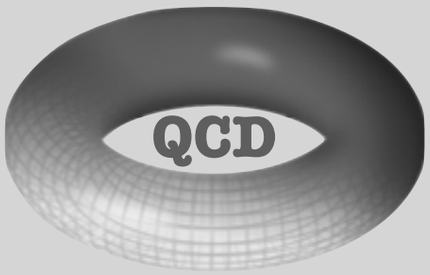
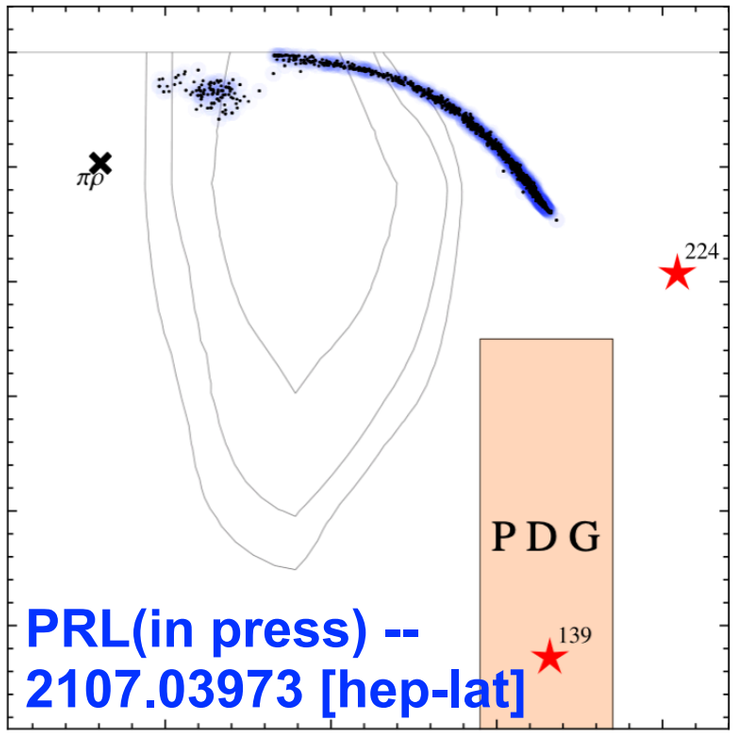
$a_1(1260)$ from lattice QCD

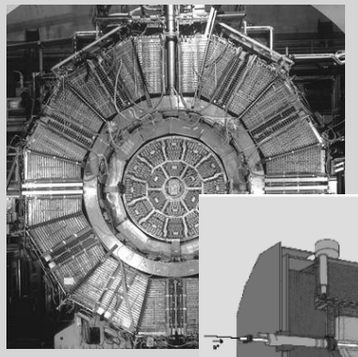
Unitarity



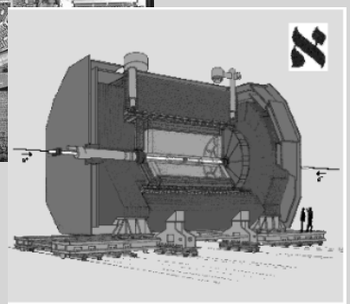
ChPT

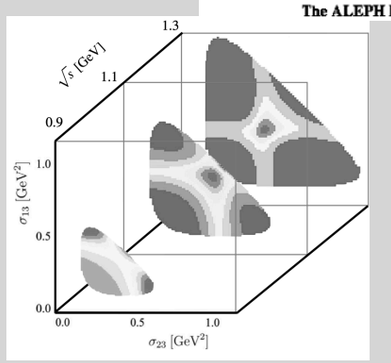


LHC



GlueX





THEORY

EXPERIMENT

Maxim Mai

with A. Alexandru, R. Brett*, C. Culver*, M. Döring,*

F. Lee, D. Sadasivan [*GWQCD]*

PHY-2012289

DE-SC0016582 DE-FG02-95ER40907

MA-7156

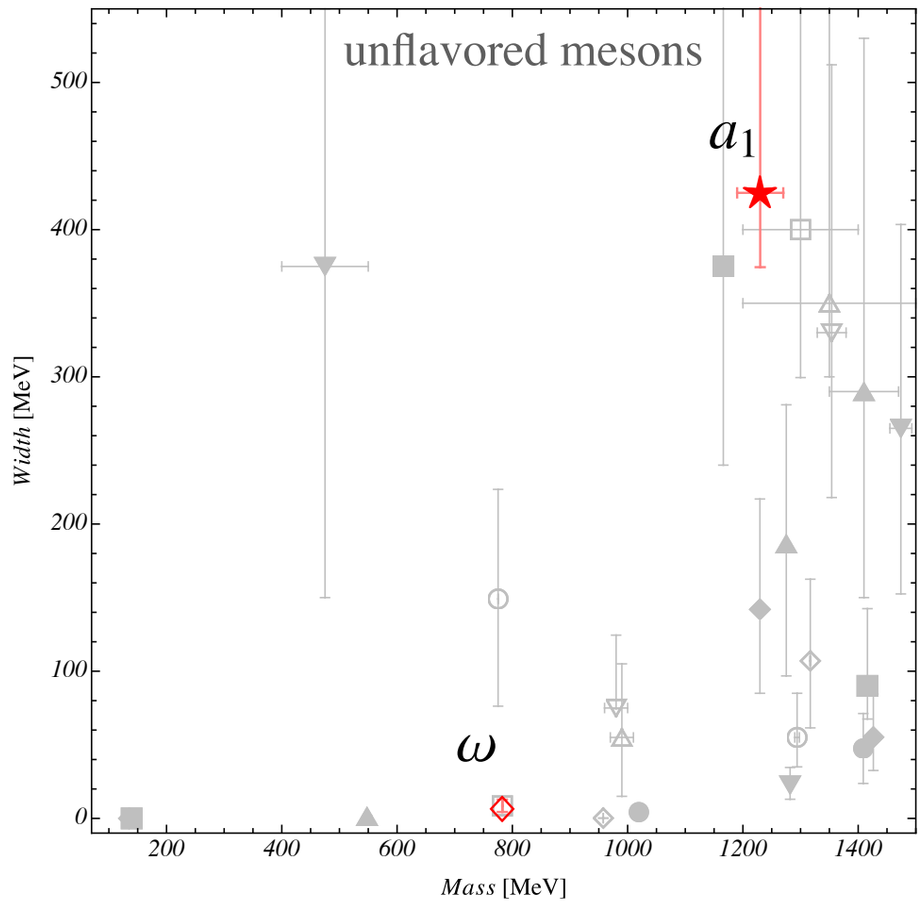
MOTIVATION

Pattern and **production mechanisms** of many excited hadrons are unresolved issues

Many questions are related to the hadronic 3-body problem

- ω(782), a_1(1260), ...
- Roper resonance *N*(1440)*
- exotic mesons: π_1(1600), ...

(experimental searches @COMPASS, GlueX)



This work: a_1(1260) from lattice QCD

MOTIVATION

Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:

MOTIVATION

Universal parameters of resonances from poles on Riemann surface

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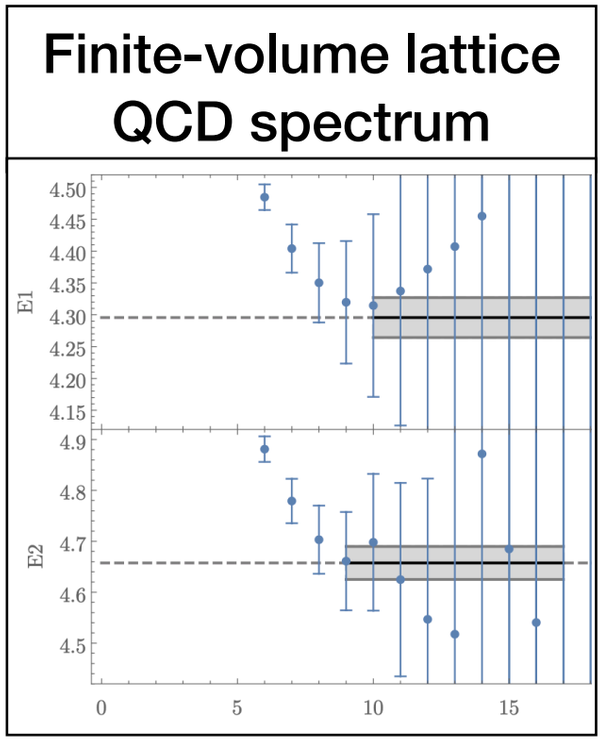

MOTIVATION

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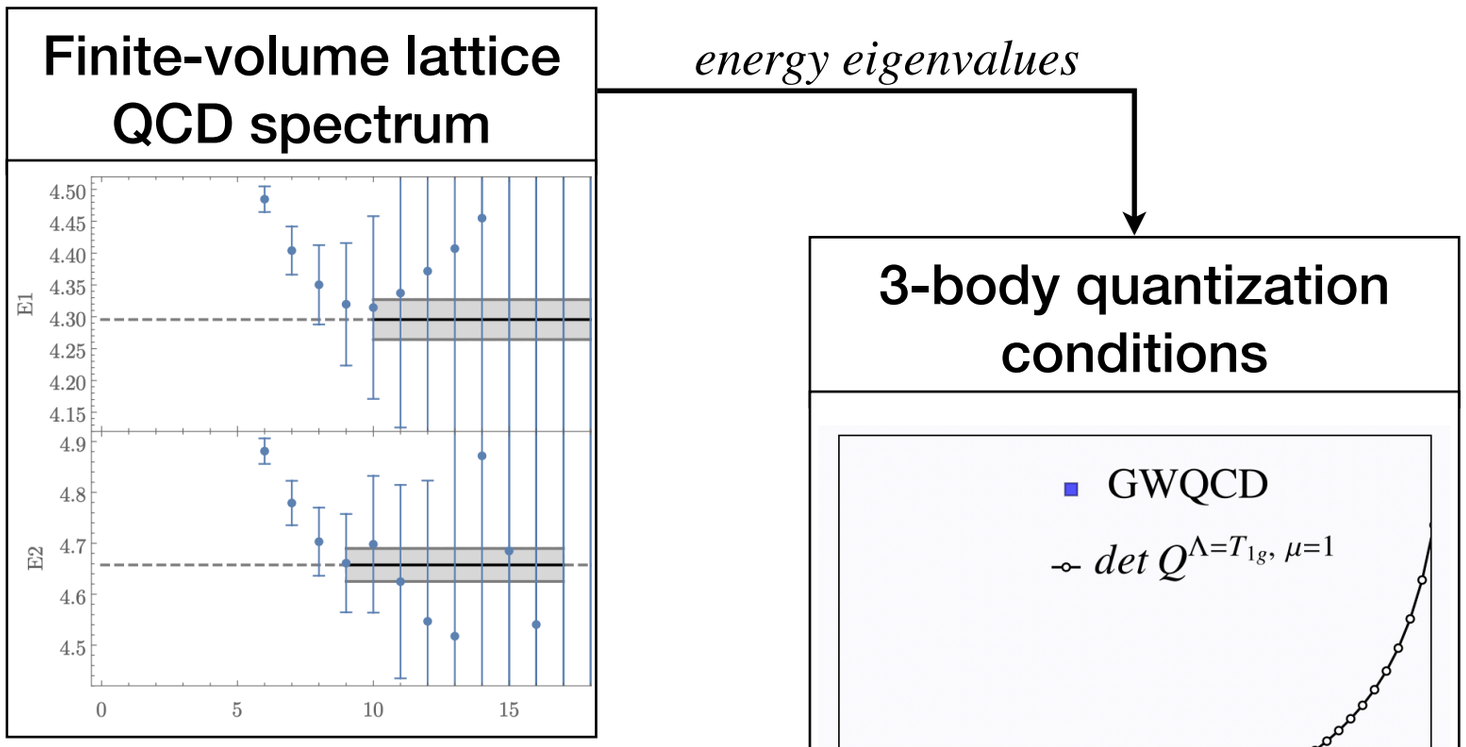
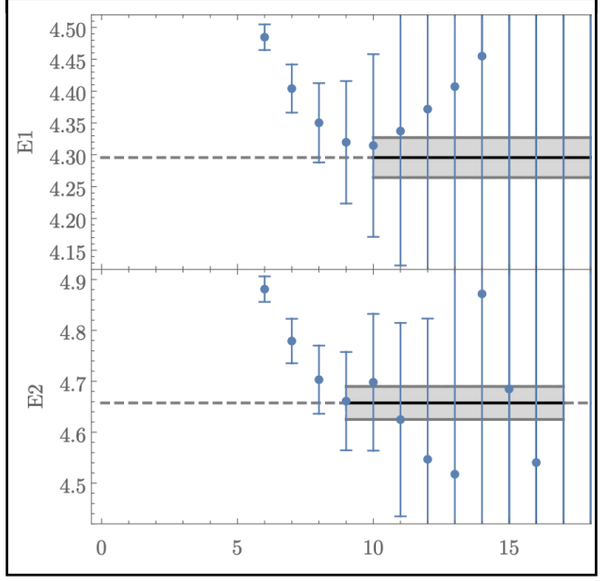


MOTIVATION

Universal parameters of resonances from poles on Riemann surface

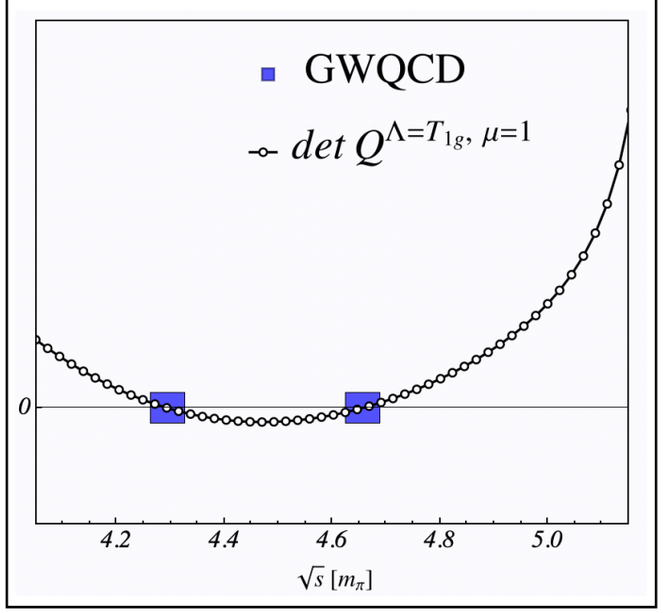
- 3-step procedure:

Finite-volume lattice QCD spectrum



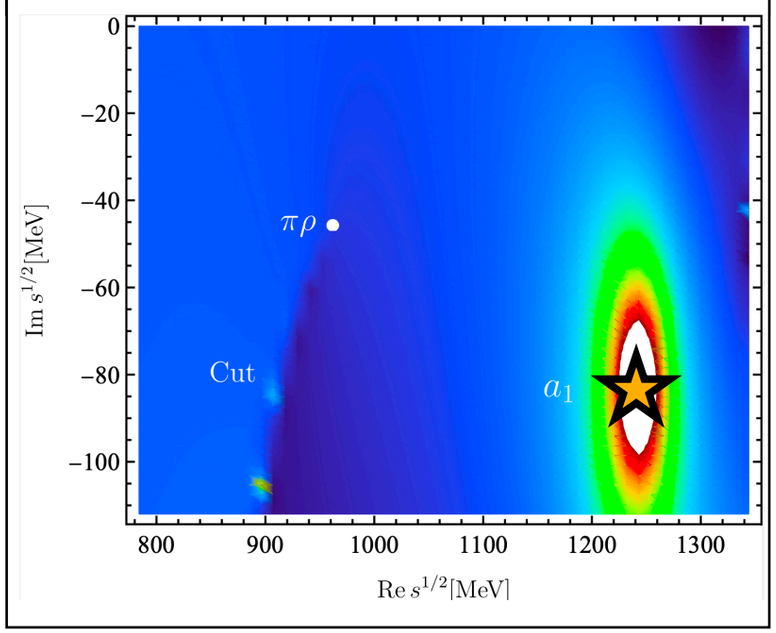
energy eigenvalues

3-body quantization conditions



volume-independent quantities

Riemann Surface: scattering amplitude

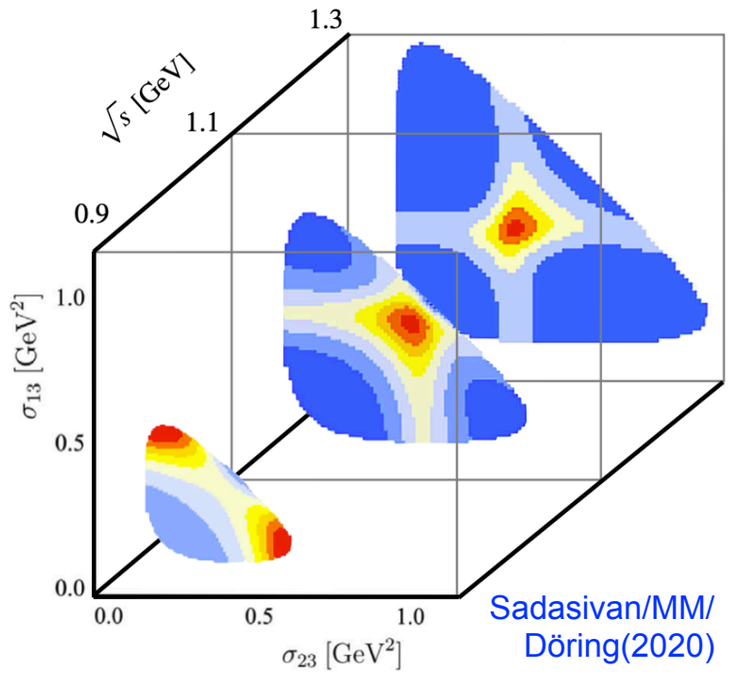


EXPERIMENT & THEORY

Phenomenological studies

- A clean way: $a_1(1260)$ from τ-decays (ALEPH@CERN)

  Mikhashenko et al. [JPAC] (2018) Sadasivan/MM/Döring(2020)



Sadasivan/MM/
Döring(2020)

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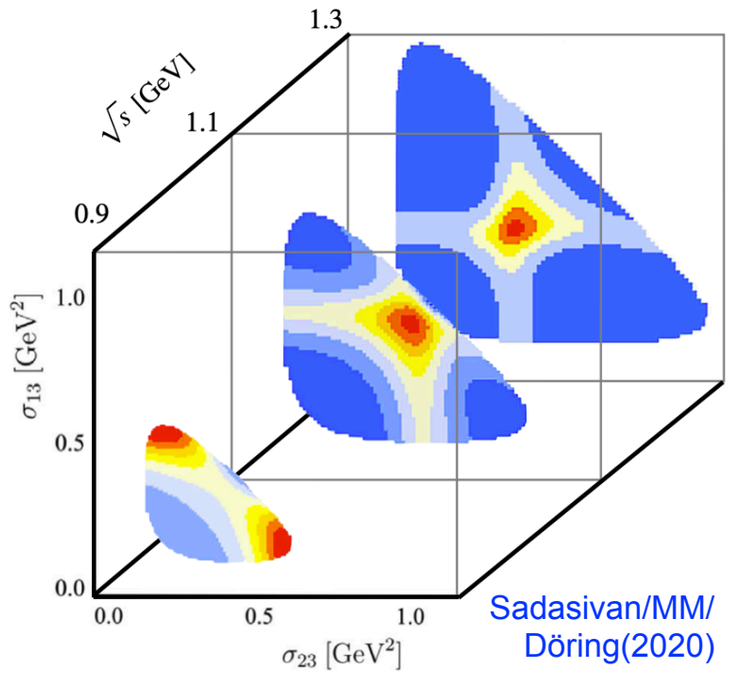

Sadasivan/MM/ Döring(2020)

Theoretical ab-initio access: lattice QCD

- full quark-gluon dynamics with some technical issues

1. Discretized Euclidean space-time >> continuum limit
2. Unphysical pion mass >> chiral extrapolations
3. Finite Volume >> quantization condition

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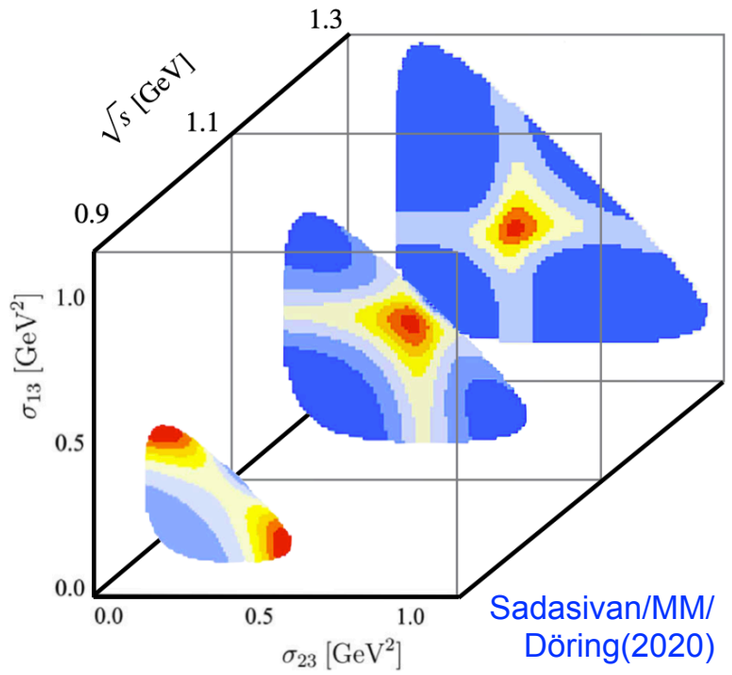

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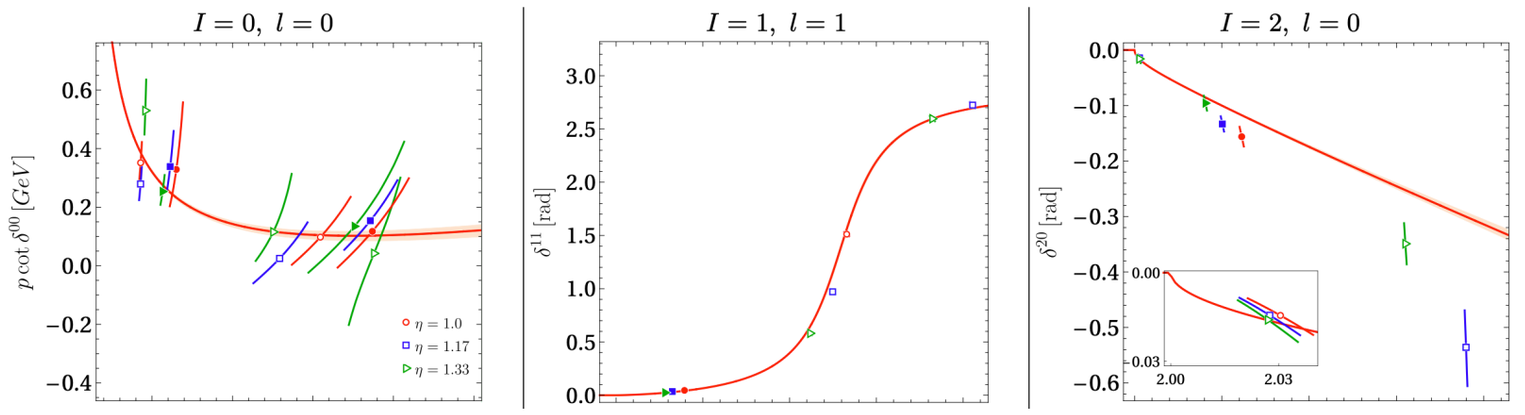
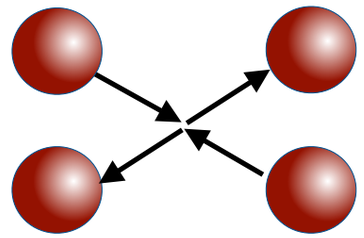
2.	Unphysical pion mass	>> chiral extrapolations
3.	Finite Volume	>> quantization condition

- many studies of 2-body systems

  [NPLQCD], [RQCD], [ETMC], [HadSpec], ... **Review: Rev.Mod.Phys. 90 (2018) 2**





e.g. MM et al. [GWQCD] (2019)

- non-resonant 3-body systems with maximal isospin: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...

  [NPLQCD]; Hörz/Hanlon; [GWQCD]; [HadSpec]; Blanton et al.

LATTICE QCD

Key details of GWQCD lattice QCD calculation

- $N_f = 2$ dynamical fermions, LapH smearing
- $\boldsymbol{P}$=(0,0,0), m_π=224 MeV, $m_\pi L$=3.3
- *GEVP with one-/two-/three-meson operators*
- *Relevant irrep(O_h) for* ***a1(1260)*** $I^G(J^{PC}) = 1^-(1^{++})$:

Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2021)
PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

$\mathbf{P}$	Λ	J^P $(I^G = 1^-)$
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	A_{1u}	$0^-, 4^-, \ldots$

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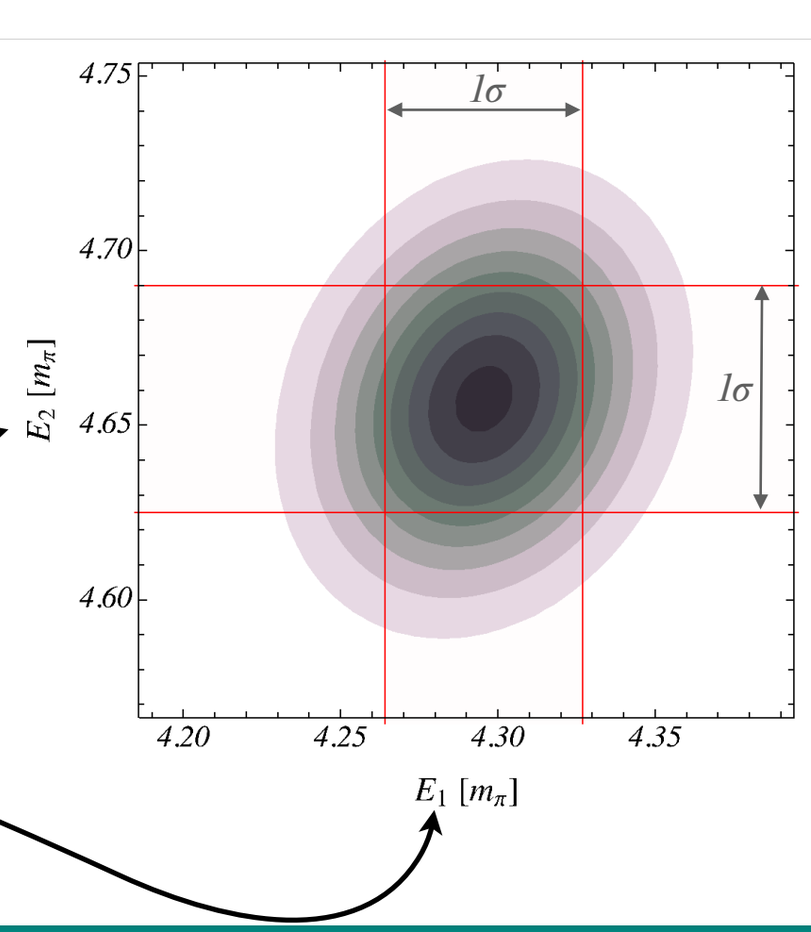
	A_{1u}	$0^-, 4^-, \ldots$

Key results:

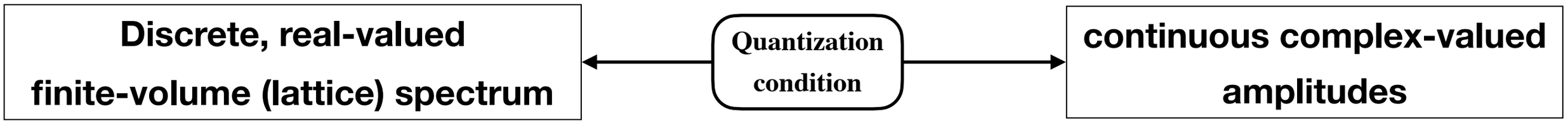
- 3π operators essential for the excited-state extraction
  c.f. $\rho\pi$ operators in previous 2-meson a_1 calculation

Lang et al. JHEP 04, 162 (2014)

- high-momentum states are required:
  $\pi(0,0,0)\pi(+1,+1,0)\pi(-1,-1,0)$ etc..
- two levels exist below 5π threshold



QUANTIZATION CONDITION



- established in 2-body: Lüscher's method, extensions... Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...

- 3-body methods matured Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu

Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

QUANTIZATION CONDITION

Discrete, real-valued finite-volume (lattice) spectrum ← **Quantization condition** → **continuous complex-valued amplitudes**

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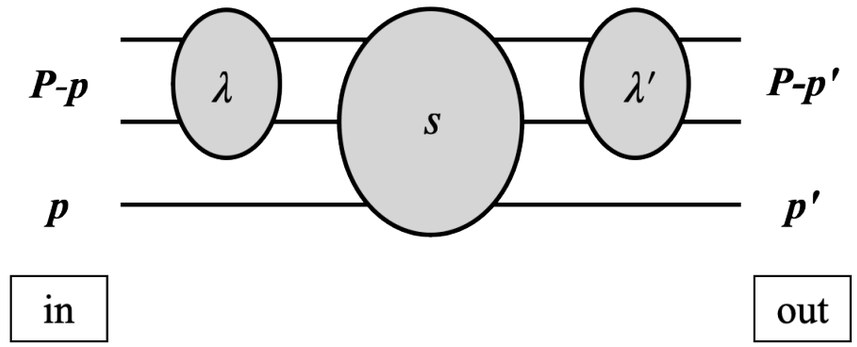
Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

Finite Volume Unitarity (FVU) MM, Döring EPJA (2017) PRL (2019)

- *extended to higher spin and coupled-channels: new degree of freedom (helicity, λ)*

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$



- *∞-dim. determinant equation in $\mathbf{p} \in \frac{2\pi}{\mathbf{L}}\mathbf{Z}^3$* → *practical applications require truncation*
  → *common to all quantization conditions*

QUANTIZATION CONDITION

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MM, Döring EPJA (2017)
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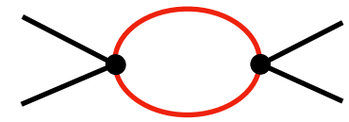
PRL (2019)

two-body self-energy

- fixed by 2b-unitarity



- no free parameters

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MM, Döring EPJA (2017)
PRL (2019)

one-particle exchange

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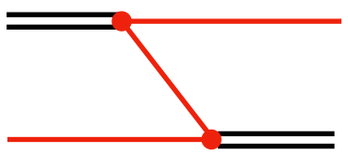

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two-body self-energy

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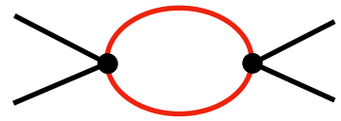

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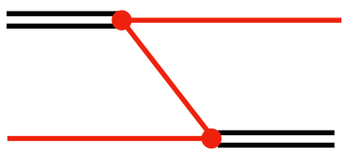
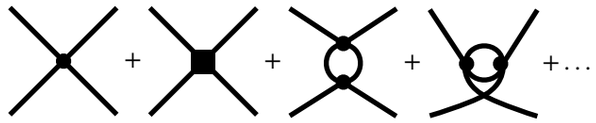


- no free parameters

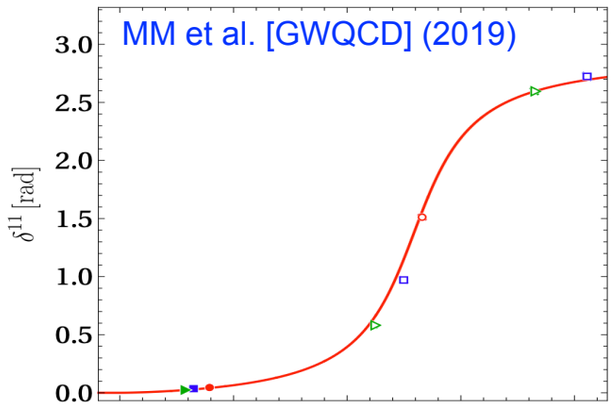
two-body kernel

- dynamics of *I=1* $\pi\pi$ system



$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

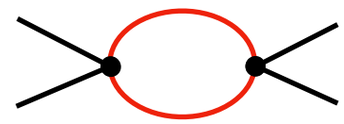
- regular function ⇒ polynomial



- parameters (a_0, a_1) from cross-channel fit to $\pi\pi$ GWQCD levels

two-body self-energy

- fixed by 2b-unitarity



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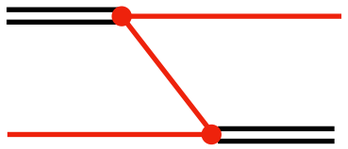
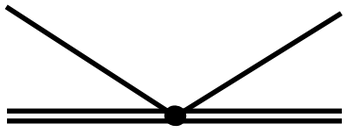


- no free parameters

three-body force

- dynamics of $\rho\pi$ system



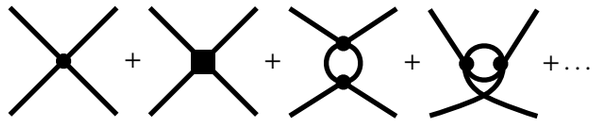
- regular function ⇒ Laurent series

$$C_{\ell'\ell}(s, \mathbf{p}', \mathbf{p}) = \sum_{i=-1}^{\infty} c^{(i)}_{\ell'\ell}(\mathbf{p}', \mathbf{p})(\mathbf{s} - \mathbf{m}^2_{\mathbf{a}_1})^{\mathbf{i}}$$

- fit to 3-body levels → *next slide*

two-body kernel

- dynamics of *I=1* $\pi\pi$ system



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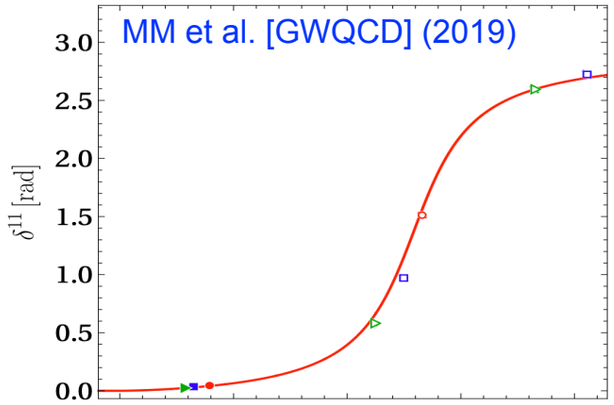

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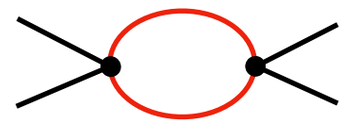


- no free parameters

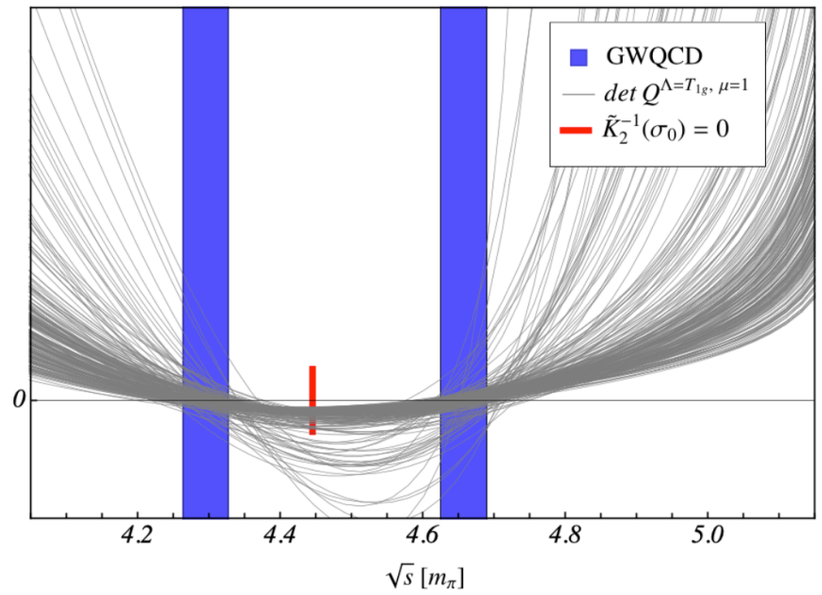
FROM LATTICE EIGENVALUES TO POLES

Last unknown (volume-independent) quantity: 3-body term C

- a_1 pole is generated *with/without explicit pole-term*
- best description (with large correlations)

$$C_{\ell'\ell} = g_{\ell'} |\mathbf{p}'|^{\ell'} \frac{\mathbf{1}}{\mathbf{s} - \mathbf{m}_{\mathbf{a_1}}^{\mathbf{2}}} \mathbf{g}_{\ell} |\mathbf{p}|^{\ell} + \mathbf{c}\, \delta_{\ell'\mathbf{0}} \delta_{\ell\mathbf{0}}$$

- full resampling... accessing statistical errors



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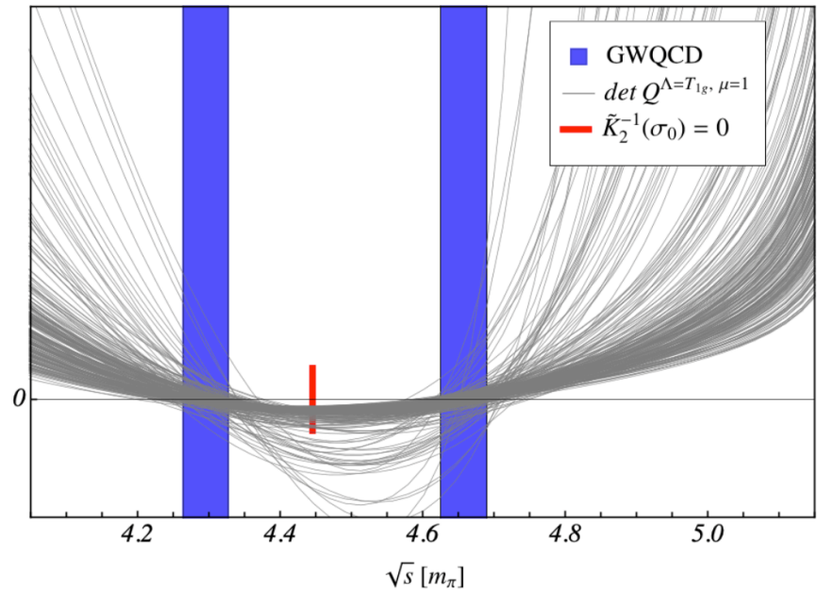
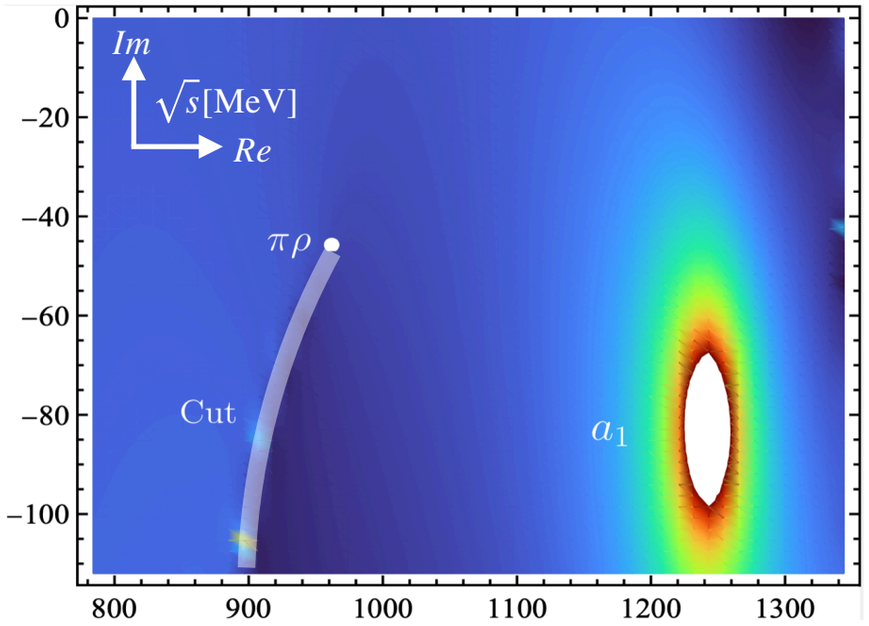


Infinite-volume scattering amplitude

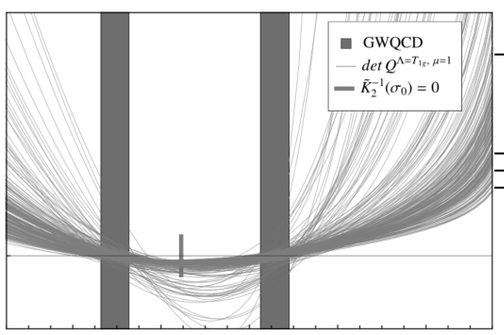
- integral equation with all inputs fixed from lattice QCD results

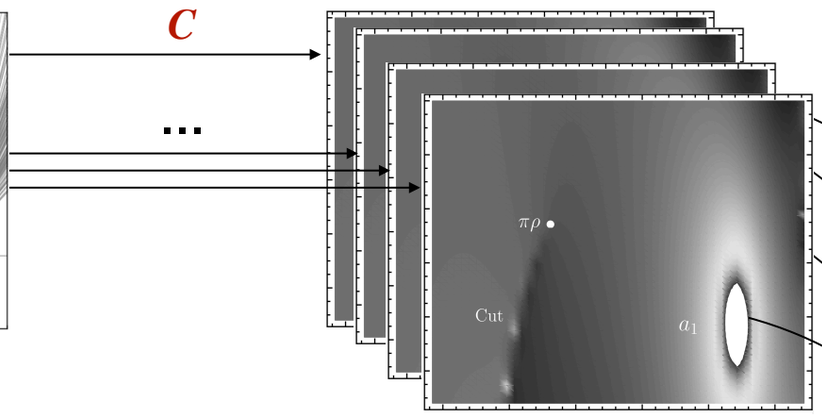
$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

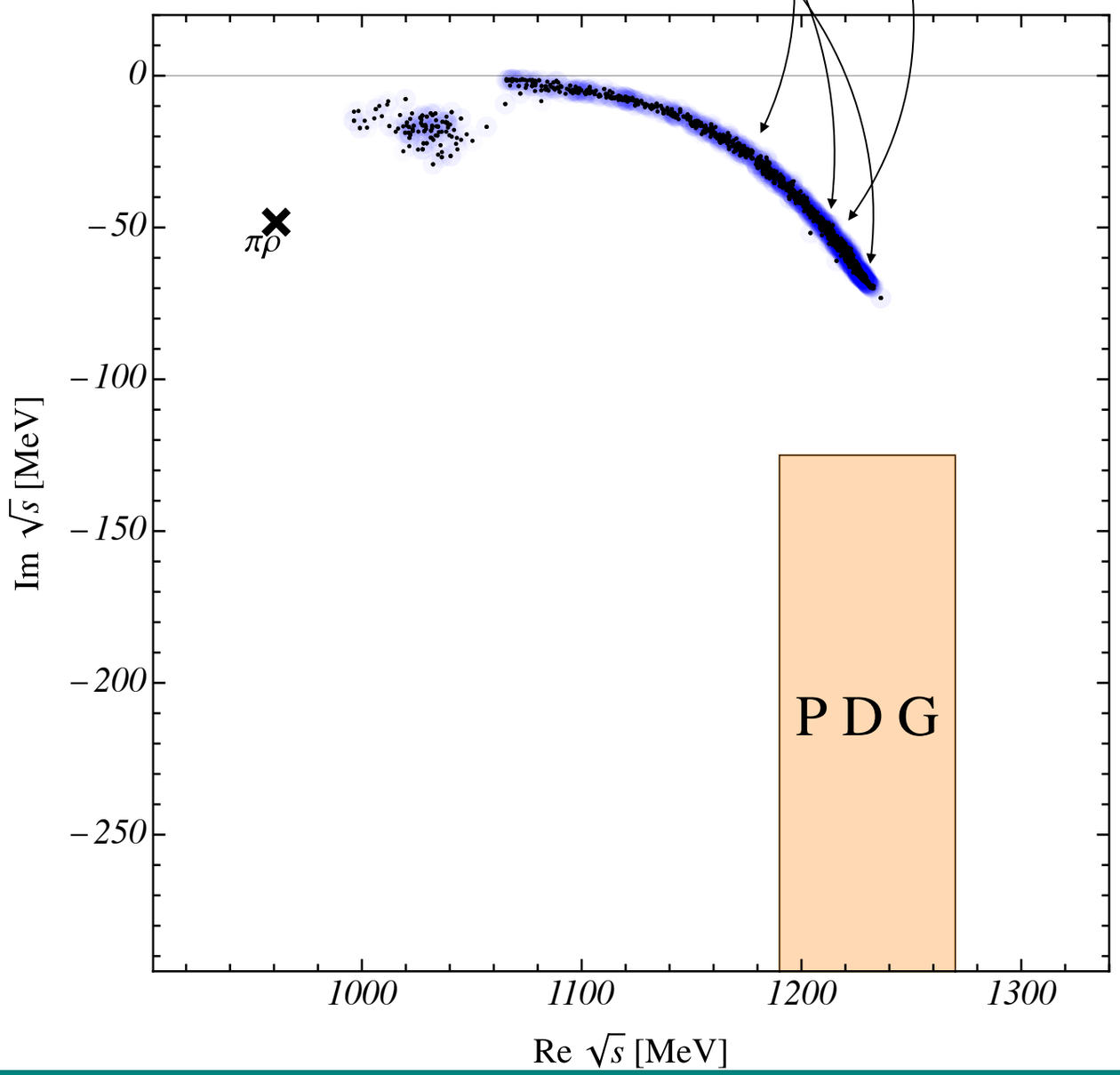
- contour deformation of spectator momenta
- analytic continuation to 2.Riemann sheet



RESULTS

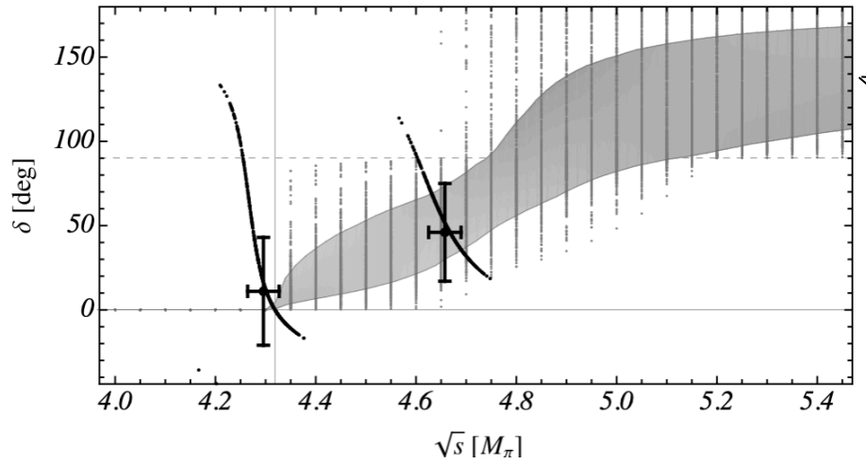




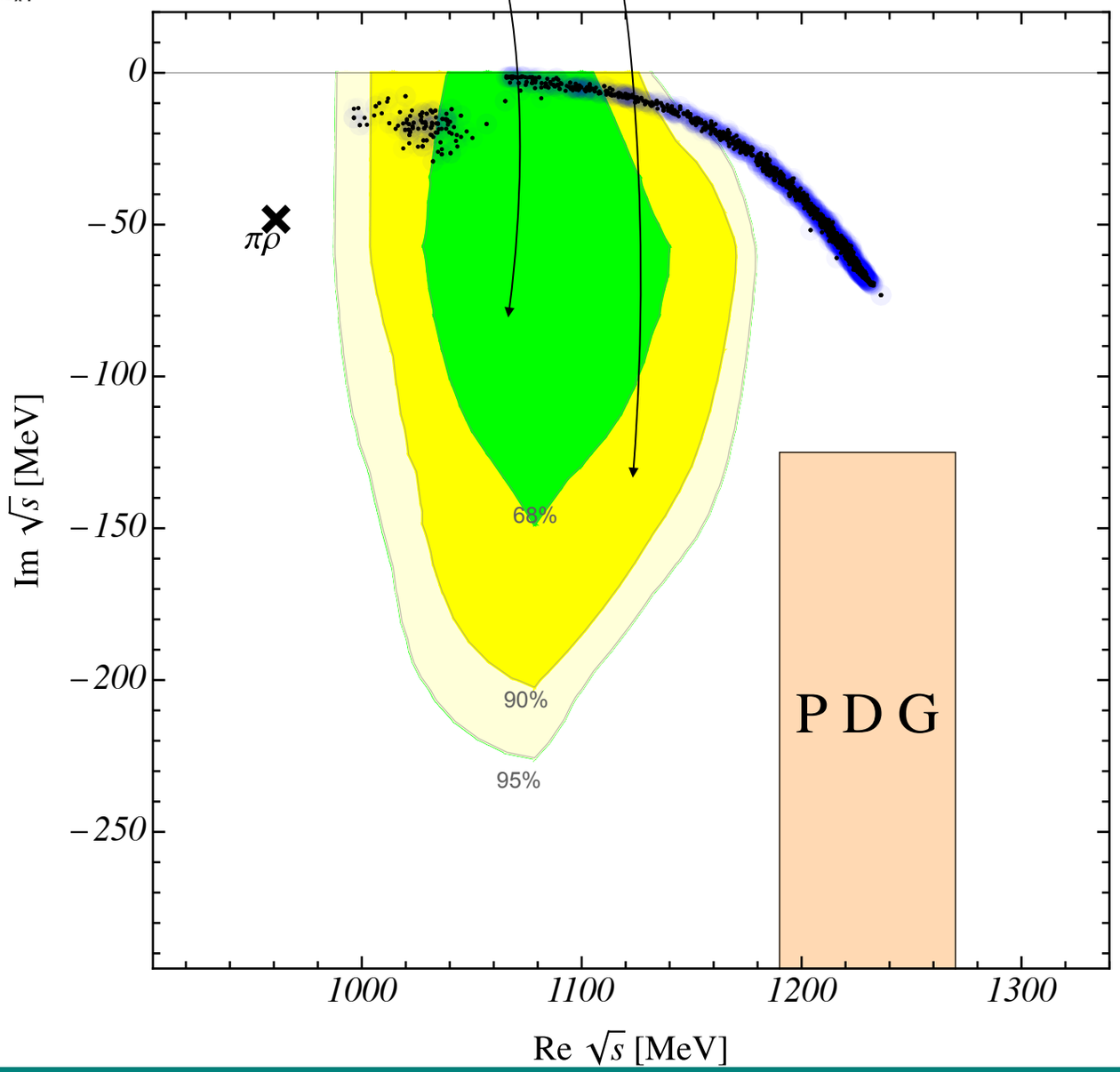


- m_π=*224* MeV
- 2000 resampling sets
- distribution of poles is finite
- 90% @ *~((1110..1210) - (20..70)i)* MeV

RESULTS & CHECKS



...

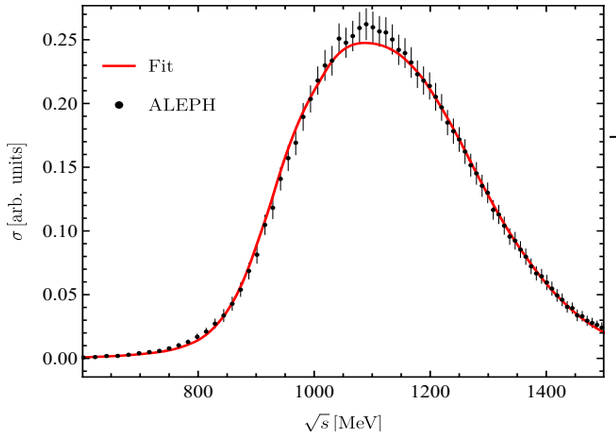


Can one use two-body approximation?

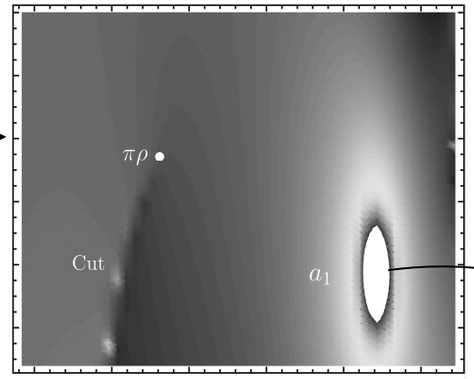
- Breit-Wigner parametrization
- 2-body Lüscher quantization condition

⇒ very crude approximation!

RESULTS & CHECKS

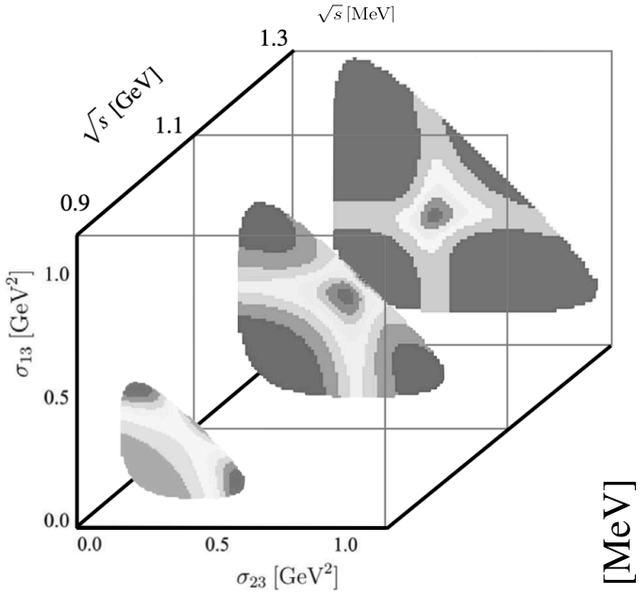


C

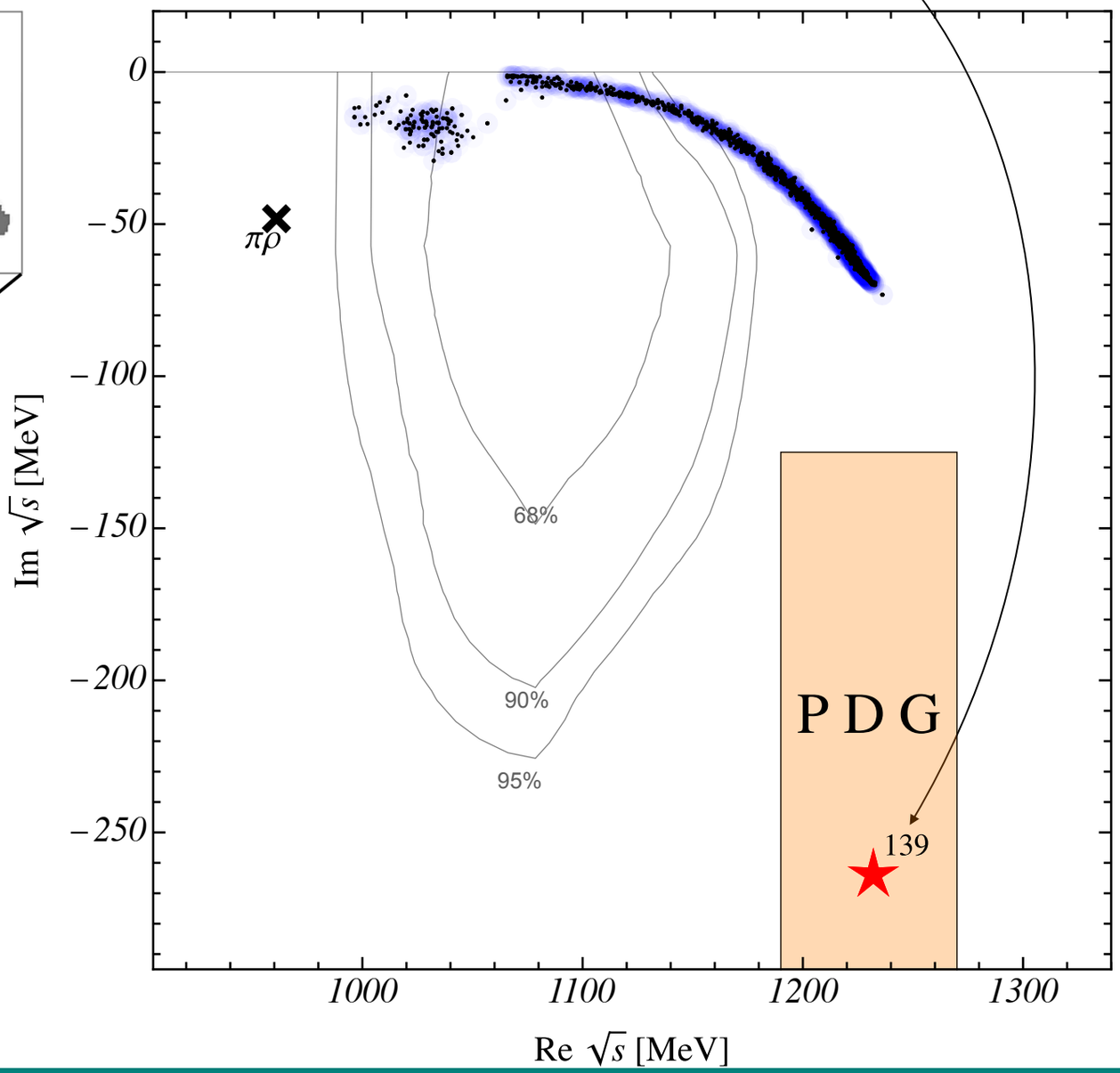


What does phenomenology says?

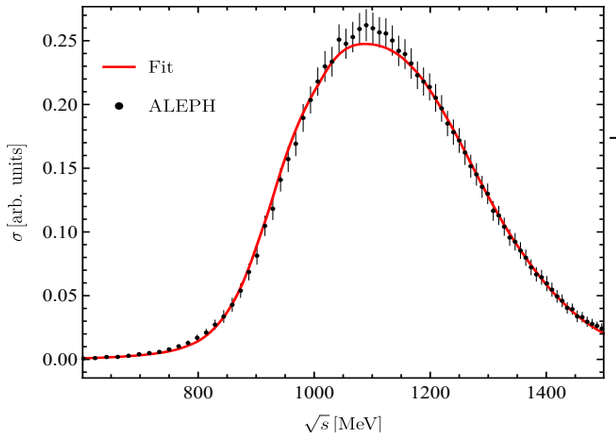
- $\tau \to (\pi\pi\pi)\nu_\tau$ from ALEPH@CERN
- fit to line shape to fix C



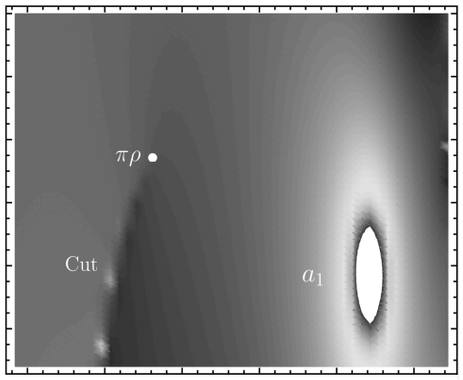
cf. Sadasivan/ MM/Döring(2020)



MOTIVATION



C



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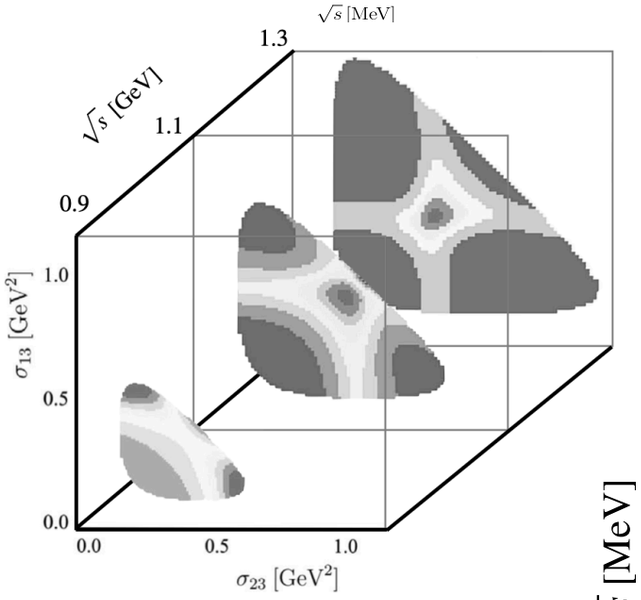

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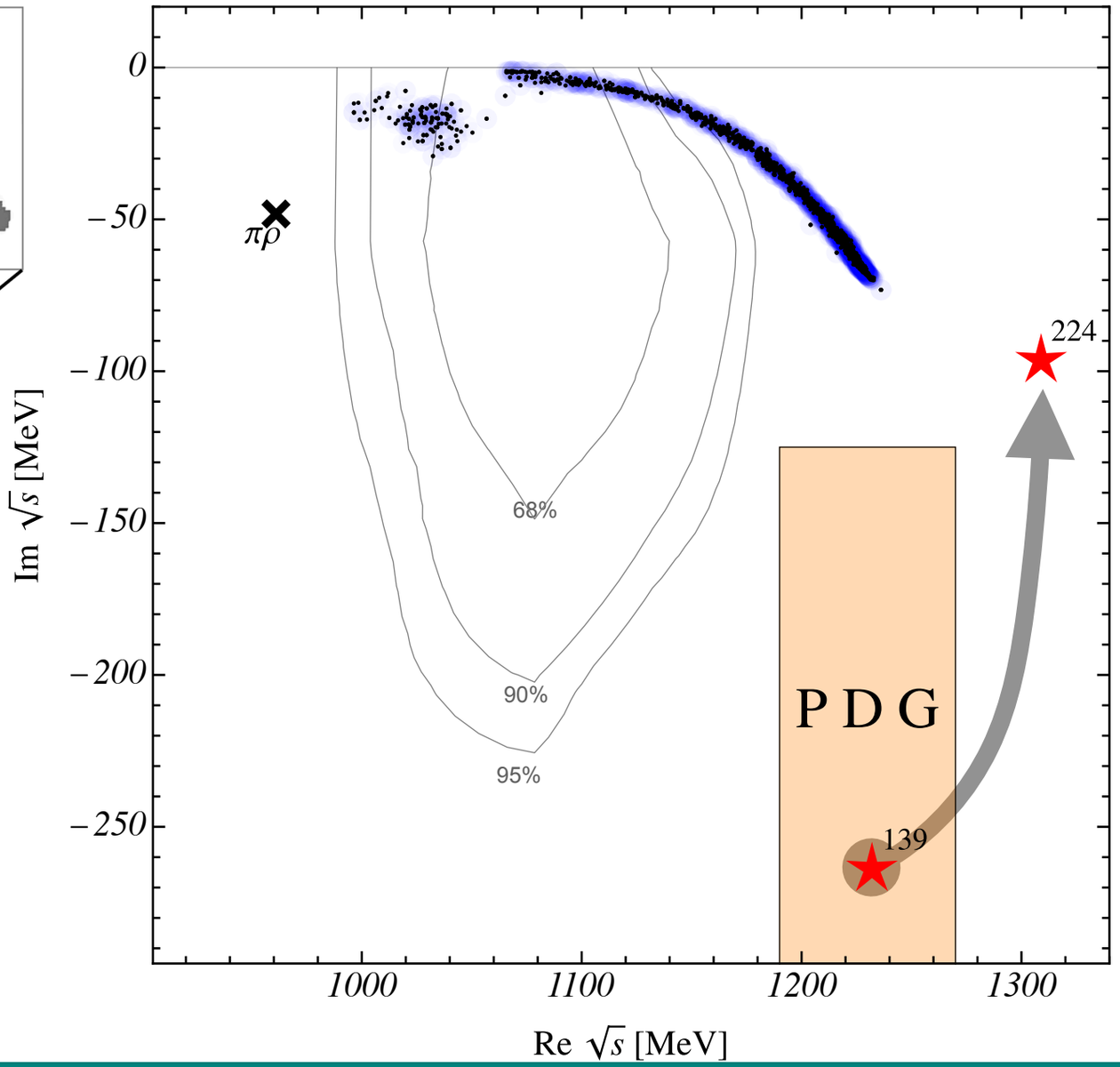
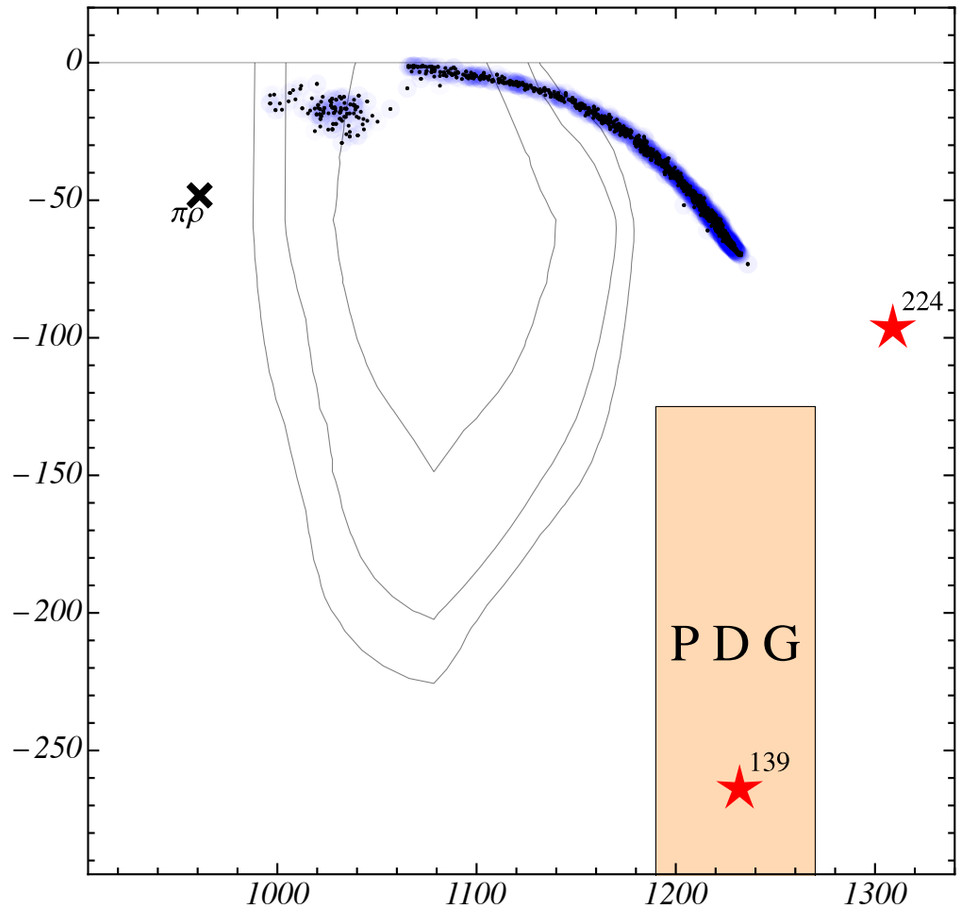


(*naive*) ***chiral extrapolation*** *confirms expectations:*

- *a_1 becomes heavier*
- *... and narrower*

Summary

- First-ever determination of a1(1260) parameters from lattice QCD
- Lattice QCD levels with 1/2/3-meson operators calculated
- 3-body quantization condition extended to spin-1 systems
- infinite-volume amplitude solved with complex integration contour and for complex energies
- pole positions and couplings are determined



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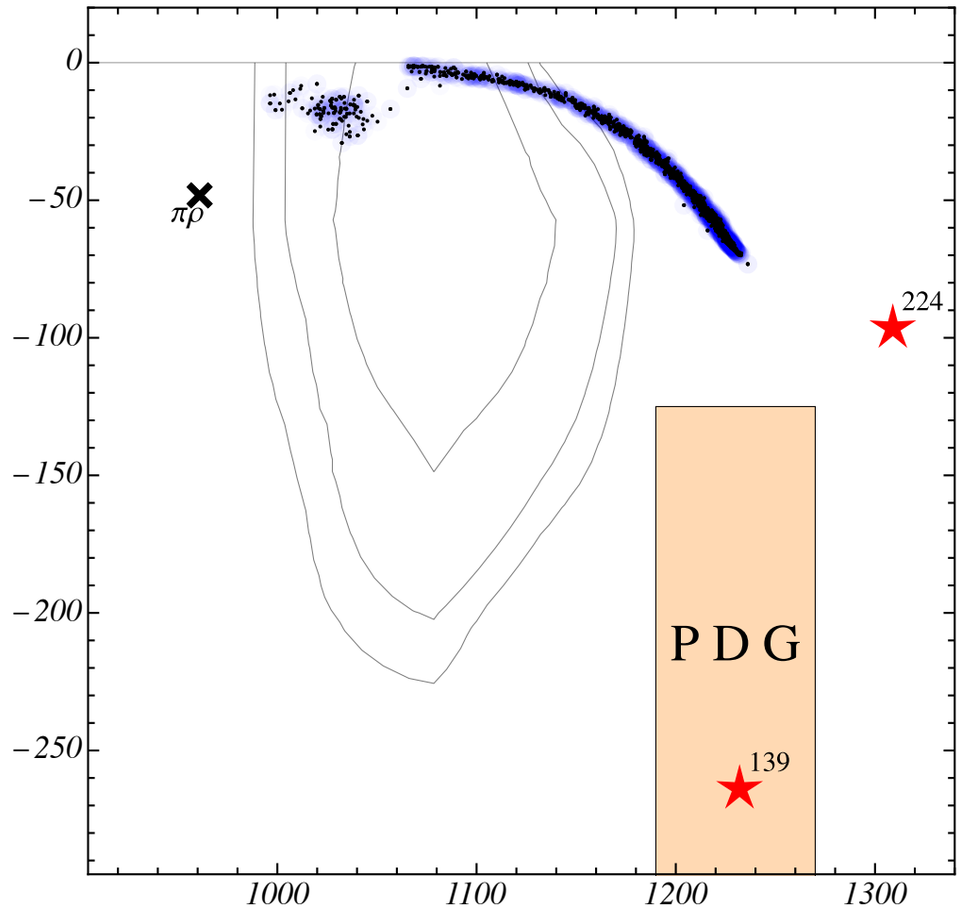
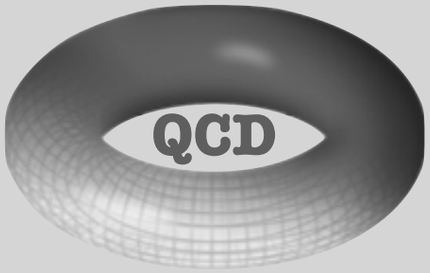


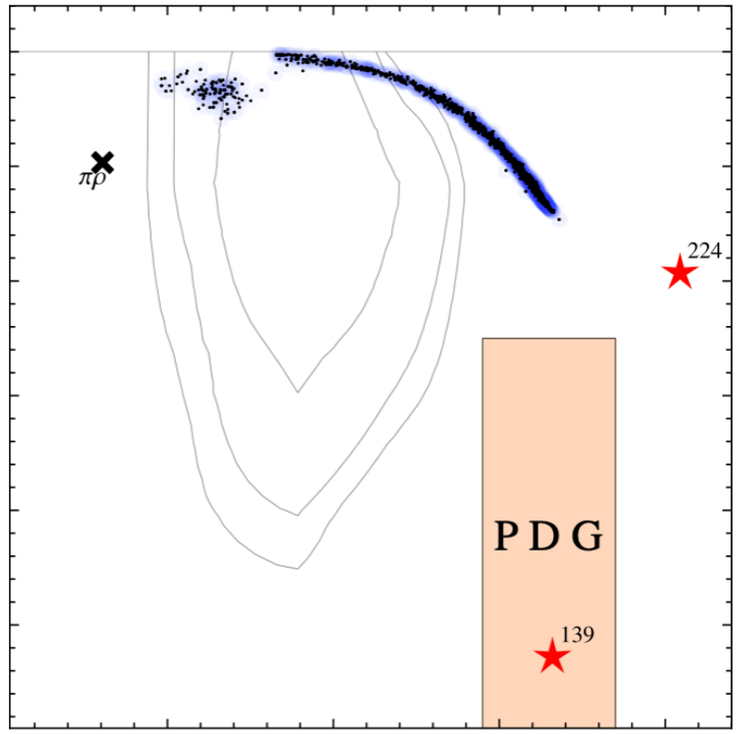
Outlook

- More lattice levels (another setup: m_π, L):
  - precise determination of a_1 parameters
  - importance of the $\sigma\pi$ channels
  - chiral trajectory of three-body term C

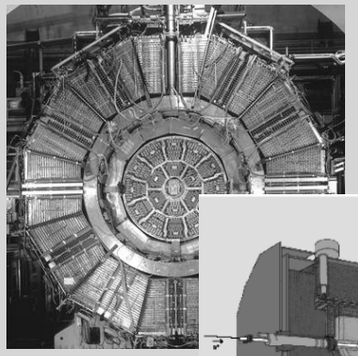
Unitarity



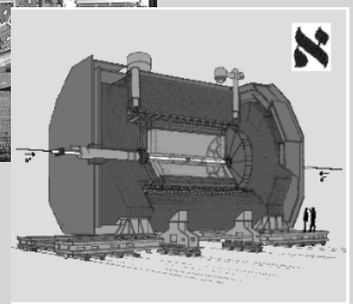
ChPT

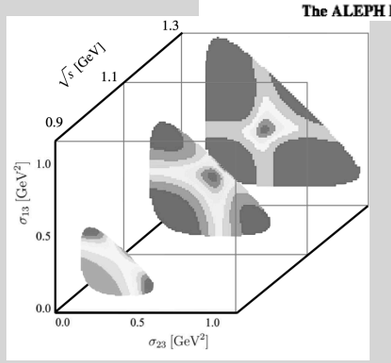


LHC



GlueX





THEORY

EXPERIMENT

THANK YOU