Preliminary results for elastic nucleon-pion scattering amplitudes from lattice QCD

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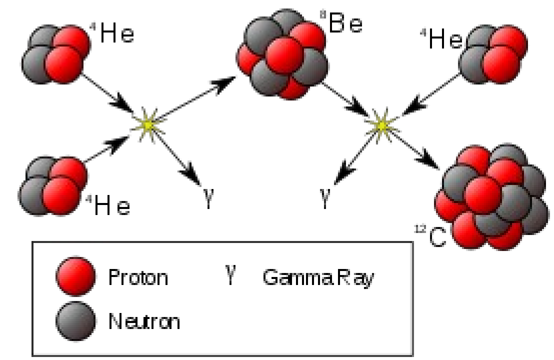
Why study nucleon-pion scattering amplitudes?

- ➔ Low energy pion, nucleon scattering:

$$\pi\pi \to \pi\pi,\ N\pi \to N\pi \quad \Rightarrow$$



- ➔ Scattering lengths provide info on $\sigma_{\pi N}$

- ➔ DUNE requires axial transition form factors:

$$N + A_\mu \to \Delta(1232) \to N + \pi$$

DEEP UNDERGROUND
NEUTRINO EXPERIMENT

Finite-volume energies from lattice QCD

- Ground state energies+overlaps from asymptotic Euclidean temporal correlators:

$$C(\tau) = \langle 0 | \hat{\mathcal{O}}_{N\pi} \, \mathrm{e}^{-\hat{H}\tau} \hat{\mathcal{O}}^{\dagger}_{N\pi} | 0 \rangle$$

$$\lim_{\tau \to \infty} C(\tau) = |\langle 0 | \hat{\mathcal{O}}_{N\pi} | E_0 \rangle_L|^2 \;\; \times \mathrm{e}^{-E_0^L \tau}$$

- Generalized Eigenvalue methods access a few excited states:

$$C_{mn}(\tau) = \langle \hat{\mathcal{O}}_m(\tau) \hat{\mathcal{O}}^{\dagger}_n(0) \rangle$$

$$C(\tau) v_n(\tau) = \lambda_n(\tau) C(\tau_0) v_n(\tau) \qquad \lim_{\tau \to \infty} \lambda_n(\tau) = \mathrm{e}^{-E_n^L \tau}$$

Scattering amplitudes from finite-volume energies

- In infinite-volume, asymptotic limit of $\langle 0|\hat{\mathcal{O}}'(\tau_1)\ldots\hat{\mathcal{O}}^\dagger(\tau_N)|0\rangle$ contains no info about on-shell amplitudes.

L. Maiani, M. Testa, *Phys. Lett.* **B245** (1990) 585

- Finite volume method: below $n \geq 3$ hadron thresholds:

$$\det[K^{-1}(E_{\mathrm{cm}}^L) - B(L\mathbf{q}_{\mathrm{cm}})] + \mathrm{O}(\mathrm{e}^{-ML}) = 0$$

$$S = (1 - iK)^{-1}(1 + iK)$$

M. Lüscher, *Nucl. Phys.* **B354** (1991) 531

- Determinant over all partial waves, channels

  → Truncate at some ℓ_{max}

- Block-diagonal in finite-volume irreps.

Difficulties with nucleon-pion scattering

- Computation of quark propagators for correlation functions

  → efficient algorithm: Stochastic LapH

  C. Morningstar, et al. PRD 83 (2011)

- Additional partial wave for each J due to nucleon spin

  → exhaustive determination of elements of B-matrix elements

  C. Morningstar, et al. NPB 910 (2016)

- Exponential signal-to-noise problem in baryon correlation functions

  → high-statistics on CLS ensemble D200

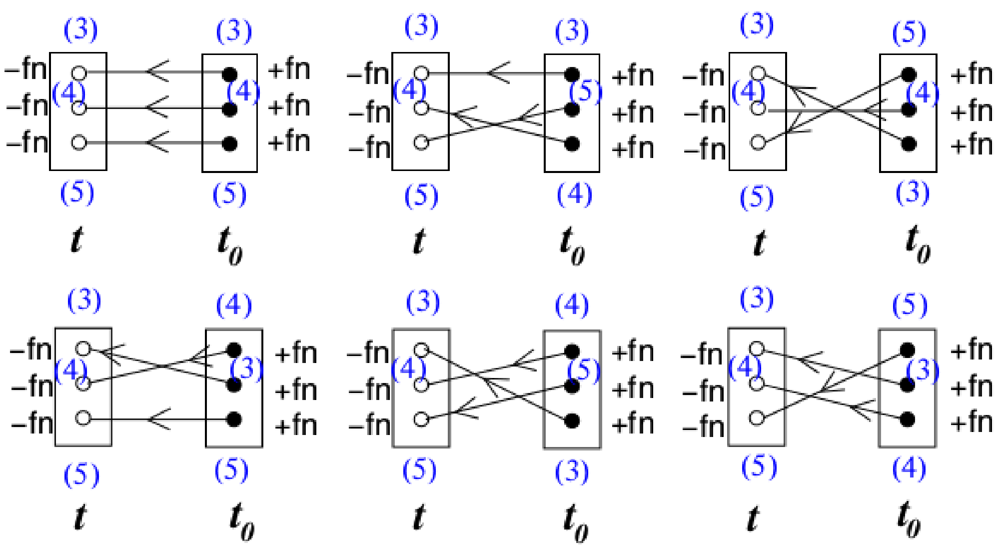
$$64^3 \times 128,\ a = 0.064\mathrm{fm},\ m_\pi = 200\mathrm{MeV}, N_{\mathrm{meas}} = 2000$$

  M. Bruno, et al. JHEP 02 (2015)

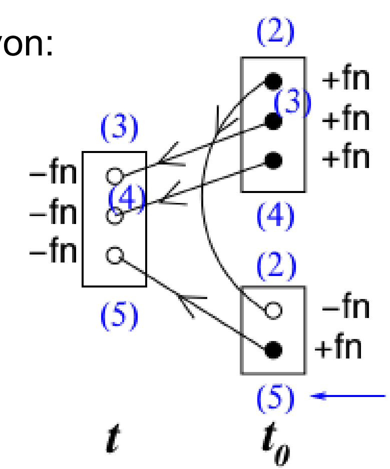
- Signal is tiny in weakly-interacting I=1/2 channel

Correlation functions constructed by tensor contraction

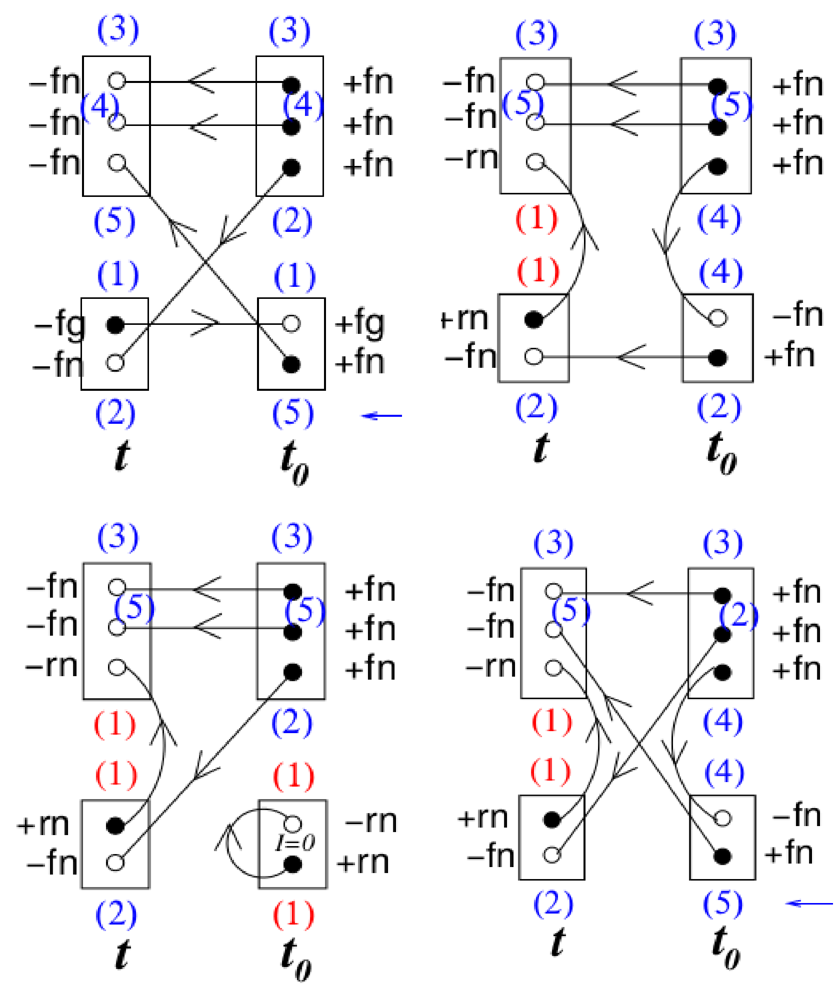
Single Baryon – Single Baryon:



Single Baryon – Meson+Baryon:



Meson+Baryon – Meson+Baryon:



Correlation functions constructed by tensor contraction

- Optimizations familiar to DFT simulations:
  - 'Path' optimization: find best contraction order
  - Common sub-expression elimination

B. Hörz, et al. PRC 103 (2021)

- Tensor contractions now require leadership class computing:

  → large Frontera (TACC) allocation

- Part of a broad program to compute meson-baryon and baryon-baryon scattering amplitudes:

C. Morningstar, S. Skinner (CMU); A. Nicholson (UNC), A. Walker-Loud (LBL), A. Hanlon (BNL), B. Hörz (Intel), JB (DESY), D. Mohler (GSI), H. Wittig, P. Madanagopalan (Mainz), J. Green (TCD)

Correlation functions constructed by tensor contraction

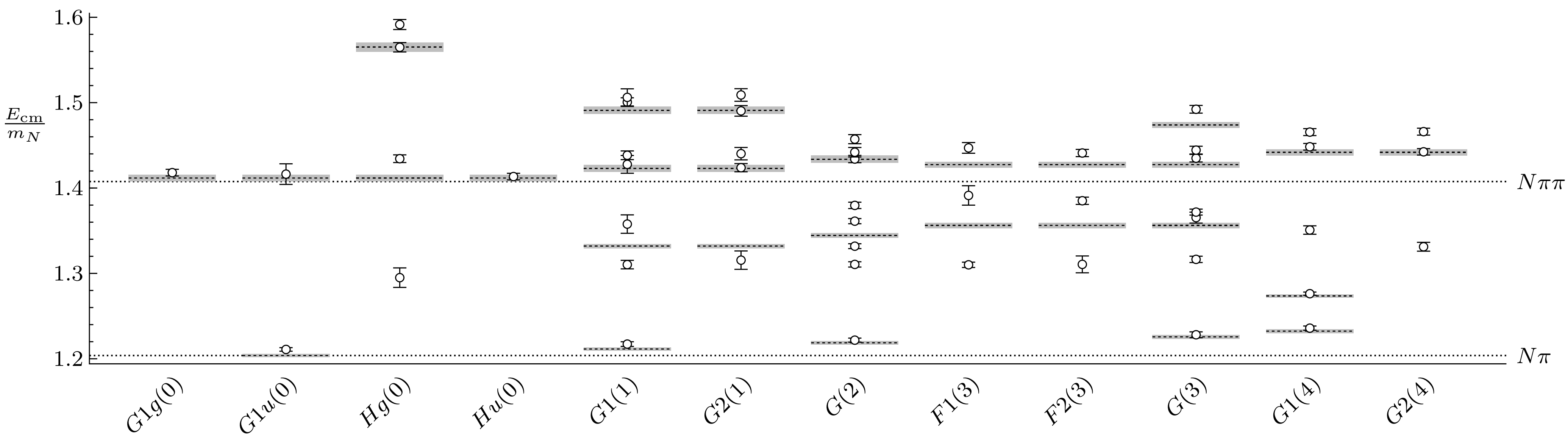
Isospin channel	D200 Number of Correlators
$I=0,\ S=0,\ NN$	8357
$I=0,\ S=-1,\ \Lambda, N\overline{K}, \Sigma\pi$	8143
$I=\frac{1}{2},\ S=0,\ N\pi$	696
$I=\frac{1}{2},\ S=-1,\ N\Lambda, N\Sigma$	17816
$I=1,\ S=0,\ NN$	7945
$I=\frac{3}{2},\ S=0,\ \Delta, N\pi$	3218
$I=\frac{3}{2},\ S=-1,\ N\Sigma$	23748
$I=0,\ S=-2,\ \Lambda\Lambda, N\Xi, \Sigma\Sigma$	16086
$I=2,\ S=-2,\ \Sigma\Sigma$	4589
Single hadrons (SH)	33

Finite volume → Reduced symmetry

- Irreps where $\ell(J^P) = 1(3/2^+)$ is the lowest partial wave for $\Delta(1232)$
- Irreps where $0(1/2^-)$ contributes without p-waves good for scattering lengths.

mom.	irrep	$\ell(J^p)$
$(0,0,0)$	H_g	$1(3/2^+),\ 3(5/2^+),\ \ldots$
	H_u	$2(3/2^-),\ 2(5/2^-),\ \ldots$
	G_{1u}	$0(1/2^-),\ 4(7/2^-),\ \ldots$
	G_{1g}	$1(1/2^+),\ 3(7/2^-),\ \ldots$
$(0,0,n)$	G_2	$1(3/2^+),\ 2(3/2^-), 2(5/2^-),\ \ldots$
$(0,n,n)$	G	$0(1/2^-), 1(1/2^+), 1(3/2^+), \ldots$
(n,n,n)	F_1	$1(3/2^+),\ 2(3/2^-),\ 2(5/2^-),\ \ldots$
	F_2	$1(3/2^+),\ 2(3/2^-),\ 2(5/2^-),\ \ldots$
	G	$0(1/2^-), 1(1/2^+), 1(3/2^+), \ldots$

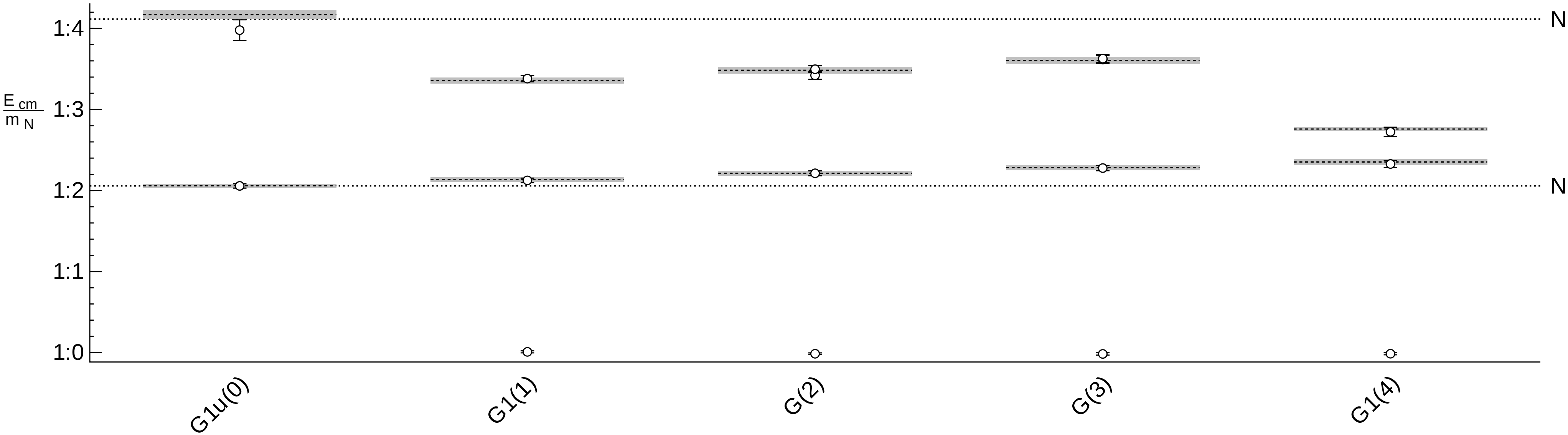
Finite-volume energies: I=3/2



- Solve GEVP, fit ratios directly:

$$R(t) = \frac{C^{4\rm pt}_{N(p_1)\pi(p_2)}(t)}{C^{2\rm pt}_{N(p_1)}(t)C^{2\rm pt}_{\pi(p_2)}(t)} = A\mathrm{e}^{-\Delta E t}$$

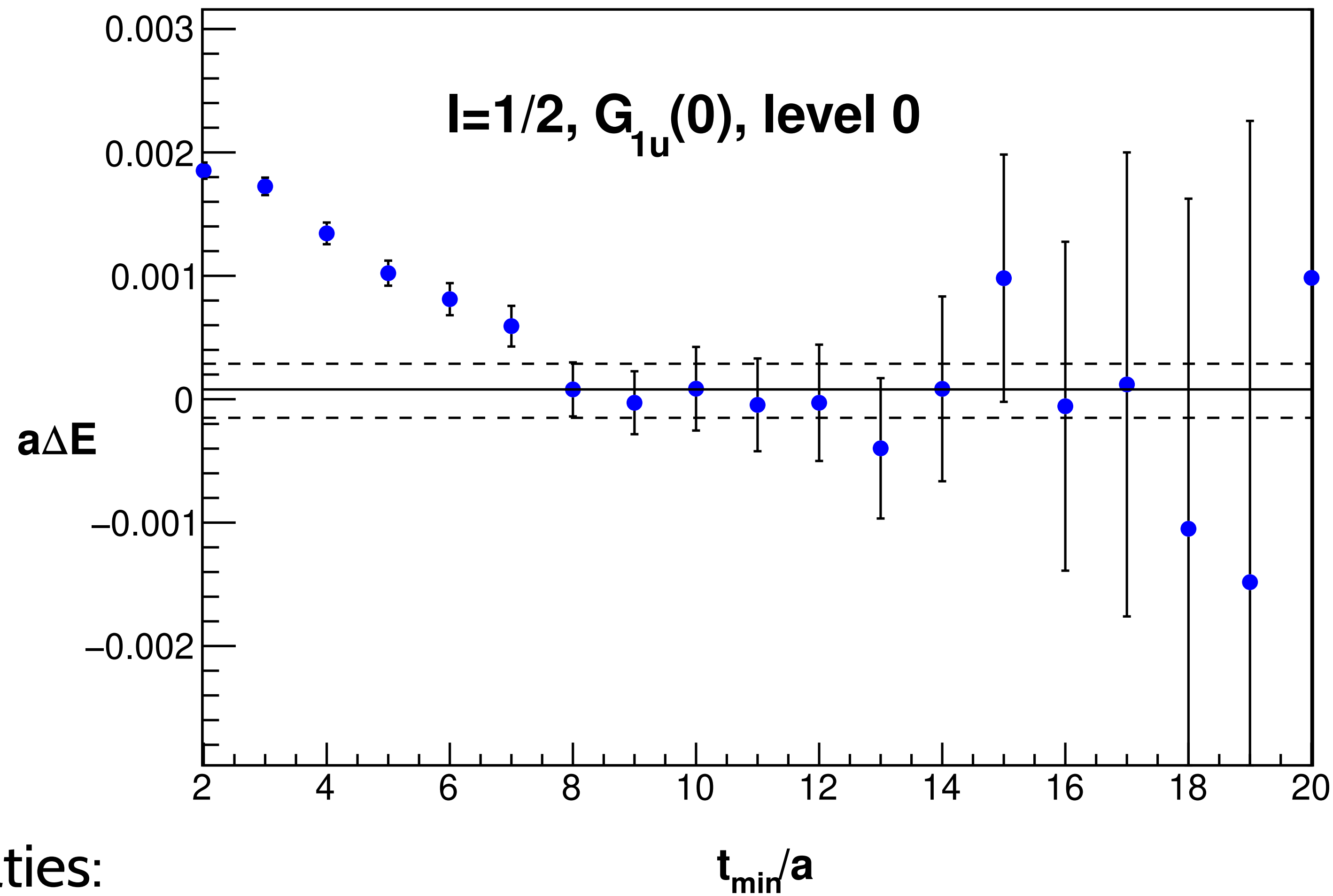
Finite-volume energies: I=1/2



Difficulties:

- Energy shifts much smaller
- Contamination from ground state N(939)

Finite-volume energies: I=1/2



Difficulties:

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I=3/2 amplitudes fit

- resonant *p*-wave:

$$(\tilde{K}^{-1})_{31} = \left(\frac{m_\Delta^2}{m_\pi^2} - \frac{E_{\rm cm}^2}{m_\pi^2}\right)\frac{6\pi}{g_{\Delta N\pi}^2}\frac{E_{\rm cm}}{m_\pi}$$

- *s*-wave:

$$(\tilde{K}^{-1})_{20} = (m_\pi\, a_0)^{-1} + (m_\pi^2 r_0/2)(\mathbf{q}_{\rm cm}/m_\pi)^2$$

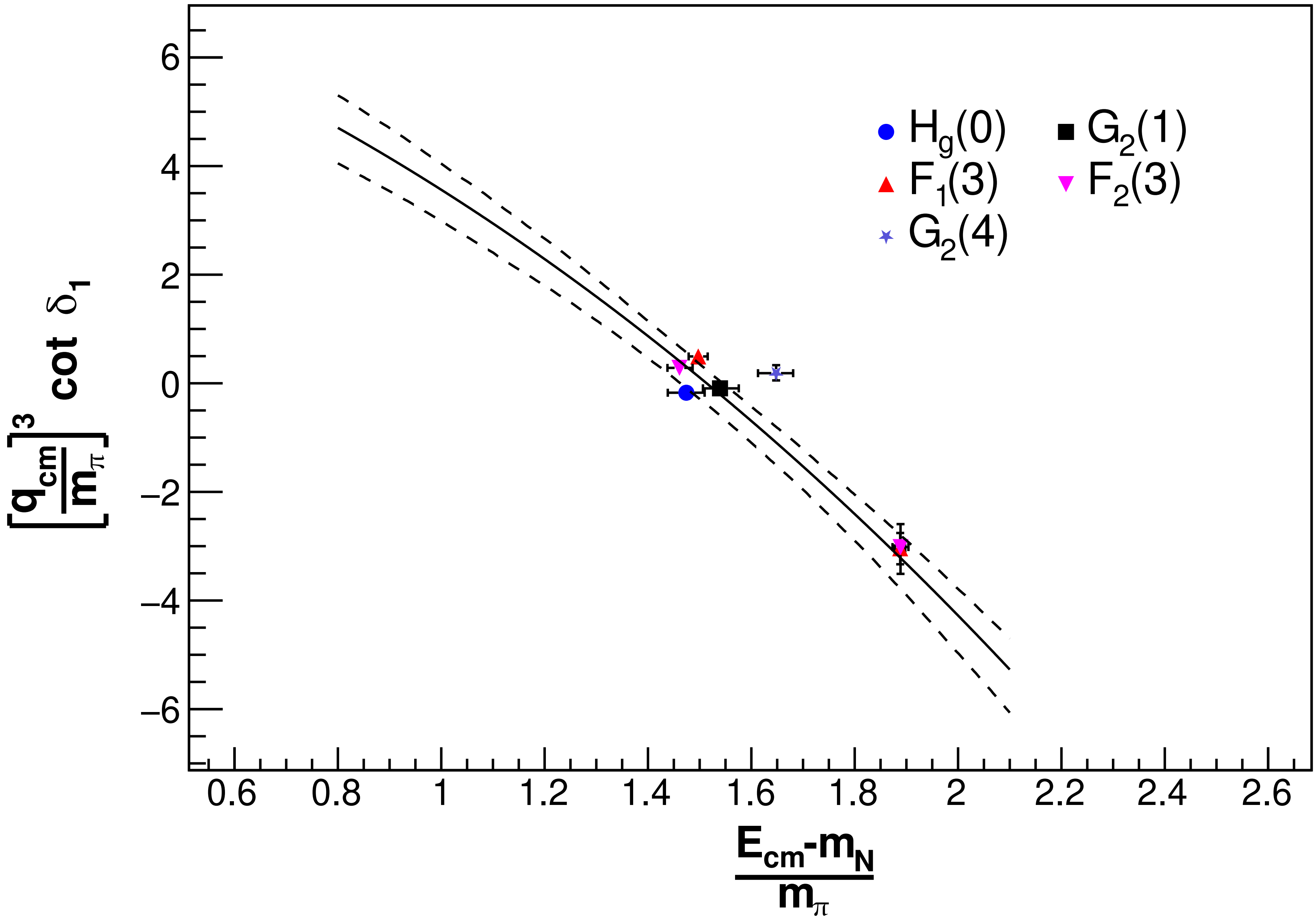
- Non-resonant *p*-wave:

$$(\tilde{K}^{-1})_{11} = (m_\pi^3\, a_{11})^{-1}$$

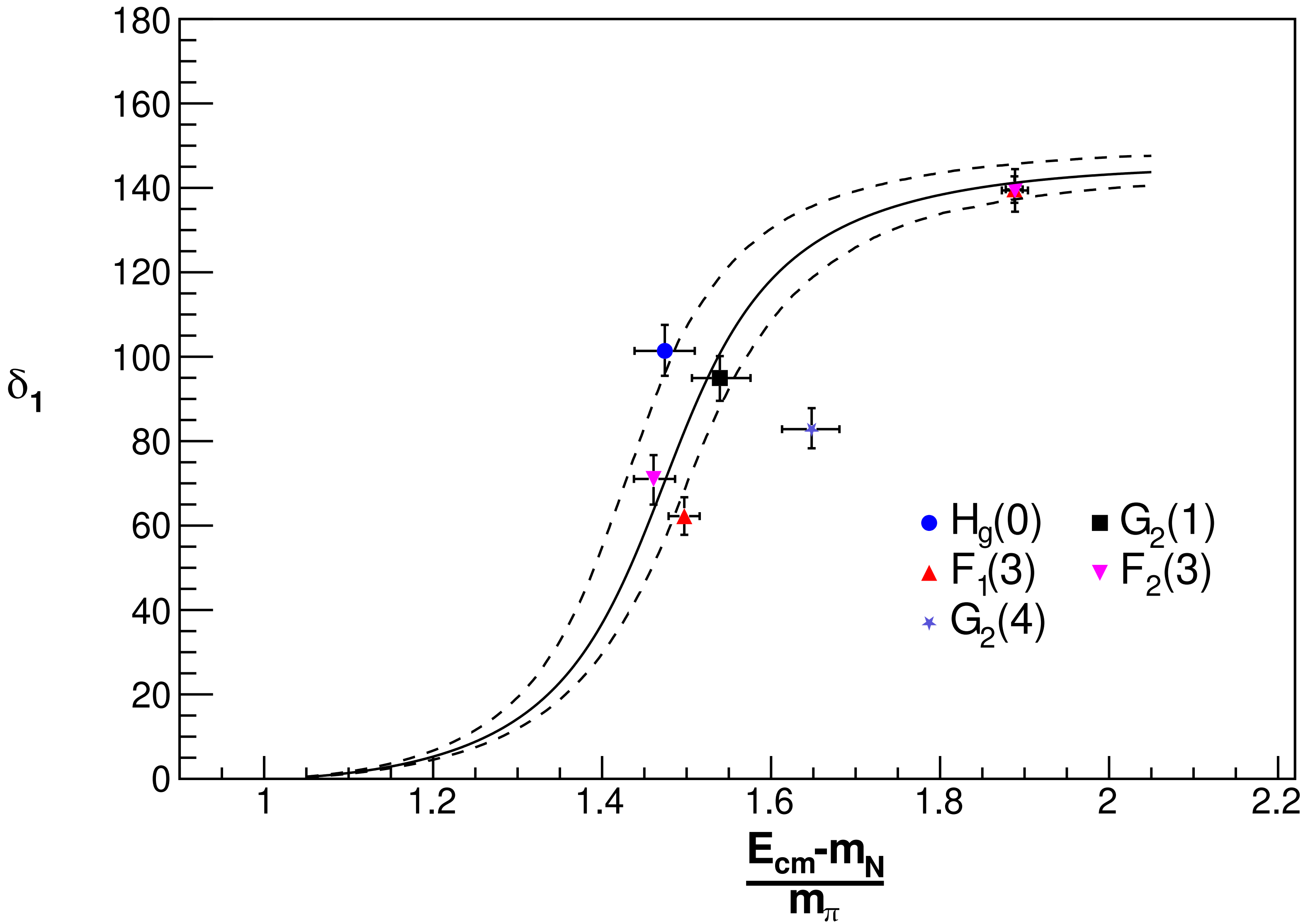
- Fit to 20 levels:

$$\frac{m_\Delta}{m_\pi} = 6.364(66), \qquad g_{\Delta N\pi}^{\rm BW} = 13.932(74),$$
$$m_\pi a_0^{3/2} = -0.271(13), \quad m_\pi^2 r_0^{3/2} = 5.74(42),$$
$$m_\pi^3 a_{11}^{3/2} = 0.094(59), \quad \chi^2/{\rm d.o.f.} = 0.55$$

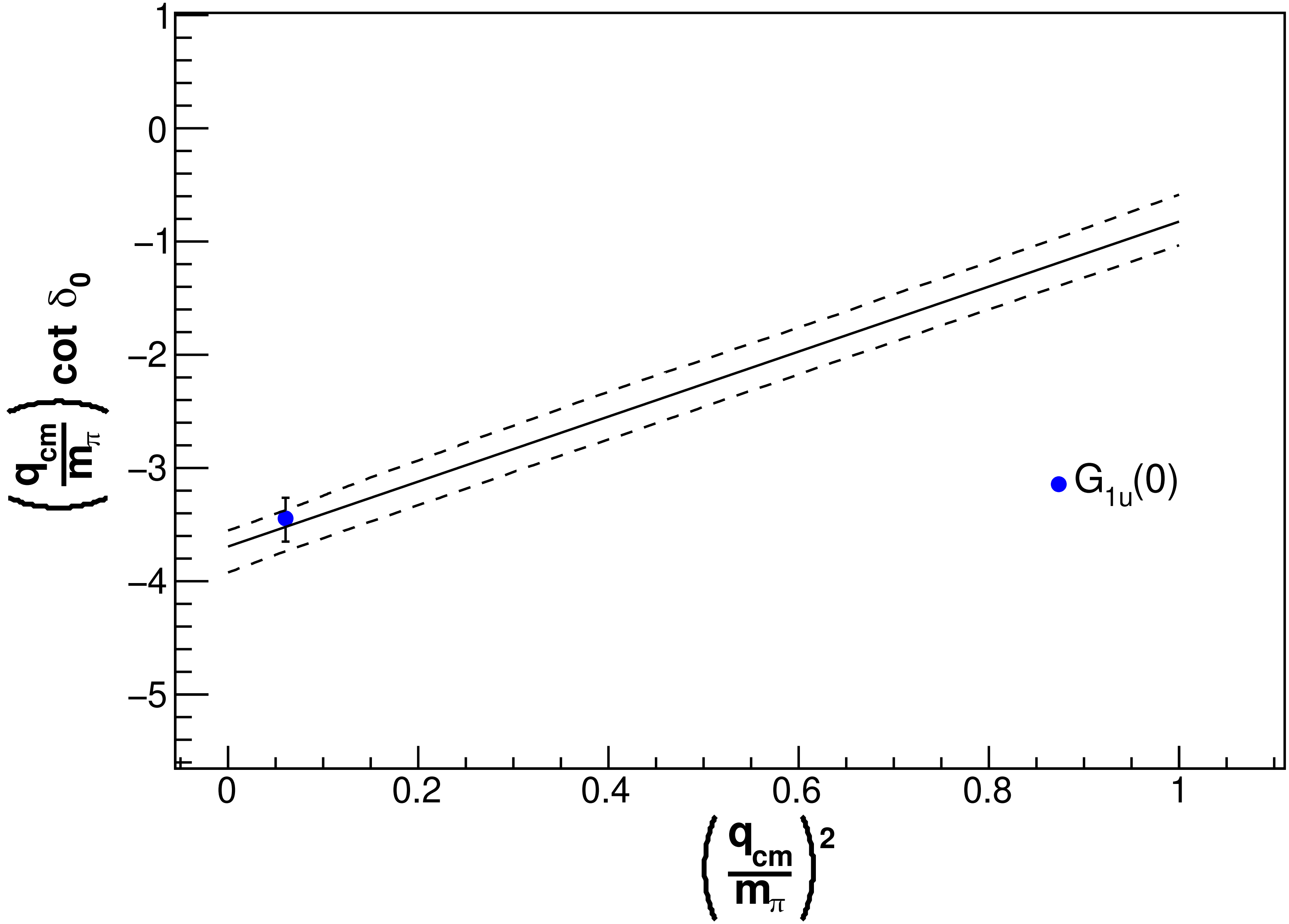
I=3/2 amplitudes fit



I=3/2 amplitudes fit



I=3/2 amplitudes fit



Conclusions

- Precise I=3/2 amplitudes at the physical point difficult, but within reach
- I=1/2 amplitudes currently poorly determined.
- Energy resolution limited by $m_\pi L,$ larger volumes needed
- Cutoff effects must be investigated. Relevant in Lambda-Lambda scattering

  J. R. Green, et al., 2103.01054 [hep-lat]
- 4x increase in statistics coming soon!
- Other meson-baryon and baryon-baryon channels coming soon!