π^0-η-η' mixing from $V \to P\gamma$ and $P \to V\gamma$ decays

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What's the goal of this analysis?

To estimate the admixtures of the η and η' mesons to the physical π^0

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon\,|\eta\rangle + \epsilon'\,|\eta'\rangle$$

where π_3 is the I_3=0 state of the pseudoscalar isospin triplet

The responsible of this mixing is isospin breaking

T. Feldmann, P. Kroll and B. Stech, Phys. Lett. B449 (1999) 339

$$\epsilon = 1.4\,\% \qquad \epsilon' = 0.37\,\%$$

η-η' mixing: a reminder

octet-singlet basis

$$|\eta\rangle = \cos\theta_P\,|\eta_8\rangle - \sin\theta_P\,|\eta_0\rangle$$

$$|\eta'\rangle = \sin\theta_P\,|\eta_8\rangle + \cos\theta_P\,|\eta_0\rangle$$

$$|\eta_8\rangle = \frac{1}{\sqrt{6}}\,|u\bar{u}+d\bar{d}-2s\bar{s}\rangle$$

$$|\eta_0\rangle = \frac{1}{\sqrt{3}}\,|u\bar{u}+d\bar{d}+s\bar{s}\rangle$$

quark-flavour basis

$$|\eta\rangle = \cos\phi_P\,|\eta_{\mathrm{NS}}\rangle - \sin\phi_P\,|\eta_{\mathrm{S}}\rangle$$

$$|\eta'\rangle = \sin\phi_P\,|\eta_{\mathrm{NS}}\rangle + \cos\phi_P\,|\eta_{\mathrm{S}}\rangle$$

$$|\eta_{\mathrm{NS}}\rangle = \frac{1}{\sqrt{2}}\,|u\bar{u}+d\bar{d}\rangle$$

$$|\eta_{\mathrm{S}}\rangle = |s\bar{s}\rangle$$

$$\boxed{\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ}$$

η-η' is heavily influenced by the $U(1)_A$ of QCD

Previous estimates

Kroll based on the FKS scheme

$$\epsilon(z) = \cos\phi\left[\frac{1}{2}\frac{m_{dd}^2 - m_{uu}^2}{m_\eta^2 - m_{\pi^0}^2} + z\right]$$

$$\epsilon'(z) = \sin\phi\left[\frac{1}{2}\frac{m_{dd}^2 - m_{uu}^2}{m_{\eta'}^2 - m_{\pi^0}^2} + z\right]$$

$$\phi = 39.3^\circ$$

$$\hat{\epsilon} = \epsilon(z=0) = (1.7 \pm 0.2)\%$$

$$\hat{\epsilon}' = \epsilon'(z=0) = (0.4 \pm 0.1)\%$$

P. Kroll, Mod. Phys. Lett. A20 (2005) 2667

Escribano et al. based on ChPT with resonances

$$\epsilon_{\pi\eta} = c\phi_{\eta\eta'}\frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_\eta^2 - m_{\pi^-}^2}\left[1 - \frac{m_\eta^2 - m_{\pi^-}^2}{M_S^2}\right]$$

$$\epsilon_{\pi\eta'} = s\phi_{\eta\eta'}\frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\eta'}^2 - m_{\pi^-}^2}\left[1 - \frac{m_{\eta'}^2 - m_{\pi^-}^2}{M_S^2}\right]$$

$$\phi_{\eta\eta'} = (41.4 \pm 0.5)^\circ$$

$$(9.8 \pm 0.3) \times 10^{-3}$$

$$(2.5 \pm 1.5) \times 10^{-4}$$

R. Escribano, S. Gonzàlez-Solís and P. Roig, Phys. Rev. D94 (2016) 034008

The experimental data

$V \to P\gamma$ and $P \to V\gamma$ decays is

the most extensive and exhaustive set of data

	Transition	Γ_{exp} (keV)	
	$\rho^0 \to \eta\gamma$	44 ± 3	7%
	$\rho^0 \to \pi^0\gamma$	69 ± 9	13%
	$\rho^+ \to \pi^+\gamma$	67 ± 7	10%
	$\omega \to \eta\gamma$	3.8 ± 0.3	8%
SND 2013	$\omega \to \pi^0\gamma$	713 ± 20	3%
SND 2000	$\phi \to \eta\gamma$	55.4 ± 1.1	2%
KLOE 2007	$\phi \to \eta'\gamma$	0.26 ± 0.01	4%
SND 2000	$\phi \to \pi^0\gamma$	5.5 ± 0.2	4%
BESIII 2018	$\eta' \to \rho^0\gamma$	57 ± 3	5%
BES 2019	$\eta' \to \omega\gamma$	5.1 ± 0.3	6%
	$K^{*0} \to K^0\gamma$	116 ± 10	9%
	$K^{*+} \to K^+\gamma$	46 ± 4	9%

The theoretical model

The most general $SU(3)_F$-symmetric effective Lagrangian consistent with Lorentz, P and C invariance

$$\mathscr{L}_{VP\gamma} = g_e \epsilon_{\mu\nu\alpha\beta} \partial^\mu A^\nu \mathrm{Tr}[Q(\partial^\alpha V^\beta P + P \partial^\alpha V^\beta)]$$

supplemented with conventional quark model ideas to introduce flavour and isospin breaking

- **Magnetic dipole transitions**

$$\mu_q = e_q/2m_q \qquad 1 - s_e \equiv \overline{m}/m_s$$

- **Relative overlap between the P and V wavefunctions**

OZI-rule: $Z_\pi = \langle \pi | \omega_{\mathrm{NS}} \rangle = \langle \pi | \rho \rangle \quad Z_{NS} = \langle \eta_{\mathrm{NS}} | \omega_{\mathrm{NS}} \rangle = \langle \eta_{\mathrm{NS}} | \rho \rangle \quad Z_S = \langle \eta_{\mathrm{S}} | \omega_{\mathrm{S}} \rangle$

The theoretical model

π^0-η-η' mixing in the quark-flavour basis

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}\mathrm{c}\phi_{23} + \epsilon_{13}\mathrm{s}\phi_{23} & \mathrm{c}\phi_{23} & -\mathrm{s}\phi_{23} \\ -\epsilon_{13}\mathrm{c}\phi_{23} - \epsilon_{12}\mathrm{s}\phi_{23} & \mathrm{s}\phi_{23} & \mathrm{c}\phi_{23} \end{pmatrix} \begin{pmatrix} \pi_3 \\ \eta_{\mathrm{NS}} \\ \eta_{\mathrm{S}} \end{pmatrix}$$

To compare to Kroll's results:

$$\begin{pmatrix} \epsilon_{12} \\ \epsilon_{13} \end{pmatrix} = \begin{pmatrix} \mathrm{c}\phi_P & \mathrm{s}\phi_P \\ -\mathrm{s}\phi_P & \mathrm{c}\phi_P \end{pmatrix} \begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix}$$

To compare to Escribano et al.'s results:

$$\begin{pmatrix} \epsilon_{12} \\ \epsilon_{13} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \mathrm{c}\theta_P - \sqrt{2}\,\mathrm{s}\theta_P & \mathrm{s}\theta_P + \sqrt{2}\,\mathrm{c}\theta_P \\ -\mathrm{s}\theta_P - \sqrt{2}\,\mathrm{c}\theta_P & \mathrm{c}\theta_P - \sqrt{2}\,\mathrm{s}\theta_P \end{pmatrix} \begin{pmatrix} \epsilon_{\pi\eta} \\ \epsilon_{\pi\eta'} \end{pmatrix}$$

The theoretical model

Couplings of the enhanced phenomenological model

$$g_{\rho^0\pi^0\gamma} = g\left(\frac{1}{3} + \epsilon_{12} z_{\rm NS}\right), \quad g_{\rho^+\pi^+\gamma} = g\frac{z_+}{3},$$

$$g_{\rho^0\eta\gamma} = g\left[\left(z_{\rm NS} - \frac{\epsilon_{12}}{3}\right)c\phi_{23} + \frac{\epsilon_{13}}{3}s\phi_{23}\right],$$

$$g_{\omega\pi^0\gamma} = g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{\rm NS}\right)c\phi_V + \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}\epsilon_{13}s\phi_V\right],$$

$$g_{\eta'\rho^0\gamma} = g\left[\left(z_{\rm NS} - \frac{\epsilon_{12}}{3}\right)s\phi_{23} - \frac{\epsilon_{13}}{3}c\phi_{23}\right],$$

$$g_{\omega\eta\gamma} = g\left\{\left[\left(\frac{z_{\rm NS}}{3} - \epsilon_{12}\right)c\phi_{23} + \epsilon_{13}s\phi_{23}\right]c\phi_V - \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}s\phi_{23}s\phi_V\right\},$$

$$g_{\eta'\omega\gamma} = g\left\{\left[\left(\frac{z_{\rm NS}}{3} - \epsilon_{12}\right)s\phi_{23} - \epsilon_{13}c\phi_{23}\right]c\phi_V + \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}c\phi_{23}s\phi_V\right\},$$

$$g_{\phi\pi^0\gamma} = g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{\rm NS}\right)s\phi_V - \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}\epsilon_{13}c\phi_V\right],$$

$$g_{\phi\pi^0\gamma} = g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{\rm NS}\right)s\phi_V - \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}\epsilon_{13}c\phi_V\right],$$

$$g_{\phi\eta\gamma} = g\left\{\left[\left(\frac{z_{\rm NS}}{3} - \epsilon_{12}\right)c\phi_{23} + \epsilon_{13}s\phi_{23}\right]s\phi_V + \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}s\phi_{23}c\phi_V\right\},$$

$$g_{\phi\eta'\gamma} = g\left\{\left[\left(\frac{z_{\rm NS}}{3} - \epsilon_{12}\right)s\phi_{23} - \epsilon_{13}c\phi_{23}\right]s\phi_V - \frac{2}{3}z_{\rm S}\frac{\overline{m}}{m_s}c\phi_{23}c\phi_V\right\},$$

$$g_{K^{*0}K^0\gamma} = -\frac{1}{3}g\left(1 + \frac{\overline{m}}{m_s}\right)z_{\rm K^0} = -\frac{1}{3}g\left(1 + z_{\rm S}\frac{\overline{m}}{m_s}\right)z'_{\rm K^0},$$

$$g_{K^{*+}K^+\gamma} = \frac{1}{3}g\left(2 - \frac{\overline{m}}{m_s}\right)z_{\rm K^+} = \frac{1}{3}g\left(2 - z_{\rm S}\frac{\overline{m}}{m_s}\right)z'_{\rm K^+},$$

$$z_{\rm NS} = Z_{\rm NS}/Z_3 \quad z_{\rm S} = Z_{\rm S}/Z_3 \quad z_+ = Z_+/Z_3$$

$$z_{K^0} = Z_{K^0}/Z_3 \quad z_{K^+} = Z_{K^+}/Z_3 \quad g = Z_3\, g_e$$

Results

η-η' mixing revisited

Table 1
Comparison between estimations for the seven free parameters from the model presented in Ref. [6], using the PDG 2000 and the most up-to-date experimental data.

Parameter	Estimation from [6]	Current Estimation
g	$0.70 \pm 0.02\ \text{GeV}^{-1}$	$0.70 \pm 0.01\ \text{GeV}^{-1}$
$\frac{m_s}{\overline{m}}$	1.24 ± 0.07	1.17 ± 0.06
ϕ_P	$(37.7 \pm 2.4)^\circ$	$(41.4 \pm 0.5)^\circ$
ϕ_V	$(3.4 \pm 0.2)^\circ$	$(3.3 \pm 0.1)^\circ$
z_{NS}	0.91 ± 0.05	0.84 ± 0.02
z_{S}	0.89 ± 0.07	0.76 ± 0.04
z_{K}	0.91 ± 0.04	0.89 ± 0.03
$\chi^2_{\text{min}}/\text{d.o.f.}$	0.7	4.6

R. Escribano and E. Royo, Phys. Lett. B807 (2020) 135534

Results

FIT 1

$$g = 0.69 \pm 0.01\ \text{GeV}^{-1}\ , \qquad z_+ = 0.95 \pm 0.05\ ,$$

$$\phi_{23} = (41.5 \pm 0.5)^\circ\ , \qquad \phi_V = (4.0 \pm 0.2)^\circ\ ,$$

$$\epsilon_{12} = (2.3 \pm 1.0)\%\ , \qquad \epsilon_{13} = (2.5 \pm 0.9)\%\ ,$$

$$z_{\rm NS} = 0.89 \pm 0.03\ , \qquad z_S \overline{m}/m_s = 0.65 \pm 0.01\ ,$$

$$z'_{\rm K^0} = 1.01 \pm 0.04\ , \qquad z'_{\rm K^+} = 0.76 \pm 0.04\ .$$

$$\chi^2_{\rm min}/\text{dof} \simeq 4.6/2 = 2.3$$

ϕ_{23}, ϕ_V **very good agreement with recent published results**

$\epsilon_{12}, \epsilon_{13}$ **very small but not compatible with zero, with a CL of 2.3 and 2.8 sigmas, respectively**

→ $\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)\%$ $\quad$ $\epsilon' = \epsilon_{\pi\eta'} = (3.4 \pm 0.9)\%$ **CL 3.8 sig.**

Results

FIT 2

Turning off secondary mechanism of isospin breaking

$$\boxed{z_+ = 1 \ \& \ z_{K^0} = z_K^+}$$

$$g = 0.69 \pm 0.01 \text{ GeV}^{-1} \ , m_s/\overline{m} = 1.17 \pm 0.06 \ ,$$

$$\phi_{23} = (41.5 \pm 0.5)^\circ \ , \qquad \phi_V = (4.0 \pm 0.2)^\circ \ ,$$

$$\epsilon_{12} = (2.4 \pm 1.0)\% \ , \qquad \epsilon_{13} = (2.5 \pm 0.9)\% \ ,$$

$$z_{\text{NS}} = 0.89 \pm 0.03 \ , \qquad z_{\text{S}} = 0.77 \pm 0.04 \ ,$$

$$z_{\text{K}} = 0.90 \pm 0.03 \ ,$$

$$\chi^2_{\text{min}}/\text{dof} \simeq 5.6/3 = 1.9$$

The z's are still different from zero

→ **Secondary mechanism of $SU(3)_F$ breaking is still required**

Results

FIT 3

Using Kroll's results: $\epsilon_{12} = (1.6 \pm 0.2)\%$ $\epsilon_{13} = (-0.8 \pm 0.1)\%$

$$g = 0.69 \pm 0.01 \text{ GeV}^{-1} \,, m_s/\overline{m} = 1.17 \pm 0.06 \,,$$

$$\phi_{23} = (41.4 \pm 0.5)^\circ \,, \qquad \phi_V = (3.1 \pm 0.1)^\circ \,,$$

$$z_{\rm NS} = 0.86 \pm 0.0 \,, \qquad z_{\rm S} = 0.77 \pm 0.04 \,,$$

$$z_{\rm K} = 0.90 \pm 0.03 \,,$$

$$\chi^2_{\rm min}/\text{dof} \simeq 22.0/5 = 4.4$$

Results

FIT 4

Using Escribano et al's results:

$$\epsilon_{12} = (7.5 \pm 0.2) \times 10^{-3} \qquad \epsilon_{13} = (-6.3 \pm 0.2) \times 10^{-3}$$

$$g = 0.70 \pm 0.01\ \mathrm{GeV}^{-1}\ , m_s/\overline{m} = 1.17 \pm 0.06\ ,$$

$$\phi_{23} = (41.4 \pm 0.5)^\circ\ , \qquad \phi_V = (3.2 \pm 0.1)^\circ\ ,$$

$$z_{\mathrm{NS}} = 0.85 \pm 0.02\ , \qquad z_{\mathrm{S}} = 0.77 \pm 0.04\ ,$$

$$z_{\mathrm{K}} = 0.90 \pm 0.03\ ,$$

$$\chi^2_{\mathrm{min}}/\mathrm{dof} \simeq 24.0/5 = 4.8$$

Theoretical estimations by Kroll and Escribano et al.
do not appear to agree with the most recent experimental data

Results

FIT 5

Charged and neutral kaon transitions are not considered

$$g = 0.69 \pm 0.01 \text{ GeV}^{-1} , z_S \overline{m}/m_s = 0.65 \pm 0.01 ,$$

$$\phi_{23} = (41.5 \pm 0.5)^\circ , \qquad \phi_V = (4.0 \pm 0.2)^\circ ,$$

$$\epsilon_{12} = (\quad 2.4 \pm 1.0)\% , \qquad \epsilon_{13} = (\quad 2.5 \pm 0.9)\% ,$$

$$z_{\rm NS} = 0.89 \pm 0.03 .$$

$$\chi^2_{\rm min}/\text{dof} \simeq 5.6/3 = 1.9$$

$\epsilon_{12}, \epsilon_{13}$ **again incompatible with zero,**
with a CL of 2.4 and 2.8 sigmas, respectively

Results

Summary of Fits

Table 2
Summary of fitted values for the Fit 1, Fit 2, Fit 3, Fit 4 and Fit 5, corresponding to Eqs. (13), (14), (15), (16), and (17), respectively.

Parameter	Fit 1	Fit 2	Fit 3	Fit 4	Fit 5
g (GeV^{-1})	0.69 ± 0.01	0.69 ± 0.01	0.69 ± 0.01	0.70 ± 0.01	0.69 ± 0.01
ϵ_{12}	$(2.3 \pm 1.0)\%$	$(2.4 \pm 1.0)\%$	-	-	$(2.4 \pm 1.0)\%$
ϵ_{13}	$(2.5 \pm 0.9)\%$	$(2.5 \pm 0.9)\%$	-	-	$(2.5 \pm 0.9)\%$
ϕ_{23} ($^\circ$)	41.5 ± 0.5	41.5 ± 0.05	41.4 ± 0.5	41.4 ± 0.5	41.5 ± 0.5
ϕ_V ($^\circ$)	4.0 ± 0.2	4.0 ± 0.2	3.1 ± 0.1	3.2 ± 0.1	4.0 ± 0.2
$m_s/\overline{m}$	-	1.17 ± 0.06	1.17 ± 0.06	1.17 ± 0.06	-
$z_S\overline{m}/m_s$	0.65 ± 0.01	-	-	-	0.65 ± 0.01
z_{NS}	0.89 ± 0.03	0.89 ± 0.03	0.86 ± 0.02	0.85 ± 0.02	0.89 ± 0.03
z_+	0.95 ± 0.05	-	-	-	-
z_S	-	0.77 ± 0.04	0.77 ± 0.04	0.77 ± 0.04	-
z_K	-	0.90 ± 0.03	0.90 ± 0.03	0.90 ± 0.03	-
z'_{K^0}	1.01 ± 0.04	-	-	-	-
z'_{K^+}	0.76 ± 0.04	-	-	-	-
$\chi^2_{min}/\mathrm{d.o.f.}$	2.3	1.9	4.4	4.8	1.9

Conclusions

- The quality of the most up-to-date experimental data enables a small amount of isospin-symmetry breaking that is incosistent with zero with a CL of aprox. 2.5 sigmas
- The quality of the performed fits is good
- The estimations for the fit parameters appear to be robust
- Our estimates for

$$\epsilon_{12} = (2.4 \pm 1.0)\%$$
$$\epsilon_{13} = (2.5 \pm 0.9)\%$$

→

$$\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)\%$$
$$\epsilon' = \epsilon_{\pi\eta'} = (3.5 \pm 0.9)\%$$

are not in accordance with theoretical estimates