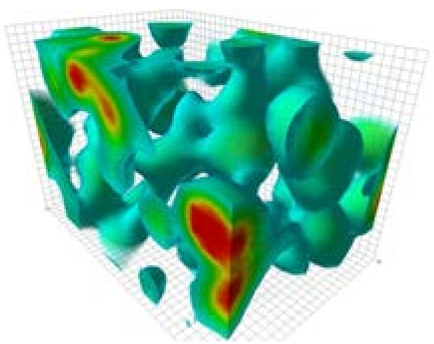
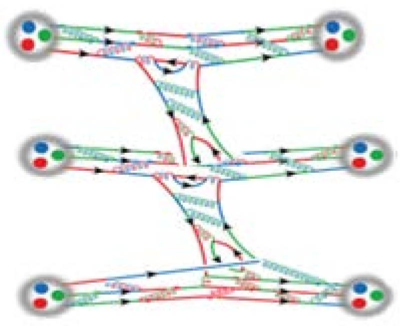
Three-Nucleon Forces from Lattice QCD

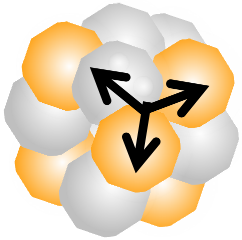
Takumi Doi

(RIKEN Nishina Center / iTHEMS)

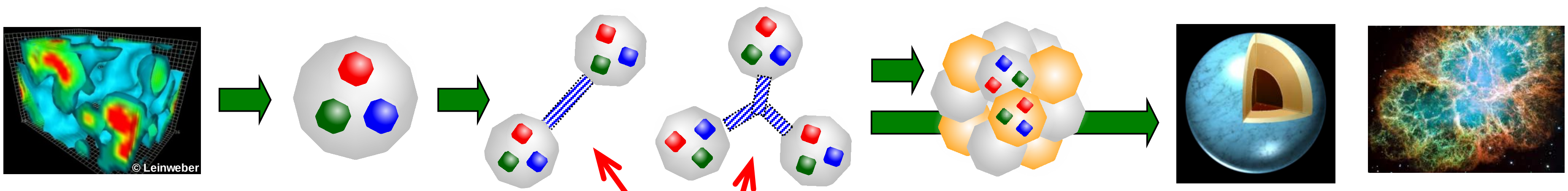
for HAL QCD Collaboration







The Odyssey from Quarks to Universe



QCD vacuum

Baryons

Nuclei

Neutron Stars / Supernovae

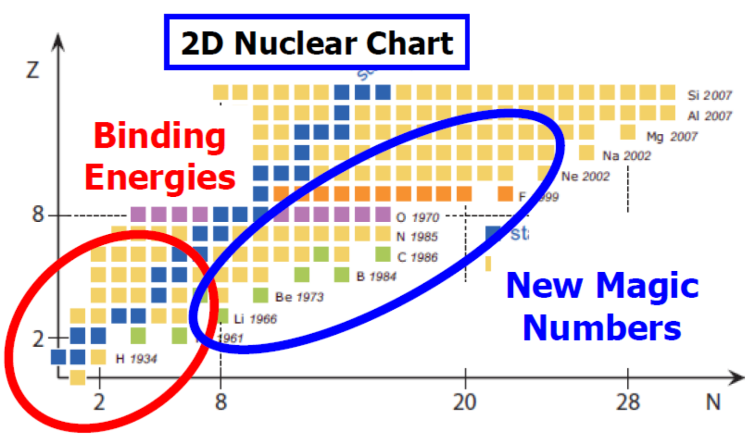
Nucleosynthesis

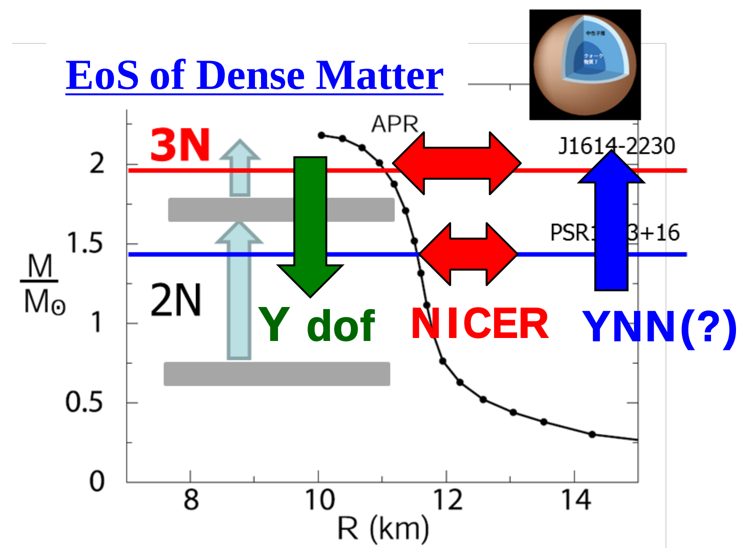
QCD

Baryon Forces

1st-principle Lattice QCD

ab-initio nuclear calc.

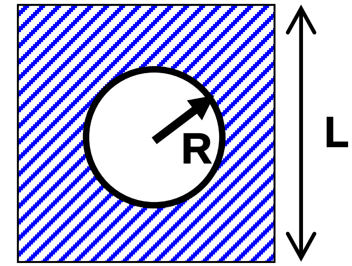




[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\boxed{\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k});W\rangle}$$



$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

  - phase shift at asymptotic region

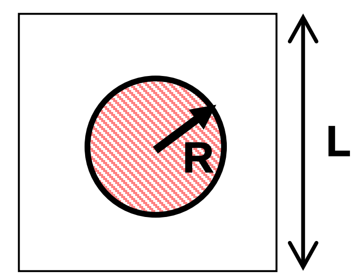
M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
N.Ishizuka, PoS LAT2009 (2009) 119
CP-PACS Coll., PRD71(2005)094504

$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

Phase shift in 2-body scattering

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  - U(r,r'): faithful to the phase shift by construction
    - U(r,r'): E-independent, while non-local in general
      - Non-locality ➔ derivative expansion

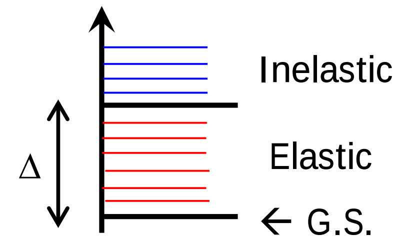
Aoki-Hatsuda-Ishii PTP123(2010)89

[Time-dependent HAL QCD method]

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

$$\int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}') R(\boldsymbol{r}', t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r}, t)$$

FV spectrum



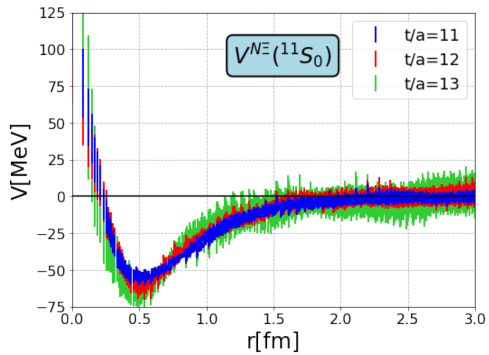
	Ground state	Excited (elastic)	Excited (inelastic)
HAL method	Signal	Signal	Noise
Direct method	Signal	Noise	Noise

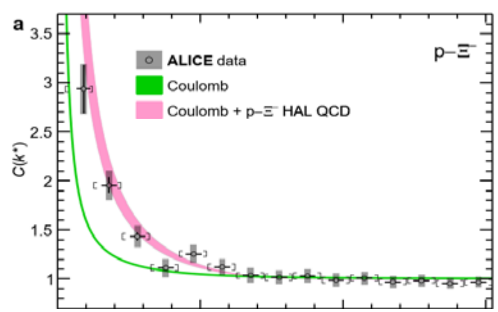
HAL method → Exponentially better S/N

Direct method = Plateau fit w/ GS saturation
+ Luscher's formula

Two-Baryon Forces:

near physical point simulation successfully performed!





and confirmed by ALICE

S. Gongyo et al.. (HAL) PRL120(2018)212001
T. Iritani et al. (HAL) PLB792(2019)284
K. Sasaki et al. (HAL) NPA998(2020)121737
Y. Lyu, H. Tong et al. PRL127(2021)072003

Calc of charmed int:
➔ Y. Lyu's talk

Extension to multi-particle systems (n>=3)

- Unitarity of S-matrix

S.Aoki et al. (HAL Coll.), PRD88(2013)014036

$$T^\dagger - T = iT^\dagger T$$

Hyper-spherical func in D=3(n-1) dim

$$T([\boldsymbol{q}^A]_n, [\boldsymbol{q}^B]_n) = \sum_{[L],[K]} T_{[L][K]}(Q_A, Q_B) Y_{[L]}(\Omega_{Q_A}) \overline{Y_{[K]}(\Omega_{Q_B})}$$

$$[L] = L, M_1, M_2, \ldots$$

diagonalization

$$T_{[L][K]}(Q,Q) = \sum_{[N]} U_{[L][N]}(Q) T_{[N]}(Q) U^\dagger_{[N][K]}(Q)$$

$$(Q = Q_A = Q_B)$$

$$T_{[L]}(Q) = -\frac{2n^{3/2}}{mQ^{3n-5}} \boxed{e^{i\delta_{[L]}(Q)} \sin\delta_{[L]}(Q)}$$

c.f. R.B. Newton (1974) for n = 3

Similar formula to 2-body system

(w/ diagonalization matrix U which includes dynamics)

(non-rela approx.)

Extension to multi-particle systems (n>=3)

- NBS wave function

S.Aoki et al. (HAL Coll.), PRD88(2013)014036

$$\psi_\alpha([\boldsymbol{x}]) =_{\text{in}} \langle 0|\phi([\boldsymbol{x}])|\alpha\rangle_{\text{in}} =_{\text{in}} \langle 0|N(\vec{x}_1)N(\vec{x}_2)\cdots N(\vec{x}_n)|\alpha\rangle_{\text{in}}$$

Lippmann-Schwinger eq.

$$\psi_\alpha([\boldsymbol{x}]) =_{\text{in}} \langle 0|\phi([\boldsymbol{x}])|\alpha\rangle_0 + \int d\beta \frac{{}_{\text{in}}\langle 0|\phi([\boldsymbol{x}])|\beta\rangle_0 T_{\beta\alpha}}{E_\alpha - E_\beta + i\epsilon}$$

Expansion w/ hyper-coordinate

$$\psi(\boldsymbol{R}, \boldsymbol{Q}_A) = \sum_{[L],[K]} \psi_{[L][K]}(R, Q_A) Y_{[L]}(\Omega_R) \overline{Y_{[K]}(\Omega_{Q_A})}$$

$$\psi_{[L],[K]}(R, Q_A) \propto \sum_{[N]} U_{[L][N]}(Q_A) \boxed{e^{i\delta_{[N]}(Q_A)} \frac{\sin(Q_A R - \Delta_L + \delta_{[N]}(Q_A))}{(Q_A R)^{(D-1)/2}}} U^\dagger_{[N][K]}(Q_A)$$

Similar asymptotic behavior to 2-body system

(non-rela approx.)

Extension when subsystem is bound also established !

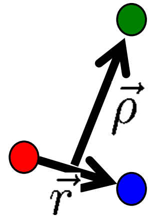
S. Gongyo, S. Aoki, PTEP2018(2018)093B03

c.f. Finite V spectrum, n=3 only, relativistic: Hansen, Sharpe, Briceno, Rusetsky, Mai, ...

Genuine 3NF from HAL QCD method

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r},\vec{\rho}) = \langle 0|\boxed{N(\vec{x}+\vec{r})}\boxed{N(\vec{x})}\boxed{N(\vec{x}+\vec{r}/2+\vec{\rho})}|3N\rangle$$



- Obtain 3NF through

$$(E-H_0^r-H_0^\rho)\psi(\vec{r},\vec{\rho}) = \left[\underbrace{\sum_{i<j} V_{ij}(\vec{r}_{ij})}_{\text{by 2N calc}} + V_{3NF}(\vec{r},\vec{\rho})\right]\psi(\vec{r},\vec{\rho})$$

- NBS is obtained by 6pt. correlator

$$G(\vec{r},\vec{\rho},t-t_0) = \sum_{\vec{x}}\langle 0|N(\vec{x}+\vec{r},t)N(\vec{x},t)N(\vec{x}+\vec{r}/2+\vec{\rho},t)\overline{NNN}(t_0)|0\rangle$$

In practical calculation,
we employ <u>**time-dependent HAL QCD method**</u>

$$\left(-H_0 - \frac{\partial}{\partial t}\right) R(\boldsymbol{r},t) = V(r)R(\boldsymbol{r},t)$$

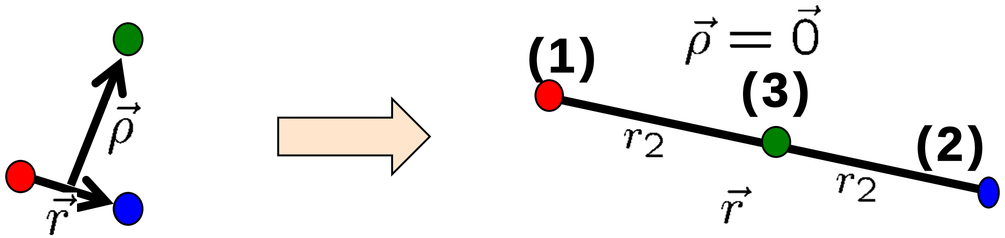
N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

➔ Ground state saturation is NOT necessary !

3NF calculation in Lat QCD

- We fix the geometry of 3N (← this is not an approximation)
- We study **linear setup**

We consider Triton channel



$(\vec{r}_2 \equiv \vec{r}/2)$

  - → $L^{(1,2)\text{-pair}} = L^{\text{total}} = 0$ or 2 only
  - → **Bases are only three**, labeled by **$^1S_0$, $^3S_1$, $^3D_1$ for (1,2)-pair**

$$|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

→ **L=even for any 2N pair** automatically guaranteed

2NF contribution can be obtained only from 2NF(P=even)

(1) Identification of Genuine 3NF

- **Genuine 3NF** can be extracted from **3x3 coupled channel**
  - Both of parity-even 2NF and parity-odd potential required

$$\hat{H}_0 \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} + \left(\underset{(V_{2N}+V_{3NF})}{V} \right) \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix} = E \begin{pmatrix} \psi(^1S_0) \\ \psi(^3S_1) \\ \psi(^3D_1) \end{pmatrix}$$

$V_C^{I,S=1,0}, V_C^{I,S=0,1}, V_T^{I,S=0,1}$: (P = even)

$\boxed{V_C^{I,S=0,0}, V_C^{I,S=1,1}, V_T^{I,S=1,1} : (\mathrm{P} = \mathrm{odd})}$

Target to be determined

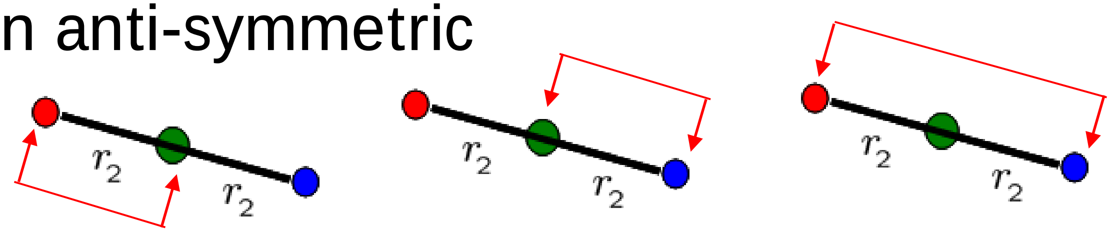
- S/N : parity-even 2NF > parity-odd 2NF in Lat QCD
  - ➔ **Desirable to extract 3NF w/ parity-even 2NF only**

c.f. K. Murano et al. (HAL Coll.) PLB735(2014)19

Solution using “symmetric” wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric

$$\psi_S =$$



  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
  - Bases are rotated as $|\psi_{^1S_0}\rangle, |\psi_{^3S_1}\rangle, |\psi_{^3D_1}\rangle \longrightarrow |\psi_S\rangle, |\psi_M\rangle, |\psi_{^3D_1}\rangle$

$$\boxed{|\psi_S\rangle = 1/\sqrt{2}\left(-|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)} \qquad |\psi_M\rangle = 1/\sqrt{2}\left(+|\psi_{^1S_0}\rangle + |\psi_{^3S_1}\rangle\right)$$

All pair P=even

$$\hat{H}_0 \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} + \hat{V}_{3NF} \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix} = E \begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{^3D_1} \end{pmatrix}$$

No V(P=odd)

Solution using "symmetric" wave function

- We can construct the wave function in which <u>any 2N pair</u> is spin/isospin anti-symmetric
  - ➔ **<u>L=even for any 2N pair</u>** automatically guaranteed
- 3x3 coupled channel is reduced to
  - **<u>one channel with only 3NF unknown</u>**
  - two channels with $V_C^{I,S=0,0}$, $V_C^{I,S=1,1}$, $V_T^{I,S=1,1}$,(3NF) unknown

$$\begin{pmatrix} & & \\ & H_0 & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{2N} & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} + \begin{pmatrix} & & \\ & V_{3NF} & \\ & & \end{pmatrix}\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix} = E\begin{pmatrix} \psi_S \\ \psi_M \\ \psi_{3D_1} \end{pmatrix}$$

No V(P=odd)

Target to be determined

- ➔ Even without parity-odd V, we can determine one 3NF
  - This method works for any fixed 3D-geometry other than linear

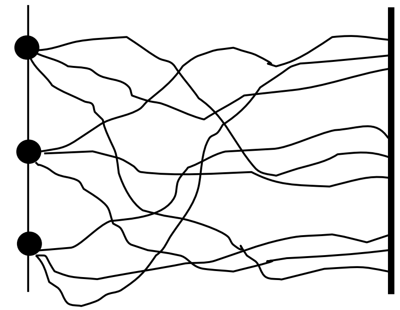
Computational Challenge

- **Enormous comput. cost for multi-baryon correlators**
  - Wick contraction (permutations)

    $$\sim [\,(\tfrac{3}{2}A)!\,]^2 \qquad \text{(A: mass number)}$$

  - color/spinor contractions

    $$\sim 6^A \cdot 4^A \quad \text{or} \quad 6^A \cdot 2^A$$



See also T. Yamazaki et al., PRD81(2010)111504

  - **Unified Contraction Algorithm (UCA)**
    - A novel method which unifies two contractions

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

Drastic Speedup

$$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$$ (x add'l. speedup)

TD and M.Endres, CPC184(2013)117

Lattice simulation setup

- Nf=2 dynamical clover fermion + RG improved gauge action
  - a^{-1}=1.269GeV, a=0.1555fm (beta=1.95)
  - 16^3 X 32 lattice, L=2.5fm

CP-PACS Coll. S. Aoki et al.,
Phys. Rev. D65 (2002) 054505

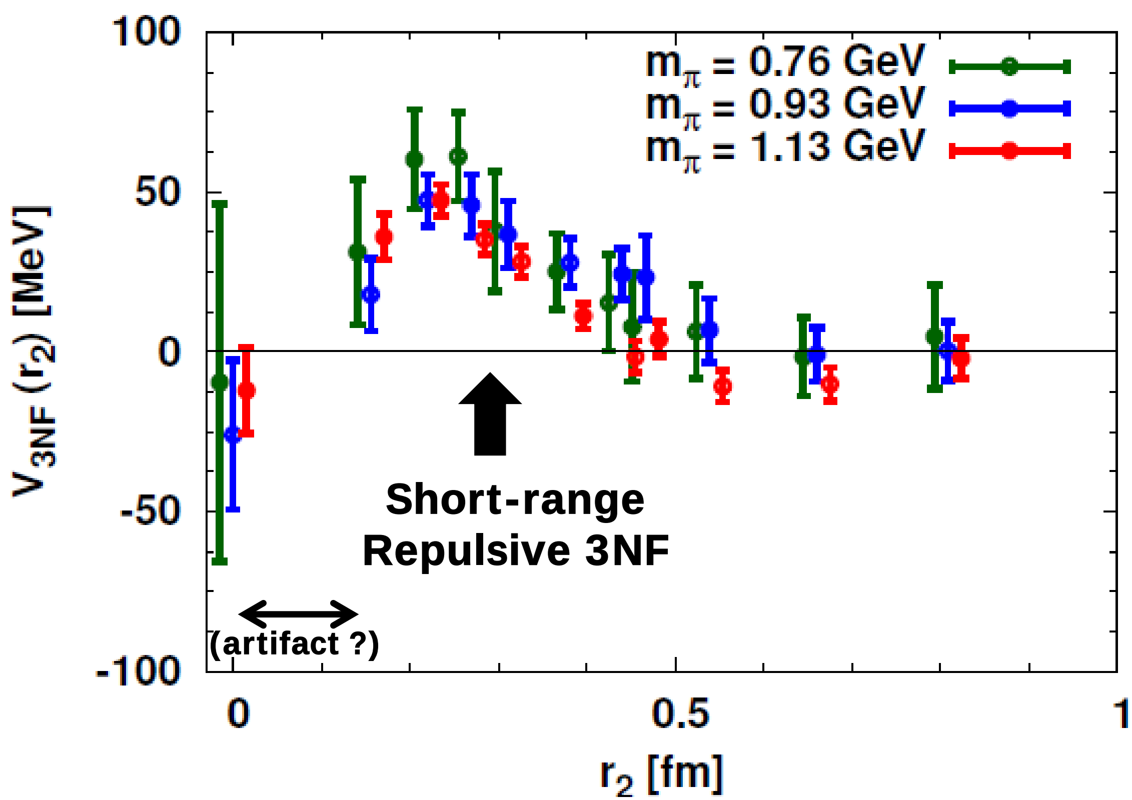
- Masses: (π, N, Δ) = (1.13, 2.15, 2.31) GeV
  - Kappa(ud)=0.13750
  - 599 configs x 32 measurements, t+1=[5,12]
- Masses: (π, N, Δ) = (0.925, 1.85, 2.02) GeV
  - Kappa(ud)=0.13900
  - 686 configs x 32 measurements, t+1=[5,12]
- Masses: (π, N, Δ) = (0.757, 1.61, 1.81) GeV
  - Kappa(ud)=0.14000
  - 686 configs x 32 measurements, t+1=[5,12]

Wall source w/ Coulomb gauge

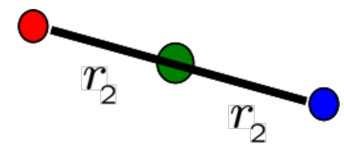
3N-forces (3NF) in m_π >=0.76 GeV

T.D. et al. (HAL QCD Coll.) PTP127(2012)723 + t-dep method updates etc.

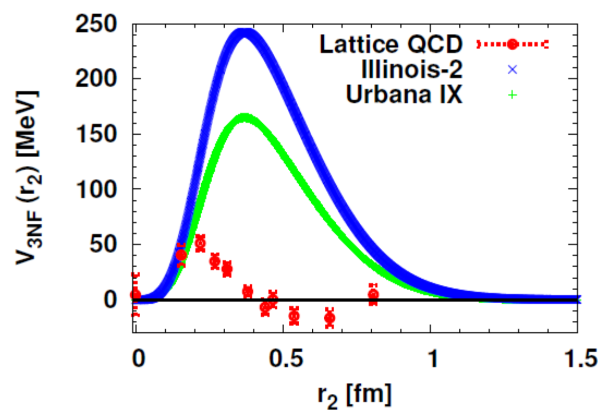
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=0.76-1.1GeV, m_N=1.6-2.1GeV



Triton channel



Naïve comparison to phenomenological 3NF

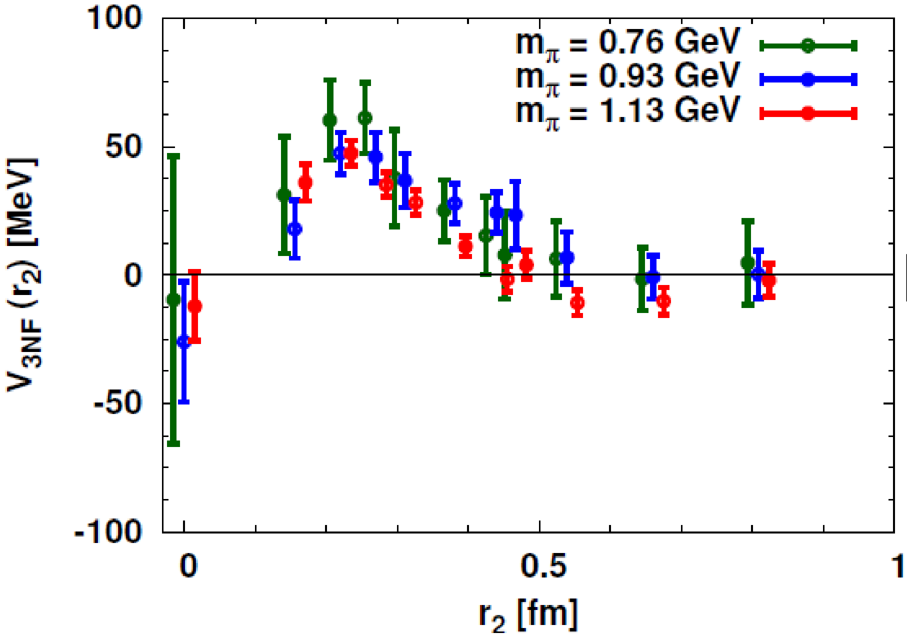


(t-t0 = 7.5)

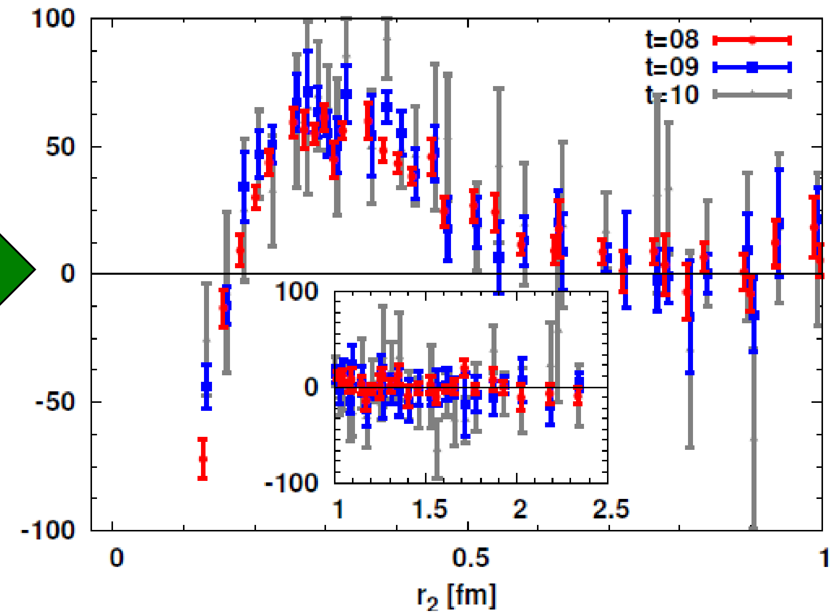
➔ Lighter quark mass important ?

Towards lighter quark masses

Nf=2, mπ=0.76-1.1 GeV



Nf=2+1, mπ=0.51 GeV



Magnitude of 3NF is similar for all masses
Range of 3NF tend to be enlarged for m(pi)=0.5GeV

Next challenge: **Calc of P-wave 2BF** : better subtraction of 2BF in 3-body systems
YNN (w/o or w/ P-wave 2BF) : gauge conf generation on Fugaku

Aoki-Doi Front.Phys.8 (2020) 307

P-wave 2NF int by all-to-all method

LapH method

M. Peardon et al., PRD80(2009)054506

$$\mathcal{S}^{ab}(x,y) = \sum_{l=0}^{N_l} \omega_l \, v_l^a(x) v_l^{*b}(y)$$

$$\Delta^{ab}(x,y;U) = \sum_{k=1}^{3} \left\{ U_k^{ab}(x)\delta(y,x+\hat{k}) + U_k^{ba}(y)^*\delta(y,x-\hat{k}) - 2\delta(x,y)\delta^{ab} \right\}$$

$$= \left(\boxed{\boldsymbol{v}_1 \quad \boldsymbol{v}_2} \quad \cdots \quad \boldsymbol{v}_N \right) \begin{pmatrix} \lambda_1 & & & \mathbf{0} \\ & \lambda_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \lambda_N \end{pmatrix} \begin{pmatrix} \boldsymbol{v}_1^\dagger \\ \boldsymbol{v}_2^\dagger \\ \vdots \\ \boldsymbol{v}_N^\dagger \end{pmatrix}$$

Approximate all-to-all prop by N_l low-modes in gauge covariant Laplacian

New improvement: Free LapH method

gauge cov. Laplacian → free Laplacian

$$\Delta(x,y) = \sum_{k=1}^{3} \left\{ \delta(y,x+\hat{k}) + \delta(y,x-\hat{k}) - 2\delta(x,y) \right\}$$

Comput. Cost can be reduced from O(Nc^4 x N_l^4) → Nc! Nc O(N_l^3)

→ Typically O(100) speedup!

Explicit calc for NN in progress

[T. Sugiura]

➔ P-wave int, LS-forces, better systematics

Summary

- Hadron Int.: Bridge between particle/nuclear/astro-physics

- HAL QCD method for LQCD determination of Hadron Int.
  - The "potential" can be defined in QFT even for three- (and multi-) body forces

- Three-Nucleon Forces
  - Method to extract 3NF in I=1/2 channel
  - Short-range repulsion observed for m(pi) = 0.51GeV, 0.76-1.13GeV

- Prospects
  - P-wave 2NF in progress ➔ 3NF in I=3/2 channel
  - 3BF w/ hyperons ➔ EoS of dense matter

Fugaku (440PFlops)