



## **Motivation**

Motivation

Conditions on Bounds



## The curse of $\chi$ PT (and other EFTs)

Many LECs Limited observables LO LECs (2): NLO LECs (10):

NNLO LECs (90): N<sup>3</sup>LO LECs (1233):

(J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64 (2014) 149)

high precision %-level precision educated guesses unknown



## Whence bounds?

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#### Linear Constraint

Basic Definition

New Bounds on YPT

#### 2.6----- NI O

2-Havour NEO

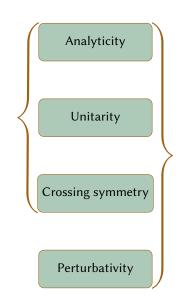
3-flavour NLO

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Higher flavou



Guaranteed for "ordinary" QFT



Not necessarily consistent for EFT ⇒ bounds!



## **Recent Developments**

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2-flavour NNLO 3-flavour NLO 3-flavour NLO

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## ■ Principles go back to the 60's; renewed interest in the 90's

A. Martin, Springer-Verlag, 1 ed., 1969

B. Ananthanarayan, D. Toublan and G. Wanders, *Phys. Rev. D* 51 (1995) 1093 [hep-ph/9410302] etc., etc.

## Our method is based on work by Manohar & Mateu

A. V. Manohar and V. Mateu, *Phys. Rev. D* 77 (2008) 094019 [0801.3222]

V. Mateu, Phys. Rev. D 77 (2008) 094020 [0801.3627]

 This talk is based on ongoing work by Alvarez, Bijnens & MS (extension of Benjamin Alvarez' 2018 master thesis)

B. Alvarez, J. Bijnens and M. Sjö, [2111.XXXXX]

## ■ Some recent (2020) extensions in similar directions

Y.-J. Wang, F.-K. Guo, C. Zhang and S.-Y. Zhou, JHEP 07 (2020) 214 [2004.03992]

A. J. Tolley, Z.-Y. Wang and S.-Y. Zhou, JHEP 05 (2021) 255 [2011.02400]

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Higher flavou

# Positivity Bounds



## **Decomposition and Crossing**

### Deriving Bounds

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## (Generalised) Isospin Decomposition

$$T(s,t) = \sum_{\mathcal{I}} a_{\mathcal{I}} T^{\mathcal{I}}(s,t)$$

 $\begin{cases} \text{isospin } 0, 1, 2 \\ \text{representation } I, A, S, AS, SS, (AA) \end{cases}$ in 2-flavour, in 3(4+)-flavour

Example:  $\pi^{\pm}\pi^{\pm} \to \pi^{\pm}\pi^{\pm}$  is purely isospin 2 (or SS)

## $s \leftrightarrow u$ Crossing Symmetry

$$T^{I}(u,t) = C_{u}^{IJ}T^{J}(s,t)$$

Matrix  $C_{\nu}^{IJ}$  determined entirely by group structure



## **Deriving Bounds**

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2 - flavour NL (

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## Dispersion relations (following Manohar & Mateu)

$$a_{\mathcal{I}} \frac{\mathrm{d}^{k}}{\mathrm{d}s^{k}} T^{\mathcal{I}}(s,t) = \underbrace{\frac{k!}{2\pi i}}_{f} \oint_{\gamma} \mathrm{d}z \underbrace{\frac{a_{\mathcal{I}} T^{\mathcal{I}}(z,t)}{(z-s)^{k+1}}}_{f} \xrightarrow{\text{Contour manipulation}}_{f}$$

$$=\frac{k!}{\pi}\int_4^\infty \mathrm{d}z \left(\frac{a_{\mathcal{I}}}{(z-s)^{k+1}}+\frac{(-1)^k a_{\mathcal{I}} C_u^{I\mathcal{I}}}{(z-u)^{k+1}}\right) \mathrm{Im} \ T^{\mathcal{I}}(z+\varepsilon i,t)$$

## Partial-wave expansion, optical theorem

$$\operatorname{Im} T^{\mathcal{I}}(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1) s \beta(s) \sigma_{\ell}^{I} P_{\ell} \left(1 + \frac{2t}{s-4}\right) \geq 0$$



## **Bounds!**

## Deriving Bounds



## **Positivity bounds**

$$a_{\mathcal{I}} \frac{\mathrm{d}^k}{\mathrm{d}s^k} T^{\mathcal{I}}(s,t) \ge 0$$

Linear in all LECs up to NNLO

## **Conditions**

 $k \geq 2$  (convergence), k even in 2-flavour

$$a_{I}\left\{\delta^{IJ}\left[\frac{z-u}{z-s}\right]^{k+1}+(-1)^{k}C_{u}^{IJ}\right\}\geq0\quad\text{for all }z\geq4$$

$$t\in[0,4]\qquad s\in[-t,4]$$

$$t\in[0,4], \qquad s\in[-t,4]$$



## Conditions on s, t, u

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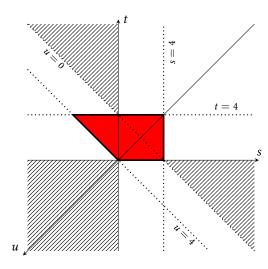
2. flavour NLO

2-flavour NNL

3-flavour NLO

Higher flavo







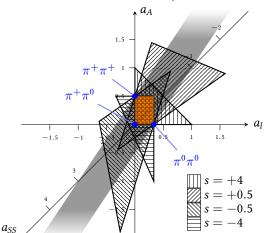
# Conditions on $a_7$

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New Bounds on YPT

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No need to restrict to mass eigenstates No need to consider bounds valid for all s, t - specific values is enough (also sufficient to check z = 4 and  $z = \infty$ )





## **Improving Bounds**

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3-Havour NEC

Higher flavou

Conclusions



- Move up to NNLO amplitude at equal meson masses
   I. Bijnens and I. Lu. JHEP 03 (2011) 028 [1102.0172]
- Most general choice of  $a_{ij}$  (previous slide)
- Integration above threshold

$$a_{\mathcal{I}} \frac{\mathrm{d}^{k}}{\mathrm{d}s^{k}} T^{\mathcal{I}} = \int_{4}^{\infty} \mathrm{d}z[\ldots] \operatorname{Im} T^{\mathcal{I}}$$

$$\to a_{\mathcal{I}} \left( \frac{\mathrm{d}^{k}}{\mathrm{d}s^{k}} T^{\mathcal{I}} - \int_{4}^{\lambda} \mathrm{d}z[\ldots] \operatorname{Im} T^{\mathcal{I}} \right) = \int_{\lambda}^{\infty} \mathrm{d}z[\ldots] \operatorname{Im} T^{\mathcal{I}}$$

- Strengthens bounds
- Reliant on low-energy approximation
- Breakdown scale is  $\lambda \sim 70/n$  at n flavours

  R. S. Chivukula, M. J. Dugan and M. Golden, *Phys. Rev. D* 47 (1993) 2930 [hep-ph/9206222]
- We have performed analytic integral up to NNLO
- Mathematical framework for reducing sets of bounds

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#### **Linear Constraints**

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Danie Barrilla

#### New Bounds on $\chi$ P $^{\circ}$

#### 2-flavour N

2-flavour NNI

3-flavour NLC

3-Havour IVE

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# Linear Constraints



## **Definition of a Linear Constraint**

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3-flavour NL

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## **Linear Constraints**

Parameters  $b_i$ , constants  $\alpha_i$ , c

$$\alpha_1 b_2 + \alpha_2 b_2 + \ldots + \alpha_N b_N - c \ge 0 \quad \Leftrightarrow \quad \boldsymbol{\alpha} \cdot \boldsymbol{b} \ge c$$

Expressed as

"The (linear) constraint  $\langle \alpha, c \rangle$  is satisfied by  ${m b}$ "



## **Combination and Comparison**

Conditions on Bounds

#### Basic Definitions



## **Combined constraints**

 $\langle \alpha, c \rangle + \langle \beta, d \rangle$  satisfied if both  $\langle \alpha, c \rangle$  and  $\langle \beta, d \rangle$  satisfied. Generally:

$$\Omega = \sum_i \langle \boldsymbol{\alpha}_i, c_i \rangle$$

## Stronger and weaker constraints

If  $\Omega'$  satisfied by all points satisfying  $\Omega$ :

$$\Omega' \leq \Omega$$

## Basic examples

$$\langle \boldsymbol{\alpha}, -1 \rangle \le \langle \boldsymbol{\alpha}, 0 \rangle \le \langle \boldsymbol{\alpha}, 1 \rangle, \qquad \Omega \le (\Omega + \Omega') \ge \Omega'$$



## **Deep Results**

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- 3-flavour N
- 3-flavour NI

Higher fla

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## Result 1

 $\langle m{lpha}, c 
angle$  weaker than  $\Omega = \sum_i \langle m{lpha}_i, c_i 
angle$  if and only if

$$oldsymbol{lpha} = \sum_i \lambda_i oldsymbol{lpha}_i, \qquad \sum_i \lambda_i c_i \geq c, \qquad \lambda_i \geq 0$$

## Result 2

Equivalently, if and only if  $\alpha$  satisfies  $\sum_{j} \langle \mathbf{n}_{j}, r_{j} \rangle$ 

Straightforward algorithm to generate  $\langle \boldsymbol{n}_j, r_j \rangle$  based on convex hulls.

## Result 3

Of all possible sets  $\mathcal S$  such that  $\Omega=\sum_{\langle {m lpha},c \rangle \in \mathcal S} \langle {m lpha},c \rangle$ , there is a (nearly) unique *smallest* such set.

Obtained as side-effect of above algorithm.

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# New Bounds on $\chi$ PT



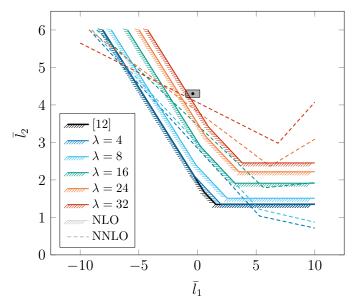
## 2-flavour NLO

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2-flavour NLO







## 2-flavour NNLO

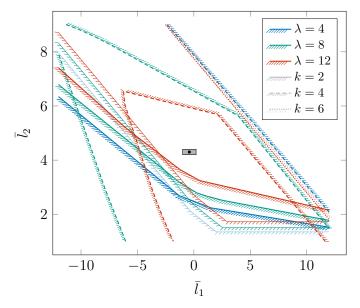
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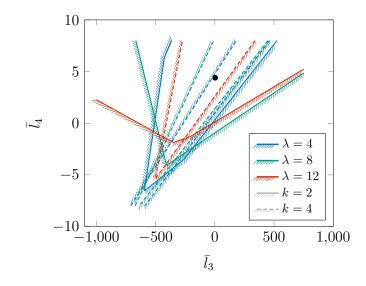
## 2-flavour NNLO

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2-flavour NNLO







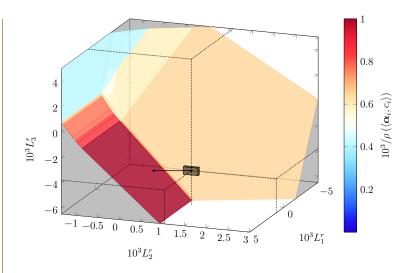
## 3-flavour NLO

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3-flavour NLO







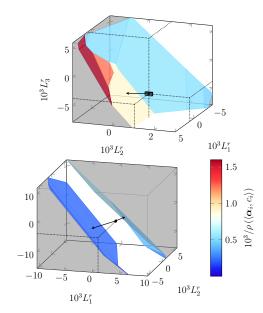
## 3-flavour NNLO

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## 3-flavour NNLO

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2-flavour N1

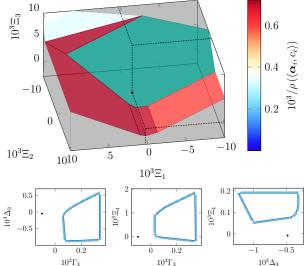
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3-flavour NLO

Higher flavo



NNLO LECs appearing through combinations  $\Xi_i, \Gamma_i, \Delta_i$ 





## **Higher Flavour**

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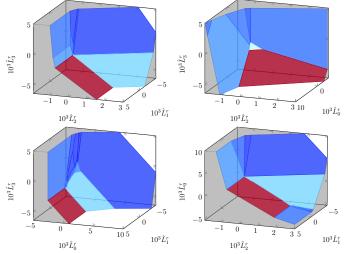
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4-flavour NLO, no great difference at even higher flavour





## **Summary & Outlook**

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## Summary

- First full NNLO bounds
- Stronger and more general bounds produced
- Powerful mathematical framework

## **Outlook**

- Multiple imporvements possible (general masses, etc.)
- Mostly plug-and-play for new amplitudes
  - N³LO, higher-point, etc.
  - Other EFTs

# Thank you!