Precision calculations of charge radii of light nuclei

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in collaboration with

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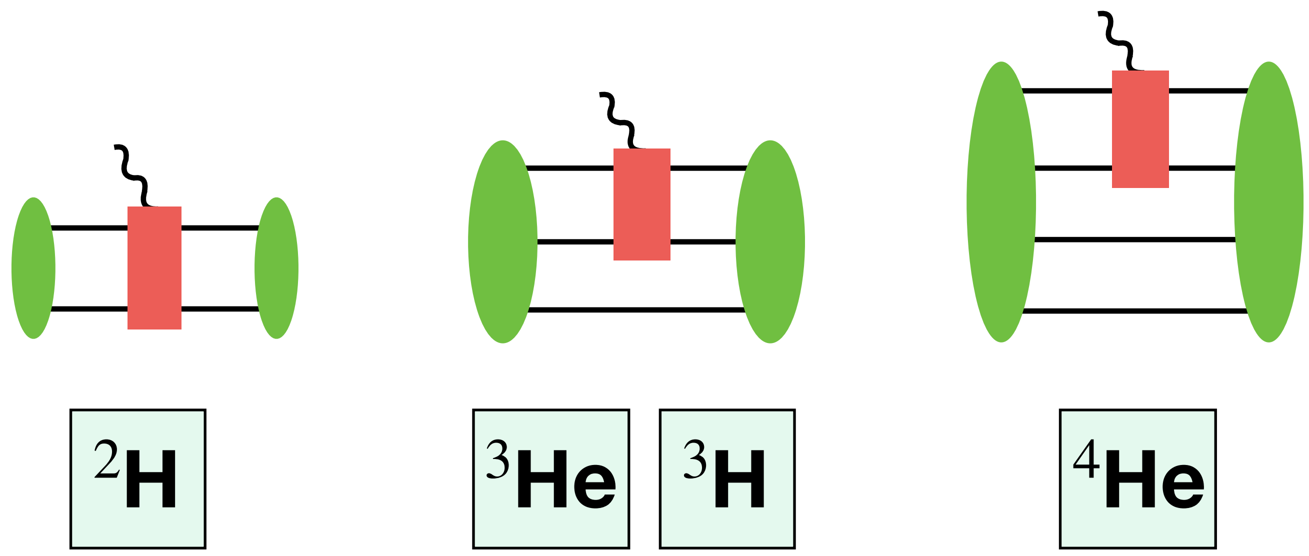
PRL 124 082501 (2020)

Phys.Rev.C 103 024313 (2021)

High-accuracy chiral EFT calculation of charge radii



Motivation:

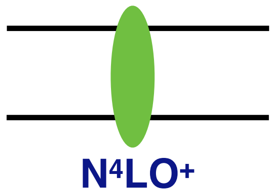
- Precision **tests of nuclear chiral EFT**

- Help to resolve long-standing issue with **underpredicted radii of medium-mass and heavy nuclei**

- A new way to **extract the neutron charge radius** from few-nucleon data

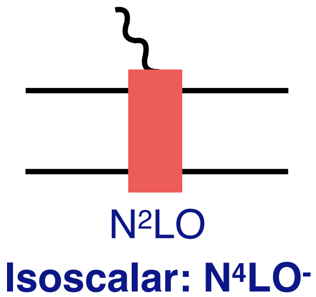
- Search for **Beyond-Standard-Model** physics

Chiral effective field theory - **precise, accurate** and **consistent**



New **high-precision chiral NN forces** (N^{4}LO$^+$) Reinert et al. PRL 126, 092501 (2021)

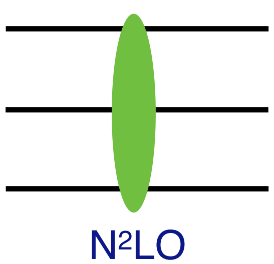
- Nearly perfect description of pp and pn scattering data up to pion production threshold



2N Chiral electromagnetic currents (general N^{2}LO; isoscalar N^{4}LO$^-$)

Kolling:2009iq
Kolling:2012cs
Krebs:2019aka
Krebs:2020pii (Review)

- N^{2}LO (**isoscalar N^{4}LO$^-$**) is derived and regularised consistently with the chiral NN forces
- Consistent regularisation of N^{3}LO (isovector) is in progress, see **talk by H. Krebs**



Chiral 3N forces (general N^{2}LO; selected terms at N^{4}LO) Epelbaum:2019kcf

- LECs cD and cE (N^{2}LO) are fitted to RIKEN Nd DCS data (**talk by K. Sekiguchi)** and ^{3}He binding energy
- **Consistent** regularisation of N^{3}LO is also in progress, **talk by H. Krebs**
- **Strong correlations** between BE of ^{3}He and ^{4}He known as the Tjon line/band **talk by D. Phillips**

Reliable methods to quantify truncation uncertainty of the EFT expansion

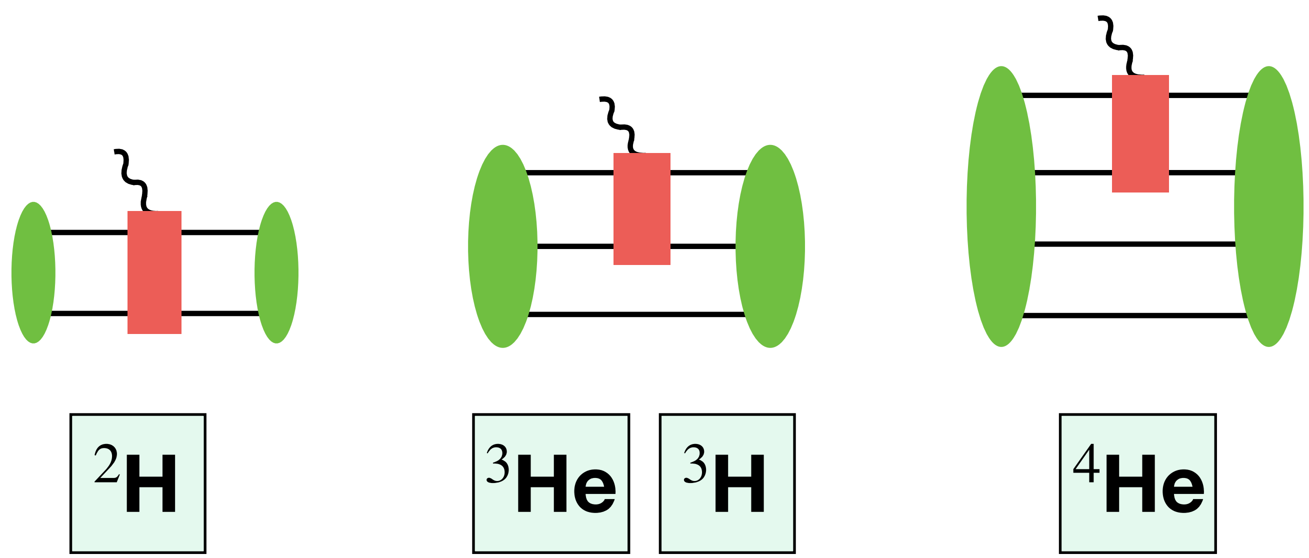
Epelbaum et al. EPJA 51 (2015); Furnstahl et al. PRC 92, 024005 (2015); Melendez et al. PRC 96, 024003 (2017),
Wesolowski et al. J. Phys. G 46, 045102 (2019); Melendez et al. PRC 100, 044001 (2019), ...

talks by D. Phillips and D. Furnstahl

High-accuracy chiral EFT calculation of charge radii

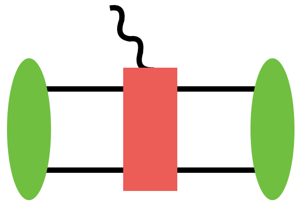
Goals of this study:

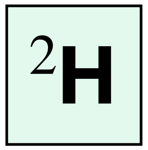
- **consistent** calculation of **isoscalar** charge radii of A = 2, 3, 4 nuclei
- **aim at N^{4}LO level of accuracy** even in the incomplete calculation
- careful estimation of uncertainties (truncation, statistical, incompleteness of 3NFs, ...)



Deuteron structure radius

& extraction of the neutron charge radius



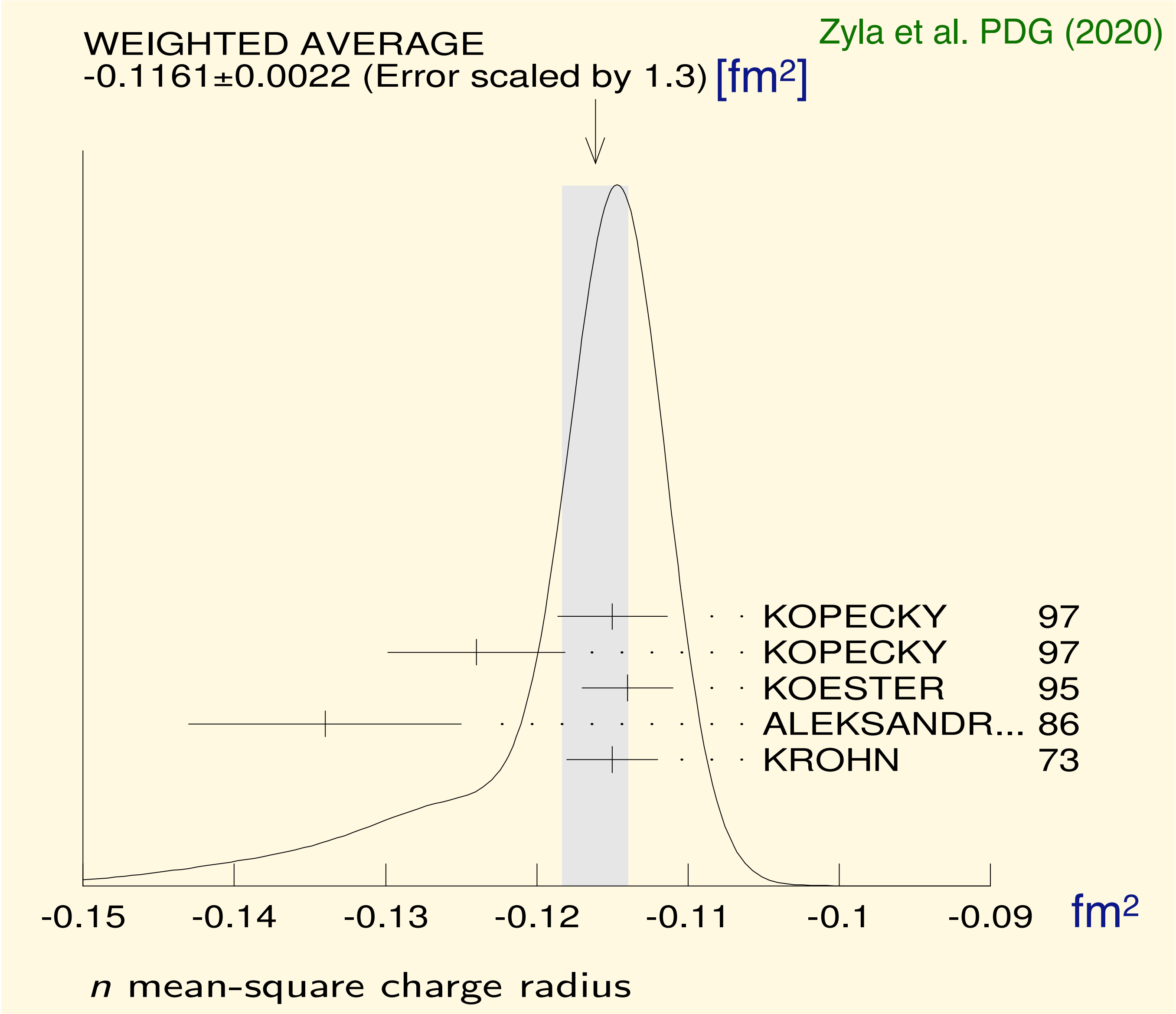


PRL124 082501 (2020)
Phys.Rev.C 103, 024313 (2021)

Neutron charge radius determination

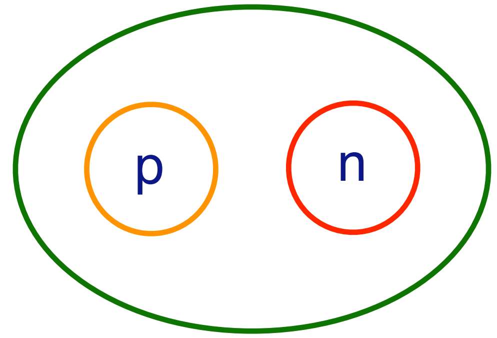
Neutron charge radius PDG (2020) weighted average $r_n^2 = -0.1161(22) fm^2$ Zyla et al. [PDG] (2020)

- Based on neutron-electron scattering length measurements on ^{208}Pb, ^{209}Bi, and other heavy nuclei
- Spread between determination on Pb and Bi is larger than (scaled) error



Accurate chiral EFT calculation of deuteron structure allows us to get a new extraction of neutron radius

Neutron charge radius determination



Deuteron charge radius: $r_d^2 = r_{str}^2 + r_p^2 + \frac{3}{4m_p^2} + r_n^2$

Accurate **χEFT calculation** of the **deuteron structure** radius

Deuteron-proton radius difference

$(r_d^2 - r_p^2) = 3.82070(31) fm^2$

Atomic spectroscopy

Hydrogen-deuterium 1S-2S isotope shift

+ QED corrections

Pachucki et al., PRA 97, 062511 (2018)
Jentschura et al. PRA 83 (2011)

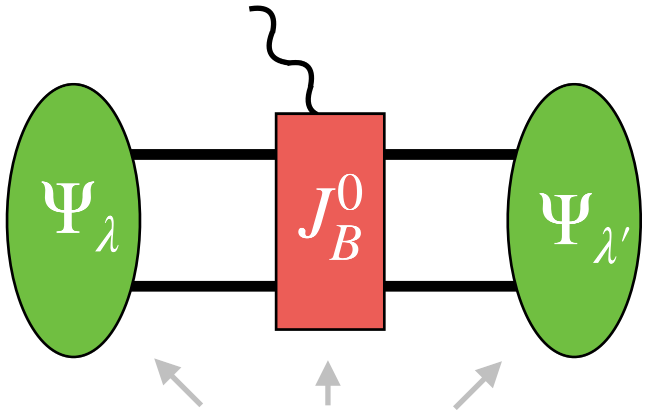
New method of determination of the neutron charge radius

$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$

Chiral EFT calculation of the deuteron charge radius

Deuteron charge radius r_d is related to the deuteron charge form factor $G_C(Q)$

$$r_d^2 = (-6)\frac{\partial}{\partial Q^2} G_C(Q^2)\Big|_{Q=0}$$

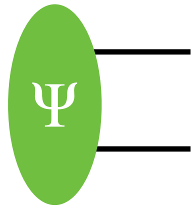


Charge form factor G_C can be computed (in the Breit frame) as

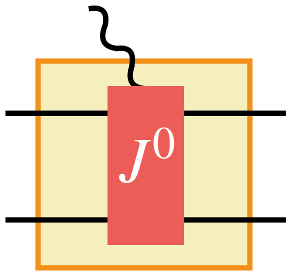
$$G_C(Q^2) = \frac{1}{3e}\frac{1}{2P_0}\sum_{\lambda} < P', \lambda | J_B^0 | P, \lambda >$$

The matrix element is a convolution of deuteron wave function and charge density operator

$$\frac{1}{2P_0} < P', \lambda_d' | J_B^\mu | P, \lambda_d > = \int \frac{d^3 l_1}{(2\pi)^3}\frac{d^3 l_2}{(2\pi)^3} \psi^\dagger_{\lambda_d'}\left(\vec{l}_2 + \frac{\vec{k}}{4}, \vec{v}_B\right) J_B^\mu \psi_{\lambda_d}\left(\vec{l}_1 - \frac{\vec{k}}{4}, -\vec{v}_B\right)$$



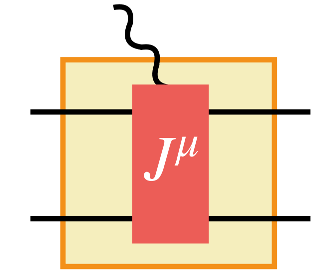
Deuteron wave function - based on high-precision chiral EFT interactions (N^4LO^+)

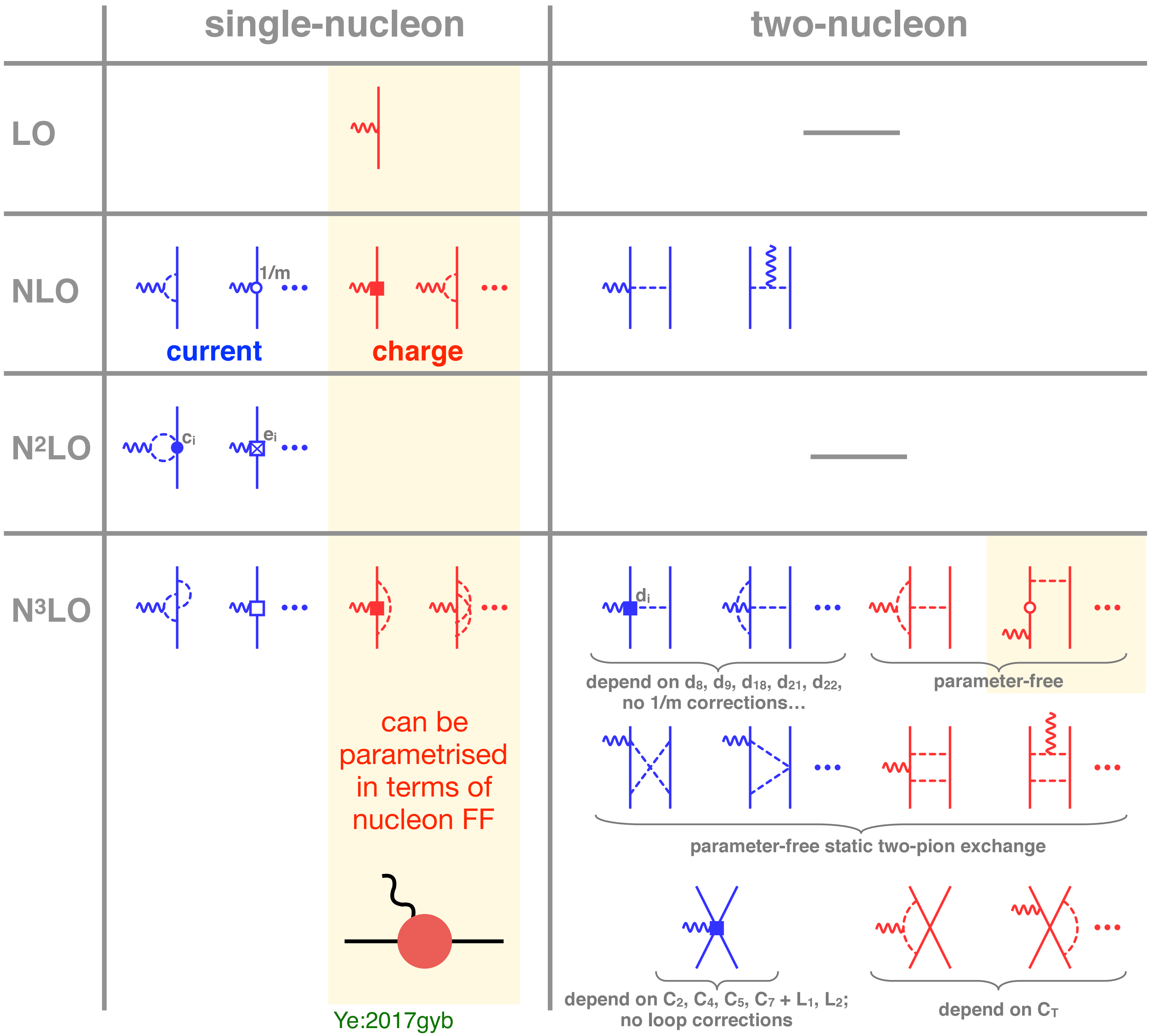


Charge density operator - consistent with chiral NN potential (isoscalar N^4LO^-)

Nuclear electromagnetic currents

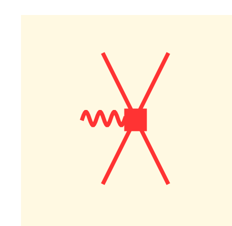
Kolling:2009iq, Kolling:2012cs, Krebs:2019aka





Ye:2017gyb
Belushkin:2006qa
Lin:2021umz
Talk by Y. Lin

Review:
H. Krebs, EPJA 56 (2020) 240



depend on **3 LECs** (A,B,C)
$^3S_1$-$^3S_1$ - can be fitted to deuteron FF data
$^3S_1$-$^3D_1$ - this one too
$^1S_0$-$^1S_0$ - can be fitted to ^{4}He FF data

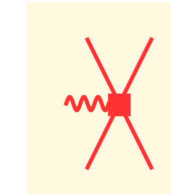
Chen, Rupak, Savage '99;
Phillips '07
AF et al. '20

Deuteron charge and quadrupole form factors

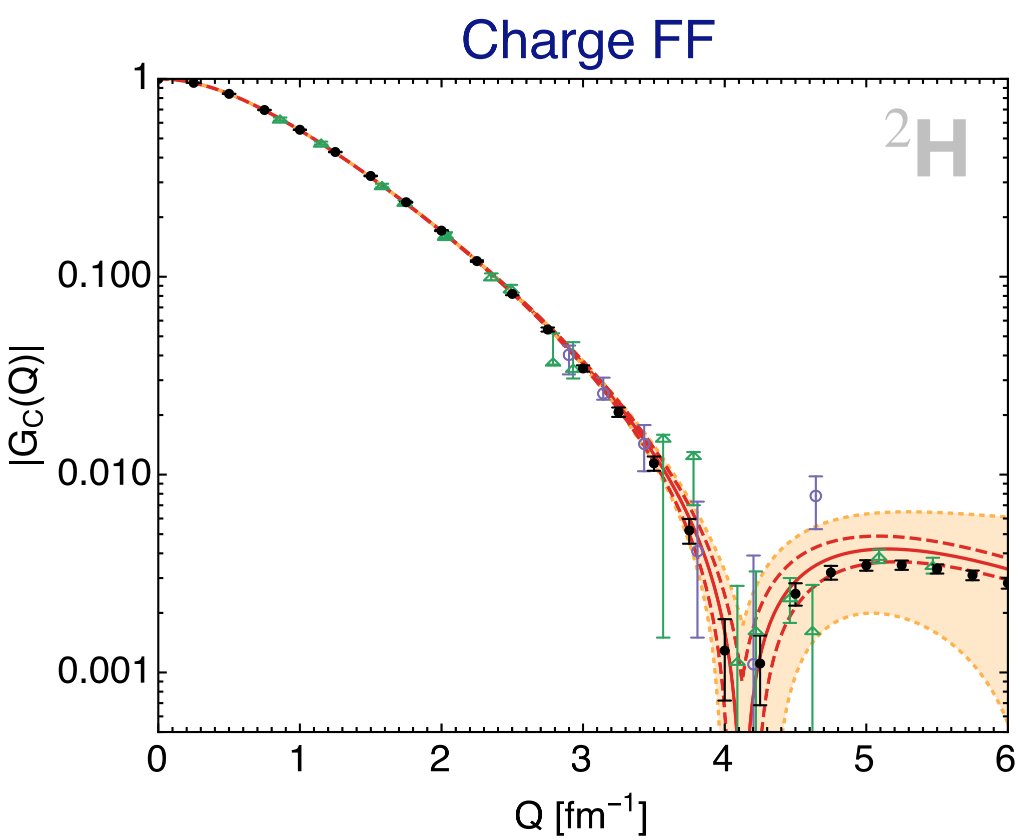
AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

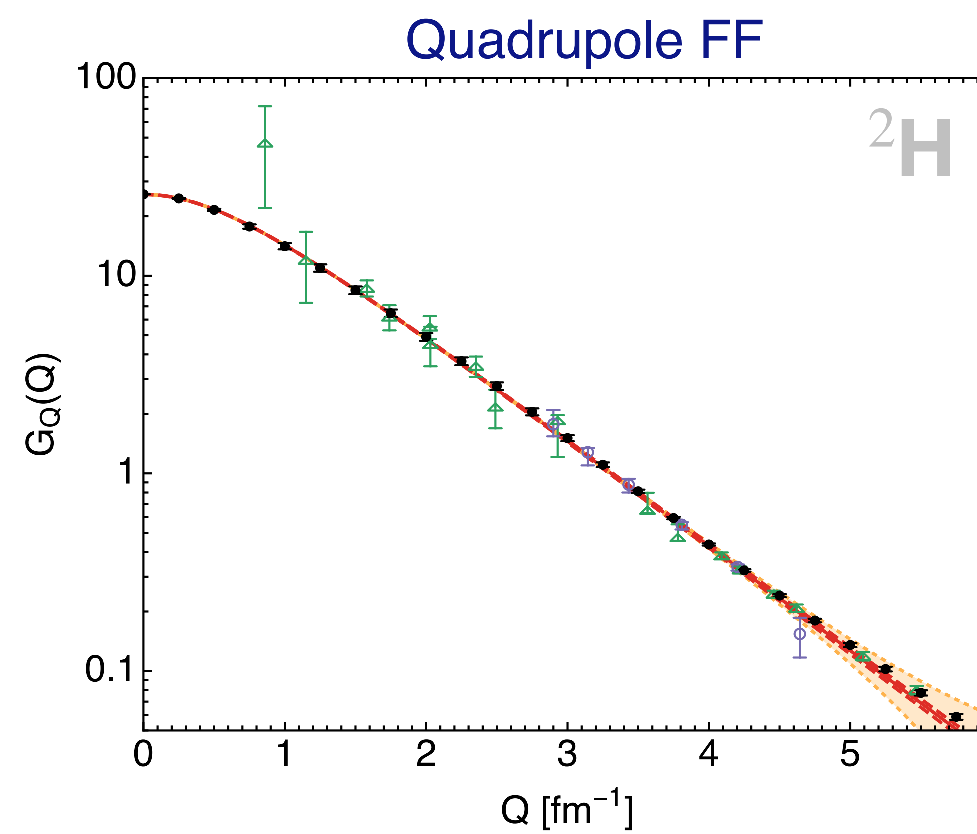
2 of 3 combinations of **N³LO LECs are fitted to deuteron charge and quadrupole FF data**

Fit takes into account theoretical uncertainties (from truncation and from nucleon form factors)



$^3S_1$-$^3S_1$ (A+B+C/3)
$^3S_1$-$^3D_1$ (C)





our result + N⁴LO truncation uncertainty

statistical uncertainty from LEC fit

World data collection: JLABt20:2000qyq + Nikolenko:2003zq;

Parameterisation (black dots) by I. Sick: Marcucci:2015rca

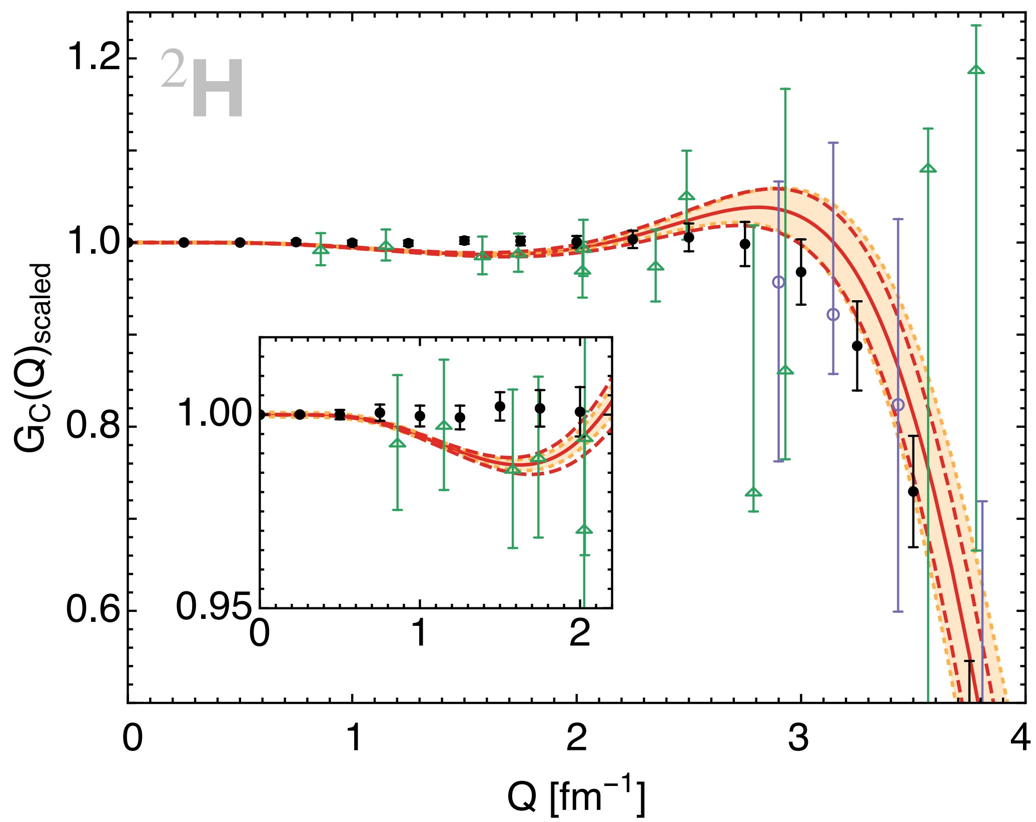
Parameterisation is not used in the fits

Deuteron charge and quadrupole form factors

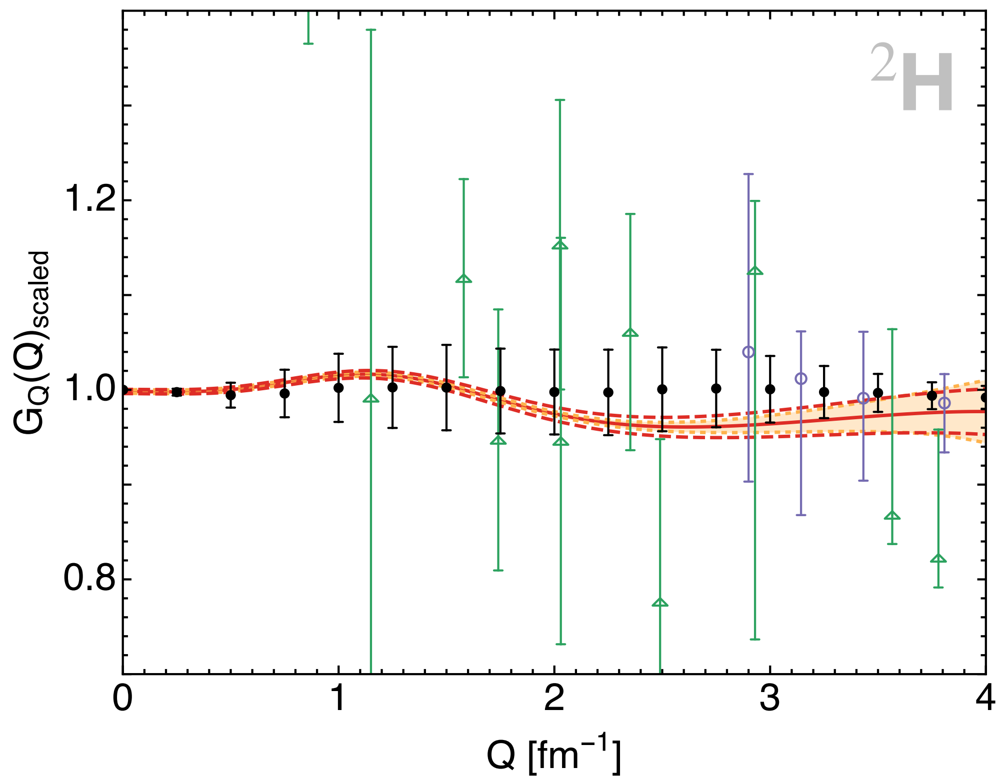
AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Deuteron form factors at low Q

Charge FF zoomed at low Q



Quadrupole FF zoomed at low Q



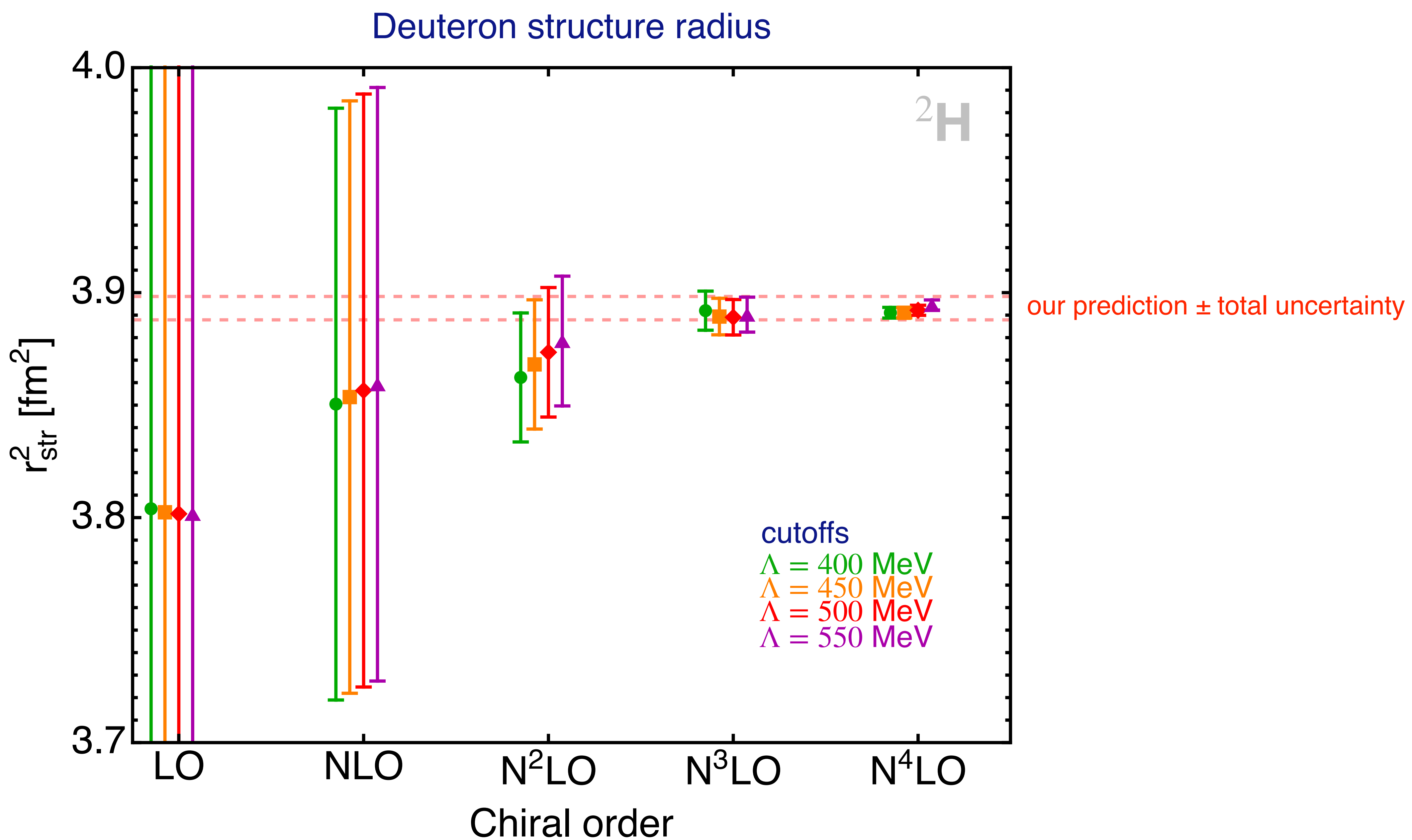
Deuteron charge FF can be used to extract neutron charge FF

Extraction of deuteron structure radius and quadrupole moment

$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm \qquad Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

Truncation uncertainty of ^{2}H structure radius

Using Bayesian model to estimate truncation uncertainty at each order Epelbaum et al. EPJA 56, 92 (2020)

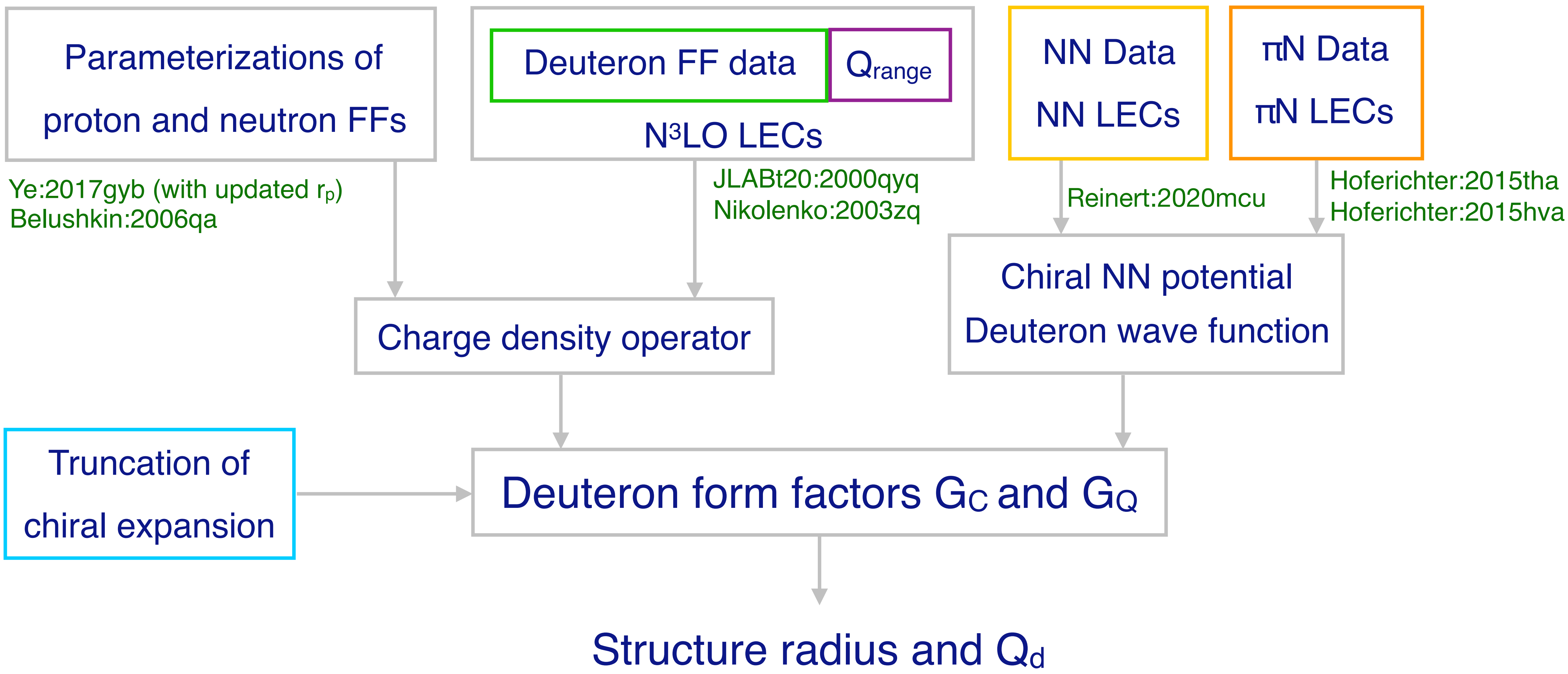


Chiral EFT expansion converges well

Cutoff dependence is smaller than the truncation uncertainty

Extensive uncertainty analysis

We propagate uncertainties from multiple sources



	Central	Truncation	$\rho^{\text{reg}}_{\text{Cont}}$	πN LECs RSA	2N LECs and f_i^2	Q range	Total
r^2_{str} (fm^2)	3.8925	± 0.0030	± 0.0024	± 0.0003	± 0.0025	$^{+0.0035}_{-0.0005}$	$^{+0.0058}_{-0.0046}$
Q_d (fm^2)	0.2854	± 0.0005	± 0.0007	± 0.0003	± 0.0016	$^{+0.0035}_{-0.0005}$	$^{+0.0038}_{-0.0017}$

Deuteron and neutron results

Predictions for the deuteron structure radius and quadrupole moment:

$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm \qquad Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

Prediction for Q_d can be compared with:

$$Q_d^{exp} = 0.285699(15)(18) fm^2$$

Puchalski et al. PRL125 253001 (2020)

Deuteron and neutron results

Predictions for the deuteron structure radius and quadrupole moment:

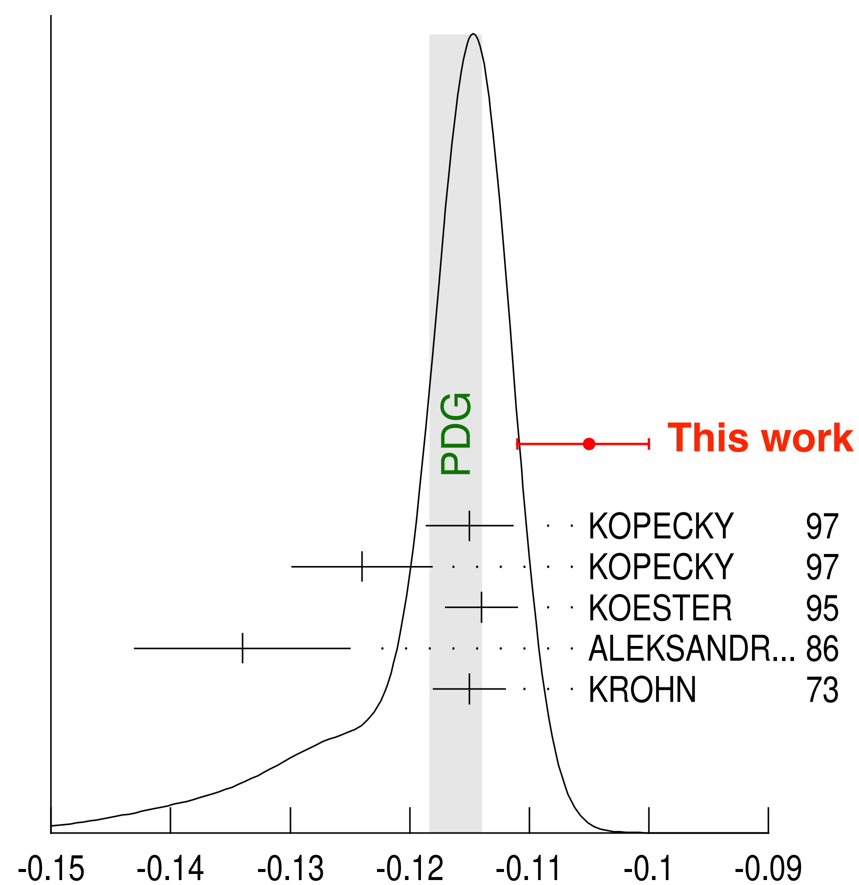
$$r_{str} = 1.9729^{+0.0015}_{-0.0012} fm \qquad Q_d = 0.2854^{+0.0038}_{-0.0017} fm^2$$

Extraction of the neutron radius from $(r_d^2 - r_p^2) = 3.82070(31) fm^2$ (atomic spectroscopy + QED corrections)

$$r_n^2 = -0.105^{+0.005}_{-0.006} fm^2$$

$$r_n^2 = (r_d^2 - r_p^2) - \frac{3}{4m_p^2} - r_{str}^2$$

~2σ deviation from the PDG (2020) weighted average $r_n^2 = -0.1161(22) fm^2$



Neutron charge radius squared [fm²]

Deuteron and neutron results

Predictions for the deuteron structure radius and quadrupole moment:

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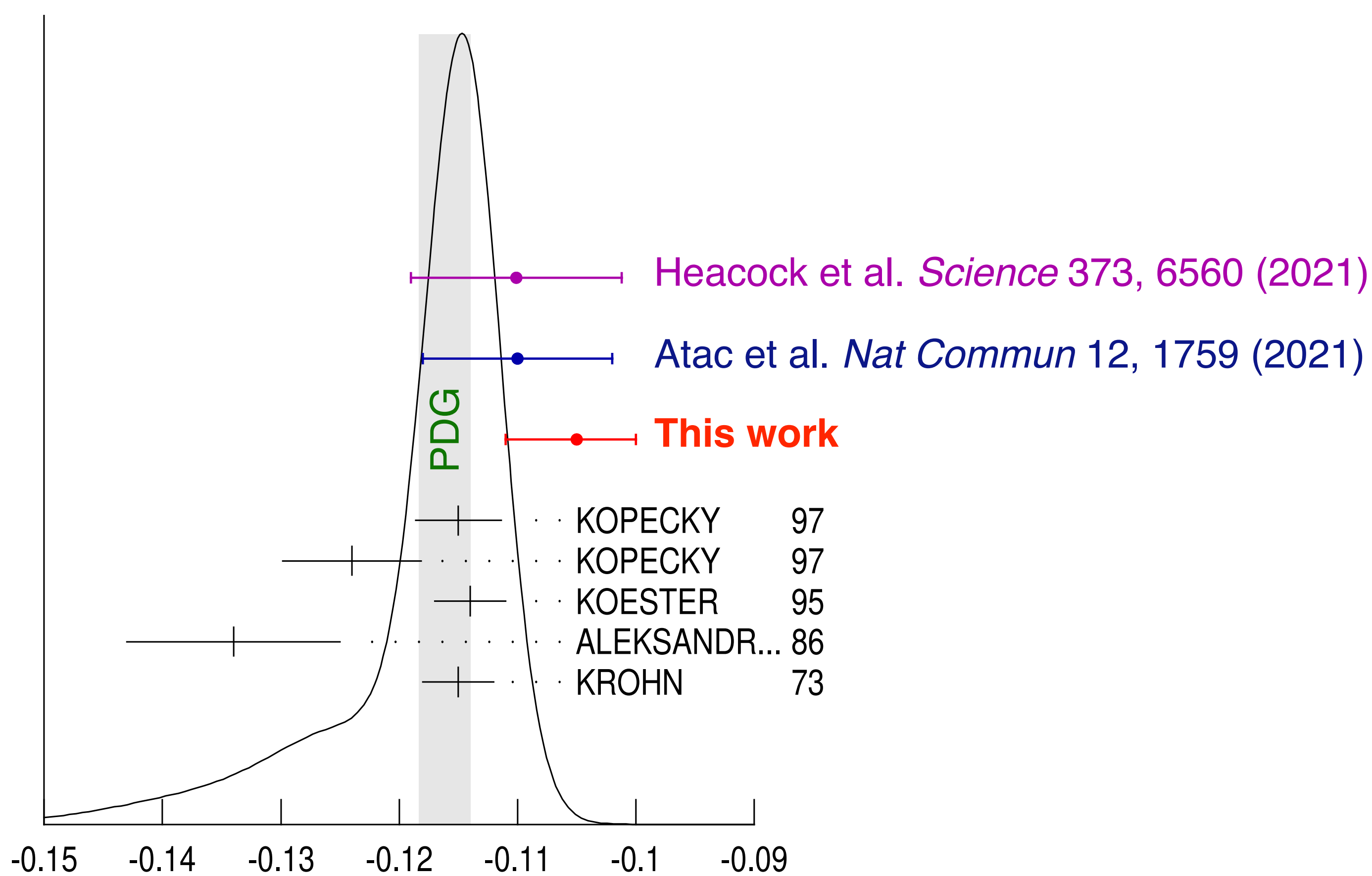


Neutron charge radius squared [fm²]

Summary of the deuteron and neutron charge radii calculations

PRL124 082501 (2020), Phys.Rev.C 103, 024313 (2021)

Comprehensive analysis of the deuteron charge and quadrupole form factors in χEFT

- Pushed to N^4LO for the first time
- Inclusion of **2N charge density** operators **consistent** with NN forces
- Determined r_{str} accurate at the permille (!) level and Q_d with a percent-level accuracy
- Detailed analysis of various sources of uncertainty

Extraction of the neutron charge radius

New method using hydrogen-deuterium 1S-2S isotope shift data

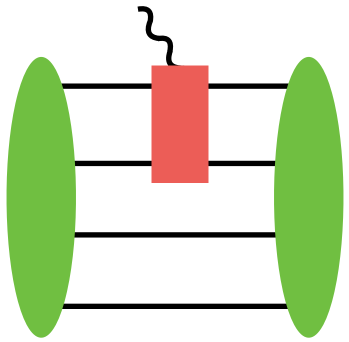
~2σ-deviation from the value from PDG (2020)

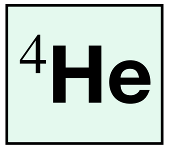
Supported by very recent determinations Atac et al. *Nat Commun* 12, 1759 (2021)
Heacock et al. *Science* 373, 6560 (2021)

Preliminary results

^{4}He charge radius

Precision test of the chiral EFT for ^{4}He



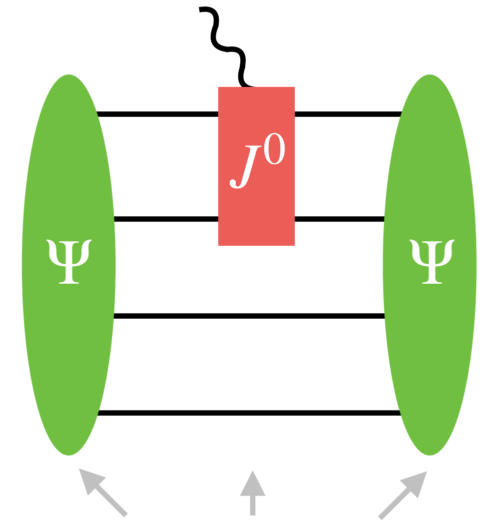


Chiral EFT calculation of the ^{4}He charge radius

^{4}He charge radius r_C is related to the ^{4}He charge form factor $F_C(Q)$

$$r_C^2 = (-6)\frac{\partial}{\partial Q^2} F_C(Q^2)\Big|_{Q=0}$$

$$F_C(Q^2) =$$

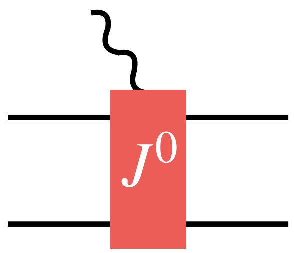


calculated **consistently**

in chiral EFT

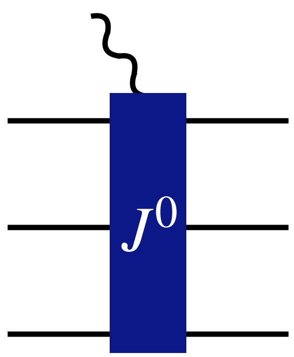
^{4}He **structure radius** (point charge radius) is defined as:

$$r_{str}^2 = r_C^2 - r_p^2 - r_{DF}^2 - r_n^2$$

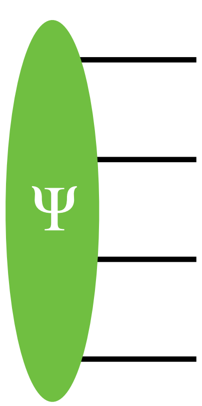


Isoscalar charge density operator - same as in deuteron

- derived consistently with chiral NN potential at **N^{3}LO + (short-range) N^{4}LO**
- one more N^{3}LO LEC should be fitted ($^1S_0$-$^1S_0$ is not fixed from deuteron)

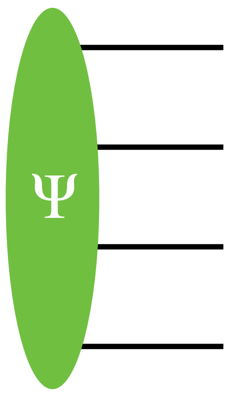


Isoscalar 3N charge density is beyond our accuracy (starts at **N^{5}LO**)

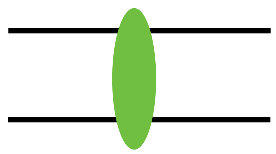


^{4}He wave function - based on high-precision chiral EFT 2NF+3NF

Chiral EFT calculation of the ^{4}He charge radius

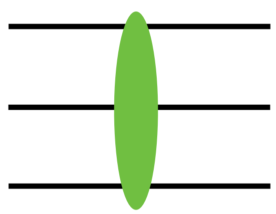


^{4}He wave function - solution of the Faddeev eq. with high-precision chiral EFT 2NF+3NF



N^4LO^+

High-precision chiral NN forces (**N^4LO^+**) Reinert:2020mcu



N^2LO

Chiral 3N forces (N^2LO + selected terms at N^4LO) **talk by H. Krebs**

N^2LO LECs cD and cE are fitted to

- RIKEN Nd DCS data (**talk by K. Sekiguchi)** and
- ^{3}He binding energy

Strong correlations between BE of ^{3}He and ^{4}He (**talk by D. Phillips)**

- **^{4}He BE is almost reproduced even with N^2LO 3NF**

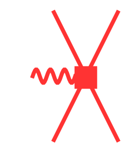
^{4}He **charge radius** and **binding energy** are also strongly **correlated**! (universality Braaten:2004rn)

-> possibility to reach N^4LO level of accuracy even in the incomplete calculation

To get maximal accuracy of ^{4}He charge radius the binding energy of ^{4}He should be reproduced (e.g. via cE1, cE3, etc)

Uncertainty due to missing 3NFs / 4NFs is estimated separately

^{4}He structure radius calculation

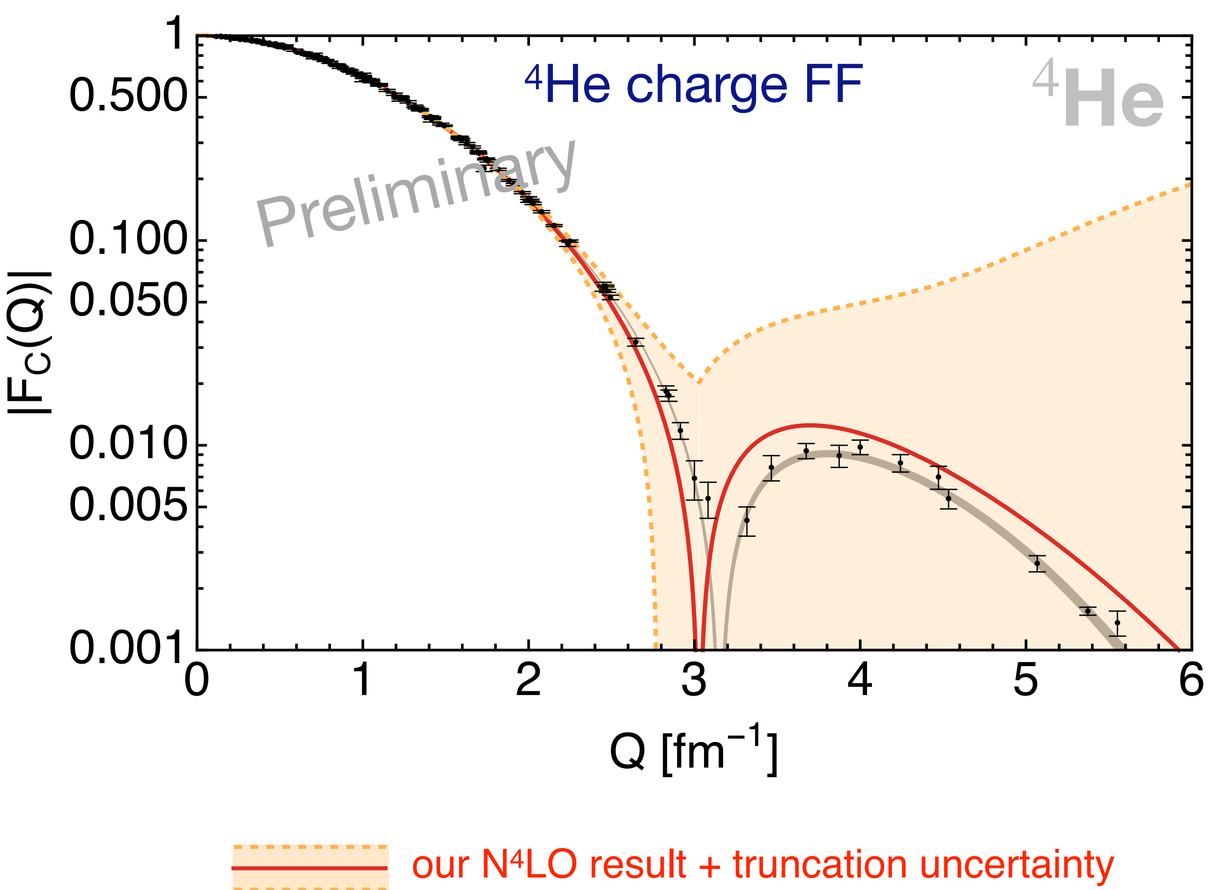


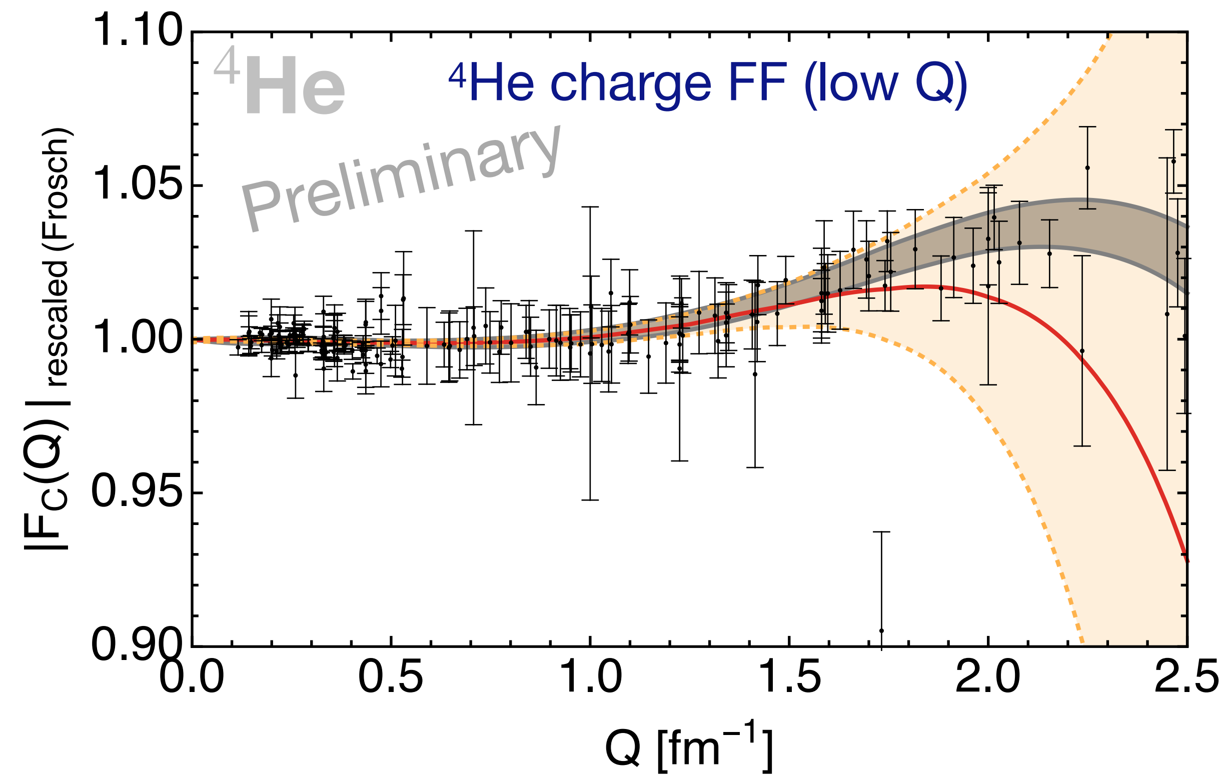
$^1S_0$-$^1S_0$ (A-3B-C)

We fit the $^1S_0$ LEC (in N^{3}LO contact charge density) to ^{4}He $F_C(Q)$ data

Fit up to Q=6 fm^{-1} including theoretical uncertainty

Result for $F_C(Q)$ with truncation uncertainty:





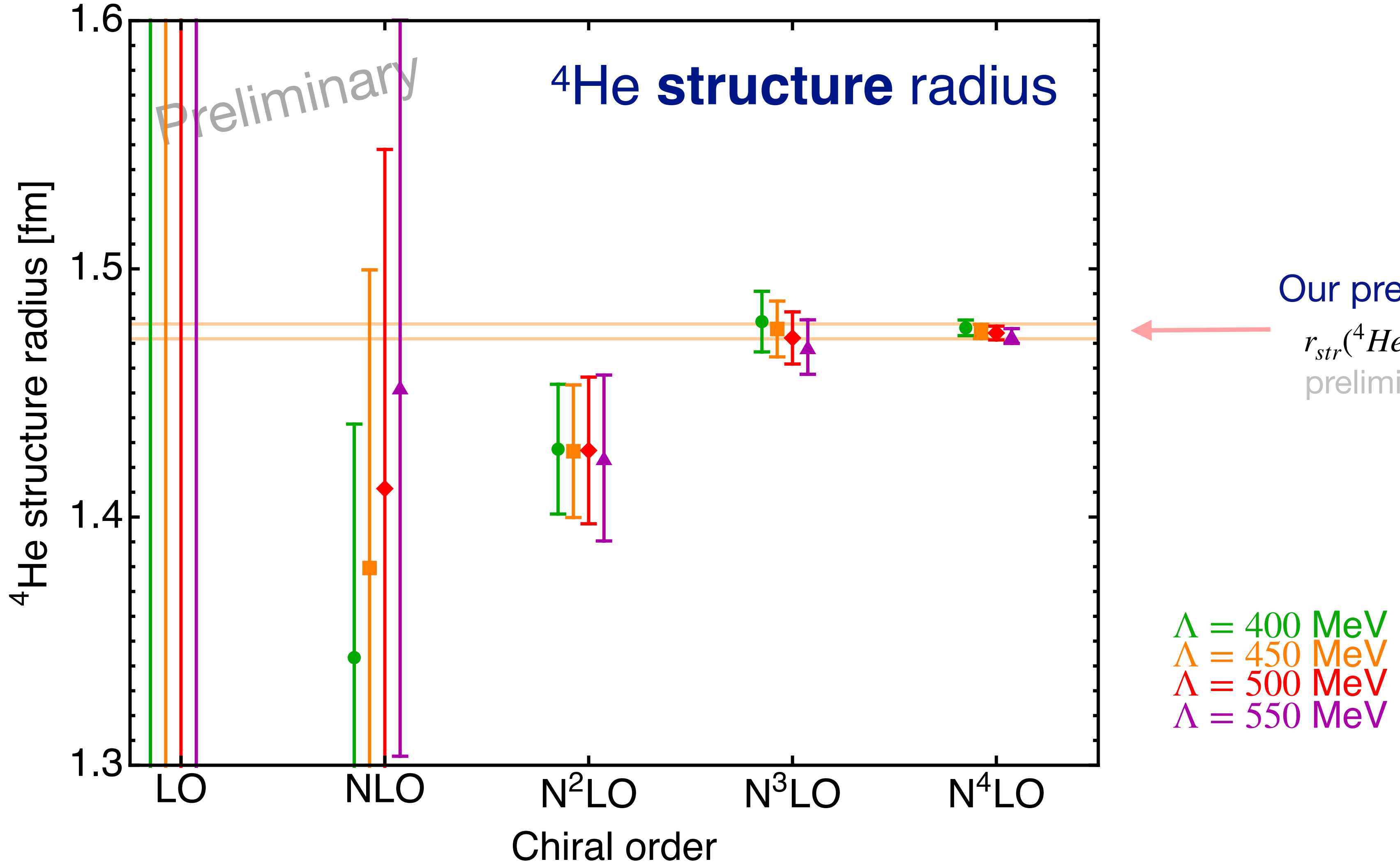
Black dots = exp. data
Gray band = parameterisation by I.Sick [Marcucci:2015rca]

Our prediction for ^{4}He structure radius:

$$r_{str}(^4He) = (1.4748 \pm 0.0030_{trunc} \pm 0.0013_{stat})\, fm$$

preliminary

Chiral EFT truncation uncertainty of the ^{4}He **structure** radius



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$r_{str}(^4He) = (1.4748 \pm 0.0030_{trunc} \pm 0.0013_{stat})\,fm$

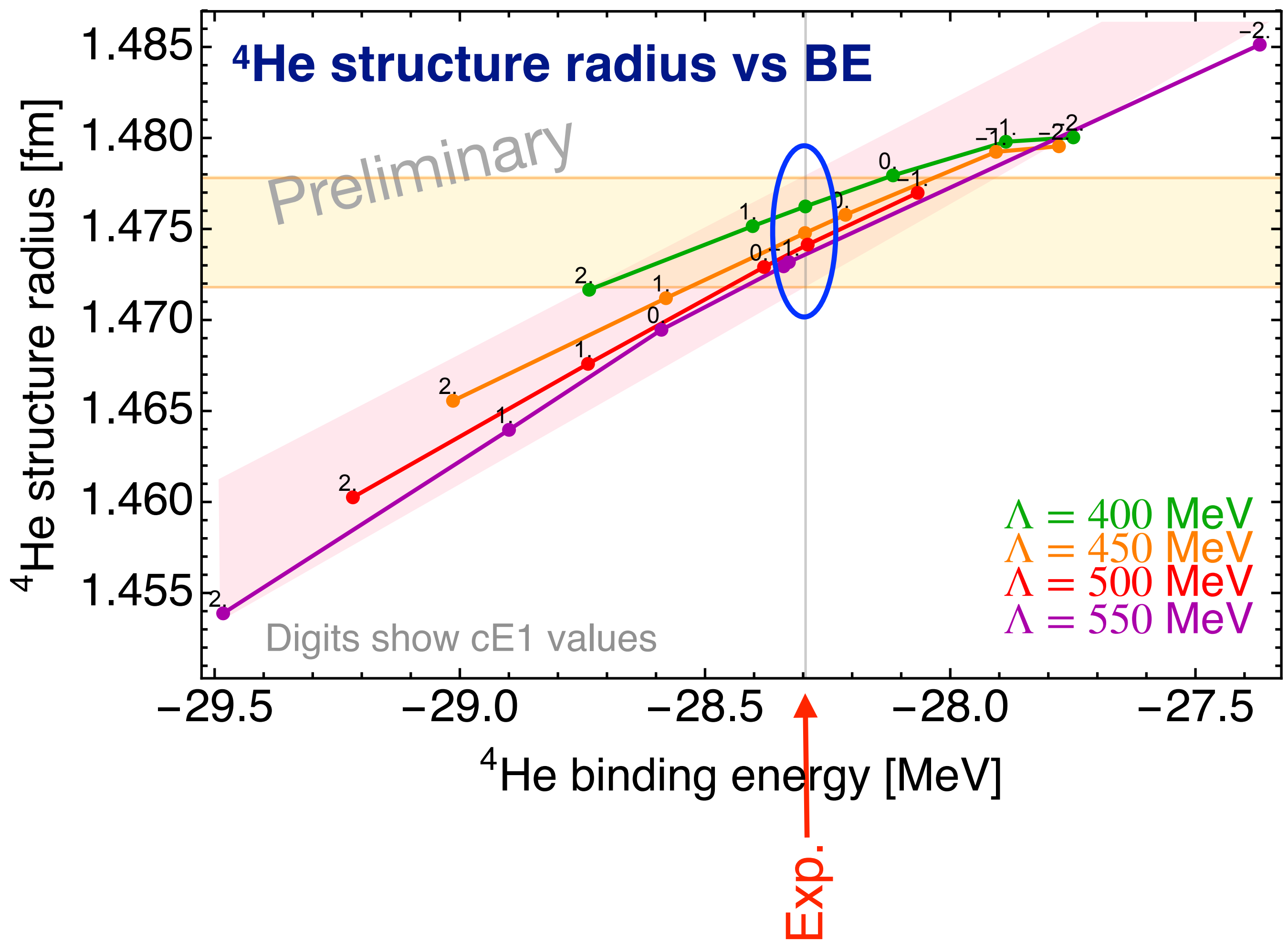
preliminary

Chiral EFT expansion converges well

Cutoff dependence is smaller than the truncation uncertainty

Correlation between ^{4}He structure radius and binding energy

^{4}He **binding energy** and $\mathbf{r_{str}}$ are **strongly correlated**!



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$$r_{str}(^4He) = (1.4748 \pm 0.0030_{trunc} \pm 0.0013_{stat})\, fm$$

preliminary

Test 1: variation of N^{4}LO 3NF **cE1**

Test 2: variation of N^{4}LO 3NF **cE3**

Test 3: variation of **N^{2}LO** 3NF **cD**

In all tests, when physical BE is reproduced the prediction for r_{str} is consistent with our error estimation (fits inside the orange band)

Correlations help to precisely extract $\mathbf{r_{str}}$ even with incomplete 3NF!

Results for ^{4}He structure and charge radii

Prediction for ^{4}He **structure** radius:

$$r_{str}(^4He) = (1.4748 \pm 0.0030_{trunc} \pm 0.0013_{stat})\, fm$$

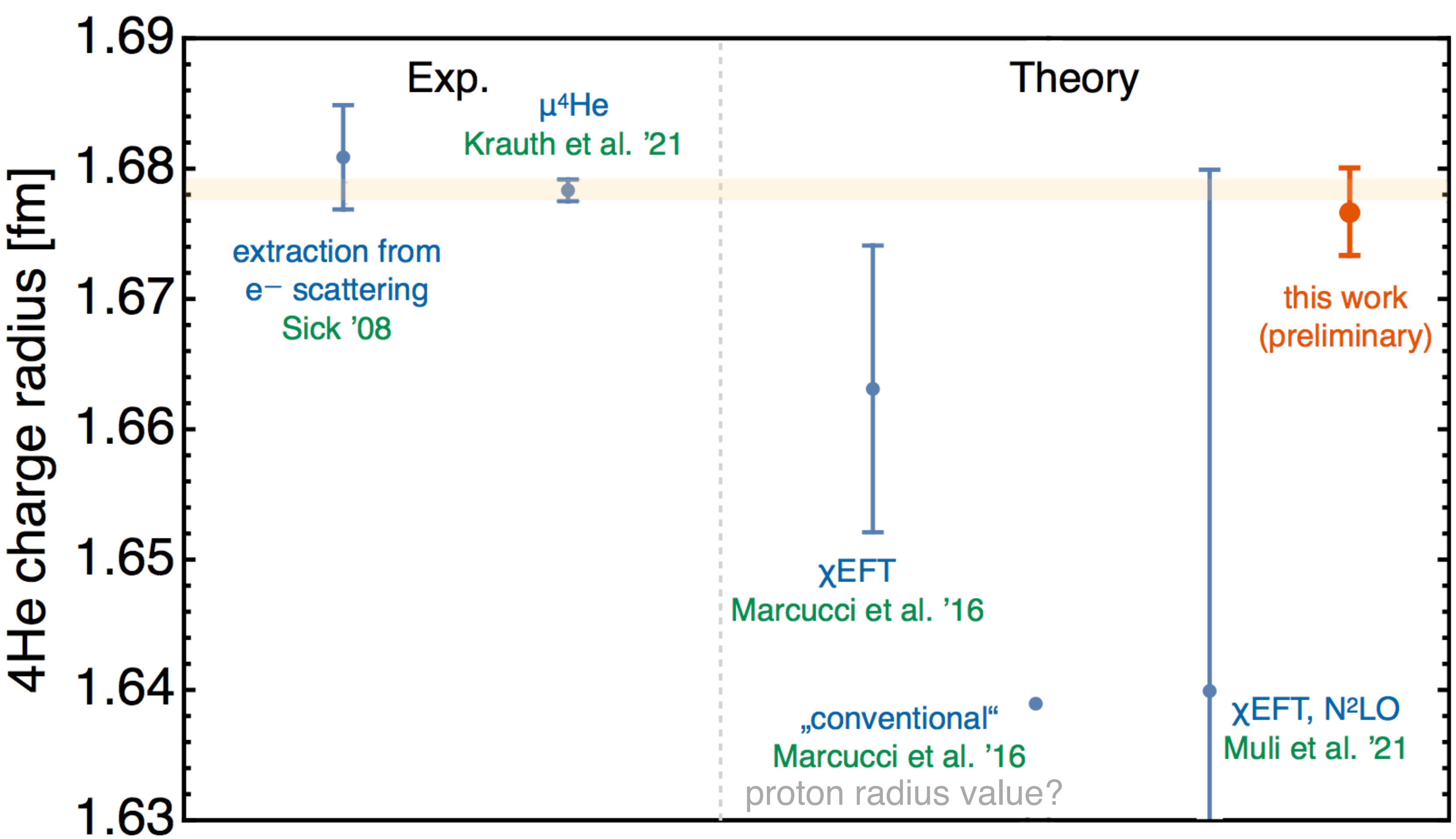
preliminary, further uncertainty sources under investigation

Prediction for ^{4}He **charge** radius

$$r_C(^4He) = (1.6766 \pm 0.0034)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

$$r_C^2 = r_{str}^2 + r_p^2 + r_{DF}^2 + r_n^2$$



The μ ^{4}He exp. value is

$$r_C^{exp}(^4He) = (1.67824 \pm 0.00083)\, fm$$

Krauth et al., Nature 589 (2021) 7843, 527-531

Indications of BSM physics?

All data used to constrain Chiral EFT parameters

are from strong interaction / electron-based experiments:

- π N Roy-Steiner analysis Hoferichter:2015tha, Hoferichter:2015hva
- NN pn and pp scattering data, deuteron BE Reinert:2020mcu
- Deuteron charge and quadrupole FF data JLABt20:2000qyq, Nikolenko:2003zq
- Deuteron-proton radii difference from atomic spectroscopy Pachucki:2018yxe, Jentschura et al. PRA 83 (2011)
- Proton charge radius CODATA2018
- ^{4}He form factor data Erich:1971rhg, Mccarthy:1977vd, VonGunten:1982yna, Ottermann:1985km, Frosch:1967pz, Arnold:1978qs, Camsonne:2013df
- Binding energies of ^{3}He and ^{4}He
- Nd DCS minimum @ 70 MeV RIKEN data (see talk by Kimiko Sekiguchi)

No muonic data is used in our chiral EFT predictions

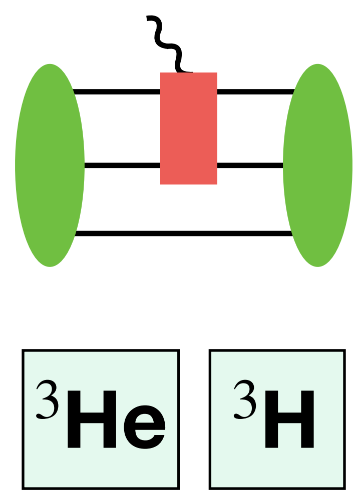
Our prediction for ^{4}He charge radius is consistent with the muonic experiment

No indication of BSM physics at this accuracy level

Preliminary results

3N charge radii

Theoretical prediction for the isoscalar ^{3}H-^{3}He charge radius
10 times more accurate than the current exp. value - to be tested by CREMA soon!

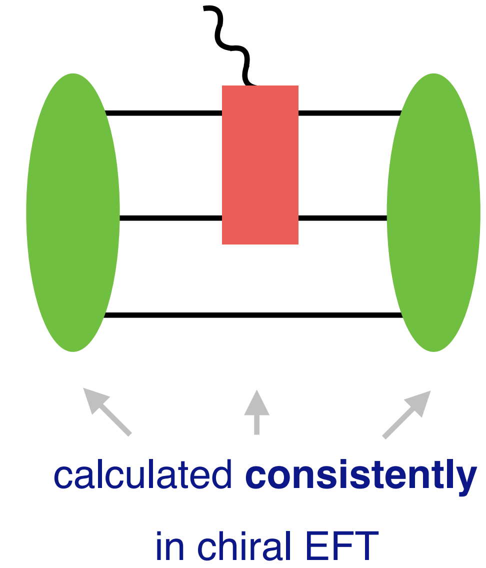


Chiral EFT calculation of the isoscalar 3N charge radius

3N charge radii r_C are related to the 3N charge form factor $F_C(Q)$

$$r_C^2 = (-6)\frac{\partial}{\partial Q^2} F_C(Q^2)\Big|_{Q=0}$$

$$F_C(Q^2) =$$

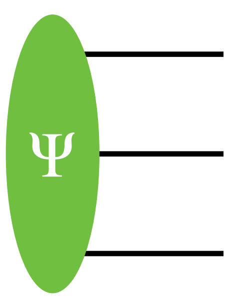


Structure radii (point charge radii) are defined as:

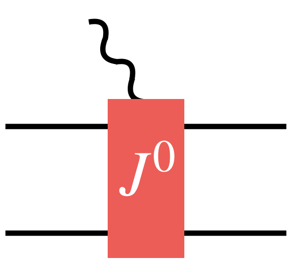
$$r_{str}^2(^3H) = r_C^2 - r_p^2 - r_{DF}^2 - 2r_n^2 \qquad r_{str}^2(^3He) = r_C^2 - r_p^2 - r_{DF}^2 - \frac{1}{2}r_n^2$$

Isoscalar 3N structure radius:

$$r_{str}^{2(isoscalar3N)} = \frac{r_{str}^2(^3H) + 2\, r_{str}^3(^3He)}{3}$$



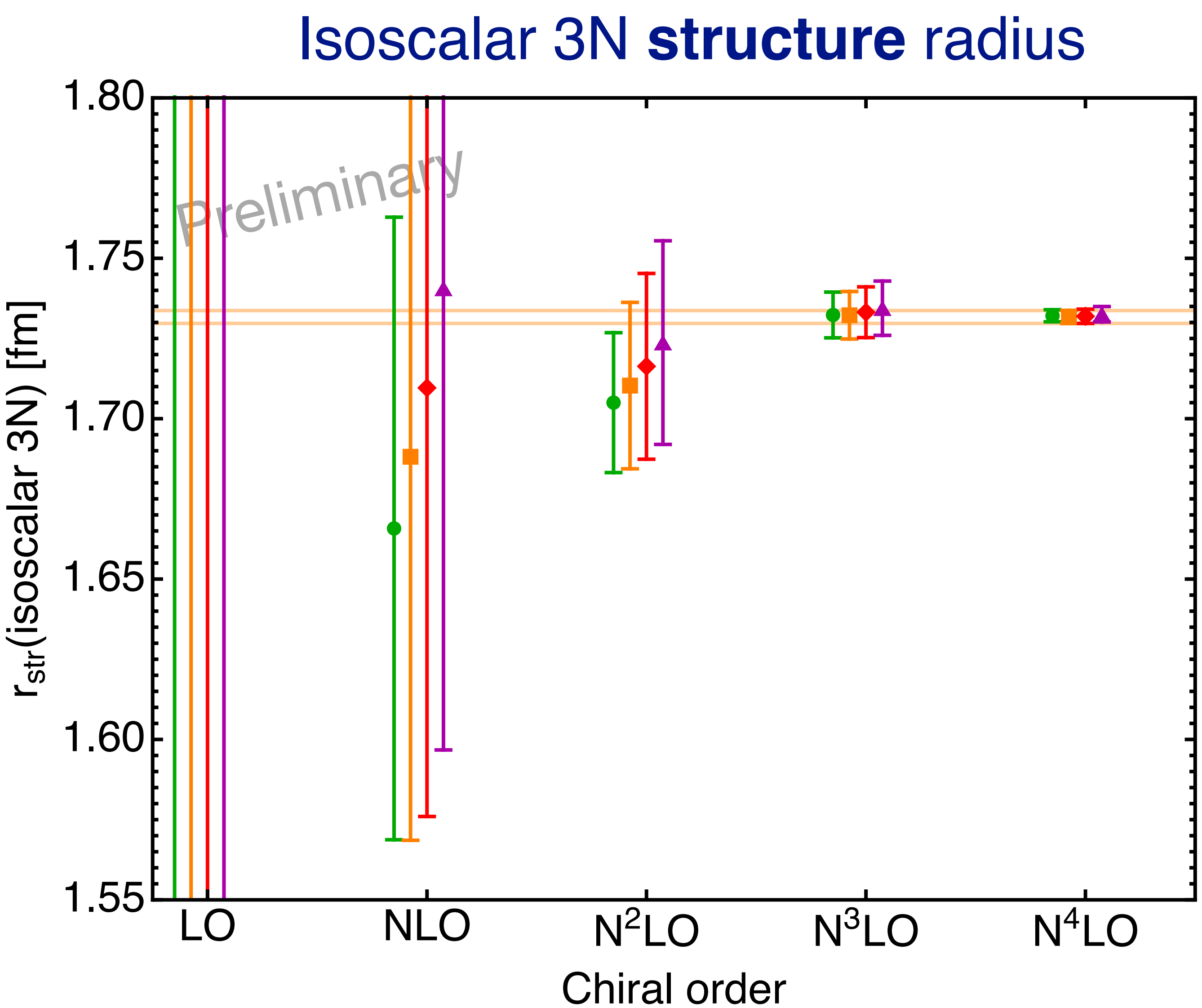
^{3}H and ^{3}He wave functions - based on high-precision chiral EFT 2NF + 3NF



Isoscalar charge density operator - same as in ^{2}H and ^{4}He

- all three N^{3}LO LEC are already fitted to ^{2}H and ^{4}He form factors
- **parameter-free prediction for isoscalar 3N structure radius**

Isoscalar 3N structure radius convergence pattern and truncation uncertainty



Prediction for isoscalar 3N **structure radius**:

$$r_{str}^{isoscalar3N} = (1.7317 \pm 0.0020_{trunc} \pm 0.0004_{stat})\, fm$$

preliminary, incompleteness of 3NF $\delta r \sim \pm 0.0013$fm

$\Lambda = 400$ MeV
$\Lambda = 450$ MeV
$\Lambda = 500$ MeV
$\Lambda = 550$ MeV

Prediction for isoscalar 3N **charge** radius

With all LECs being fixed, we can predict the isoscalar 3N **charge** radius: $r_C^{isoscalar3N} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{isoscalar3N} = (1.9065 \pm 0.0026)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Comparison with the experimental data:

the ^{3}**H charge radius** from e$^-$ scattering experiments: $r_C^{3H} = (1.7550 \pm 0.0860)\, fm$ (5%) Amroun et al. '94 (world average)

the ^{3}**He charge radius**:

e$^-$ scattering experiments: $r_C^{3He} = (1.9730 \pm 0.0160)\, fm$ Sick '15 (world average)

muonic ^{3}He (preliminary): $r_C^{3He} = (1.9687 \pm 0.0013)\, fm$ Pohl '20 (preliminary)

3N isoscalar charge radius: (using muonic ^{3}He and old ^{3}H) $r_{C,\,exp.}^{isoscalar3N} = (1.9030 \pm 0.0290)\, fm$

T-REX experiment in Mainz [Pohl et al.] aims at measuring r_C^{3H} within ±0.0002 fm (400x more precise)

The isoscalar 3N radius will be then known within ±0.0009 fm

⇒ **precision tests of nuclear chiral EFT!**

Estimation of ^{3}H charge radius

Our preliminary prediction for **isoscalar** 3N **charge** radius:

$$r_C^{isoscalar3N} = (1.9065 \pm 0.0026)\, fm$$

preliminary, using CODATA 2018 r_p and own determination of r_n

Preliminary ^{3}He charge radius [Pohl et al.]

$$r_C^{3He} = (1.9687 \pm 0.0013) fm$$

Isoscalar 3N charge radius definition:

$$(r_C^{isoscalar3N})^2 = \frac{(r_C^{3H})^2 + 2\,(r_C^{3He})^2}{3}$$

Expression for **^{3}H radius:**

$$(r_C^{3H})^2 = 3(r_C^{isoscalar3N})^2 - 2\,(r_C^{3He})^2$$

Coefficients 2 and 3 amplify both theoretical and experimental uncertainties

Our ^{3}H radius estimation:

$$r_C^{(3H)} = (1.7757 \pm 0.0088) fm$$

preliminary

This estimation is 10x more precise than e$^-$ data $r_C^{3H} = (1.7550 \pm 0.0860)\, fm$ Amroun et al. '94 (world average)

But it suffers from parametric amplification of uncertainties (both from theory and from ^{3}He data)

=> isoscalar 3N charge radius should be used for precision tests

Summary

Comprehensive analysis of A = 2, 3, 4 charge and quadrupole form factors in chiral EFT

Few-body calculations with sub-percent accuracy!

Deuteron:

- determined structure radius (0.1% accuracy) and quadrupole moment (1.4% accuracy)
- combined with isotope-shift data, extracted the neutron charge radius (2σ tension with PDG)

preliminary **^{4}He:**

- the extracted charge radius (0.2% accuracy) agrees with the new μ^{4}He measurement

preliminary **^{3}H-^{3}He:**

- predicted the isoscalar charge radius r_C (0.1% accuracy)
- our r_C is in agreement with the current exp. value (which has 10x larger errors)
- the ongoing T-REX (^{3}H) exp. in Mainz will allow for a precision test of nuclear chiral EFT

Outlook

- **Consistent** inclusion of **N^3LO, N^4LO 3NFs**
- **Consistent** inclusion of **isovector currents** (predictions separately for ^3H and ^3He)

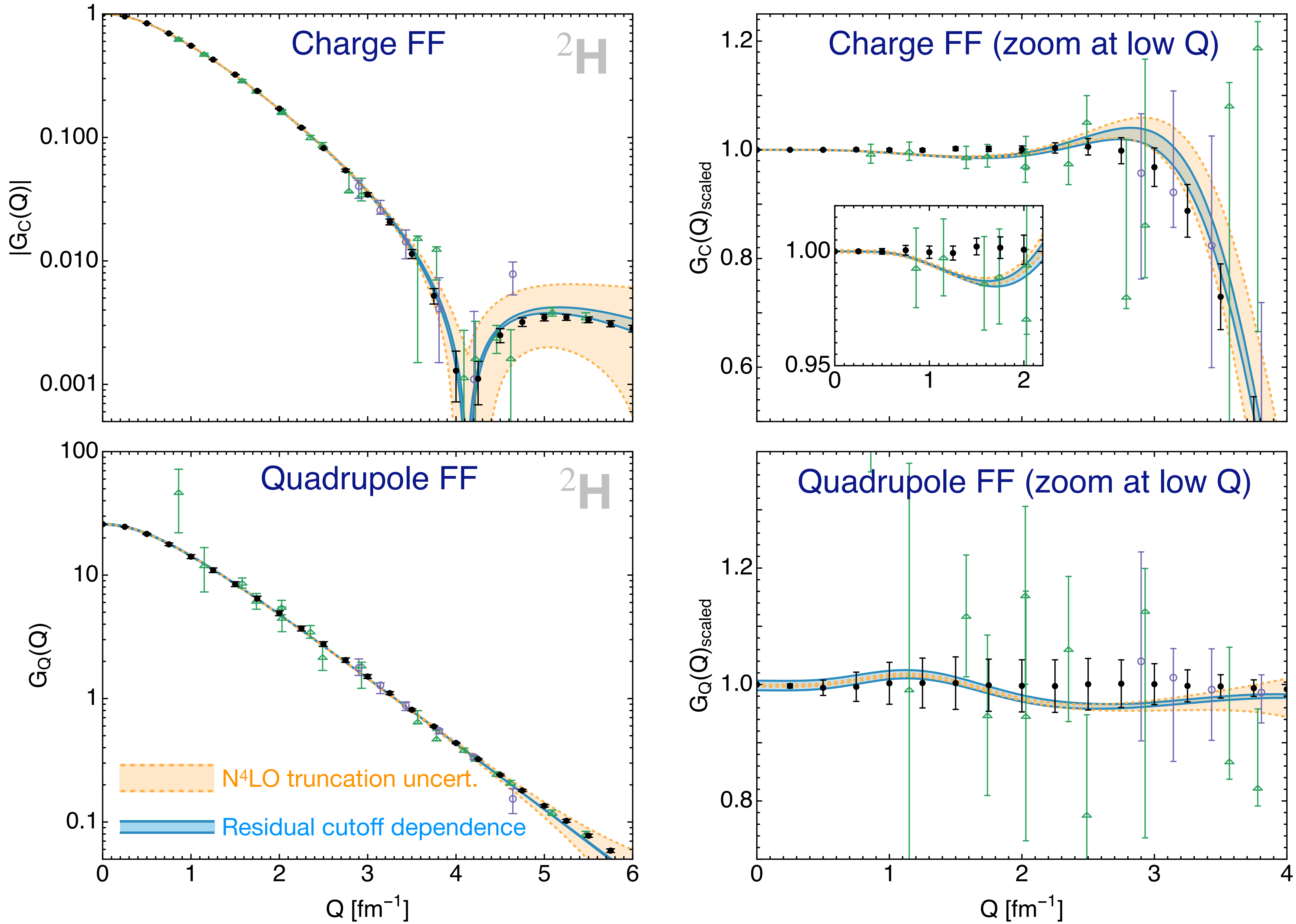
- Analysis of **magnetic form factors** of ^2H, ^3H and ^3He
- Application to processes with two photons (**polarizabilities**, ...)

- Isoscalar 2N charge-density can be used to predict **charge radii of heavy nuclei** (LECs are fixed):
  work in progress by LENPIC

Spares

Deuteron charge and quadrupole form factors

AF, Möller, Baru, Epelbaum, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313



Cutoff range 400-550 MeV