The 10th International Workshop on Chiral Dynamics

Nuclear forces in a manifestly Lorentz-invariant formulation of chiral EFT

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Beijing, 2021

OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary and perspective

Nuclear force

Nuclear force

- Acts between two or more nucleons
- Binds protons and neutrons into atomic nuclei
- Plays an **important role in whole nuclear physics**

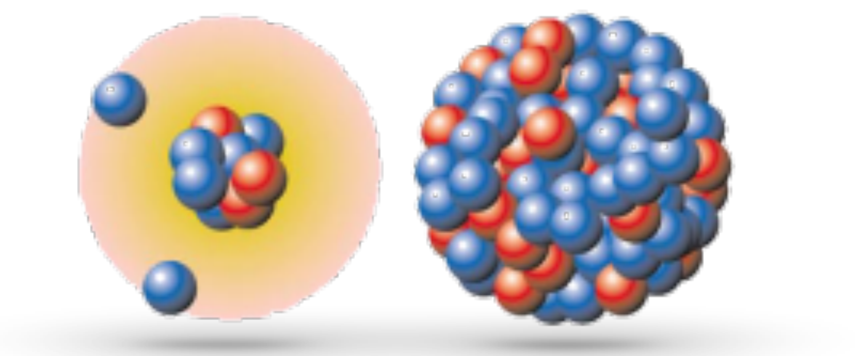
✓ ***Ab-initio calculation***

ONLY input: **Realistic Nuclear force**

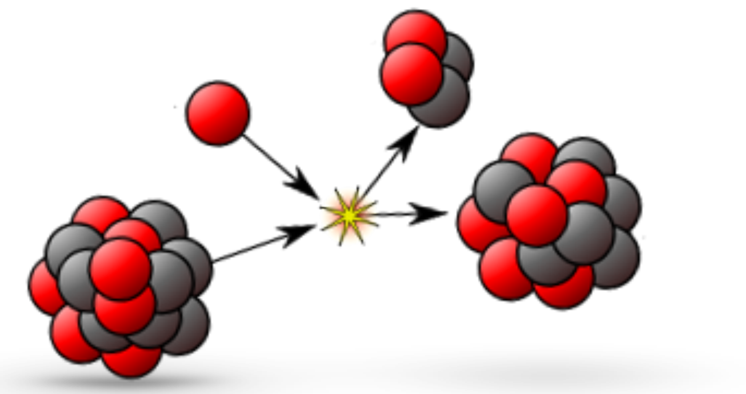
e.g. NN: AV18, CD-Bonn, Reid93, N^3LO…
3N: Tucson-Melbourne, NNLO…

Exactly Solve: **Many-Body Hamiltonian**

e.g. no-core shell model,
Green's function MC method,
Nuclear Lattice EFT…



Nuclear structure

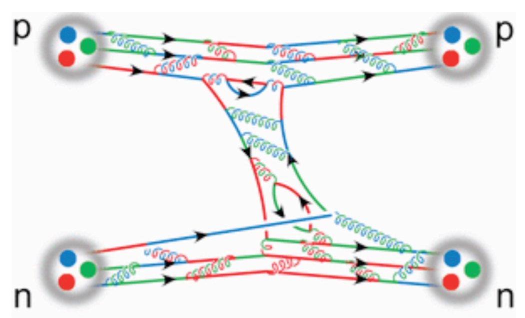


Nuclear reaction

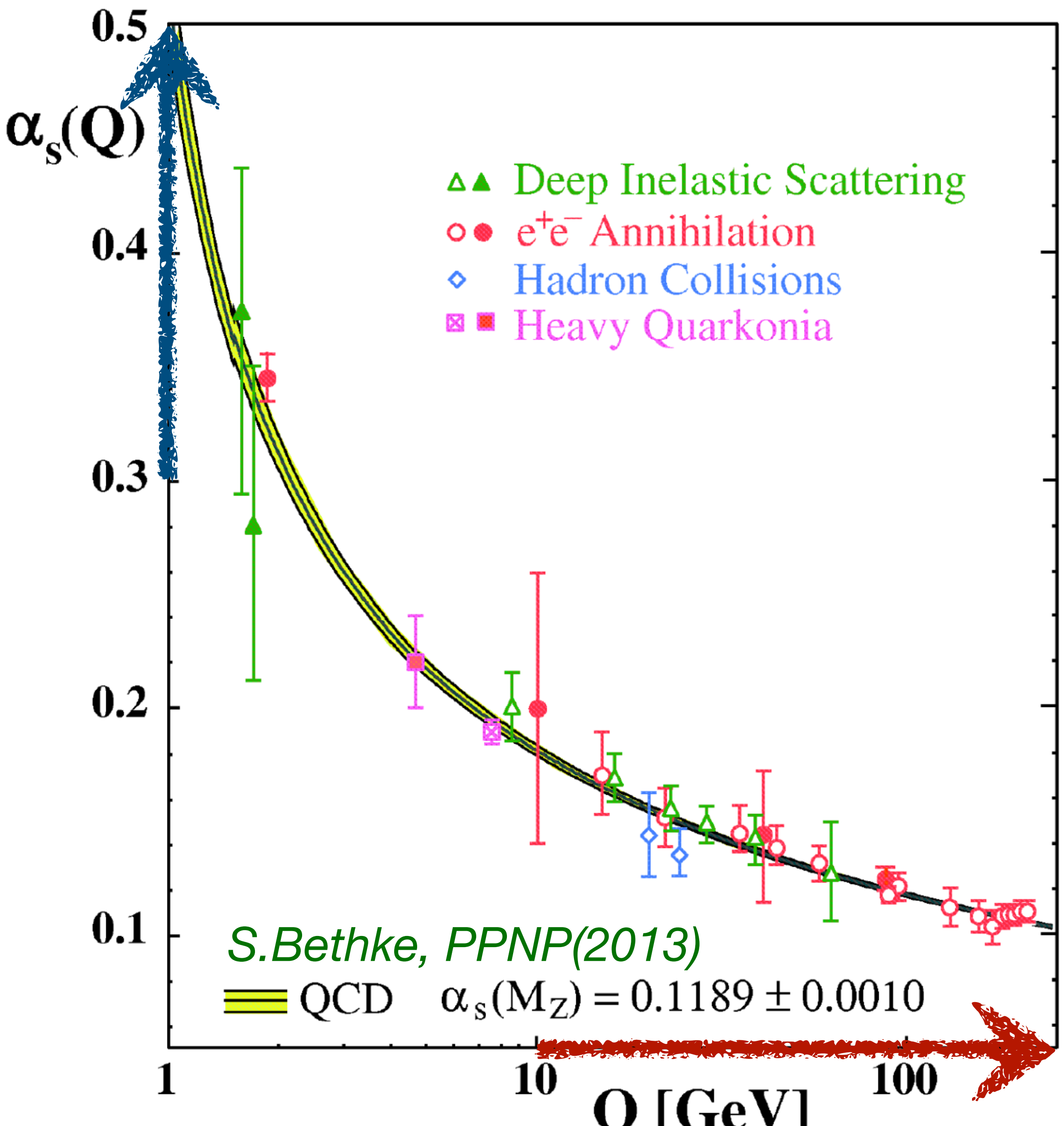
Detailed understanding of the strong nuclear force is essential!

Nuclear force (NF) from QCD

- Residual quark-gluon strong interaction
- Understood from Quantum Chromo-Dynamics
  - Fundamental theory for strong interactions



- **In the low-energy region**
  - Running coupling constant $\alpha_s > 1$
  - **Non-perturbative QCD**



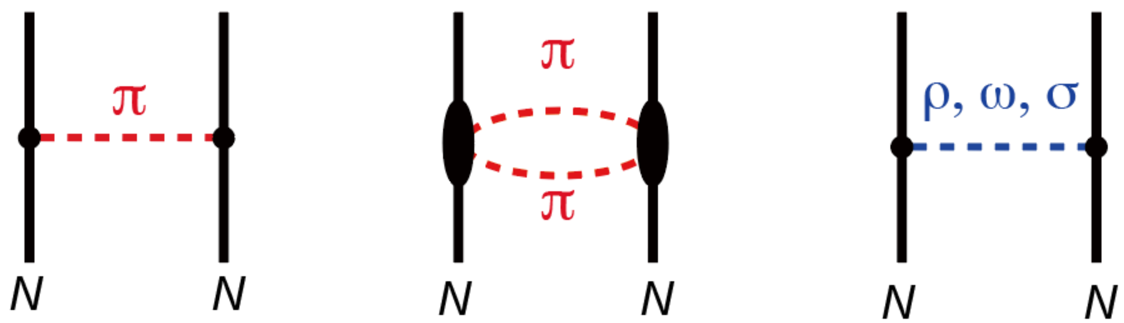
Low-energy phenomena

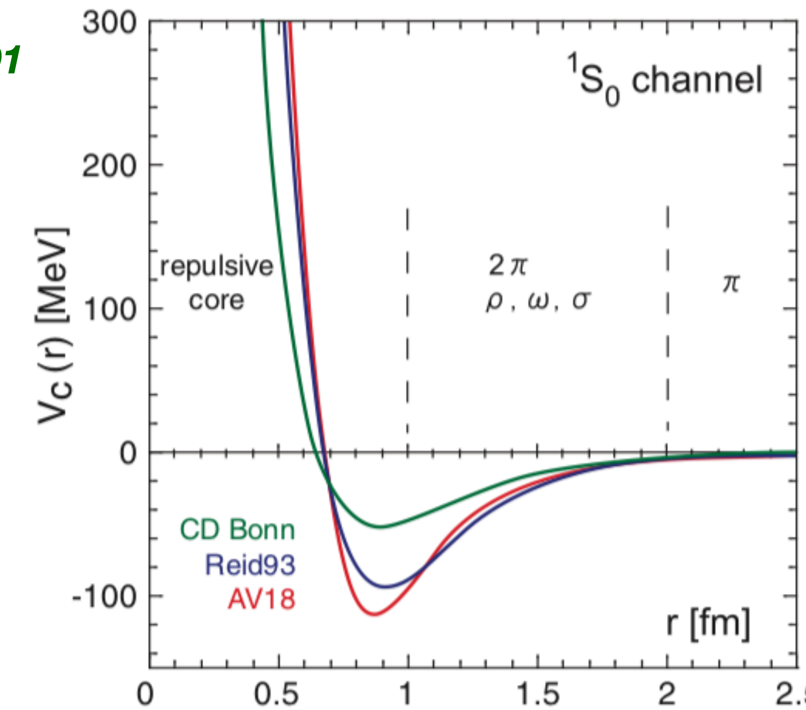
- **Phenomenological models**
- **Lattice QCD simulations**
- **Chiral effective field theory**

Nuclear force studies (I)

NF from phenomenological models (since 1935, Yukawa OPE)

- Meson-exchange theory: CD-Bonn *R. Machleidt, PRC2001*





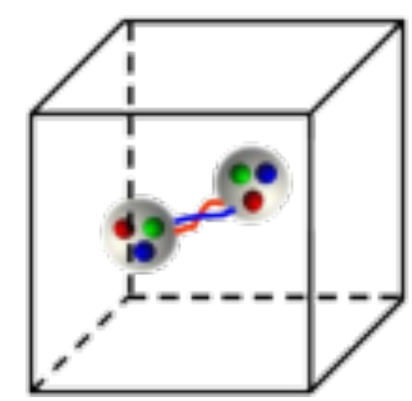
N. Ishii et al. PRL(2007)

- Operator parameterization: Reid93, *V. Stoks, PRC(1994)*
  AV18, *R. Wiringa,PRC(1994)*

$$V_{NN} = V_c(r)\,\hat{1} + V_\sigma(r)\,\sigma_1 \cdot \sigma_2 + V_{LS}(r)\,L \cdot S + V_T(r)\,\sigma_1 \cdot q\sigma_2 \cdot q + \cdots$$

NF from lattice QCD simulations (since 2006)

- HAL QCD
- NPLQCD
- CalLat / sLapHnn
- T. Yamazaki et al.
- Mainz



T. Doi, Wed 18:50
A. Walker-Loud, Wed 23:10

Nuclear force studies (II)

NF from chiral effective field theory (since 1990, Weinberg)

- Hierarchy of chiral forces in non-relativistic scheme (Weinberg p.c.)

	2NF	3NF	4NF
LO (Q^0)	*S. Weinberg, PLB 1990, NPB1990*	—	—
NLO (Q^2)	*U. van Kolck et al, PLB1992, PRL1994* *N. Kaiser et al., NPA1997*	—	—
N^2LO (Q^3)	*U. van Kolck et al., PRC1994* *E. Epelbaum et al., NPA1998, 2000*	*U. van Kolck et al., PRC1994*	—
N^3LO (Q^4)	••• *R. Machleidt et al., PRC2003* *E. Epelbaum et al., NPA2005*	••• *S. Ishikwas et al., PRC2007* *V. Bernard et al., PRC2007*	••• *E. Epelbaum, PLB2006, EPJA2007*
N^4LO (Q^5)	••• *R. Machleidt et al., PRC2015* *E. Epelbaum et al., PRL2015*	••• *H. Krebs et al, PRC2012,2013* *Short-range loop contrib. still missing*	••• *Not yet…*

1990	**LO**
2003	**N^3LO: 2N**
2007	**N^3LO: 3N**
2015	**N^4LO: 2N**
In future	**N^4LO: 3N & 4N**

P. F. Bedaque, U. van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339
E. Epelbaum, H.-W. Hammer, Ulf-G. Meißner, Rev. Mod. Phys. 81 (2009) 1773
R. Machleidt, D. R. Entem, Phys. Rept. 503 (2011) 1

High precision chiral force

Description of neutron-proton scattering observables

	Phenomenological forces			Non-Rel. Chiral nuclear force				
	Reid93	**AV18**	**CD-Bonn**	LO	NLO	NNLO	**N^{3}LO**	**N^{4}LO$^+$** include 4 ct. in F-waves
No. of para.	**50**	**40**	**38**	2+2	9+2	9+2	24+2 (3 redundant)	**24+3+4** (3 redundant)
χ^2/datum *np 0-300 MeV*	**1.03**	**1.04**	**1.02**	94	36.7	5.28	1.27	**1.10**
				D. Entem, et al., PRC96(2017)024004				
				75	14	4.2	2.01	**1.06**
				P. Reinert, et al., EPJA54(2018)86				

Non-Rel. Chiral NF has achieved a great success!

- **NNLO/N^{3}LO chiral forces** have been extensively applied in the study of nuclear structure and reactions within the few/many-body theories.

E. Epelbaum, et al., PRL 106(2011) 192501, PRL109(2012)252501, PRL112(2014)102501; S. Elhatisari, et al., Nature 528 (2015) 111; G. Hagen, et al., PRL109(2012)032502; H. Hergert, et al., PRL110(2013)24501; G.R. Jansen, et al., PRL113(2014)102501; S.K.Bogner, et al., PRL113(2014)142501; J.E. Lynn, et al., PRL113(2014)192501; V. Lapoux, et al.,PRL117(2016)052501

S. Pastore, Thu. 23:20
A. Filin, Fri. 19:00
C. Drischler, Fri. 21:20

Recent reviews/collections

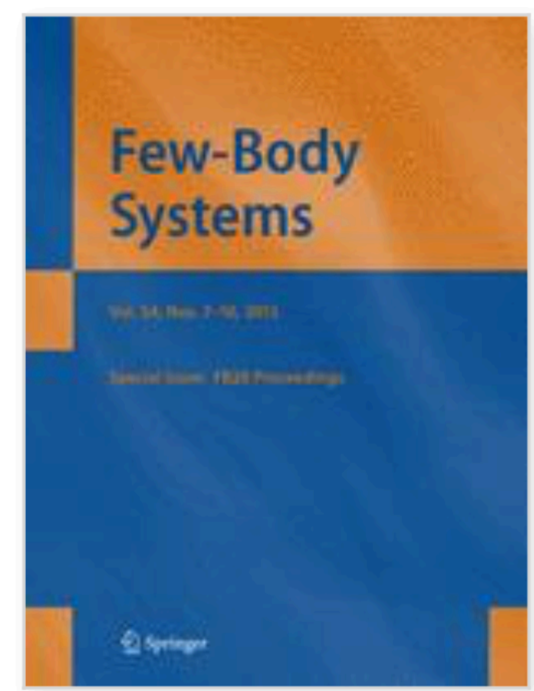
frontiers

Research Topics

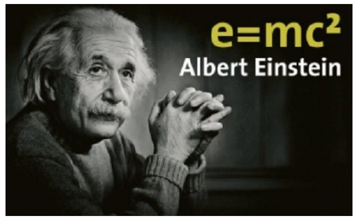
Few-Body Systems
All Volumes & Issues

Celebrating 30 years of the Steven Weinberg's paper Nuclear Forces from Chiral Lagrangians

ISSN: 0177-7963 (Print) 1432-5411 (Online)



Why reformulate chiral NF in Lorentz invariant scheme?

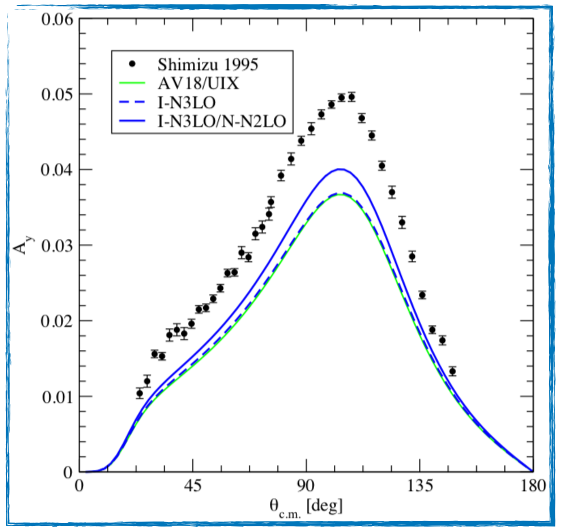


- **The principle of relativity: law of nature**
  - Elastic NN scattering (0--300 MeV)

    $$\sqrt{m_N^2 + p^2} \leq m_N\sqrt{1+0.16}$$

    ✓Non-relativistic is a good approximation
  - Small relativistic effects could be detectable

    ✓via the ongoing/future high precision experiments



J. Golak et al. EPJA50(2014)177

- Investigate the relativistic effects
  - Unsolved problems: Ay puzzle in N-d scattering**?**
  - Could improve the description of polarization observables
- Relativistic chiral force could provide a new view
  - Renormalization group invariance: still controversial …
  - Convergence of chiral expansion: relatively faster convergence?
- Key inputs for the relativistic many-body calculations
  - Relativistic Breuckner-Hartree-Fock theory

S. Shen, H. Liang, W.H. Long, J. Meng, P. Ring, PPNP109, 103713 (2019)

Chiral forces in Lorentz invariant framework

Relativistic chiral potential (covariant form)

- Based on the Lorentz invariant chiral Lagrangians
- Proposed covariant power counting
- Formulate chiral force up to **next-to-next-to-leading order**

L.-S. Geng, Tue 18:30

Y. Xiao, C-X Wang, J.-X. Lu, L.-S. Geng, PRC102,054001(2020)
C.-X. Wang, J.-X. Lu, Y. Xiao, L.-S. Geng, 2110.05278
J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, R. Peter, 2111.07766

Modified Weinberg approach *E. Epelbaum and J. Gegelia, PLB716(2012)338-344*

- Based on the Lorentz invariant chiral Lagrangians
- Adopt Weinberg power counting to expand the NN potential and the relativistic corrections are perturbatively included

$$V(p', p) = \bar{u}_1 \bar{u}_2 \mathscr{A} u_1 u_2 \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

- Use the Kadyshevsky equation to calculate the scattering T-matrix

$$T(p', p) = V(p', p) + \int \frac{k^2 dk}{(2\pi)^3} V(p', k) \frac{m^2}{2(k^2 + m^2)} \frac{1}{\sqrt{p^2 + m^2} - \sqrt{k^2 + m^2} + i\epsilon} T(k, p) \,.$$

✓ Milder ultraviolet behavior than in Lippmann-Schwinger equation

In our work

- We proposed a systematic framework within the time-ordered perturbation theory using the Lorentz invariant chiral Lagrangians
  - Derive the rules of time-ordered diagrams, especially for the rules with spin-1/2 fermion *(as far as we know, there was no such rules in the literature)*
  - Formulate the nucleon-nucleon interaction up to **next-to-next-to-leading order**
    - ✓ Calculate the two-pion-exchange contributions at one-loop level
    - ✓ Describe the partial wave phase shifts

V. Baru, E. Epelbaum, J. Gegelia, XLR*, Phys. Lett. B 798 (2019) 134987

XLR*, E.Epelbaum, J.Gegelia, Phys. Rev. C 101 (2020) 034001

XLR*, E. Epelbaum, J. Gegelia, in preparation

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- Introduction
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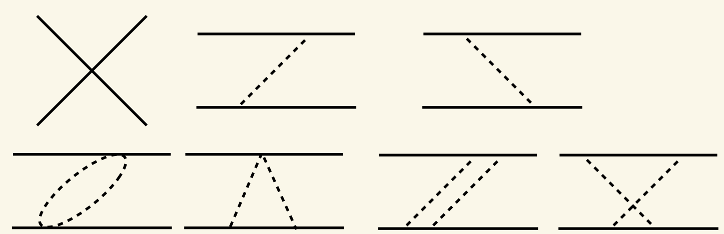
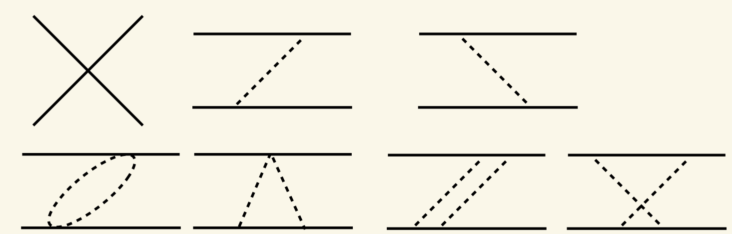
Theoretical framework

- Follow the standard procedure of formulating chiral forces in time-ordered perturbation theory

S. Weinberg, PLB1990, NPB1991; C. Ordóñez, U. van Kolck PLB(1992) …

	Non-relativistic (Heavy-baryon)	**Manifestly Lorentz invariant**
Chiral Lagrangians	$N^\dagger\left[i(v\cdot D)+g_A(S\cdot u)\right]N$ $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$	$\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$
Potentail TOPT diagrams		
Scattering equations (T = V + VGT)	Lippmann-Schwinger equation	Kadyshevsky equation
Power counting	Weinberg p.c.	Weinberg p.c.

$1/m_N$

Chiral Lagrangian up to NNLO

Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

- Purely pionic sector

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

J.Gasser, H. Leutwyler, Ann.Phys.(1984)

- One-nucleon sector

$f_\pi = 92.4$ MeV, $g_A = 1.267$

$c_{1,2,3,4}$ **are determined by πN scattering**

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

J. Gasser, M. E. Sainio, and A. Svarc, NPB(1988)

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left(D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[u_\mu, u_\nu \right] \right\} \Psi_N$$

N.Fettes, U.-G. Meißner, S. Steininger, NPA(1998)

- Two-nucleon sector

$$\begin{aligned}\mathcal{L}_{NN}^{(0)} = & \frac{1}{2}\left[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N) + C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right) + C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)\right.\\ & \left. + C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right) + C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\right]\end{aligned}$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N \mathcal{O}_i \Psi_N \Psi_N$$

L.Girlanda, S. Pastore, R. Schiavilla, M. Viviani, PRC(2010)

Yang Xiao, Li-Sheng Geng, XLR, PRC(2019)

Diagrammatic rules in TOPT

External lines

- Incoming (outgoing) nucleon lines: $u(p)\ [\bar{u}(p')]$ *Dirac spinors*

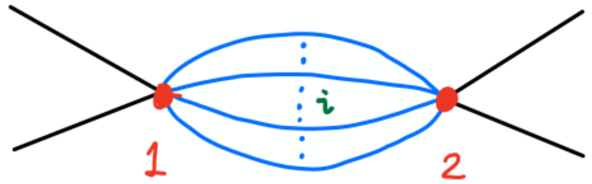
Internal lines

- Pseudo-scalar meson lines: $\dfrac{1}{2\omega(q, M_\pi)}$ $\omega_q = \sqrt{q^2 + M_\pi^2}$
- Baryon lines: $\dfrac{m_N}{\omega(p, m_N)} \sum u(p)\bar{u}(p)$ $\omega_p = \sqrt{p^2 + m_N^2}$
- Anti-baryon lines: $\dfrac{m_N}{\omega(p, m_N)} \sum u(p)\bar{u}(p) - \gamma_0$ $\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}$

Interaction vertices: Follow the standard Feynman rules

- Take care of zeroth components of integration momenta
  - ✓ Replaced as $\omega(p, m)$ for particle
  - ✓ Replaced as $-\omega(p, m)$ for antiparticle

Intermediate state: **a set of lines between any two vertices**

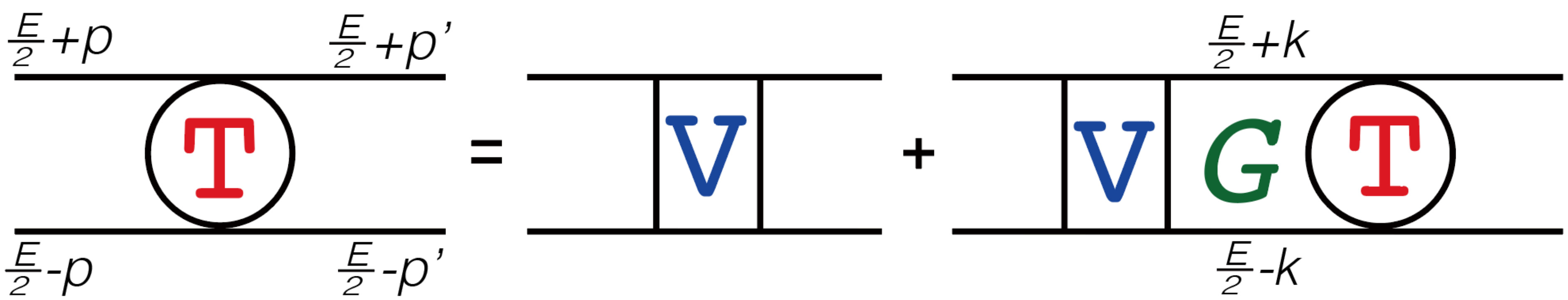


$$[E - \sum_i \omega(p_i, m_i) + i\epsilon]^{-1}$$

E is the total energy of the system

Scattering equations in TOPT

- Following the above diagrammatic rules of TOPT, we obtain the scattering equations



- Two-nucleon Green function

$$G(E) = \frac{m_N^2}{\omega^2(k, m_N)} \frac{1}{E - 2\omega(k, m_N) + i\epsilon}$$

✓ This is the Kadyshevsky propagator of NN scattering *V. Kadyshevsky, NPB (1968)*

✓ **SELF-CONSISTENTLY obtained** in TOPT

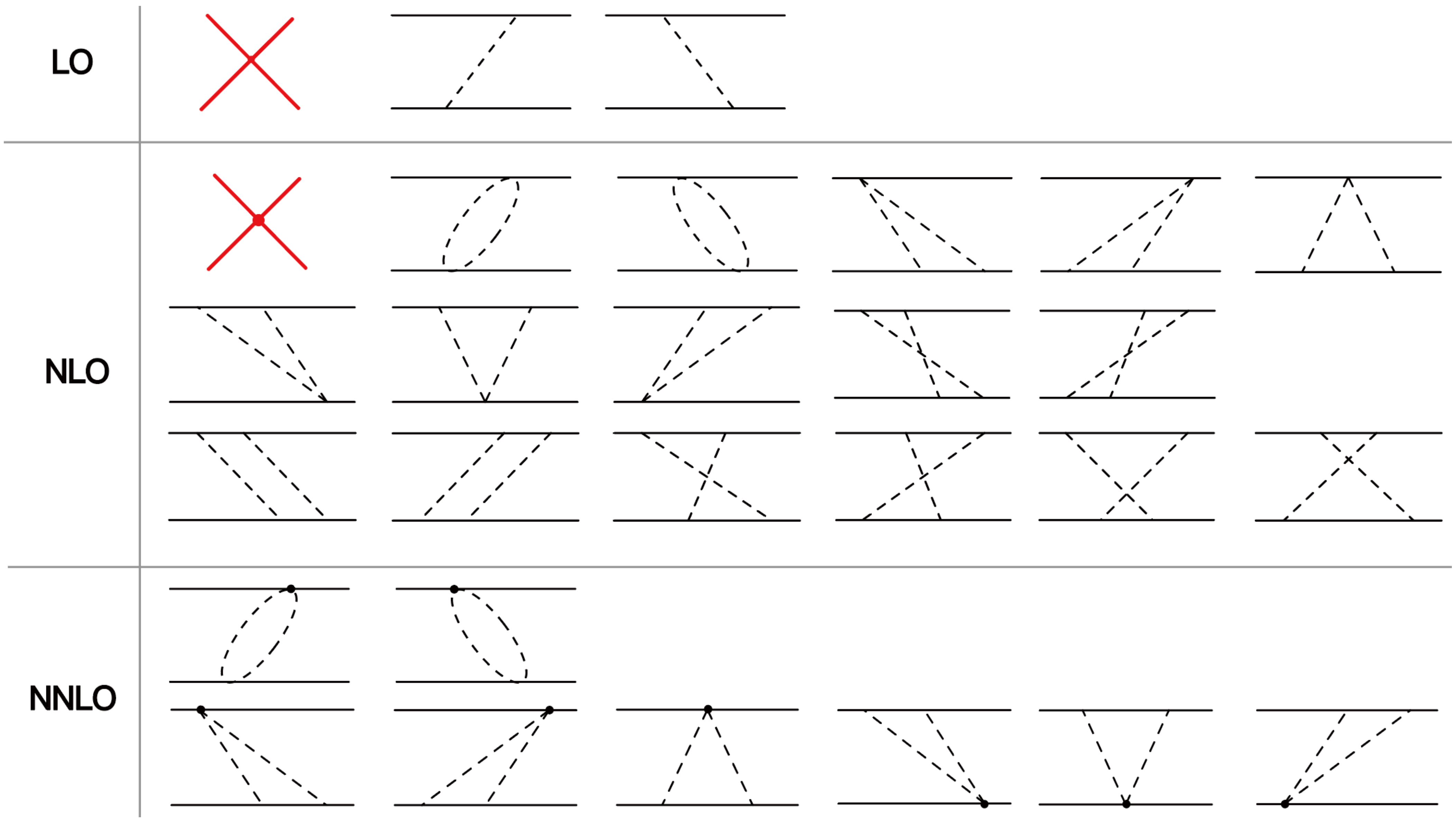
$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{2(\boldsymbol{k}^2 + m_N^2)} \frac{1}{\sqrt{\boldsymbol{p}^2 + m_N^2} - \sqrt{\boldsymbol{k}^2 + m_N^2} + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

✓ Milder UV behaviour than the Lippmann-Schwinger equation

Green function $G \xrightarrow{k\longrightarrow\infty}$ Kady. $\dfrac{1}{k^3}$ vs. LS $\dfrac{1}{k^2}$

Nucleon-nucleon potential in TOPT

Two-nucleon **irreducible time-ordered diagrams**



- Use the obtained TOPT rules to evaluate these diagrams
- Apply Weinberg power counting to organize
  - ✓ Expand the nucleon energy appearing in the numerator

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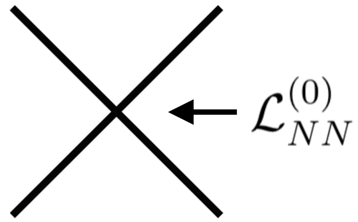
  - Leading order study Phys. Lett. B 798 (2019) 134987
  - Next-to-next-to-leading order calculation

    XLR*, E. Epelbaum, J. Gegelia, in preparation
- Summary and perspective

Leading order potentials

- Contact nucleon-nucleon interaction



  - According to our TOPT rules

$$V_{0,C} = C_S(\bar{u}_3\, u_1)(\bar{u}_4\, u_2) + C_A(\bar{u}_3\, \gamma_5\, u_1)(\bar{u}_4 \gamma_5 u_2) + C_V(\bar{u}_3\, \gamma_\mu\, u_1)(\bar{u}_4\, \gamma^\mu\, u_2)$$
$$+ C_{AV}(\bar{u}_3\, \gamma_\mu \gamma_5\, u_1)(\bar{u}_4\, \gamma^\mu \gamma_5\, u_2) + C_T(\bar{u}_3\, \sigma_{\mu\nu}\, u_1)(\bar{u}_4\, \sigma^{\mu\nu}\, u_3)$$

  - Contain higher order contributions according to Weinberg P.C.
  - Perform the expansion for the nucleon energies

$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

$$\boxed{V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}$$

✓ Two independent parameters to be fixed

✓ Consistent with the non-relativistic contact terms

S. Weinberg, PLB251(1990)288-292

Leading order potentials

- One-pion-exchange (OPE) potential

  - According to our TOPT rules

$$V_{0,\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{2\omega(q,M_\pi)}\left[\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right.$$
$$\left.+\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right]$$

  - Contains higher order contributions according to Weinberg P.C.
  - Perform the expansion for the nucleon energies in numerator

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

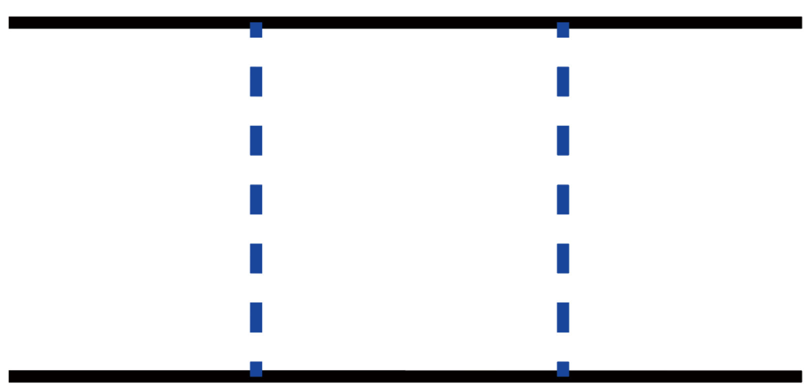
✓ Keep the nucleon energies in denominator (consistent with Kadyshevsky eq.)

$$V_{\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{4m_N^2}{\omega(q,M_\pi)\,(m_N+\omega(p,m_N))\,(m_N+\omega(p',m_N))}$$
$$\times\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}$$

It has a milder UV behaviour than the non-relativisitc OPEP

UV Behavior of the long-range potential

- One-loop integral $V\,G\,V$:



$$I_{VGV} = \int \frac{d^3k}{(2\pi)^3} V_{\rm OPE}\, G(E)\, V_{\rm OPE}$$

$$\xrightarrow{k \to \infty}$$

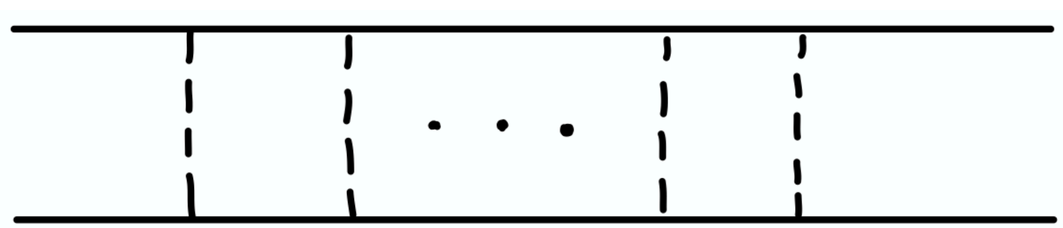
Our: $I_{VGV}^{\rm Our} \to \int dk^3 \frac{1}{k}\frac{1}{k^3}\frac{1}{k} = \int dk^3 \frac{1}{k^5}$

Ultraviolet convergent!

NR: $I_{VGV}^{\rm NR} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3 \frac{1}{k^2}$

Ultraviolet divergent!

- Iteration of our OPEP



$$\xrightarrow{k \to \infty}$$

Finite diagram!

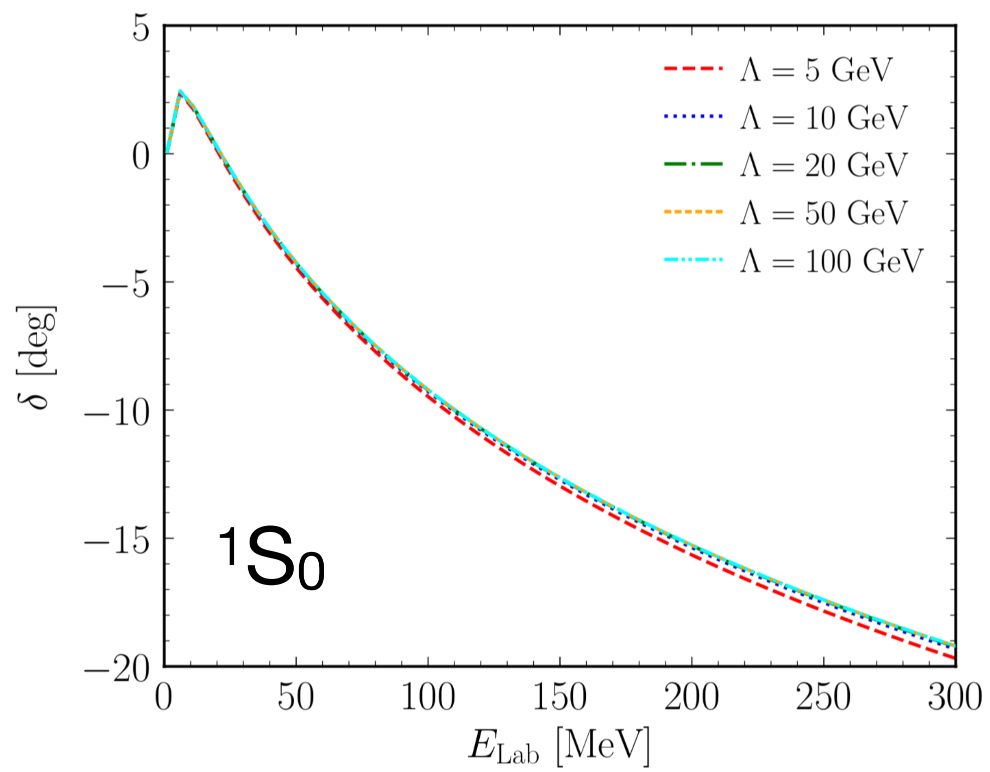
- Scattering amplitude from OPEP is cutoff independent

$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}\, G\, T_{\rm OPE}$$

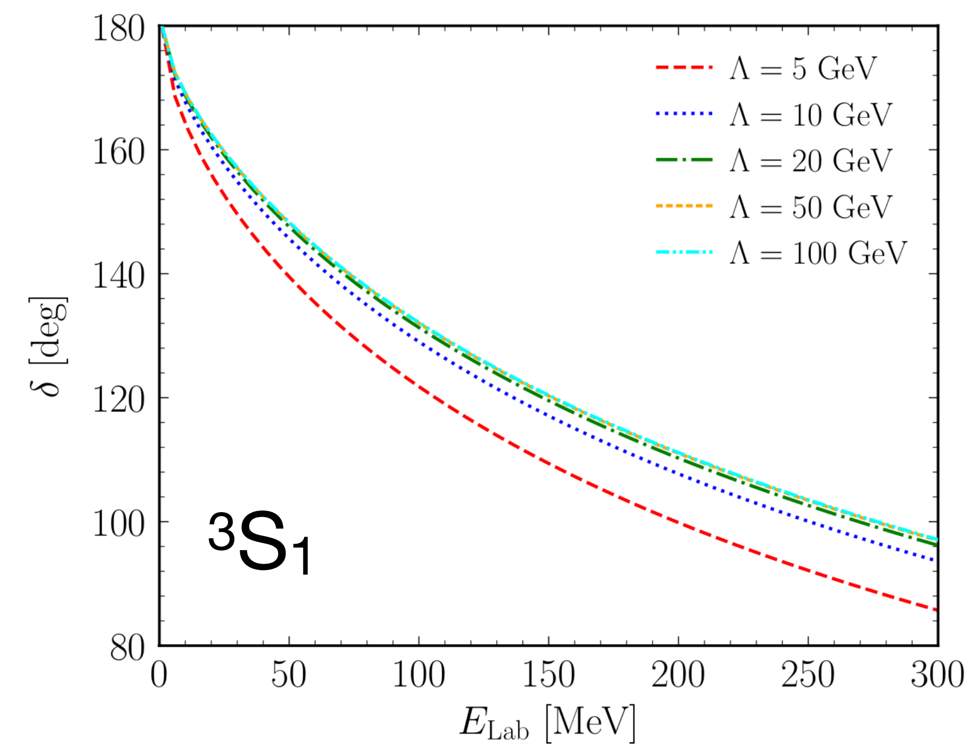
Renormalizable!

Phase shifts: cutoff-independent

- NN single channel: e.g. 1S0



- NN couple channels: e.g. 3S1

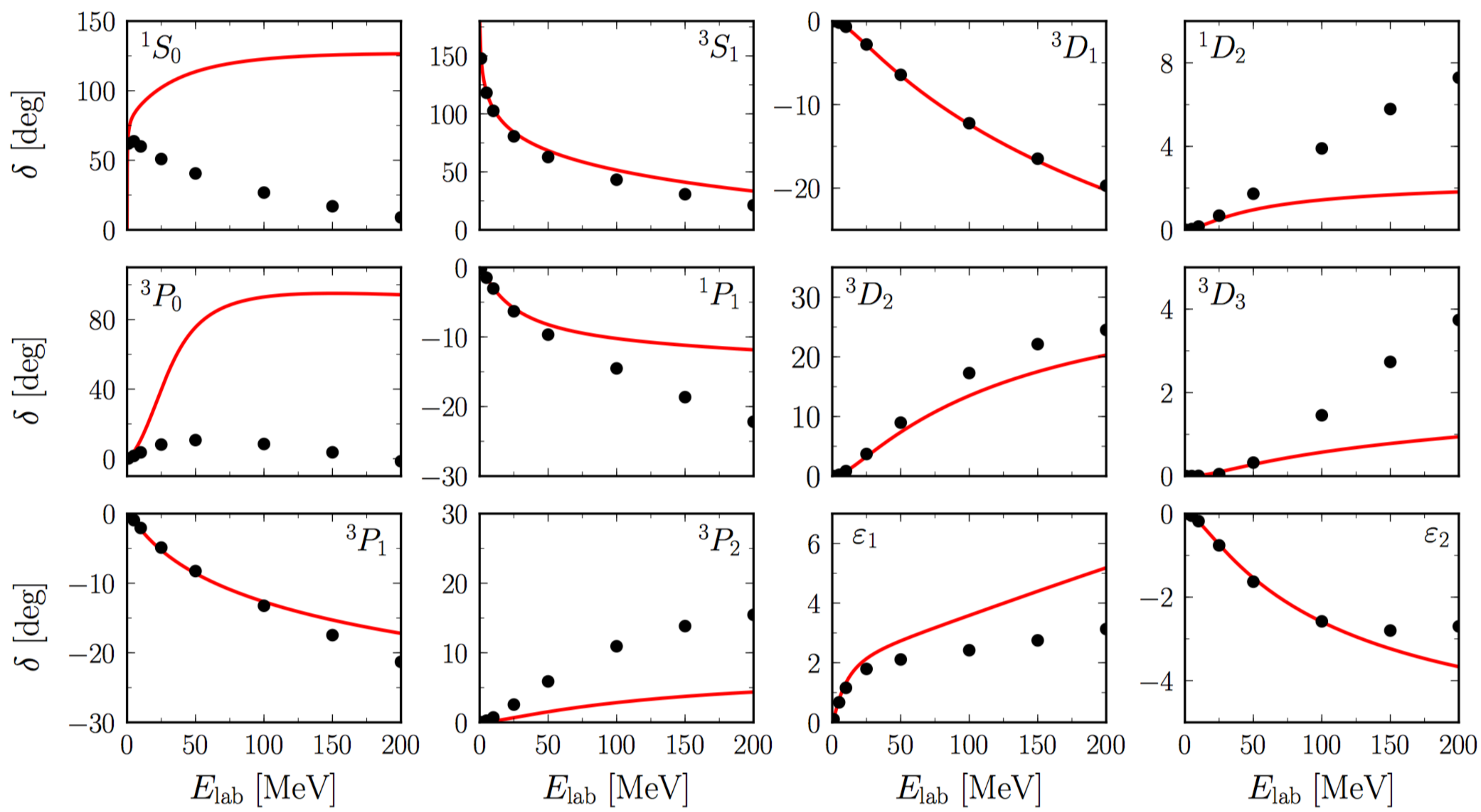


✤ **Our LO potential is perturbatively renormalizable!**

- All divergences appearing from its iterations can be absorbed in the coupling constant of the contact interaction
- Scattering equation has unique solutions for all partial waves
- Avoid finite-cutoff artefacts inherent to the conventional non-relativistic framework

Phase shifts of NN scattering

- Phase shifts at LO with cutoff → ∞
  - Two LECs are fixed by the scattering lengths of 1S0 and 3S1



- **Our LO calculation provides a reasonable description of the empirical phase shifts**
- **$^1S_0$ and $^3P_0$: Large deviation**
  - Part of the subleading corrections must be treated non-perturbatively

Beyond LO

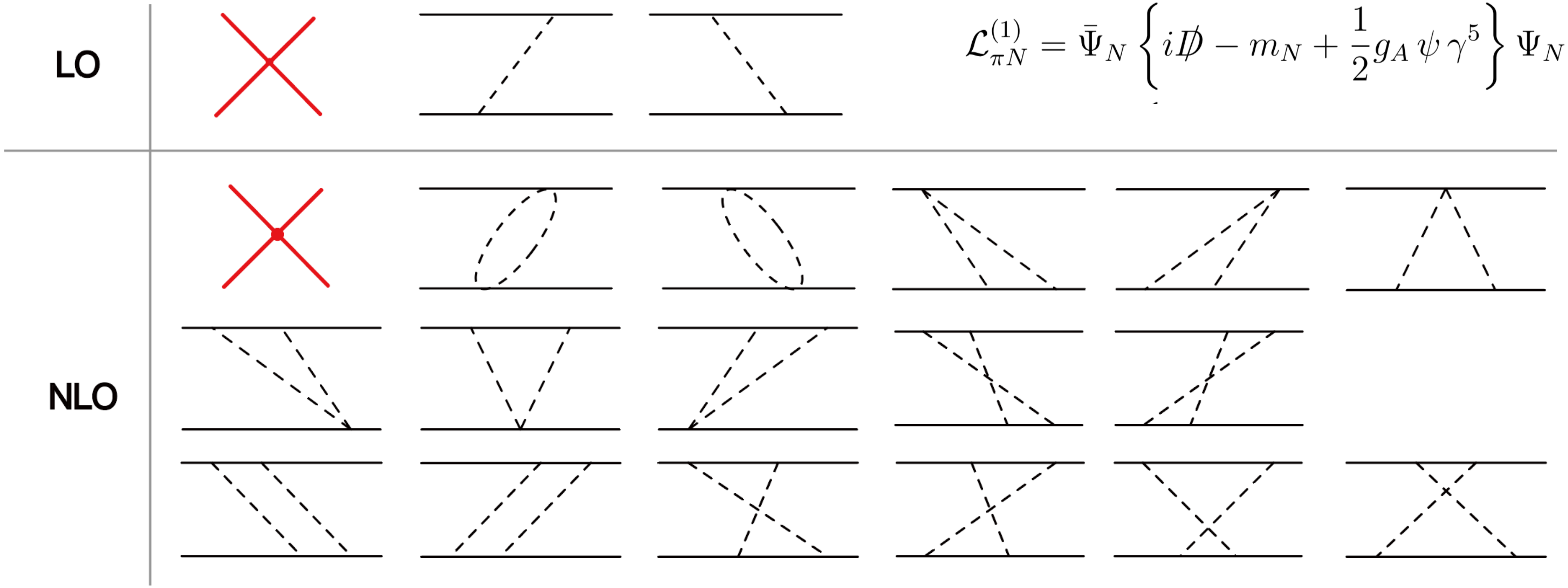
Beyond Leading order studies

- Two strategies to include higher orders
  - Restricting the non-perturbative treatment to the (non-singular) LO potential and higher-order interactions are treated perturbatively
    - ✓ Systematically remove all divergences from the amplitude
  - Full effective potential (LO + higher orders) are treated non-perturbatively
    - ✓ Milder UV behavior offers a larger flexibility regarding admissible cutoff
    - ✓ Direct input for few-/many-body problems
- Here, we focus on the second strategy (as a first step)
  - Since the derivation of higher order contributions is computationally more demanding
  - **Formulate the chiral nuclear potential up to NLO and NNLO**
  - Calculate the two-pion exchange contribution at one-loop level

XLR*, E. Epelbaum, J. Gegelia, in preparation

Study of NLO potential in TOPT

- Time ordered diagrams up to NLO



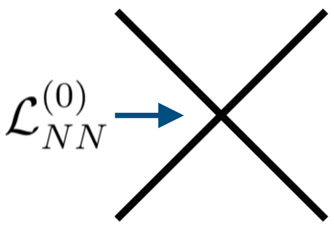
$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2} \left[C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma_5 \Psi_N \right) + C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \Psi_N \right) \right.$$
$$\left. + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N \right) \left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N \right) + C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N \right) \left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N \right) \right]$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N \mathcal{O}_i \Psi_N \Psi_N$$

Contact terms up to NLO

LO contact term (5 LECs)



$$V_{\rm LO} = C_S(\bar{u}_3\, u_1)(\bar{u}_4\, u_2) + C_A(\bar{u}_3\,\gamma_5\, u_1)(\bar{u}_4\gamma_5 u_2) + C_V(\bar{u}_3\,\gamma_\mu\, u_1)(\bar{u}_4\,\gamma^\mu\, u_2)$$
$$+ C_{AV}(\bar{u}_3\,\gamma_\mu\gamma_5\, u_1)(\bar{u}_4\,\gamma^\mu\gamma_5\, u_2) + C_T(\bar{u}_3\,\sigma_{\mu\nu}\, u_1)(\bar{u}_4\,\sigma^{\mu\nu}\, u_3)$$

- Expand the nucleon energy up to $\mathcal{O}(p^2)$ / NLO

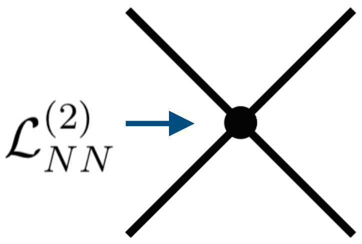
$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}\, m_N^{3/2}} + \mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for LO contact terms

➡ Keep the full form of Dirac spinors

NLO contact term

- Expand the nucleon energy $\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$
- Same form as the non-relativistic case with 7 LECs



$$V_{\rm NLO} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{P}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{P}^2\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{i}{2} C_5\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}$$
$$+ C_6\, (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1)\,(\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7\, (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1)\,(\boldsymbol{P} \cdot \boldsymbol{\sigma}_2)$$

Partial wave decomposition for contact terms

- J=0: 1S0 and 3P0 partial waves

$$V(^1S_0) = \xi_N C^{LO}_{^1S_0}\left(1+R_p^2R_{p'}^2\right) + \xi_N\left[\hat{C}^{LO}_{^1S_0} + 4m_N^2 C^{NLO}_{^1S_0}\right]\left(R_p^2+R_{p'}^2\right)$$
$$= \xi_N C^{LO}_{^1S_0}\left(1+R_p^2R_{p'}^2\right) + \xi_N \tilde{C}_{^1S_0}\left(R_p^2+R_{p'}^2\right)$$

$$V(^3P_0) = -2\xi_N\left[C^{LO}_{^3P_0} - 2m_N^2 C^{NLO}_{^3P_0}\right]R_pR_{p'}$$
$$= -2\xi_N\tilde{C}_{^3P_0}R_pR_{p'} = -2\tilde{C}_{^3P_0}\frac{p\,p'}{4m_N^2}$$

$$\xi_N = \frac{(\omega_p+m_N)(\omega_{p'}+m_N)}{4m_N^2},\quad R_p = \frac{p}{\omega_p+m_N},\ \text{and}\ R_{p'} = \frac{p'}{\omega_{p'}+m_N}$$

- J=1: 1P1, 3P1, 3S1-3D1 partial waves

$$V(^1P_1) = \frac{2}{3}\xi_N\left[-C^{LO}_{^1P_1} - 6m_N^2\,C^{NLO}_{^1P_1}\right]R_pR_{p'}$$
$$= \frac{2}{3}\xi_N\tilde{C}_{^1P_1}R_pR_{p'} = \frac{2}{3}\tilde{C}_{^1P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3P_1) = -\frac{4}{3}\xi_N\left[C^{LO}_{^3P_1} - 3m_N^2C^{NLO}_{^3P_1}\right]R_pR_{p'}$$
$$= -\frac{4}{3}\xi_N\tilde{C}_{^3P_1}R_pR_{p'} = -\frac{4}{3}\tilde{C}_{^3P_1}\frac{p\,p'}{4m_N^2}.$$

$$V(^3S_1) = \frac{\xi_N}{9}C^{LO}_{^3S_1}\left(9+R_p^2R_{p'}^2\right) + \frac{\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 36m_N^2C^{NLO}_{^3S_1}\right]\left(R_p^2+R_{p'}^2\right)$$
$$= \frac{\xi_N}{9}C^{LO}_{^3S_1}\left(9+R_p^2R_{p'}^2\right) + \xi_N\tilde{C}_{^3S_1}\left(R_p^2+R_{p'}^2\right)$$

$$V(^3D_1) = \frac{8\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

$$V(^3S_1-{}^3D_1) = \frac{2\sqrt{2}\xi_N}{9}\left[\hat{C}^{LO}_{^3S_1} + 9\sqrt{2}m_N^2C^{NLO}_{^3D_1-{}^3S_1}\right]R_p^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$
$$= \xi_N\tilde{C}_{^3D_1-{}^3S_1}R_p^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

$$V(^3D_1-{}^3S_1) = \xi_N\tilde{C}_{^3D_1-{}^3S_1}R_{p'}^2 + \frac{2\sqrt{2}\xi_N}{9}C^{LO}_{^3S_1}R_p^2R_{p'}^2$$

- J=2: 3P2 partial wave $V(^3P_2) = C^{NLO}_{^3P_2}\,p\,p' = \tilde{C}_{^3P_2}\frac{p\,p'}{4m_N^2}$

- Finally, we have **9 LECs to be fixed:** $C^{LO}_{^1S_0}, C^{LO}_{^3S_1},\ \tilde{C}_{^1S_0}, \tilde{C}_{^3P_0}, \tilde{C}_{^1P_1}, \tilde{C}_{^3P_1}, \tilde{C}_{^3S_1}, \tilde{C}_{^3D_1-{}^3S_1}, \tilde{C}_{^3P_2}$

Same number of contact terms as the non-relativistic NLO case

One–Pion exchange potential up to NLO

OPE potential

$$V_{\rm OPE} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{\omega_q}\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega_p+\omega_{p'}+\omega_q-E-i\epsilon}$$

- Expand the nucleon energy expansion for OPEP at NLO

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N}+\frac{p^2}{4\sqrt{2}\,m_N^{3/2}}+\mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for OPE potential

➡ Keep the full form of Dirac spinors

- Eliminate the energy dependence of OPEP (avoid the pole contribution)

✓ Expand E at $\omega_p+\omega_p'$, then, we obtain contribution of OPEP at NLO

$$V_{\rm OPE}^{\not E} = -\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\frac{1}{\omega_q^2}(\bar{u}_3\gamma_\mu\gamma_5 q^\mu u_1)(\bar{u}_4\gamma_\nu\gamma_5 q^\nu u_2) \longrightarrow$$

LO correction

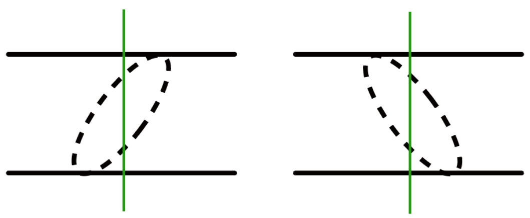
NLO correction

$$+\frac{1}{2}\left(\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\right)^2\int\frac{d^3k}{(2\pi)^3}\frac{m_N^2}{k^2+m_N^2}\frac{\omega_{p'-k}+\omega_{p-k}}{\omega_{p'-k}^3\omega_{p-k}^3}$$
$$\times\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}-\boldsymbol{p})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}-\boldsymbol{p})\right].$$

Two-pion exchange potential at NLO

- Follow our TOPT rules:
  - Football diagram

$$V_F = \frac{1}{16 f_\pi^4}\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \int \frac{d^3k}{(2\pi)^3} \frac{(\omega_k + \omega_{k+q})(\omega_p + \omega_{p'}) + 4\omega_k\omega_{k+q} - E(\omega_k + \omega_{k+q})}{2\omega_k\omega_{k+q}\,\underline{(\omega_k + \omega_{k+q} + \omega_p + \omega_{p'} - E)}}.$$

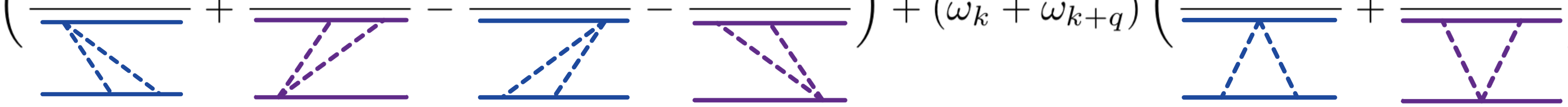


Energy denominator of football diagram

  - Triangle diagrams

$$V_{T+\tilde{T}}^{NN} = \frac{4 m_N g_A^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{128 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[(\boldsymbol{k}^2 + (\boldsymbol{p}' - \boldsymbol{p}) \cdot \boldsymbol{k}) + \frac{i}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}(a+b)\right] \frac{1}{\omega_k \omega_{k+q} \omega_{p-k}}$$

$\times \Big[(\omega_{k+q} - \omega_k)\big(\tfrac{1}{\cdots} + \tfrac{1}{\cdots} - \tfrac{1}{\cdots} - \tfrac{1}{\cdots}\big) + (\omega_k + \omega_{k+q})\big(\tfrac{1}{\cdots} + \tfrac{1}{\cdots}\big)\Big]$

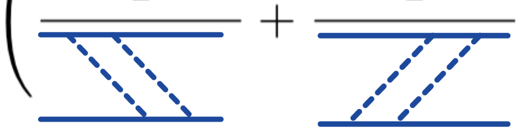


Energy denominator

  - Planar and crossed box diagrams

$$V_B = \frac{m_N^2 g_A^4 (3 - 2\,\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{64 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[X_1 + X_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + X_3 \frac{i\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}}{2} + X_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}) + X_5\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})\right]$$

$\times \dfrac{1}{\omega_k \omega_{k+q} \omega_{p-k}^2} \big(\tfrac{1}{\cdots} + \tfrac{1}{\cdots}\big)$

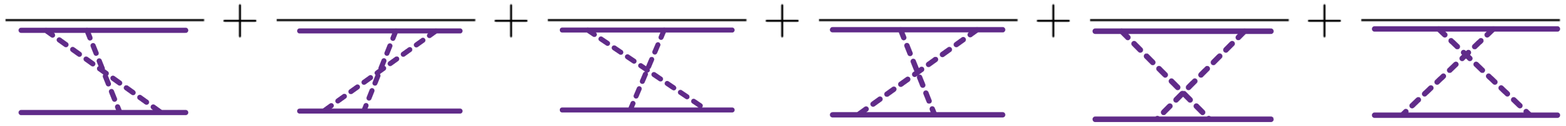


$$\boldsymbol{k} = a\,\boldsymbol{p} + b\,\boldsymbol{p}' + c\,(\boldsymbol{p}' \times \boldsymbol{p})$$

$X_1 = \left[\boldsymbol{k}^2 + \boldsymbol{q} \cdot \boldsymbol{k}\right]^2$, $X_2 = -c^2 \boldsymbol{q}^2 \left[\boldsymbol{P}^2 \boldsymbol{q}^2 - (\boldsymbol{q} \cdot \boldsymbol{P})^2\right]$, $X_3 = -2(a+b)\left(\boldsymbol{k}^2 + (\boldsymbol{p}' - \boldsymbol{p}) \cdot \boldsymbol{k}\right)$,
$X_4 = -(a+b)^2 + c^2 \boldsymbol{q}^2$, $X_5 c^2 \left[\boldsymbol{P}^2 \boldsymbol{q}^2 - (\boldsymbol{q} \cdot \boldsymbol{P})^2\right]$

$$V_{\tilde{B}} = \frac{m_N^2 g_A^4 (3 + 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{64 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[X_1 + X_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + X_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}) + X_5\,(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})\right]$$

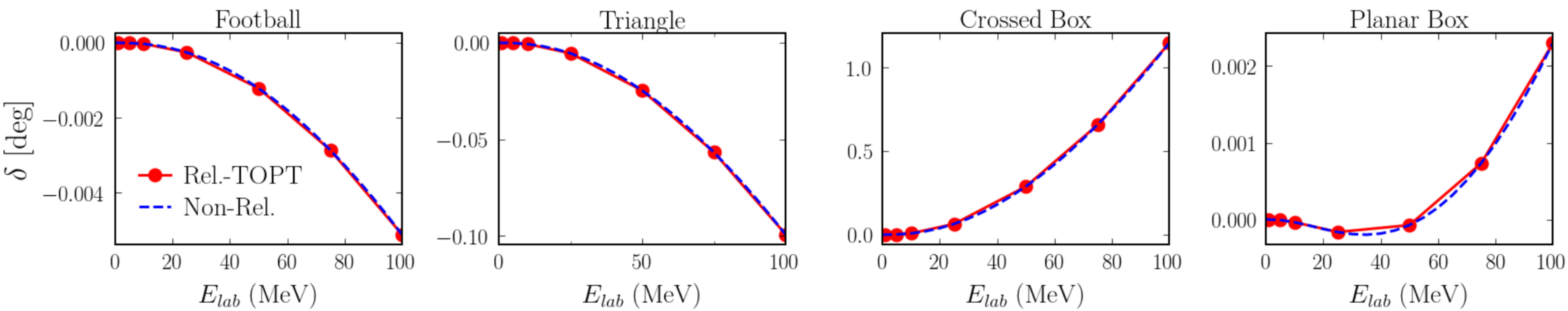
$\times \dfrac{1}{\omega_k \omega_{k+q} \omega_{p-k} \omega_{p'+k}} \big(\tfrac{1}{\cdots} + \tfrac{1}{\cdots} + \tfrac{1}{\cdots} + \tfrac{1}{\cdots} + \tfrac{1}{\cdots} + \tfrac{1}{\cdots}\big)$



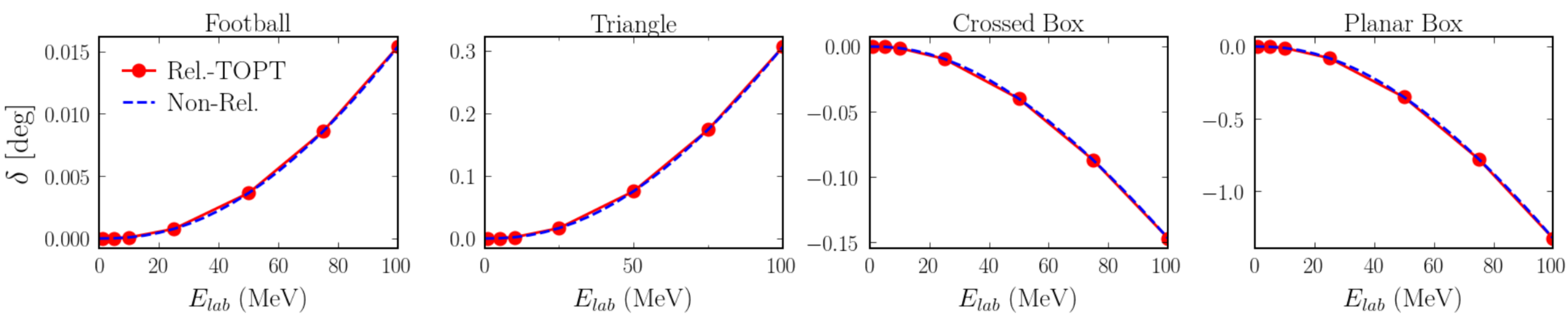
UV Divergent terms and power counting breaking terms are removed by using **the subtractive renormalization**

Consistency check in large m_N limit for TPEP

- Take $m_N = 1000 m_N^{\text{Phys.}}$ for our Two-Pion Exchange potential
  - Iterated in the Lippmann-Schwinger equation (with cutoff=600MeV)
  - Phase shifts for single channels (e.g. ${}^1D_2$)



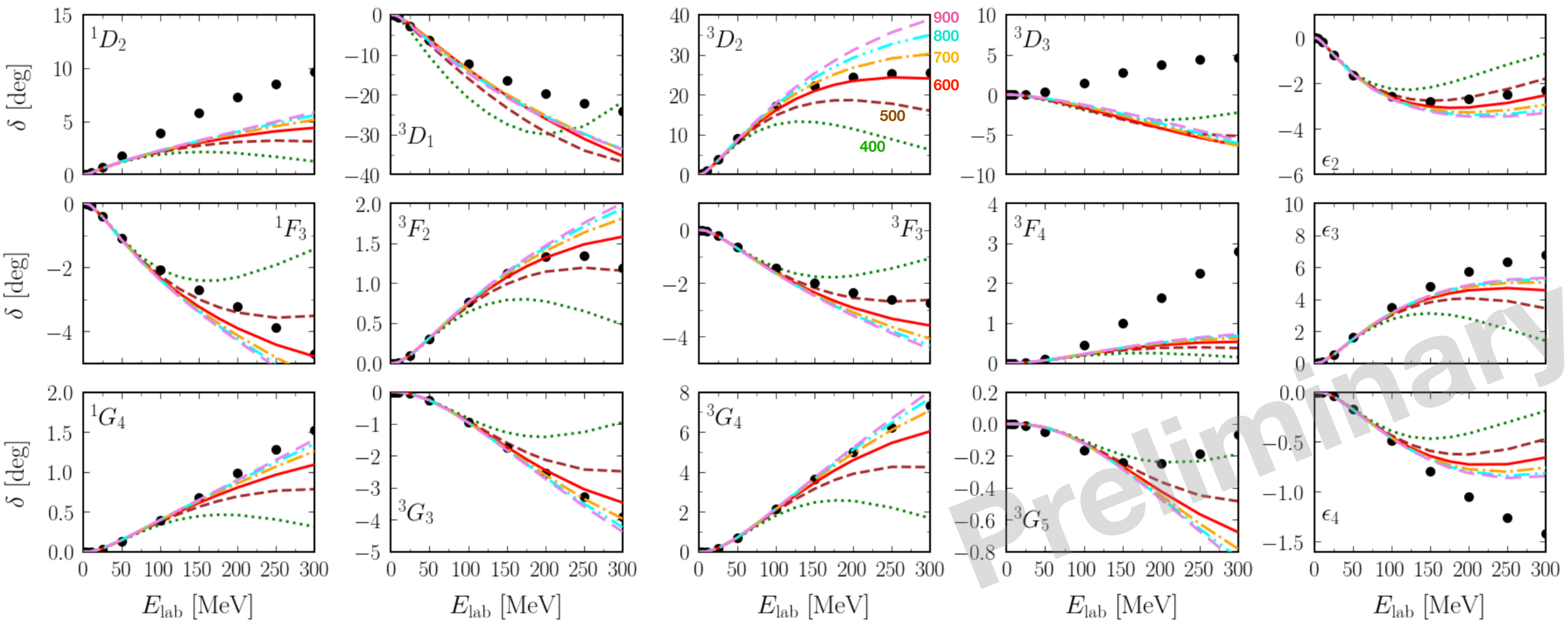
  - Phase shifts for coupled channels (e.g. ${}^3D_3$)



Our TPE potential (NLO) passed the consistency check!

Phase shifts of D, F, G waves at NLO

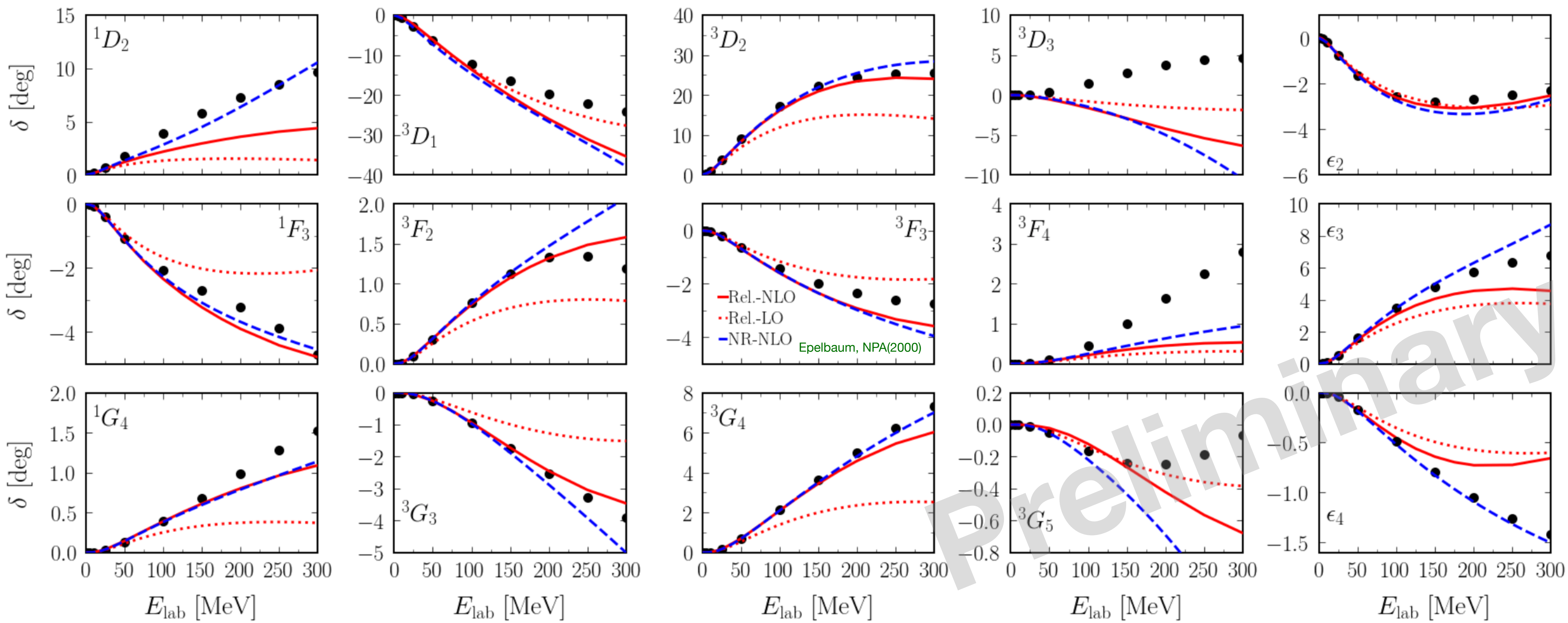
- Use OPE + TPE potentials to predict the phase shifts
  - No free parameters: $f_\pi = 92.4$ MeV, $g_A = 1.267$
  - Solve the Kadyshevsky equation with cutoff $\Lambda \sim 400 - 900$ MeV
    - ✓ Exponential form factor: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$ with n=2



Description of phase shifts with cutoff=600-900 MeV are (globally) similar!

Phase shifts of D, F, G waves at NLO

- Use OPE + TPE potentials to predict the phase shifts
  - No free parameters: $f_\pi = 92.4$ MeV, $g_A = 1.267$
  - Solve the Kadyshevsky equation with cutoff $\Lambda \sim 400 - 900$ MeV
    - ✓ Exponential form factor: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$ with n=2
  - Our NLO results is improved in comparison with our LO study and consistent with the case of non-rel. NLO.



Numerical details of our NLO fit

- 9 LECs are determined by fitting
  - **NPWA**: ***p-n*** scattering phase shifts of Nijmegen 93 *V. Stoks et al., PRC48(1993)792*
  - **7** partial waves: ${}^1S_0, {}^3P_0, {}^1P_1, {}^3P_1, {}^3S_1, \epsilon_1, {}^3P_2$
    - **35** data points: 5 data points for each partial wave **(Elab ≤ 50 MeV)**
    - LECs can be determined separately with different partial waves
  - **Fit-chi-square**:

$$\tilde{\chi}^2 = \sum_i \left(\delta_i^{\text{Theory}} - \delta_i^{\text{Nij93}}\right)^2 .$$

- Solve the Kadyshevsky equation numerically
  - Exponential regulator (first attempt)

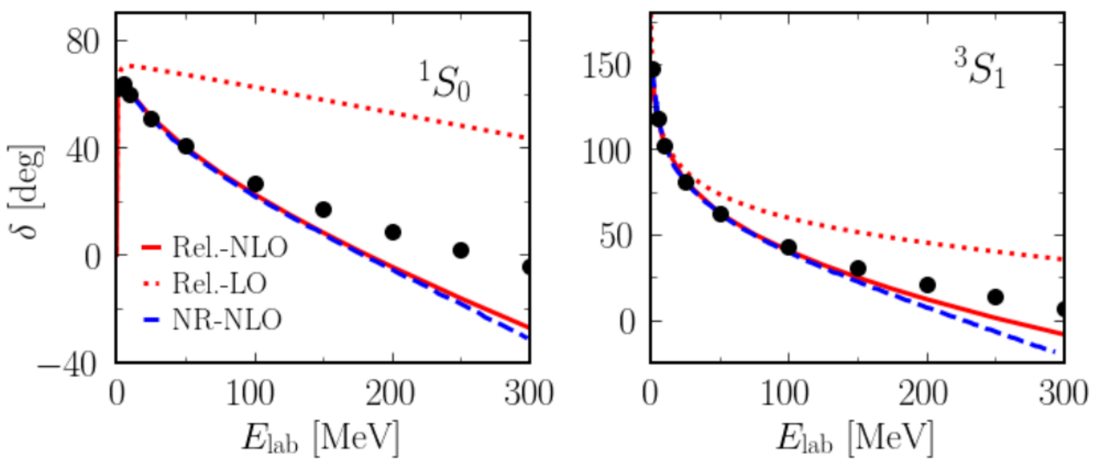
$$F(p) = \exp(-p^{2n}/\Lambda^{2n}) \text{ with n=2}$$

E.Epelbaum, W. Glockle, U.-G. Meißner, NPA671,295(2000)

  - Take cutoff $\Lambda = 600$ MeV

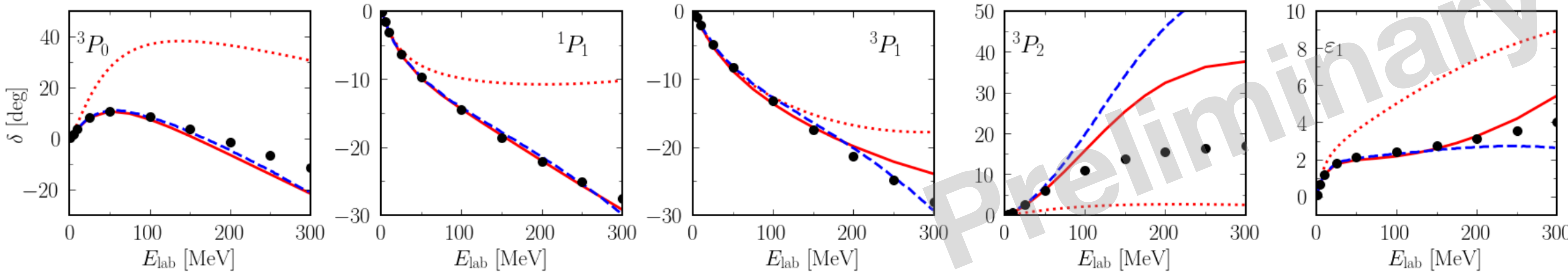
Phase shifts of S and P waves at NLO

Fitting results:



Low-energy constants with Λ = 600 MeV

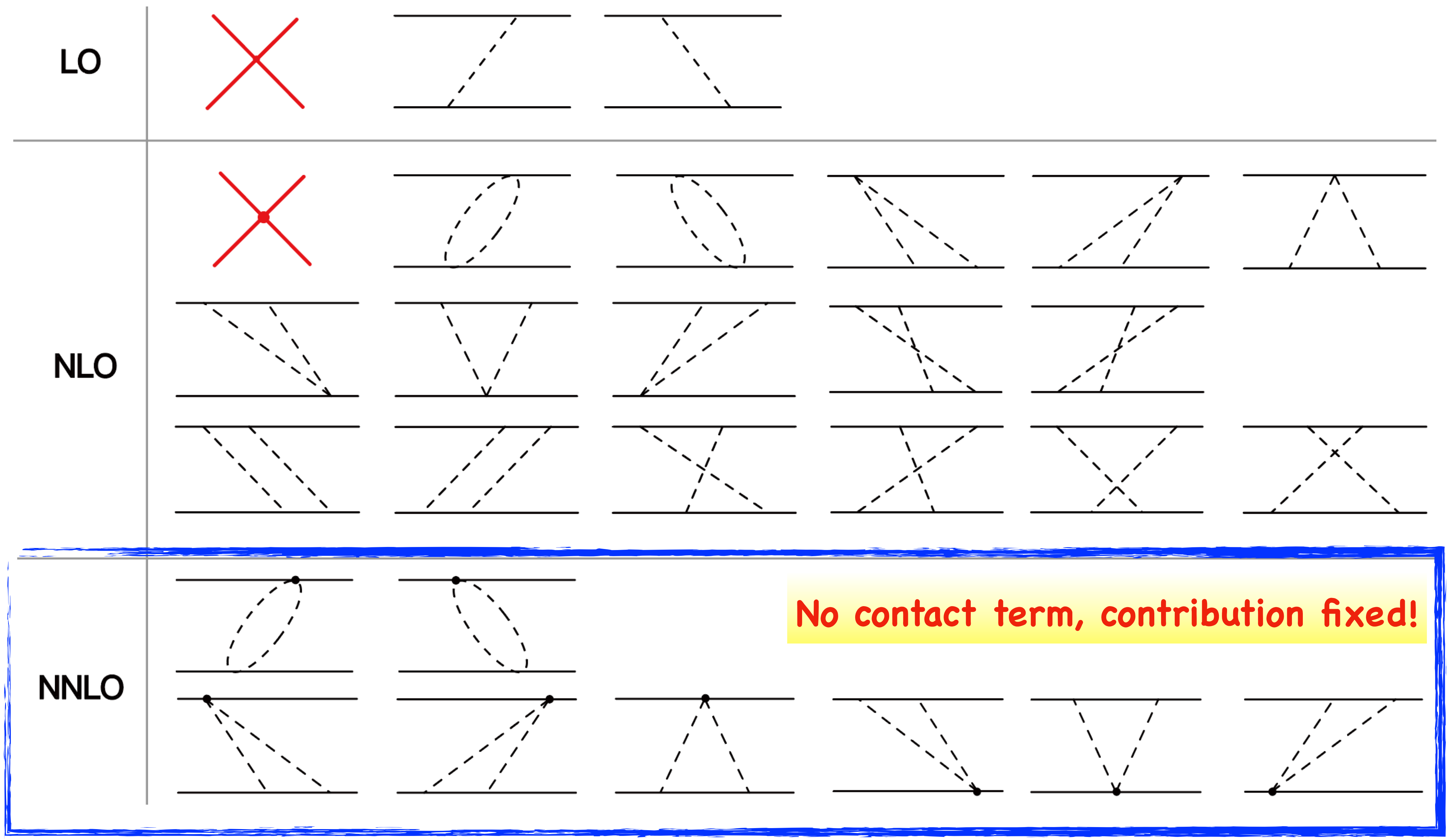
$C^{LO}_{^1S_0}$	0.892	$\tilde{C}_{^3P_0}$	-0.571
$\tilde{C}_{^1S_0}$	-8.920	$\tilde{C}_{^1P_1}$	1.482
$C^{LO}_{^3S_1}$	0.879	$\tilde{C}_{^3P_1}$	-0.014
$\tilde{C}_{^3S_1}$	6.728	$\tilde{C}_{^3P_2}$	0.909
$\tilde{C}_{^3D_1-^3S_1}$	-2.591	Unit	10^4 GeV^{-2}



- NLO calculation can achieve rather good description of phase shifts
- Consistent with the non-relativistic case

Study of NNLO potential in TOPT

- Time ordered diagrams up to NNLO

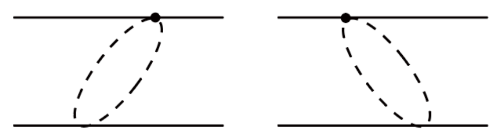


$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle (D_\mu D_\nu + \text{ h.c. }) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] \right\} \Psi_N$$

Two-pion exchange potential at NNLO

- Follow our TOPT rules:
  - Football diagrams

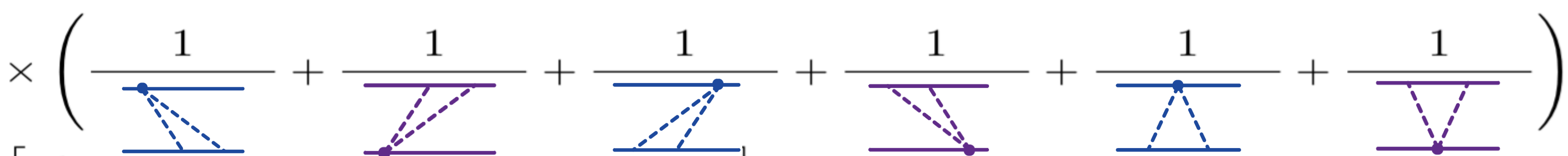


No contribution!

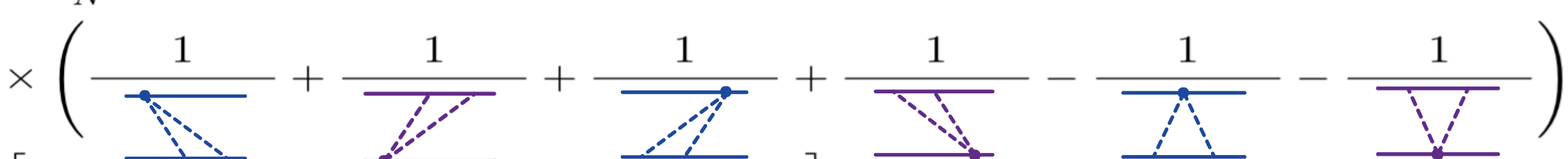
  - Triangle diagrams

$$V_{T+\tilde{T}} = \frac{3m_N g_A^2}{16 f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k}) - (a+b)\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

$$\times\left\{\left[4c_1M_\pi^2 - \frac{c_2}{m_N^2}\left(\boldsymbol{p}\cdot\boldsymbol{k}\,\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\boldsymbol{p}'\cdot\boldsymbol{k}\,\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\right)+2c_3\boldsymbol{k}\cdot(\boldsymbol{k}+\boldsymbol{q})\right]\right.$$

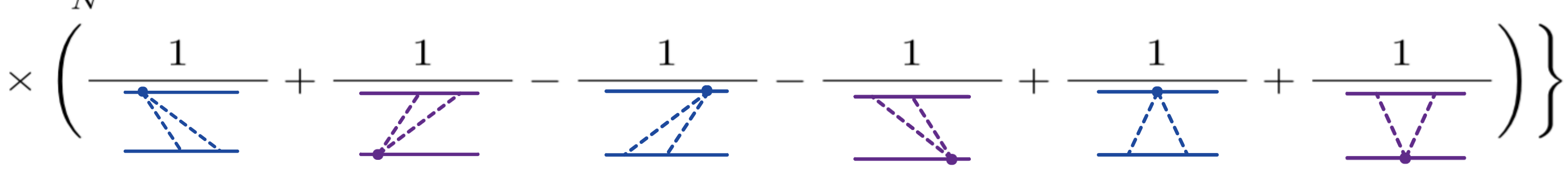


$$+\left[\frac{c_2}{m_N^2}\omega_k\omega_{k+q}(\omega_p+\omega_{p'})+2c_3\omega_k\omega_{k+q}\right]$$



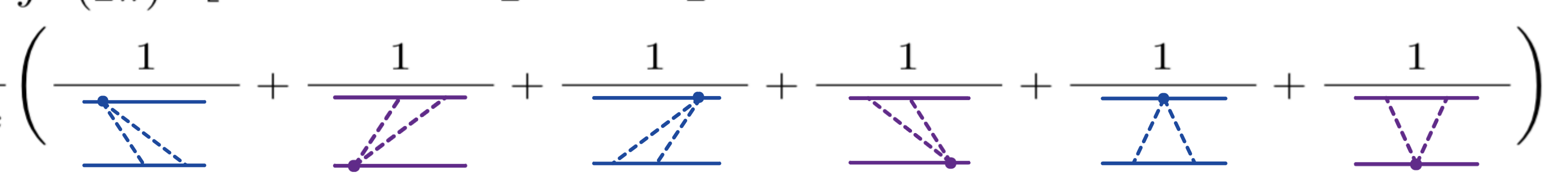
$$+\left[\frac{c_2}{m_N^2}\omega_k\left(\omega_p\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\omega_{p'}\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\right)\right]$$

$$-\left[\frac{c_2}{m_N^2}\omega_{k+q}\left(\omega_p\boldsymbol{p}\cdot\boldsymbol{k}+\omega_{p'}\boldsymbol{p}'\cdot\boldsymbol{k}\right)\right]$$



$$+\frac{c_4 m_N g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{8f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{X_3}{2}\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}+X_4(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

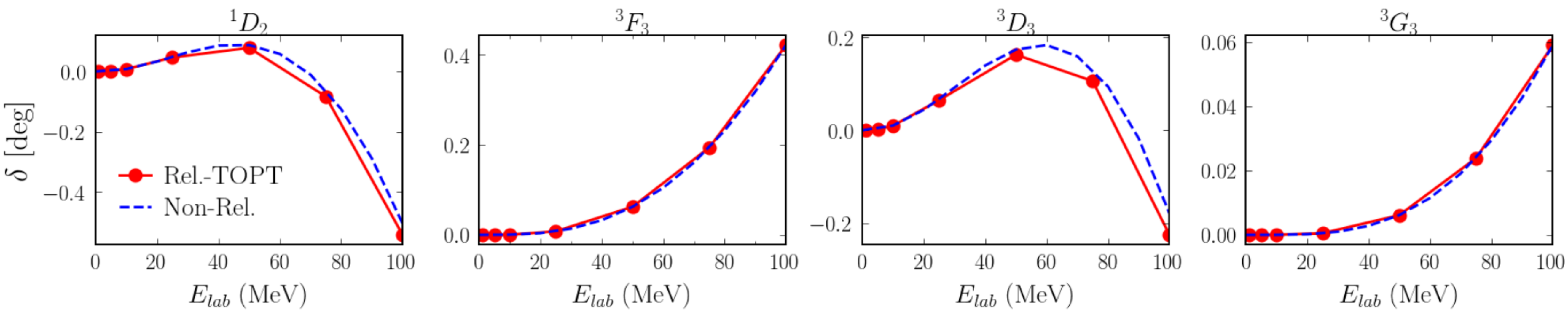
$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$



- UV Divergent terms
- Power-counting breaking terms
- are removed by using **the subtractive renormalization**

Consistency check in large m_N limit

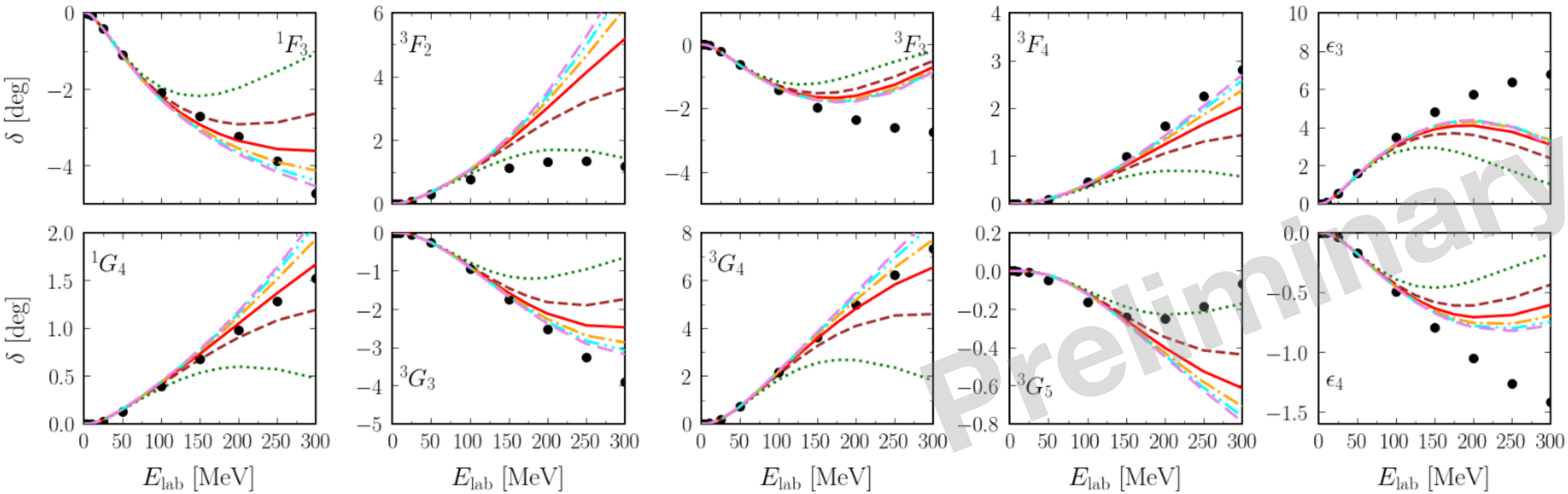
- Take $m_N = 1000 m_N^{\text{Phys.}}$ for our Two-Pion Exchange potential at NNLO
  - Fix couplings $c_1 = -1.22,\ c_2 = 3.58,\ c_3 = -6.04,\ c_4 = 3.48\ \text{GeV}^{-1}$
    D.-L.Yao et al., JHEP05(2016)038
  - Iterated in the Lippmann-Schwinger equation (with cutoff=800MeV)
  - Compare with non-relativistic TPEP at NNLO (neglecting $1/m_N$ correction terms)
  - Phase shifts from both approaches are consistent



Our TPE potential at NNLO passed the consistency check!

Phase shifts of F, G waves at NNLO

- Use OPE + TPE(NLO) + TPE(NNLO) potentials to predict the phase shifts
  - Solve the Kadyshevsky equation with cutoff $\Lambda \sim 400 - 900$ MeV



- More analyses/results will be reported soon
  - Affects of the values of $c_{1,2,3,4}$ to TPEP at NNLO
  - Regularization schemes
    - ✓ Exponential regulator: distorts the long-range part of the interaction
    - ✓ Semi-local regulator:

$$V_{\rm NN}^{\pi}(\mathbf{r}) \to \left(1-\exp\left[-\left(r^2/R^2\right)\right]\right)^n V_{\rm NN}^{\pi}(\mathbf{r})$$

$$\delta(\mathbf{r}) \to C \to \exp\left[-\left(\left(p^2+p'^2\right)/\Lambda^2\right)^n\right] C$$

E.Epelbaum, et al., PRL(2015)

Summary

- We proposed a systematic framework to formulate the NN interactions based on the time-ordered perturbation theory using the manifestly Lorentz invariant effective Lagrangian
  - Obtained the rules of time-ordered diagrams with spin-1/2 fermions
  - Derived the Kadyshevsky equation self-consistently
    - ✓ Effective potential and the scattering equation are obtained within the same framework
  - Obtained **non-singular LO potential**, which is perturbatively renormalizable
    - ✓ Avoid finite-cutoff artefacts and take cutoff $\Lambda \to \infty$
  - **Formulated the chiral potential up to NNLO**
    - ✓ Calculated the complicated two-pion-exchange potential at one-loop level
    - ✓ As a first step, NNLO potential is treated **non-perturbatively**
    - ✓ Achieved a rather reasonable description of phase shifts

Future perspectives

- Investigate the energy-independent potential at NNLO
  - Follow the idea of energy-independent OPEP

$$V(E;k,q) = V\left(\omega_k + \omega_q; p', p\right) + (E - \omega_k - \omega_q)\,\frac{\partial V(E;k,p)}{\partial E}|_{E=\omega_k+\omega_q} + \frac{(E-\omega_k-\omega_q)^2}{2!}\frac{\partial^2 V(E,k,p)}{\partial E^2}|_{E=\omega_k+\omega_q} + \cdots$$

  - Can be applied to the whole two-pion-exchange potentials
  - This will be more convenient for many-body calculations
- Perturbatively include NLO/NNLO contributions
  - Based on our non-singular LO potential, all divergences of the amplitude can be systematically removed $(\Lambda \sim \infty)$
- In the long run, apply **symmetry preserving regularization** to investigate the chiral potential
  - e.g. preserve chiral symmetry

J. Behrendt, E. Epelbaum, J. Gegelia, U.-G. Meißner and A. Nogga, Eur. Phys. J. A 52,296 (2016).

Thank you for your attention!