimental bounds are welcome
  - $\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$: test of *C*-violation
- New light BSM particles:
  - dark photon appears as a resonance in $\eta^{(\prime)} \to \ell^+\ell^-\gamma$ (REDTOP)
  - Axion-like particles searches in $\eta^{(\prime)}$ decays, *e.g.* $\eta^{(\prime)} \to 2\pi a$