Recent and Ongoing Activities by the LENPIC

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LENPIC

Low Energy Nuclear Physics International Collaboration

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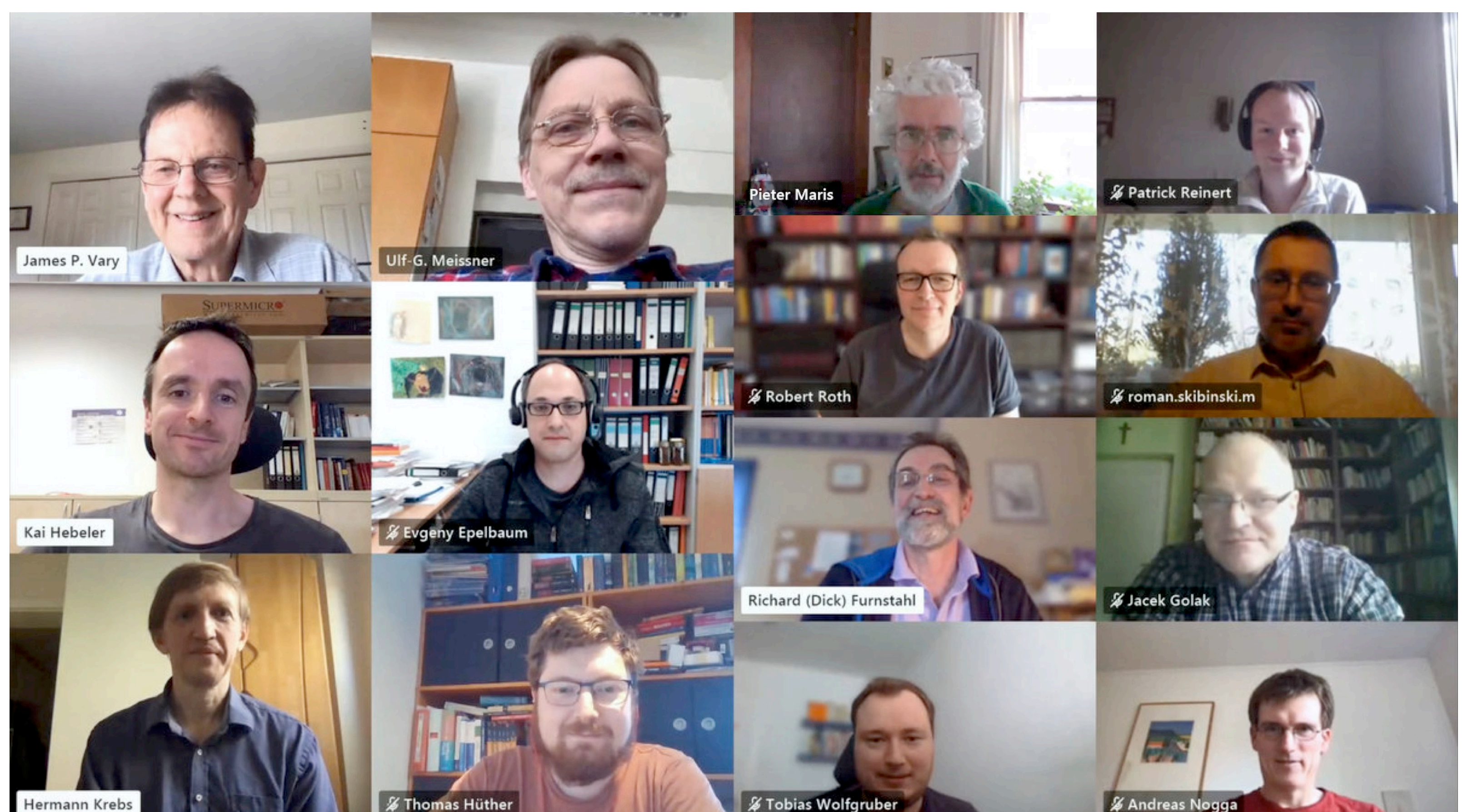
(LENPIC Collaboration)

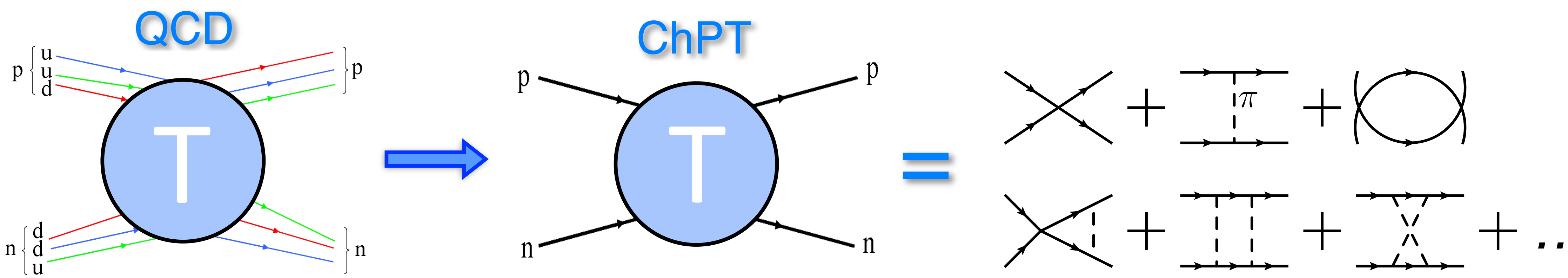
LENPIC aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents



Outline

- Nuclear forces in chiral EFT
- P-Shell nuclei with $A \leq 16$ up to N^{2}LO
- Challenges for consistent treatment of 3NF at N^{3}LO
- Symmetry preserving regularization

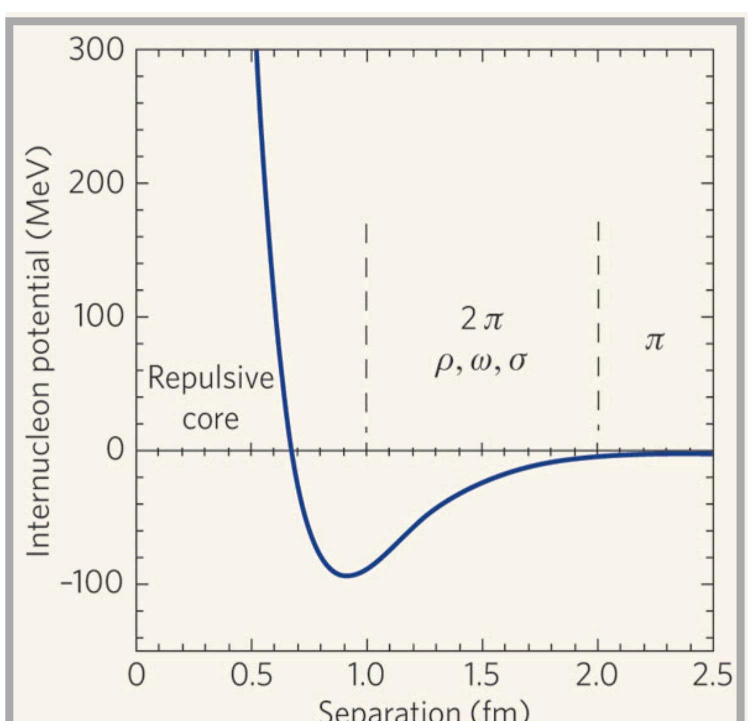
From QCD to nuclear physics

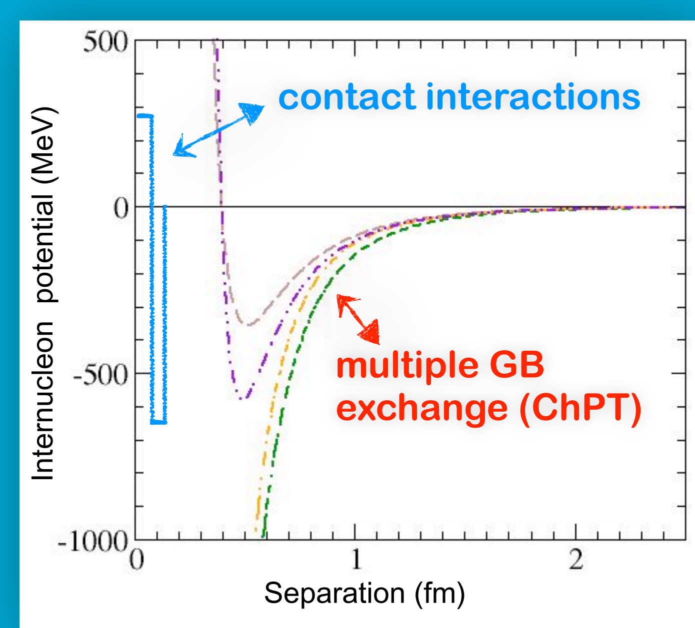


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\textit{derived within ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

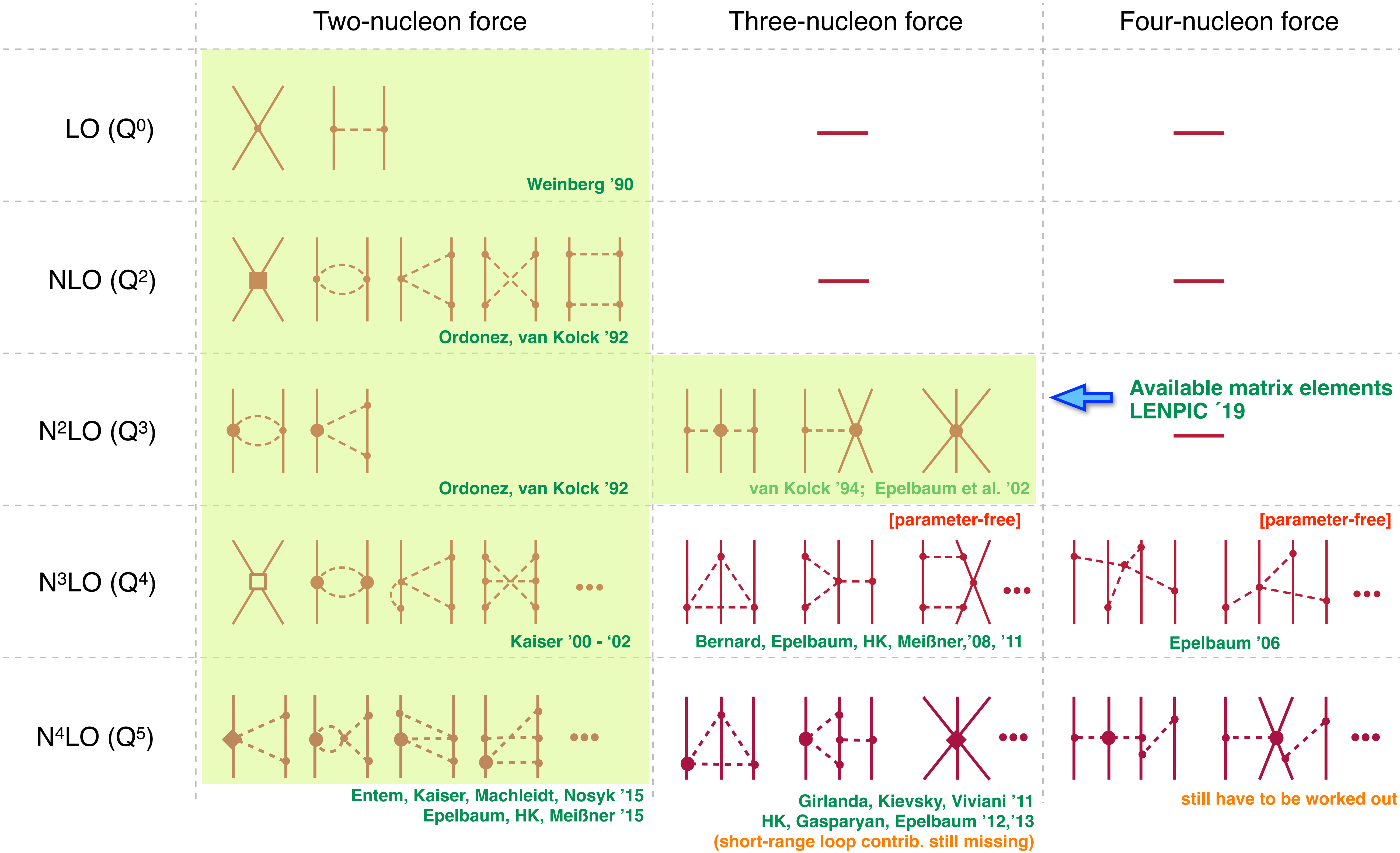
Weinberg '91





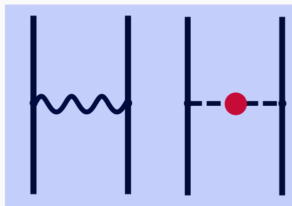
- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

Chiral Expansion of the Nuclear Forces

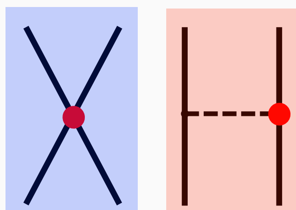


Isospin-breaking in the Nuclear Force

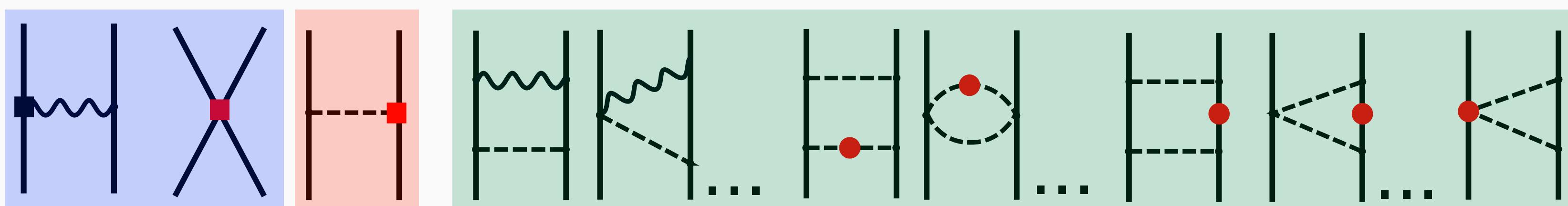
NLO



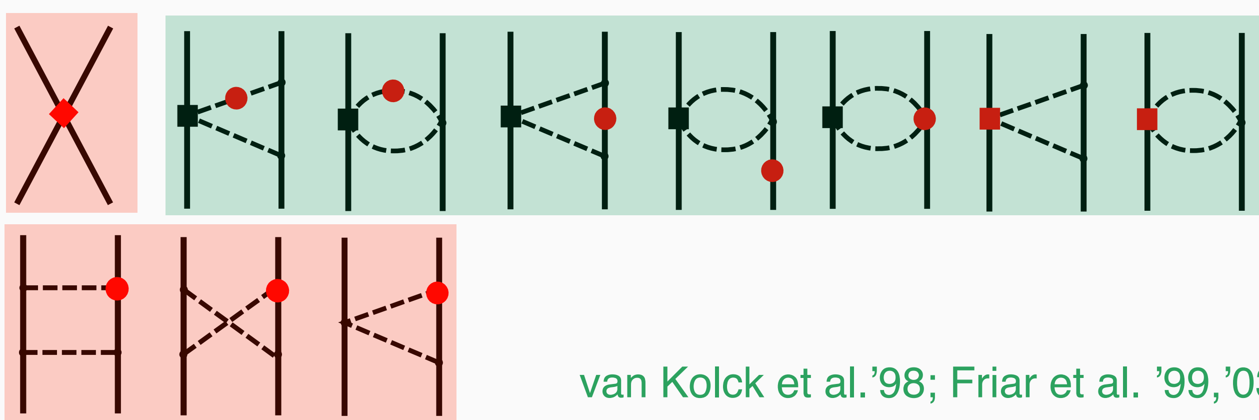
N^{2}LO



N^{3}LO



N^{4}LO



Have been employed in **Reinert, HK, Epelbaum, EPJA 54 (2018) 88**

Parameter-free: depend on δM_π, δm = 1.29 MeV and $(\delta m)^{QCD}$ = -1.87(16) MeV **[Gasser, Leutwyler, Rusetsky '21]**

Depend on 3 πN coupling constants + 3 IB contact terms in p-waves

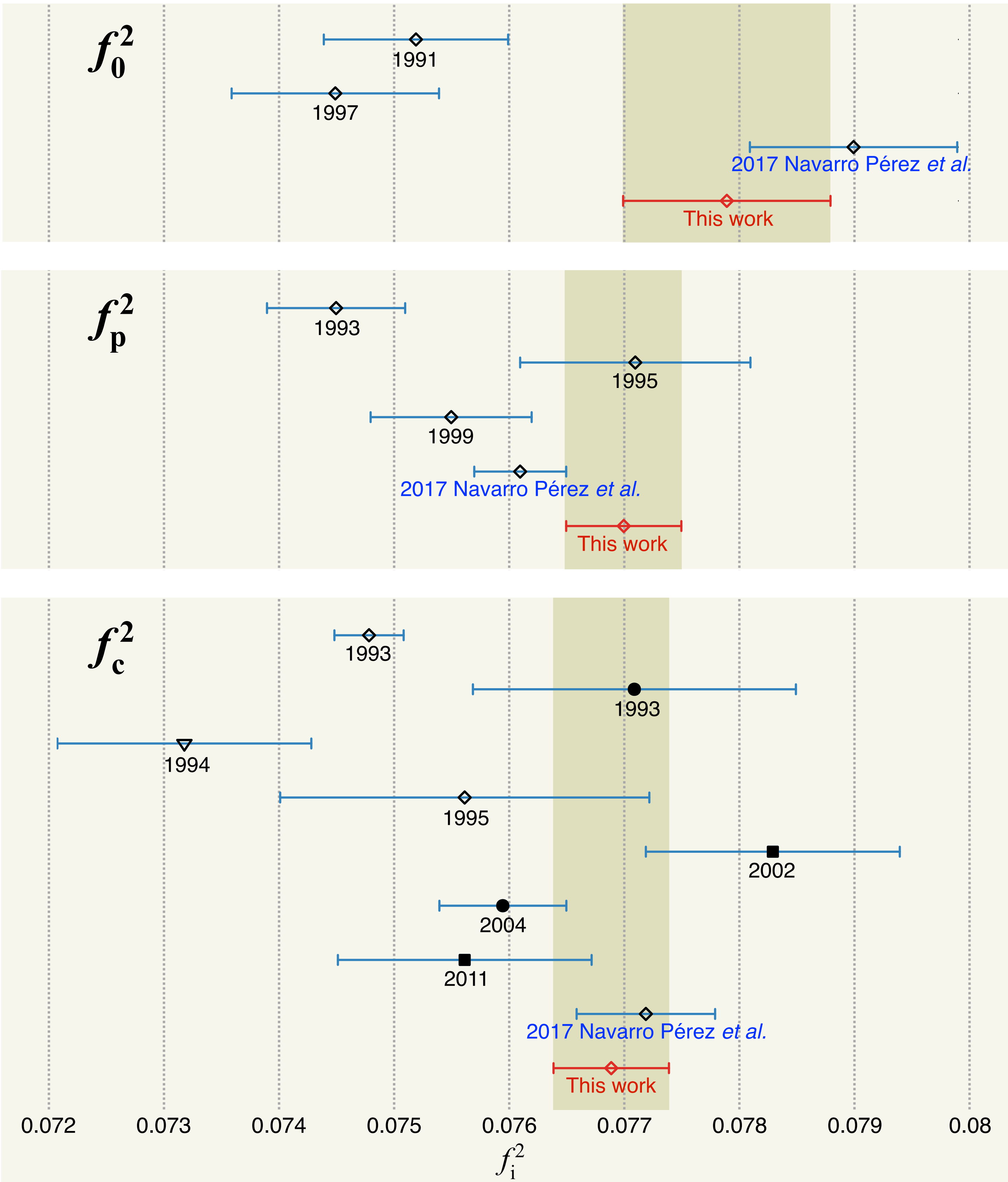
van Kolck et al.'98; Friar et al. '99,'03,'04; Niskanen '02; Epelbaum, Meißner '05

Away from the isospin limit, one introduces 3 **π**N coupling constants:

$$f_{\pi^0 \text{pp}} = \frac{M_{\pi^\pm}\, g_{\pi^0 \text{pp}}}{2\sqrt{4\pi} m_\text{p}} \qquad f_{\pi^0 \text{nn}} = \frac{M_{\pi^\pm}\, g_{\pi^0 \text{nn}}}{2\sqrt{4\pi} m_\text{n}} \qquad f_{\pi^\pm \text{pn}} = \frac{M_{\pi^\pm}\, g_{\pi^\pm \text{pn}}}{\sqrt{4\pi}(m_\text{p} + m_\text{n})}$$

$$\begin{aligned} f_0^2 &= -f_{\pi^0 \text{nn}}\, f_{\pi^0 \text{pp}} \\ f_p^2 &= f_{\pi^0 \text{pp}}\, f_{\pi^0 \text{pp}} \\ 2 f_c^2 &= f_{\pi^\pm \text{pn}}\, f_{\pi^\pm \text{pn}} \end{aligned}$$

Determination of πN constants



Reinert, HK, Epelbaum PRL126 (2021) 092501

$$f_0^2 = 0.0779(9)(1.3)$$
$$f_p^2 = 0.0770(5)(0.8)$$
$$f_c^2 = 0.0769(5)(0.9)$$

statistical **and systematic** errors due to the EFT truncation, choice of E_{max} and data selection

uncertainty in the subleading πN LECs

No evidence for charge dependence of the πN coupling constants

Our f_c^2 value is consistent with the extractions from the πN system

f_c^2 corresponds to $g_{\pi NN} = 13.23 \pm 0.04$

Pionic hydrogen exp. at PSI **Hirtl et al.´21**

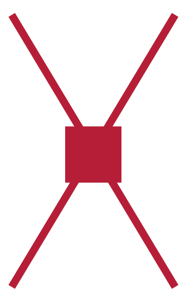
$$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D}: \quad g_{\pi NN} = 13.10 \pm 0.10$$

$$\Gamma_{1s}^{\pi H}: \quad g_{\pi NN} = 13.24 \pm 0.10$$

Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO [Q^0]: 2 operators (S-waves)

NLO [Q^2]: + 7 operators (S-, P-waves and ε_1)

N^2LO [Q^3]: no new terms

N^3LO [Q^4]: + 12 operators (S-, P-, D-waves and ε_1, ε_2)

N^4LO [Q^5]: + 5 IB operators

N^4LO^+ [Q^6]: + 4 operators (F-waves)

# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

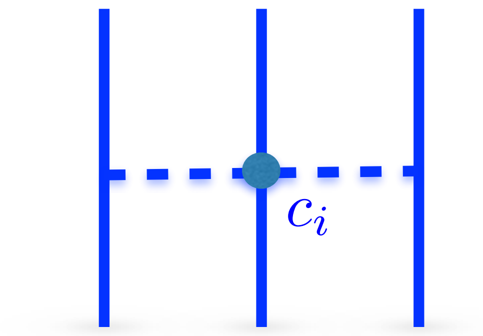
Summary on NN

- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data

  χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

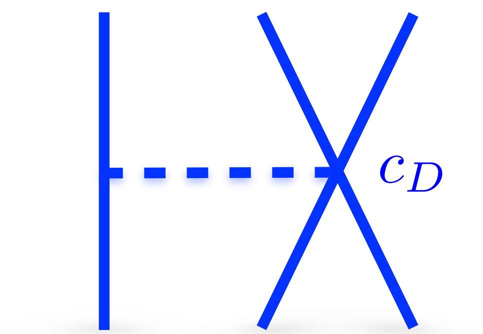
Three-Nucleon Force at N²LO

Epelbaum et al. EPJA56 (2020) 92; Maris et al. PRC103 (2021) 054001



c_i´s are extracted from solutions of Roy-Steiner equation in pion-nucleon scattering: Hoferichter et al. PRL115 (2015) 192301

$$c_1 = -0.74\,\mathrm{GeV}^{-1} \quad c_3 = -3.61\,\mathrm{GeV}^{-1} \quad c_4 = 2.44\,\mathrm{GeV}^{-1}$$

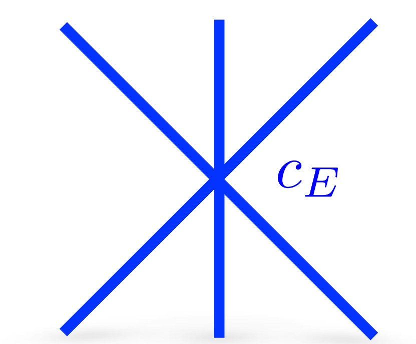


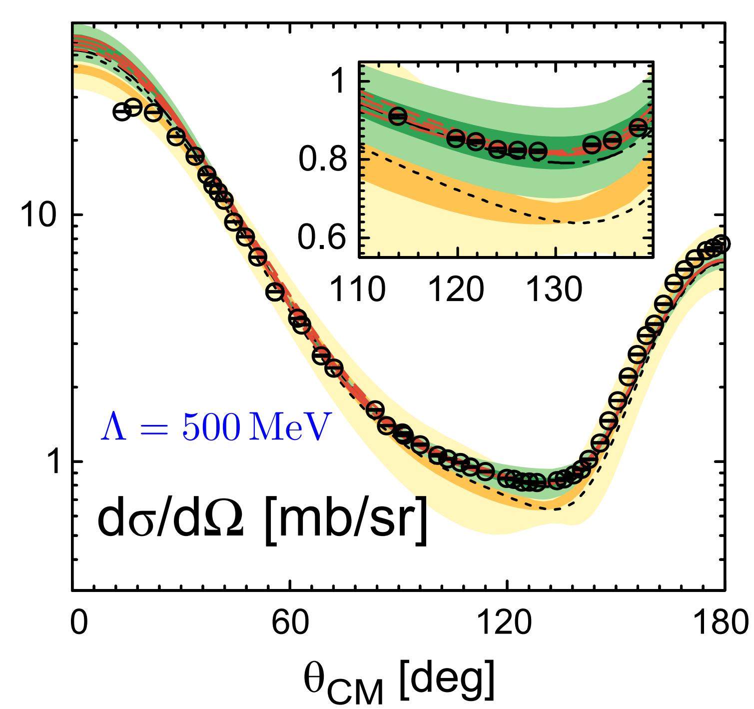
Requirement to reproduce ³H correlates c_D & c_E

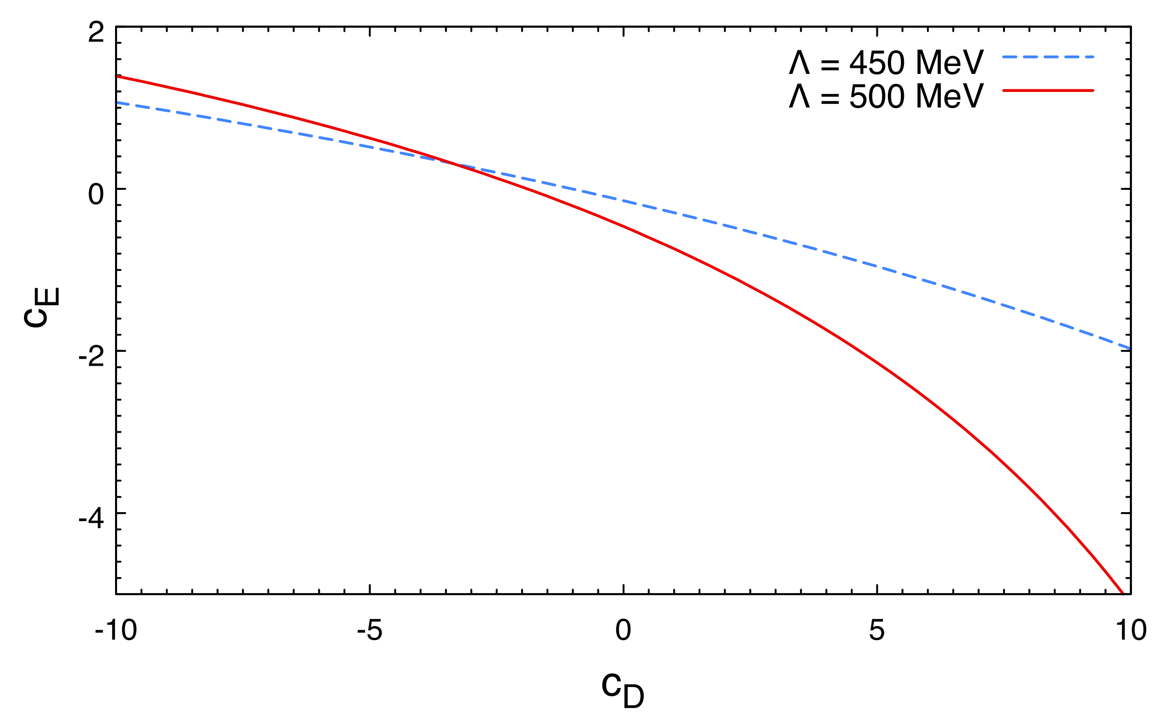
c_D is fitted to the minimum of Nd-scattering cross section at $E^N_{\mathrm{lab}} = 70\,\mathrm{MeV}$

Sekiguchi et al. PRC65 (2002) 034003

$$c_D = 2.485, \quad c_E = -0.528 \quad \text{for} \quad \Lambda = 450\,\mathrm{MeV}$$
$$c_D = -1.626, \quad c_E = -0.063 \quad \text{for} \quad \Lambda = 500\,\mathrm{MeV}$$

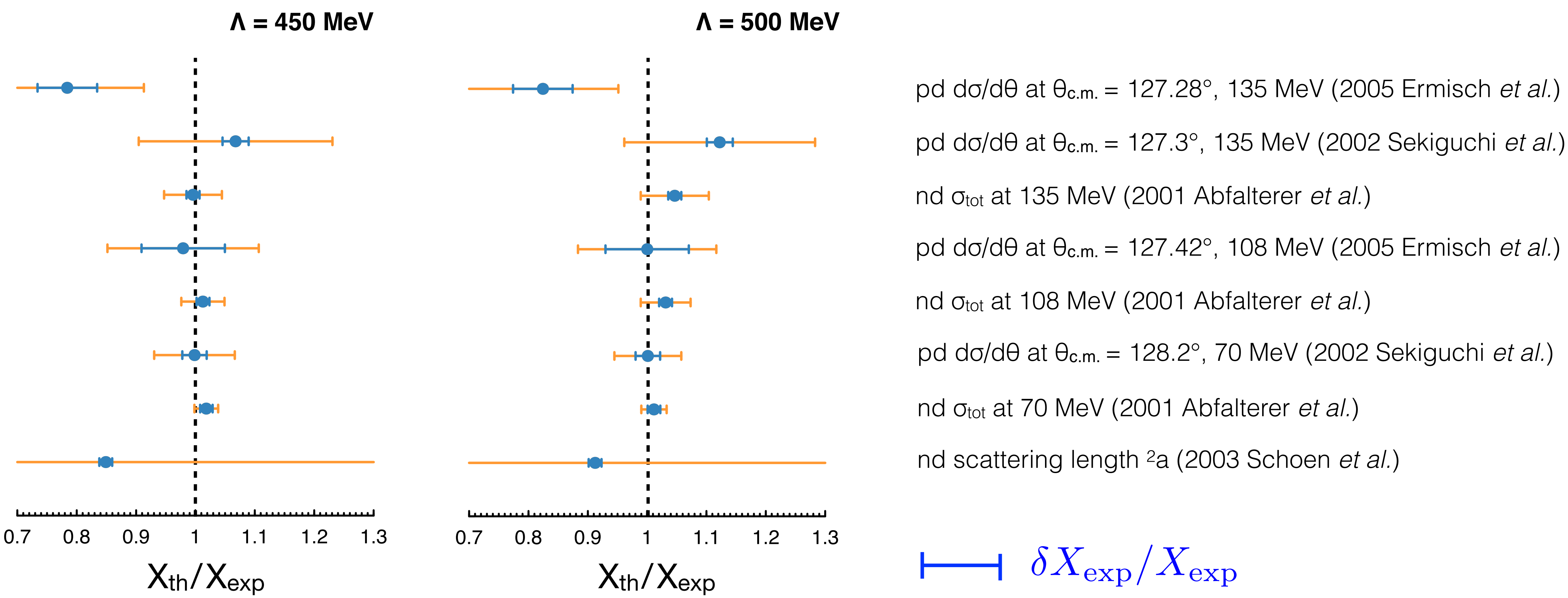






Nucleon-Deuteron Scattering at N^2LO

Maris et al. PRC103 (2021) 054001



Yellow error bar from Bayesian analysis: 68% DoB

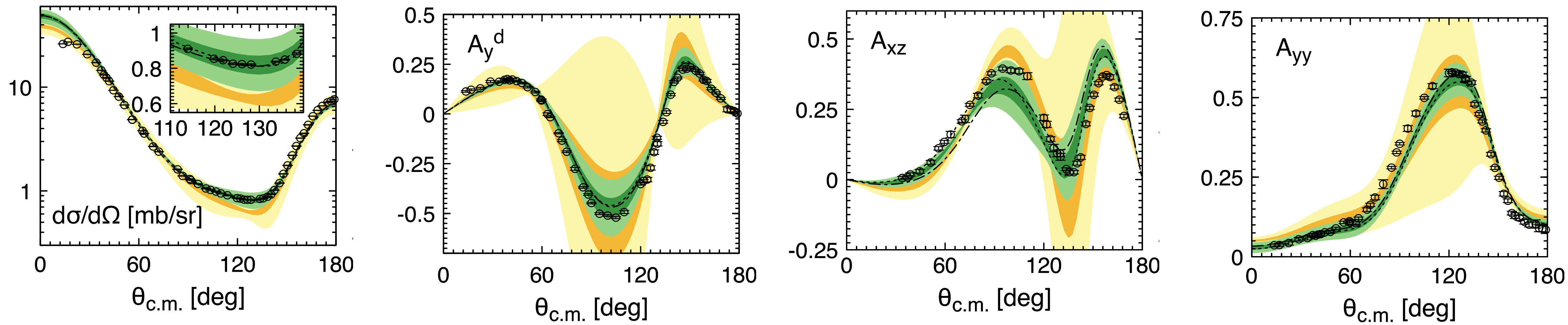
Epelbaum et al. EPJA56 (2020) 92
Furnstahl et el. PRC92 (2015) 2, 024005

$$X = X_{\rm ref}\left(c_0 + c_2 Q^2 + c_3 Q^3 + \dots\right) \qquad Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\rm eff}}{\Lambda_b}\right) \qquad \Lambda_b = 650\,{\rm MeV}$$

- c_D determination is consistent with given Nd observables
- No conclusion about discrepancies at 135 MeV is possible due to large error bars at N^2LO

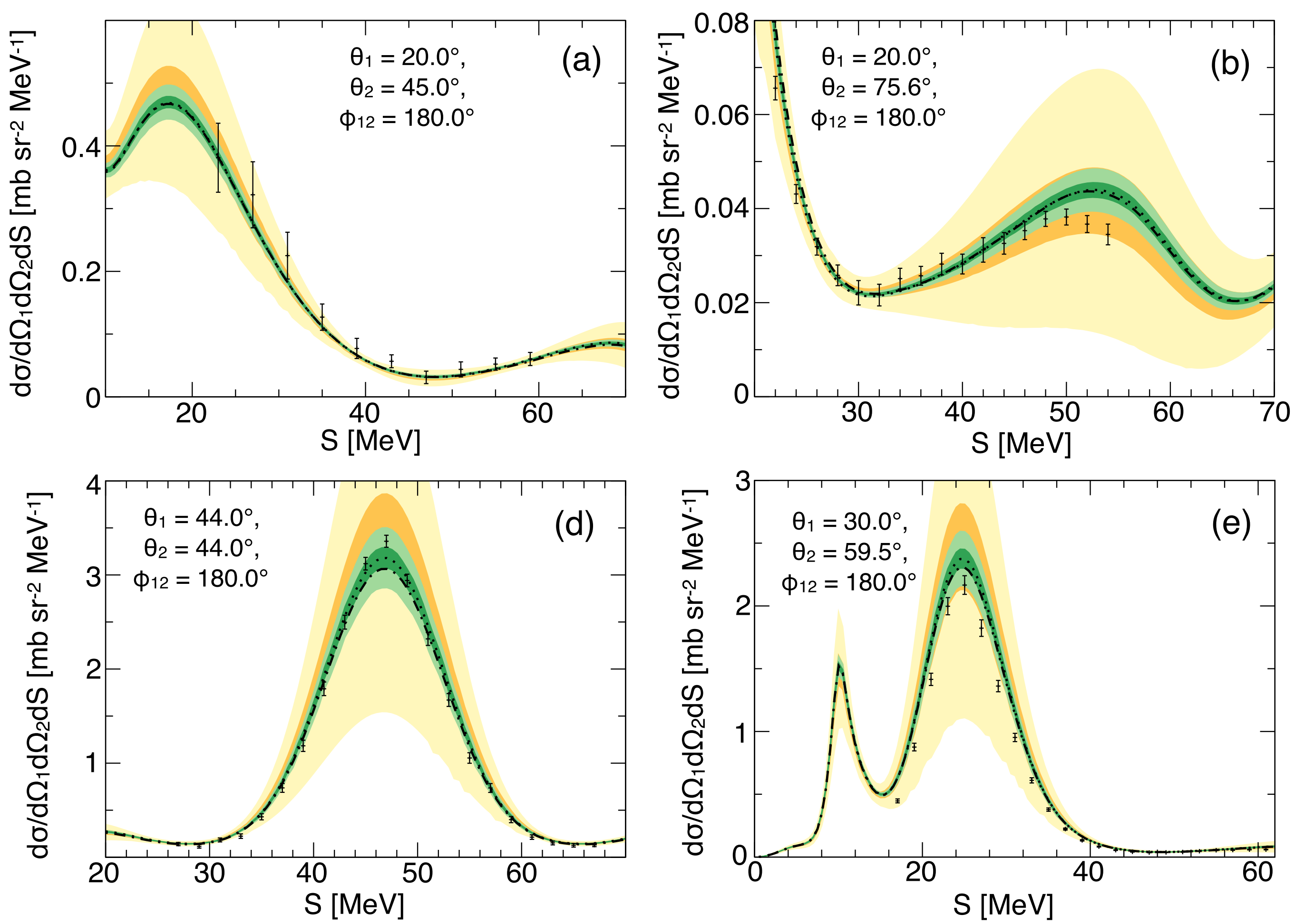
Nd Scattering at Low Energy

Differential cross section and selected analyzing powers of elastic Nd scattering at $E_N = 70$ MeV



Deuteron break up at $E_N = 65$ MeV

Light (dark) shaded bands indicate 95% (68%) DoB



Small 3NF contribution to these observables

Relativistic corrections are known to decrease (d) and (e) by 10%

Skibinski et al. EPJA30 (2006) 369

Relativistic corrections to Nd start to contribute at N^3LO which is consistent with the size of theoretical error bands

A = 3 & 4 Nuclei

Faddeev and Yakubovsky equations in momentum space: **Nogga et al. PRC65 (2002) 054003**

All angular momenta ≤ 5 of the two-body subsystem are taken into account

Numerical accuracy $\sim 1\,\mathrm{keV}$ reached for $A = 3$ binding energies and expectation values

Maris et al. PRC103 (2021) 054001

		Λ	E	$\langle T \rangle$	$\langle V_{NN} \rangle$	$\langle V_{3NF} \rangle$
^{3}H	LO	450	−12.22	52.38	−64.61	
	NLO		−8.515	34.31	−42.82	
	N^2LO (NN-only)		−8.143	34.94	−43.08	
	N^2LO+3NFs		−8.483	36.13	−44.16	−0.459
^{3}H	LO	500	−12.52	57.84	−70.36	
	NLO		−8.325	35.87	−44.19	
	N^2LO (NN-only)		−7.920	37.94	−45.86	
	N^2LO+3NFs		−8.482	40.27	−48.09	−0.660
^{3}He	LO	450	−11.34	51.45	−62.79	
	NLO		−7.751	33.55	−41.30	
	N^2LO (NN-only)		−7.397	34.15	−41.55	
	N^2LO+3NFs		−7.734	35.37	−42.65	−0.452
^{3}He	LO	500	−11.63	56.88	−68.51	
	NLO		−7.574	35.07	−42.65	
	N^2LO (NN-only)		−7.194	37.11	−44.30	
	N^2LO+3NFs		−7.739	39.44	−46.54	−0.641

Point Coulomb interaction has been included for pp system in ^{3}He calc

Experimental values:
^{3}H BE = - 8.482 MeV
^{3}He BE = - 7.718 MeV

Starting from N^2LO, the BE's are underpredicted with NN - only

Attractive contribution of 3NF brings E to its physical value

A = 3 & 4 Nuclei

For A = 4 three orbital angular momenta contribute. We need to constrain two of them to get finite number of partial waves. Sum of all orbital angular momenta is constrained to be ≤ 10

Numerical accuracy $\sim 10\,\mathrm{keV}$ reached for A = 4 binding energies and $\sim 50\,\mathrm{keV}$ for expectation values

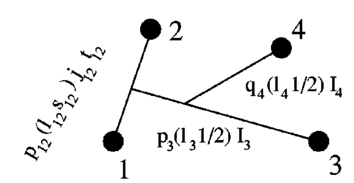
Maris et al. PRC103 (2021) 054001

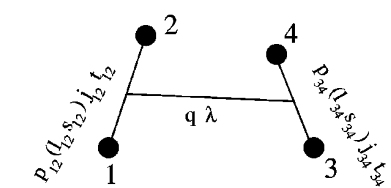
		Λ	E	$\langle T \rangle$	$\langle V_{NN} \rangle_1$	$\langle V_{NN} \rangle_2$	$\langle V_{3NF} \rangle$
^{4}He	LO	450	−49.99	124.4	−174.4	−174.4	
	NLO		−29.36	71.47	−100.8	−100.8	
	N^{2}LO, NN-only		−27.32	71.95	−99.3	−99.2	
	N^{2}LO+3NFs		−28.62	75.73	−102.0	−102.0	−2.376
^{4}He	LO	500	−51.47	139.2	−190.7	−190.7	
	NLO		−28.15	74.56	−102.7	−102.7	
	N^{2}LO, NN-only		−25.95	78.54	−104.5	−104.4	
	N^{2}LO+3NFs		−28.72	86.71	−111.9	−111.9	−3.474

Different choice of Jacobi momenta:

$\langle \ldots \rangle_1 \leftrightarrow 3+1$

$\langle \ldots \rangle_2 \leftrightarrow 2+2$





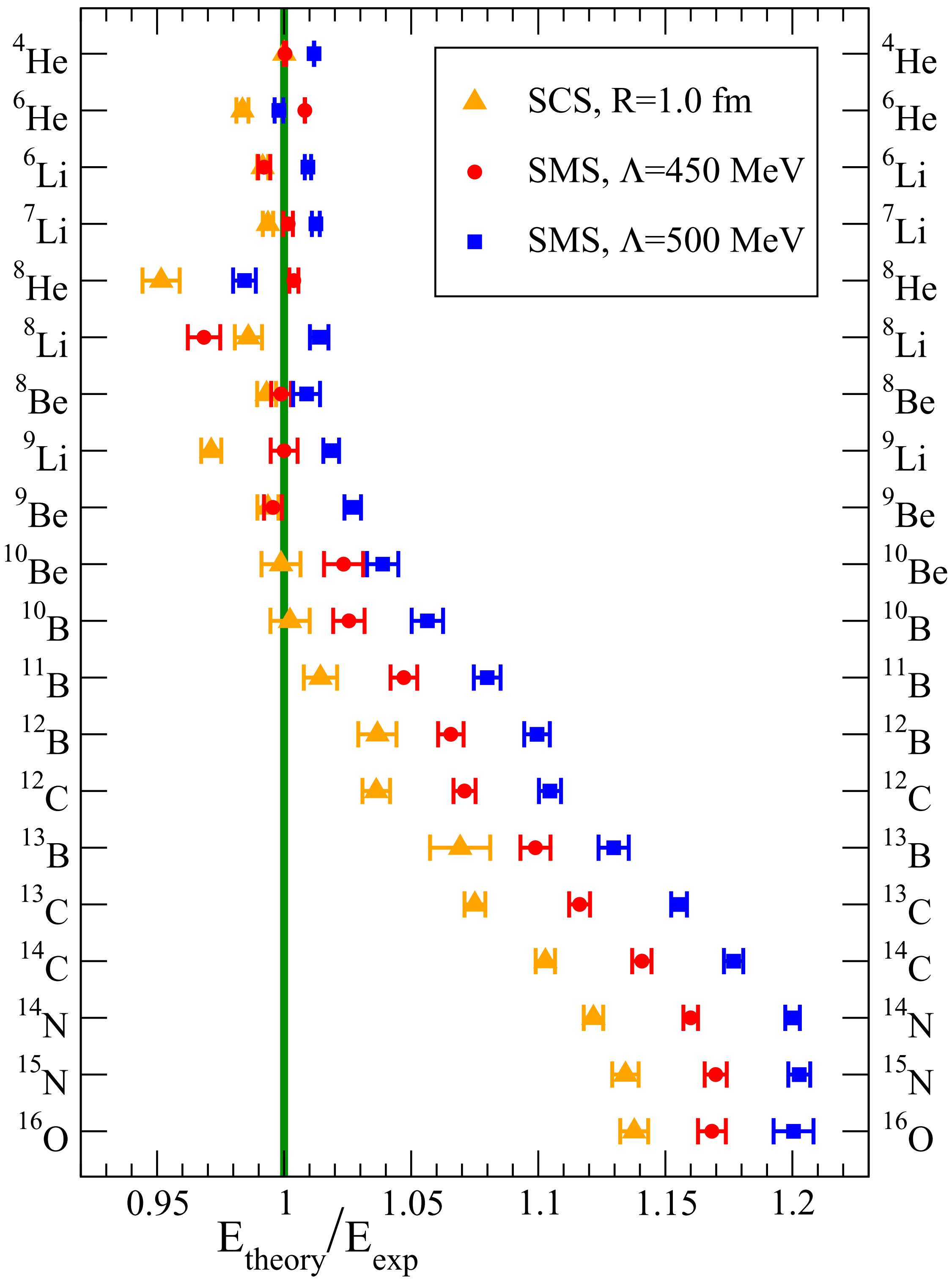
Strong overbinding at LO compared to experimental ^{4}He BE = - 28.296 MeV
Overbinding is drastically reduced at NLO. At N^{2}LO without 3NF ^{4}He is slightly underbound.

NDA estimate of 3NF contribution to BE of ^{4}He: $X_{\mathrm{ref}} = \langle V_{NN} \rangle \sim -100\,\mathrm{MeV}$

$$Q = \frac{M_\pi^{\mathrm{eff}}}{\Lambda_b} = \frac{200}{650} \simeq 0.31 \;\Rightarrow\; X_{\mathrm{ref}}\, Q^3 \simeq -2.9\,\mathrm{MeV}$$

P-Shell Nuclei

No-Core Configuration Interaction (NCCI) approach: **Barrett, Navratil, Vary, PPNP69 (2013) 131**
+ Similarity Renormalization Group (SRG): **Bogner, Furnstahl, Schwenk PPNP65 (2010) 94**
Roth et al. PRC90 (2014) 024325



Systematic truncation errors are not shown here

- SRG induced 3NF's have been included. Similar results for two SRG parameters.

Two different interactions are compared

Semilocal Coordinate Space (SCS)

Epelbaum, HK, Meißner, PRL115 (2015) 122301

$$V_{\text{long-range}}(\vec{r}\,)\left[1-\exp\left(-\frac{r^2}{R^2}\right)\right]^n$$

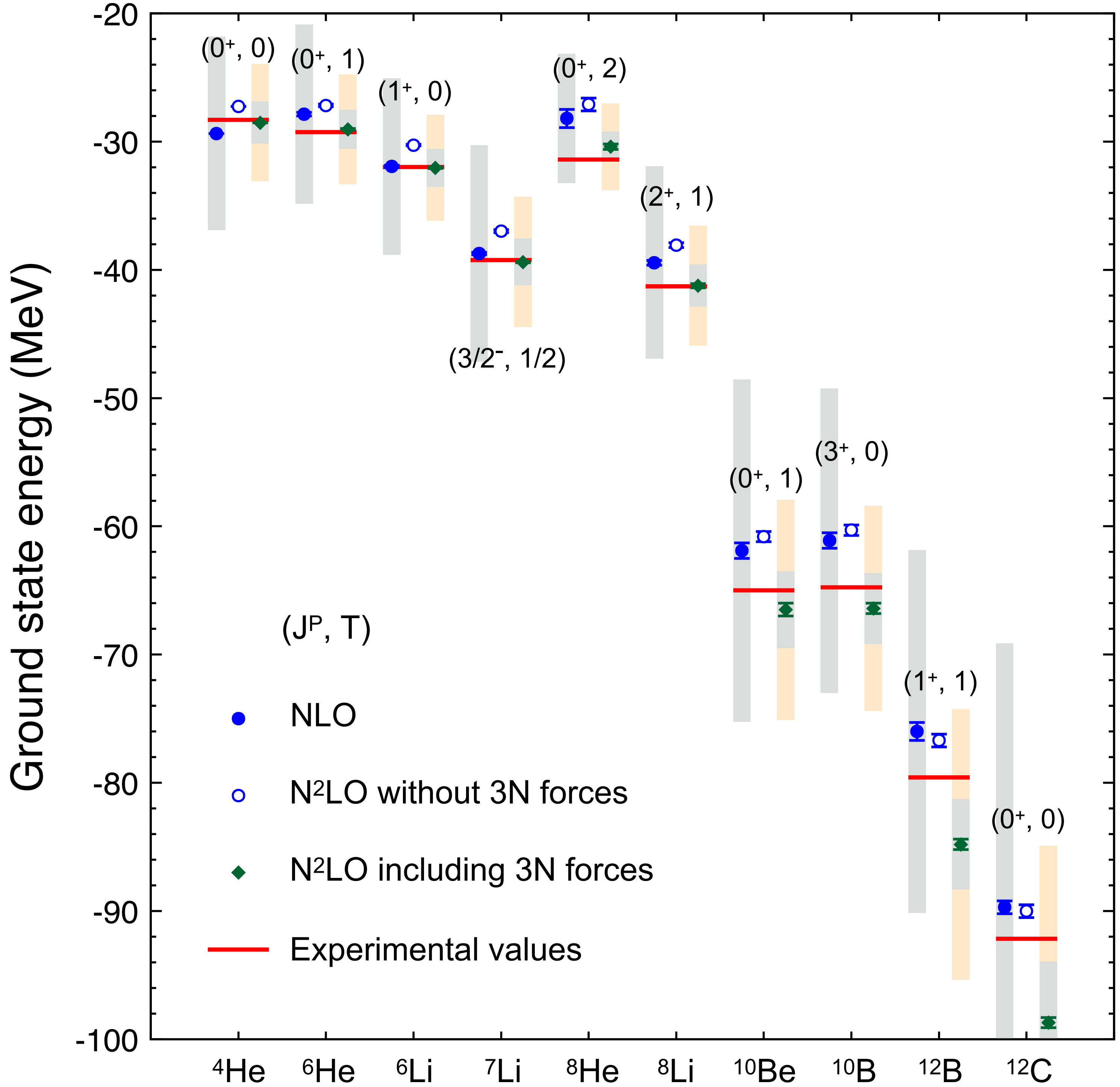
Semilocal Momentum Space (SMS)

Reinert, HK, Epelbaum, EPJA54 (2018) 5, 86

- Overbinding at N²LO increases with increasing atomic number A
- SCS are closer to experimental data than SMS predictions

P-Shell Nuclei

Systematic uncertainties with expansion parameter $Q = \frac{M_\pi^{\text{eff}}}{\Lambda_b} = \frac{200}{650} \simeq 0.31$ is assumed



Estimated Q may be too optimistic for heavier nuclei

Binder et al. PRC93 (2016) 044002

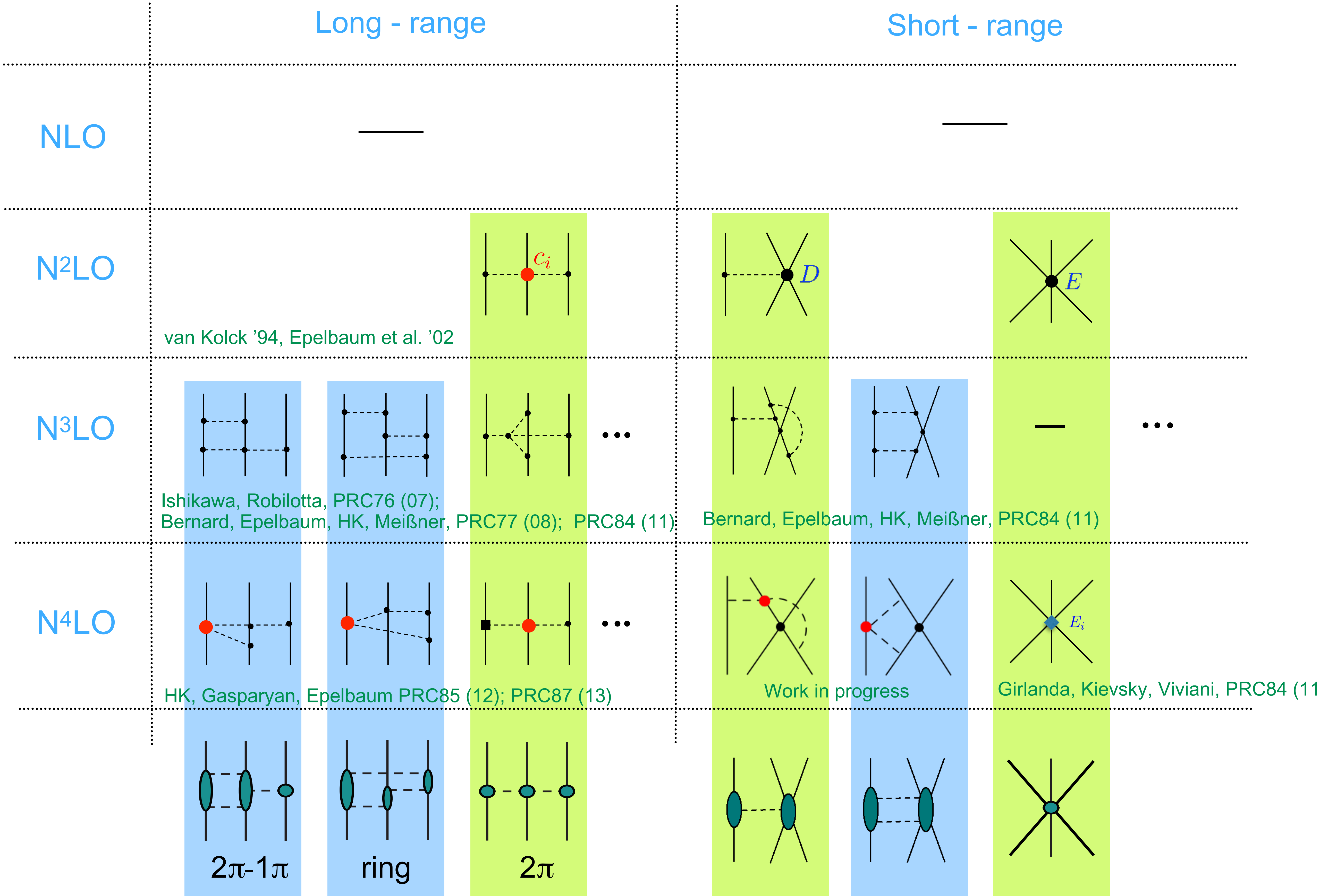
NLO left & N²LO right symbols

Light(coral)/dark(grey) error bands correspond to 95%/68% DoB

The systematic overbinding of heavier nuclei can be traced back to a deficiency of 2NF@N²LO.
It goes away when using 2NF@N>²LO

LENPIC in preparation

3NF up to N4LO



SMS Regularization of NN Force

- Regularize one-pion-exchange propagator: *Reinert, HK, Epelbaum '17 (inspired by Rijken '91)*

$$\frac{1}{q^2+M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2+M_\pi^2} = \frac{1}{q^2+M_\pi^2} - \frac{1}{\Lambda^2} + \frac{q^2+M_\pi^2}{2\Lambda^4} + \ldots$$

all 1/Λ-corrections are short-range interactions

- Implement similar regularization for two-pion exchange

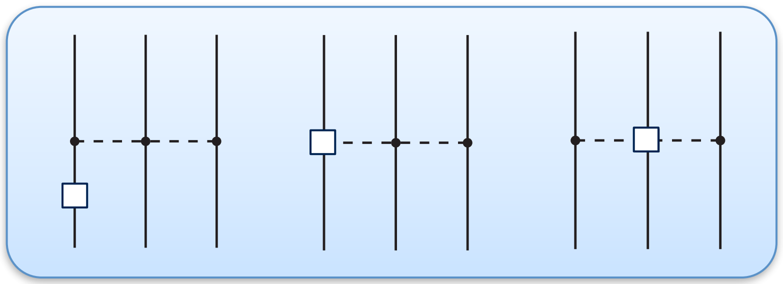
$$V(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\frac{\rho(\mu)}{q^2+\mu^2} \to V_\Lambda(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\frac{\rho(\mu)}{q^2+\mu^2}\exp\left(-\frac{q^2+\mu^2}{2\Lambda^2}\right)$$

- For a simple gaussian regulator $\exp\left(-\frac{q^2}{\Lambda^2}\right)$ πN-coupling gets quenched

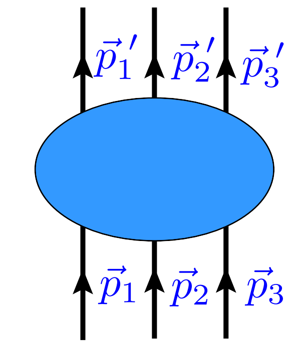
$$g_A \to g_A \exp\left(-\frac{M_\pi^2}{2\Lambda^2}\right) < g_A$$

Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.



1/m - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i\frac{g_A^2}{32mF_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\tau_1\cdot(\tau_2\times\tau_3)(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

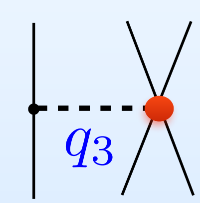
Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

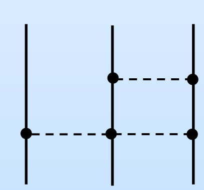
First iteration with OPE NN potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2}+\ldots$$

No such D-like term in chiral Lagrangian

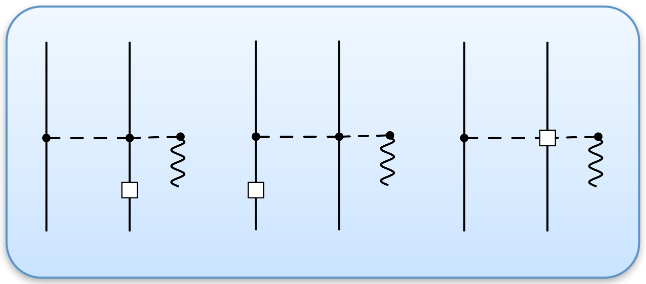


The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

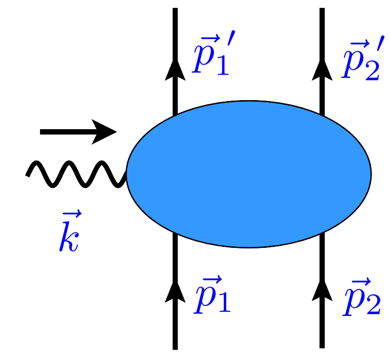
In dim. reg. $V_{2\pi-1\pi}$ =  + … is finite

Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.



1/m - corrections to pion-pole OPE current proportional to g_A



$$\vec{A}^{a,(Q:\,g_A)}_{2\text{N}:\,1\pi,1/m} = i\,[\tau_1\times\tau_2]^a\,\frac{g_A}{8F_\pi^2 m}\,\frac{\vec{k}}{(k^2+M_\pi^2)(q_1^2+M_\pi^2)}\Big(\vec{k}_2\cdot(\vec{k}+\vec{q}_1)-\vec{k}_1\cdot\vec{q}_1+i\,\vec{k}\cdot(\vec{q}_1\times\vec{\sigma}_2)\Big)+1\leftrightarrow 2$$

$$\vec{q}_i = \vec{p}_i{}'-\vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}\,(\vec{p}_i{}'+\vec{p}_i)$$

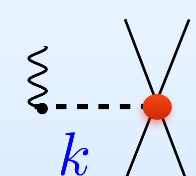
Naive local cut-off regularization of the current and potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\text{N}:\,1\pi,1/m} = \vec{A}^{a,(Q:\,g_A)}_{2\text{N}:\,1\pi,1/m}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V^{Q^0,\Lambda}_{1\pi} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

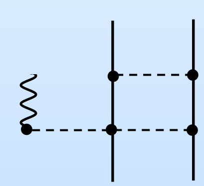
First iteration with OPE NN potential

$$\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\text{N}:\,1\pi,1/m}\frac{1}{E-H_0+i\epsilon}V^{O^0,\Lambda}_{1\pi}+V^{O^0,\Lambda}_{1\pi}\frac{1}{E-H_0+i\epsilon}\vec{A}^{a,(Q:\,g_A,\Lambda)}_{2\text{N}:\,1\pi,1/m} = \Lambda\frac{g_A^3}{32\sqrt{2}\pi^{3/2}F_\pi^4}([\tau_1]^a-[\tau_2]^a)\frac{\vec{k}}{k^2+M_\pi^2}\vec{q}_1\cdot\vec{\sigma}_1+\ldots$$

No such counter term in chiral Lagrangian



The problematic divergence is canceled by the one $\vec{A}^{a\,(Q)}_{2\text{N}:\,2\pi}$ if calculated via cutoff regularization

In dim. reg. $\vec{A}^{a\,(Q)}_{2\text{N}:\,2\pi}$ =  + … is finite

Conclusion for $N^{>2}LO$ 3NF's & Currents

- All available calculations with N^3LO 3NF's & currents violate chiral symmetry

  Naively regularized expressions for 3NF's/currents (DR multiplied with cutoff regulator) produce wrong iterative contributions/expectation values.

- Nuclear forces & currents at $N^{>2}LO$ need to be worked out within a symmetry preserving regularization scheme

  Promising regularization schemes:

  - Higher derivative regularization (work in progress)

    Based on ideas: Slavnov, NPB31 (1971) 301;
    Djukanovic et al. PRD72 (2005) 045002; Long and Mei PRC93 (2016) 044003

  - Gradient-flow regularization:

    Proposed recently in chiral EFT community by D. Kaplan and well known in LQCD:

    Lüscher, JHEP04 (2013) 123; Bär, Golteman, PRD89 (2014) 034505

Higher Derivative Regularization

Based on ideas: Slavnov, NPB31 (1971) 301;
Djukanovic et al. PRD72 (2005) 045002; Long and Mei PRC93 (2016) 044003

- Change leading order pion - Lagrangian (modify free part)

$$S_\pi^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\vec{\pi}(x) \to S_{\pi,\Lambda}^{(2)} = \int d^4x \frac{1}{2}\vec{\pi}(x)\left(-\partial^2 - M_\pi^2\right)\exp\left(\frac{\partial^2 + M_\pi^2}{\Lambda^2}\right)\vec{\pi}(x)$$

$$\frac{1}{q^2 + M_\pi^2} \to \frac{\exp\left(-\frac{q^2+M_\pi^2}{\Lambda^2}\right)}{q^2 + M_\pi^2}$$

$\mathcal{L}_{\pi,\Lambda}^{(2)}$ has to be invariant under $\mathrm{SU(2)_L \times SU(2)_R \times U(1)_V}$

- Every derivative should be covariant one
- Lagrangian $\mathcal{L}_{\pi,\Lambda}^{(2)}$ should be formulated in terms of $U(\vec{\pi}(x)) \in \mathrm{SU}(2)$

Gasser, Leutwyler ´84, ´85; Bernard, Kaiser, Meißner ´95

Building blocks $\chi = 2B(s + ip)$

$$\nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)U$$

Higher Derivative Regularization

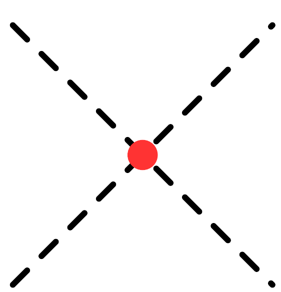
- Regularization of pion - Lagrangian will not affect nucleon Green function
  - Schrödinger or LS-equations get not modified
  - Only nuclear forces get affected

  We are not going to change pion-nucleon Lagrangian

- Not every chiral symmetric higher derivative extension of pion - Lagrangian leads to a regularized theory

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] \to \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U^\dagger \exp\left(-\vec{\partial}^2/\Lambda^2\right)\partial^\mu U\right]$$

$$= \frac{i}{q^2}\exp\left(-q^2/\Lambda^2\right) \quad ✓$$

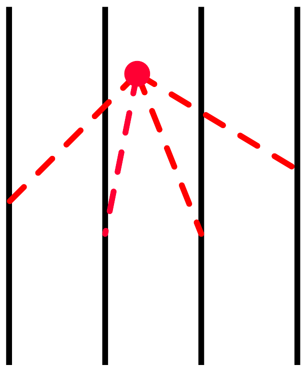


$$= \exp\left((\vec{q}_1 + \vec{q}_2)^2/\Lambda^2\right)\,\mathrm{Polynomial}(\mathrm{q_i's}) + \ldots \quad ✖$$

Unregularization of two propagators

Higher Derivative Regularization

Four-nucleon force as a regularization test



$$= \exp\left[\left(-\vec{q}_1^{\,2} - \vec{q}_2^{\,2} - \vec{q}_3^{\,2} - \vec{q}_4^{\,2} + (\vec{q}_1 + \vec{q}_2)^2\right)/\Lambda^2\right] \frac{1}{q_1^2 q_1^2 q_3^2 q_4^2} \cdots$$

$$= \exp\left[\left(-(\vec{q}_1 + \vec{q}_3)^2 - (\vec{q}_1 + \vec{q}_4)^2\right)/\Lambda^2\right] \frac{1}{q_1^2 q_2^2 q_3^2 q_4^2} \cdots$$

Only two linear combinations of momenta get regularized → Unregularized 4NF

Which additional constrain is needed to construct a regularized theory?

- All higher derivative terms of the non-linear sigma model Lagrangian in **Slavnov, NPB31 (1971) 301** are proportional to equation of motion

Generalize this idea to chiral EFT: all additional terms ~ EOM

$$\mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

$\mathrm{EOM} = 0$ ← classical equation of motion for pions

Higher Derivative Lagrangian

- To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\,p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

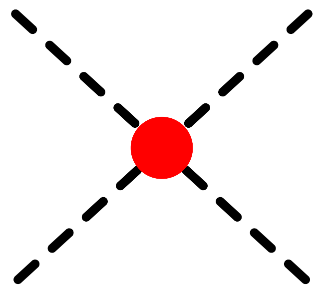
$$\mathcal{L}^{(2)}_{\pi,\Lambda} = \mathcal{L}^{(2)}_{\pi} + \frac{F^2}{4} \mathrm{Tr}\left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu} \mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM}\right]$$

$$\mathcal{L}^{(2)}_{\pi} = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

Use dimensional regularization on top of higher derivative one

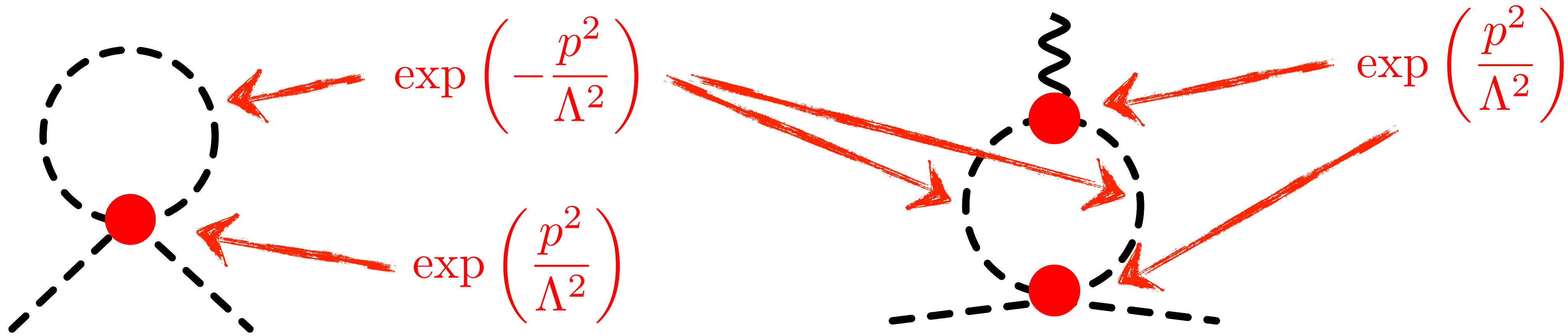
$\rightarrow$ regularization of remaining divergencies in pion sector

Modified Vertices



- Enhanced by $\exp\left(\frac{p^2}{\Lambda^2}\right)$
- Every propagator is suppressed by $\exp\left(-\frac{p^2}{\Lambda^2}\right)$

Pionic sector becomes unregularized



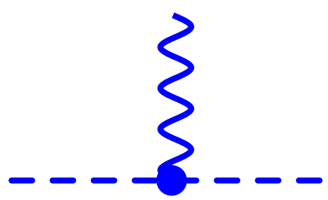
- Use dimensional on top of higher derivative regularization
- Dimensional regularization will not affect effective potential and Schrödinger or LS equations but will regularize pionic sector

Regularization of Vector Current

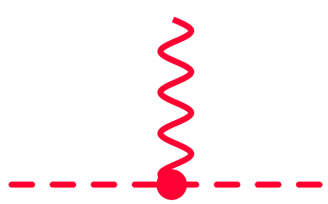
- Modify pion-propagators in a vector current

$$= \frac{1}{q^2 + M^2} \rightarrow \frac{\exp\left(-\frac{q^2+M^2}{\Lambda^2}\right)}{q^2 + M^2} =$$

- Modify two-pion-photon vertex



$$= e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2}$$

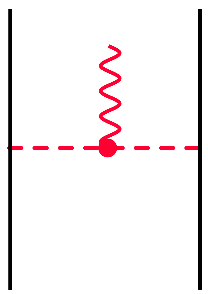


$$= e\,\epsilon_\mu (q_2^\mu - q_1^\mu)\epsilon_{3,a_1,a_2} \times \frac{1}{q_1^2 - q_2^2}\left[(q_1^2 + M^2)\exp\left(\frac{q_1^2 + M^2}{\Lambda^2}\right) - (q_2^2 + M^2)\exp\left(\frac{q_2^2 + M^2}{\Lambda^2}\right)\right]$$

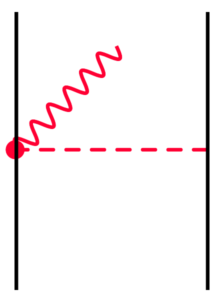
Modified two-pion-photon vertex leads to exponential increase in momenta

Regularization of Vector Current

Regularization of pion-exchange vector current



$$= \frac{i\, e\, g_A^2}{4F^2} \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_2 [\tau_1 \times \tau_2]_3 \frac{\vec{\epsilon} \cdot (\vec{q}_2 - \vec{q}_1)}{q_1^2 - q_2^2} \left[\frac{\exp\left(-\frac{q_2^2 + M^2}{\Lambda^2}\right)}{q_2^2 + M^2} - \frac{\exp\left(-\frac{q_1^2 + M^2}{\Lambda^2}\right)}{q_1^2 + M^2} \right]$$



$$= -\frac{i\, e\, g_A^2}{4F^2} \vec{\epsilon} \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_2 [\tau_1 \times \tau_2]_3 \frac{\exp\left(-\frac{q_1^2 + M^2}{\Lambda^2}\right)}{q_1^2 + M^2} + (1 \leftrightarrow 2)$$

Riska prescription: longitudinal part of the current can be derived from continuity equation

Riska, Prog. Part. Nucl. Phys. 11 (1984) 199

$$\left[H_{\text{strong}}, \boldsymbol{\rho} \right] = \vec{k} \cdot \vec{\boldsymbol{J}}$$

Application at tree level → talk by Arseniy Filin

Application to higher orders → work in progress

Summary

- Chiral two-nucleon forces are worked out up to N^4LO^+ (Q^5)
- IB corrections are implemented up to N^4LO
- Extracted πN couplings from NN data with chiral EFT
- Statistically perfect description of NN data at N^4LO^+ including IB
- P-shell nuclei studied within NCCI+SRG at N^2LO with quantified errors

- All available calculations with N^3LO 3NF's & currents violate chiral symmetry
- Higher derivative regularization respects chiral/gauge symmetries
- Construction of N^3LO 3NF's & currents within higher derivative approach is work in progress