

UAB

Universitat Autònoma
de Barcelona



IFAE

Institut de Física
d'Altes Energies



π^0 - η - η' mixing from $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays

Rafel Escribano
UAB and IFAE-BIST

CD 2021

**10th International Workshop
on Chiral Dynamics**

November 16, 2021
Beijing (China)

What's the goal of this analysis?

To **estimate** the **admixture**s of the η and η' mesons to the physical π^0

$$|\pi^0\rangle = |\pi_3\rangle + \epsilon |\eta\rangle + \epsilon' |\eta'\rangle$$

where π_3 is the **$I_3=0$ state** of the pseudoscalar **isospin triplet**

The responsible of this **mixing** is **isospin breaking**

T. Feldmann, P. Kroll and B. Stech, Phys. Lett. B449 (1999) 339

$$\epsilon = 1.4 \% \qquad \epsilon' = 0.37 \%$$

η - η' mixing: a reminder

octet-singlet basis

$$|\eta\rangle = \cos\theta_P |\eta_8\rangle - \sin\theta_P |\eta_0\rangle \qquad |\eta_8\rangle = \frac{1}{\sqrt{6}} |u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$$

$$|\eta'\rangle = \sin\theta_P |\eta_8\rangle + \cos\theta_P |\eta_0\rangle \qquad |\eta_0\rangle = \frac{1}{\sqrt{3}} |u\bar{u} + d\bar{d} + s\bar{s}\rangle$$

quark-flavour basis

$$|\eta\rangle = \cos\phi_P |\eta_{NS}\rangle - \sin\phi_P |\eta_S\rangle \qquad |\eta_{NS}\rangle = \frac{1}{\sqrt{2}} |u\bar{u} + d\bar{d}\rangle$$

$$|\eta'\rangle = \sin\phi_P |\eta_{NS}\rangle + \cos\phi_P |\eta_S\rangle \qquad |\eta_S\rangle = |s\bar{s}\rangle$$

$$\theta_P = \phi_P - \arctan\sqrt{2} \simeq \phi_P - 54.7^\circ$$

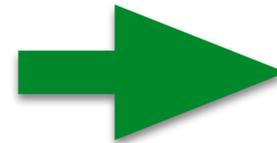
η - η' is heavily influenced by the $U(1)_A$ of QCD

Previous estimates

Kroll based on the FKS scheme

$$\epsilon(z) = \cos \phi \left[\frac{1}{2} \frac{m_{dd}^2 - m_{uu}^2}{m_{\eta}^2 - m_{\pi^0}^2} + z \right]$$

$$\epsilon'(z) = \sin \phi \left[\frac{1}{2} \frac{m_{dd}^2 - m_{uu}^2}{m_{\eta'}^2 - m_{\pi^0}^2} + z \right]$$



$$\phi = 39.3^\circ$$

$$\hat{\epsilon} = \epsilon(z=0) = (1.7 \pm 0.2)\%$$

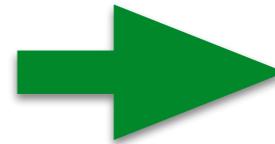
$$\hat{\epsilon}' = \epsilon'(z=0) = (0.4 \pm 0.1)\%$$

P. Kroll, Mod. Phys. Lett. A20 (2005) 2667

Escribano et al. based on ChPT with resonances

$$\epsilon_{\pi\eta} = c\phi_{\eta\eta'} \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\eta}^2 - m_{\pi^-}^2} \left[1 - \frac{m_{\eta}^2 - m_{\pi^-}^2}{M_S^2} \right]$$

$$\epsilon_{\pi\eta'} = s\phi_{\eta\eta'} \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_{\eta'}^2 - m_{\pi^-}^2} \left[1 - \frac{m_{\eta'}^2 - m_{\pi^-}^2}{M_S^2} \right]$$



$$\phi_{\eta\eta'} = (41.4 \pm 0.5)^\circ$$

$$(9.8 \pm 0.3) \times 10^{-3}$$

$$(2.5 \pm 1.5) \times 10^{-4}$$

R. Escribano, S. González-Solís and P. Roig, Phys. Rev. D94 (2016) 034008

The experimental data

$V \rightarrow P\gamma$ and $P \rightarrow V\gamma$ decays is

the most **extensive** and **exhaustive** set of data

	Transition	Γ_{exp} (keV)	
	$\rho^0 \rightarrow \eta\gamma$	44 ± 3	7%
	$\rho^0 \rightarrow \pi^0\gamma$	69 ± 9	13%
	$\rho^+ \rightarrow \pi^+\gamma$	67 ± 7	10%
	$\omega \rightarrow \eta\gamma$	3.8 ± 0.3	8%
SND 2013	$\omega \rightarrow \pi^0\gamma$	713 ± 20	3%
SND 2000	$\phi \rightarrow \eta\gamma$	55.4 ± 1.1	2%
KLOE 2007	$\phi \rightarrow \eta'\gamma$	0.26 ± 0.01	4%
SND 2000	$\phi \rightarrow \pi^0\gamma$	5.5 ± 0.2	4%
BESIII 2018	$\eta' \rightarrow \rho^0\gamma$	57 ± 3	5%
BES 2019	$\eta' \rightarrow \omega\gamma$	5.1 ± 0.3	6%
	$K^{*0} \rightarrow K^0\gamma$	116 ± 10	9%
	$K^{*+} \rightarrow K^+\gamma$	46 ± 4	9%

The theoretical model

The **most general** $SU(3)_F$ -**symmetric** effective Lagrangian consistent with **Lorentz**, **P** and **C** invariance

$$\mathcal{L}_{VP\gamma} = g_e \epsilon_{\mu\nu\alpha\beta} \partial^\mu A^\nu \text{Tr}[Q (\partial^\alpha V^\beta P + P \partial^\alpha V^\beta)]$$

supplemented with **conventional quark model** ideas to introduce **flavour** and **isospin breaking**

- **Magnetic dipole transitions**

$$\mu_q = e_q / 2m_q \quad 1 - s_e \equiv \bar{m}/m_s$$

- **Relative overlap** between the **P** and **V** wavefunctions

OZI-rule: $Z_\pi = \langle \pi | \omega_{NS} \rangle = \langle \pi | \rho \rangle$ $Z_{NS} = \langle \eta_{NS} | \omega_{NS} \rangle = \langle \eta_{NS} | \rho \rangle$ $Z_S = \langle \eta_S | \omega_S \rangle$

The theoretical model

π^0 - η - η' mixing in the quark-flavour basis

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} 1 & \epsilon_{12} & \epsilon_{13} \\ -\epsilon_{12}c\phi_{23} + \epsilon_{13}s\phi_{23} & c\phi_{23} & -s\phi_{23} \\ -\epsilon_{13}c\phi_{23} - \epsilon_{12}s\phi_{23} & s\phi_{23} & c\phi_{23} \end{pmatrix} \begin{pmatrix} \pi_3 \\ \eta_{NS} \\ \eta_S \end{pmatrix}$$

To compare to Kroll's results:

$$\begin{pmatrix} \epsilon_{12} \\ \epsilon_{13} \end{pmatrix} = \begin{pmatrix} c\phi_P & s\phi_P \\ -s\phi_P & c\phi_P \end{pmatrix} \begin{pmatrix} \epsilon \\ \epsilon' \end{pmatrix}$$

To compare to Escribano et al.'s results:

$$\begin{pmatrix} \epsilon_{12} \\ \epsilon_{13} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} c\theta_P - \sqrt{2}s\theta_P & s\theta_P + \sqrt{2}c\theta_P \\ -s\theta_P - \sqrt{2}c\theta_P & c\theta_P - \sqrt{2}s\theta_P \end{pmatrix} \begin{pmatrix} \epsilon_{\pi\eta} \\ \epsilon_{\pi\eta'} \end{pmatrix}$$

The theoretical model

Couplings of the enhanced phenomenological model

$$\begin{aligned}
 g_{\rho^0\pi^0\gamma} &= g\left(\frac{1}{3} + \epsilon_{12}z_{NS}\right), & g_{\rho^+\pi^+\gamma} &= g\frac{z_+}{3}, \\
 g_{\rho^0\eta\gamma} &= g\left[\left(z_{NS} - \frac{\epsilon_{12}}{3}\right)c\phi_{23} + \frac{\epsilon_{13}}{3}s\phi_{23}\right], \\
 g_{\omega\pi^0\gamma} &= g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{NS}\right)c\phi_V + \frac{2}{3}z_S\frac{\bar{m}}{m_S}\epsilon_{13}s\phi_V\right], \\
 g_{\eta'\rho^0\gamma} &= g\left[\left(z_{NS} - \frac{\epsilon_{12}}{3}\right)s\phi_{23} - \frac{\epsilon_{13}}{3}c\phi_{23}\right], \\
 g_{\omega\eta\gamma} &= g\left\{\left[\left(\frac{z_{NS}}{3} - \epsilon_{12}\right)c\phi_{23} + \epsilon_{13}s\phi_{23}\right]c\phi_V\right. \\
 &\quad \left.- \frac{2}{3}z_S\frac{\bar{m}}{m_S}s\phi_{23}s\phi_V\right\}, \\
 g_{\eta'\omega\gamma} &= g\left\{\left[\left(\frac{z_{NS}}{3} - \epsilon_{12}\right)s\phi_{23} - \epsilon_{13}c\phi_{23}\right]c\phi_V\right. \\
 &\quad \left.+ \frac{2}{3}z_S\frac{\bar{m}}{m_S}c\phi_{23}s\phi_V\right\}, \\
 g_{\phi\pi^0\gamma} &= g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{NS}\right)s\phi_V - \frac{2}{3}z_S\frac{\bar{m}}{m_S}\epsilon_{13}c\phi_V\right],
 \end{aligned}$$

$$\begin{aligned}
 g_{\phi\pi^0\gamma} &= g\left[\left(1 + \frac{\epsilon_{12}}{3}z_{NS}\right)s\phi_V - \frac{2}{3}z_S\frac{\bar{m}}{m_S}\epsilon_{13}c\phi_V\right], \\
 g_{\phi\eta\gamma} &= g\left\{\left[\left(\frac{z_{NS}}{3} - \epsilon_{12}\right)c\phi_{23} + \epsilon_{13}s\phi_{23}\right]s\phi_V\right. \\
 &\quad \left.+ \frac{2}{3}z_S\frac{\bar{m}}{m_S}s\phi_{23}c\phi_V\right\}, \\
 g_{\phi\eta'\gamma} &= g\left\{\left[\left(\frac{z_{NS}}{3} - \epsilon_{12}\right)s\phi_{23} - \epsilon_{13}c\phi_{23}\right]s\phi_V\right. \\
 &\quad \left.- \frac{2}{3}z_S\frac{\bar{m}}{m_S}c\phi_{23}c\phi_V\right\}, \\
 g_{K^{*0}K^0\gamma} &= -\frac{1}{3}g\left(1 + \frac{\bar{m}}{m_S}\right)z_{K^0} = -\frac{1}{3}g\left(1 + z_S\frac{\bar{m}}{m_S}\right)z'_{K^0}, \\
 g_{K^{*+}K^+\gamma} &= \frac{1}{3}g\left(2 - \frac{\bar{m}}{m_S}\right)z_{K^+} = \frac{1}{3}g\left(2 - z_S\frac{\bar{m}}{m_S}\right)z'_{K^+},
 \end{aligned}$$

$$\begin{aligned}
 z_{NS} &= Z_{NS}/Z_3 & z_S &= Z_S/Z_3 & z_+ &= Z_+/Z_3 \\
 z_{K^0} &= Z_{K^0}/Z_3 & z_{K^+} &= Z_{K^+}/Z_3 & g &= Z_3 g_e
 \end{aligned}$$

Results

η - η' mixing revisited

Table 1

Comparison between estimations for the seven free parameters from the model presented in Ref. [6], using the PDG 2000 and the most up-to-date experimental data.

Parameter	Estimation from [6]	Current Estimation
g	$0.70 \pm 0.02 \text{ GeV}^{-1}$	$0.70 \pm 0.01 \text{ GeV}^{-1}$
$\frac{m_s}{\bar{m}}$	1.24 ± 0.07	1.17 ± 0.06
ϕ_P	$(37.7 \pm 2.4)^\circ$	$(41.4 \pm 0.5)^\circ$
ϕ_V	$(3.4 \pm 0.2)^\circ$	$(3.3 \pm 0.1)^\circ$
z_{NS}	0.91 ± 0.05	0.84 ± 0.02
z_S	0.89 ± 0.07	0.76 ± 0.04
z_K	0.91 ± 0.04	0.89 ± 0.03
$\chi_{\min}^2/\text{d.o.f.}$	0.7	4.6

Results

FIT 1

$$\begin{aligned}g &= 0.69 \pm 0.01 \text{ GeV}^{-1}, & z_+ &= 0.95 \pm 0.05, \\ \phi_{23} &= (41.5 \pm 0.5)^\circ, & \phi_V &= (4.0 \pm 0.2)^\circ, \\ \epsilon_{12} &= (2.3 \pm 1.0)\%, & \epsilon_{13} &= (2.5 \pm 0.9)\%, \\ z_{\text{NS}} &= 0.89 \pm 0.03, & z_{S\bar{m}}/m_S &= 0.65 \pm 0.01, \\ z'_{\text{K}^0} &= 1.01 \pm 0.04, & z'_{\text{K}^+} &= 0.76 \pm 0.04.\end{aligned}$$

$$\chi_{\text{min}}^2/\text{dof} \simeq 4.6/2 = 2.3$$

ϕ_{23}, ϕ_V **very good agreement** with recent published results

$\epsilon_{12}, \epsilon_{13}$ **very small but not compatible with zero,**
with a CL of 2.3 and 2.8 sigmas, respectively



$$\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)\%$$

$$\epsilon' = \epsilon_{\pi\eta'} = (3.4 \pm 0.9)\% \quad \text{CL 3.8 sig.}$$

Results

FIT 2

Turning off **secondary mechanism** of **isospin breaking**

$$z_+ = 1 \ \& \ z_{K^0} = z_K^+$$

$$g = 0.69 \pm 0.01 \text{ GeV}^{-1}, m_s/\bar{m} = 1.17 \pm 0.06,$$

$$\phi_{23} = (41.5 \pm 0.5)^\circ, \quad \phi_V = (4.0 \pm 0.2)^\circ,$$

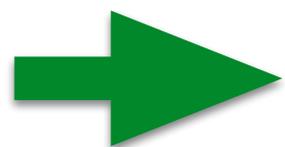
$$\epsilon_{12} = (2.4 \pm 1.0)\%, \quad \epsilon_{13} = (2.5 \pm 0.9)\%,$$

$$z_{NS} = 0.89 \pm 0.03, \quad z_S = 0.77 \pm 0.04,$$

$$z_K = 0.90 \pm 0.03,$$

$$\chi_{\min}^2/\text{dof} \simeq 5.6/3 = 1.9$$

The z's are still different from zero



Secondary mechanism of **SU(3)_F breaking** is still required

Results

FIT 3

Using Kroll's results: $\epsilon_{12} = (1.6 \pm 0.2)\%$ $\epsilon_{13} = (-0.8 \pm 0.1)\%$

$$\begin{aligned} g &= 0.69 \pm 0.01 \text{ GeV}^{-1}, m_s/\bar{m} = 1.17 \pm 0.06, \\ \phi_{23} &= (41.4 \pm 0.5)^\circ, & \phi_V &= (3.1 \pm 0.1)^\circ, \\ z_{NS} &= 0.86 \pm 0.0, & z_S &= 0.77 \pm 0.04, \\ z_K &= 0.90 \pm 0.03, \end{aligned}$$

$$\chi_{\min}^2/\text{dof} \simeq 22.0/5 = 4.4$$

Results

FIT 4

Using Escribano et al's results:

$$\epsilon_{12} = (7.5 \pm 0.2) \times 10^{-3} \quad \epsilon_{13} = (-6.3 \pm 0.2) \times 10^{-3}$$

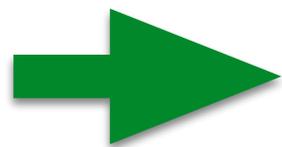
$$g = 0.70 \pm 0.01 \text{ GeV}^{-1}, m_S/\bar{m} = 1.17 \pm 0.06,$$

$$\phi_{23} = (41.4 \pm 0.5)^\circ, \quad \phi_V = (3.2 \pm 0.1)^\circ,$$

$$z_{NS} = 0.85 \pm 0.02, \quad z_S = 0.77 \pm 0.04,$$

$$z_K = 0.90 \pm 0.03,$$

$$\chi_{\min}^2/\text{dof} \simeq 24.0/5 = 4.8$$



Theoretical estimations by Kroll and Escribano et al.
do not appear to agree with the most recent experimental data

Results

FIT 5

Charged and neutral **kaon transitions** are **not considered**

$$\begin{aligned}g &= 0.69 \pm 0.01 \text{ GeV}^{-1}, z_S \bar{m}/m_S = 0.65 \pm 0.01, \\ \phi_{23} &= (41.5 \pm 0.5)^\circ, \quad \phi_V = (4.0 \pm 0.2)^\circ, \\ \epsilon_{12} &= (2.4 \pm 1.0)\%, \quad \epsilon_{13} = (2.5 \pm 0.9)\%, \\ z_{NS} &= 0.89 \pm 0.03.\end{aligned}$$

$$\chi_{\min}^2/\text{dof} \simeq 5.6/3 = 1.9$$

$\epsilon_{12}, \epsilon_{13}$ again **incompatible with zero**,
with a **CL** of 2.4 and 2.8 sigmas, respectively

Results

Summary of Fits

Table 2

Summary of fitted values for the Fit 1, Fit 2, Fit 3, Fit 4 and Fit 5, corresponding to Eqs. (13), (14), (15), (16), and (17), respectively.

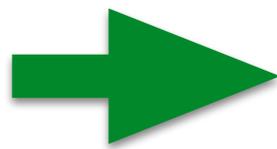
Parameter	Fit 1	Fit 2	Fit 3	Fit 4	Fit 5
g (GeV ⁻¹)	0.69 ± 0.01	0.69 ± 0.01	0.69 ± 0.01	0.70 ± 0.01	0.69 ± 0.01
ϵ_{12}	$(2.3 \pm 1.0)\%$	$(2.4 \pm 1.0)\%$	-	-	$(2.4 \pm 1.0)\%$
ϵ_{13}	$(2.5 \pm 0.9)\%$	$(2.5 \pm 0.9)\%$	-	-	$(2.5 \pm 0.9)\%$
ϕ_{23} (°)	41.5 ± 0.5	41.5 ± 0.05	41.4 ± 0.5	41.4 ± 0.5	41.5 ± 0.5
ϕ_V (°)	4.0 ± 0.2	4.0 ± 0.2	3.1 ± 0.1	3.2 ± 0.1	4.0 ± 0.2
m_s/\bar{m}	-	1.17 ± 0.06	1.17 ± 0.06	1.17 ± 0.06	-
$z_S\bar{m}/m_s$	0.65 ± 0.01	-	-	-	0.65 ± 0.01
z_{NS}	0.89 ± 0.03	0.89 ± 0.03	0.86 ± 0.02	0.85 ± 0.02	0.89 ± 0.03
z_+	0.95 ± 0.05	-	-	-	-
z_S	-	0.77 ± 0.04	0.77 ± 0.04	0.77 ± 0.04	-
z_K	-	0.90 ± 0.03	0.90 ± 0.03	0.90 ± 0.03	-
z'_{K^0}	1.01 ± 0.04	-	-	-	-
z'_{K^+}	0.76 ± 0.04	-	-	-	-
$\chi^2_{\min}/\text{d.o.f.}$	2.3	1.9	4.4	4.8	1.9

Conclusions

- The quality of the most **up-to-date experimental data** enables a small amount of **isospin-symmetry breaking** that is inconsistent with zero with a CL of approx. 2.5 sigmas
- The quality of the performed fits is **good**
- The **estimations** for the fit parameters appear to be robust
- Our **estimates** for

$$\epsilon_{12} = (2.4 \pm 1.0)\%$$

$$\epsilon_{13} = (2.5 \pm 0.9)\%$$



$$\epsilon = \epsilon_{\pi\eta} = (0.1 \pm 0.9)\%$$

$$\epsilon' = \epsilon_{\pi\eta'} = (3.5 \pm 0.9)\%$$

are not in accordance with **theoretical estimates**