Large-N_c Constraints for Beyond the Standard Model Few-Nucleon Currents in Effective Field Theory

Thomas R. Richardson

Few-Body Dynamics
10th International Workshop on Chiral Dynamics
November 17, 2021

In collaboration with M.R. Schindler, S. Pastore, R.P. Springer, X. Lin, and S.T. Nguyen

Work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics

Outline

1. Effective field theory and the large-N expansion
2. Neutrinoless double beta decay with light Majorana exchange
3. WIMP-light nucleus elastic scattering
4. Summary

Effective Field Theory

- Lagrangian organized as power series

$$\mathcal{L}_{\text{eff}} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

- Low energy coefficients dimensionless and $c_{\mathcal{O}} \sim O(1)$
- Systematic and model independent
- Pionless EFT and Chiral EFT
  - Symmetries of QCD
  - Simple inclusion of external fields

Low Energy Coefficients

- LECs must be obtained from:
  - fit to data
    - lacking for many low-energy processes
  - nonperturbative QCD calculation
    - lattice QCD
- Theoretical constraints from large-N_c QCD
  - other applications: two-nucleon potential, parity violation, time-reversal invariance violation, etc.
  - **Fierz transformations can obscure large-N_c relations**

Kaplan and Savage, 1996; Kaplan and Manohar, 1997; Banerjee et al. 2002; Schindler et al. 2016, 2018; Phillips and Schat 2013; Phillips et al. 2015

Large-N$_c$ Baryons

- Limit $N_c \to \infty$ with $g^2_{\rm strong} N_c$ fixed
  - QCD becomes expansion in $1/N_c$
- Mesons are weakly interacting with fixed mass

$$F_0 \sim \sqrt{N_c}$$

- Baryon must be made of N$_c$ quarks
- Baryon mass scales as N$_c$

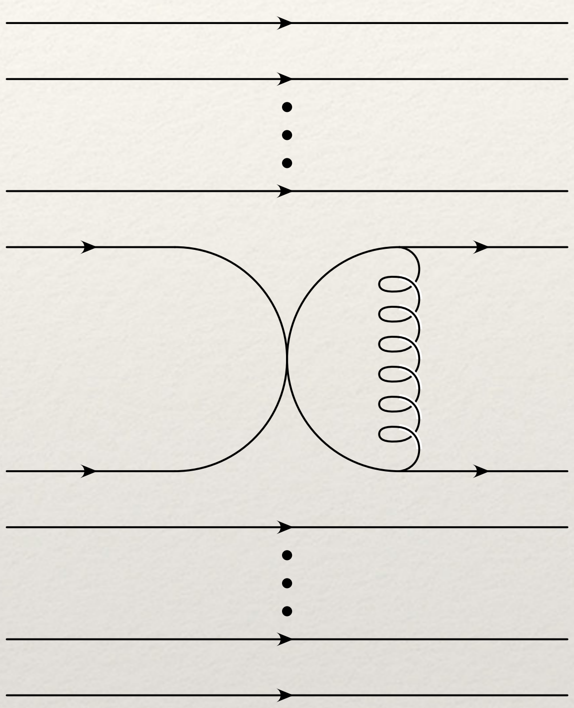


Figure: Baryon-baryon scattering
Manohar, 1998

't Hooft, 1973; Witten, 1979

Large-N$_c$ Spin-Flavor Symmetry

- Large-N$_c$ SU(4) spin-flavor symmetry

$$S^i = q^\dagger \frac{\sigma^i}{2} q \qquad I^a = q^\dagger \frac{\tau^a}{2} q \qquad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

$$\langle B' | \frac{\mathcal{O}^{(n)}}{N_c^n} | B \rangle \sim N_c^{-|I-S|} \qquad \langle B' | \hat{1} | B \rangle \sim \mathcal{O}(N_c)$$

- Factorization and Hartree Hamiltonian

(t-channel)

$$H = N_c \sum_{n,s,t} v_{stn} \left(\frac{S^i}{N_c} \right)^s \left(\frac{I^a}{N_c} \right)^t \left(\frac{G^{ia}}{N_c} \right)^{n-s-t} \qquad p_- \sim 1 \qquad p_+ \sim 1/N_c$$

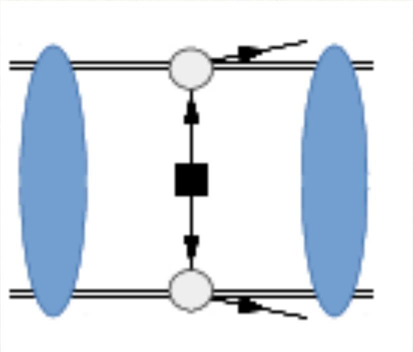
$$\langle N_3 N_4 | \mathcal{O}_1 \mathcal{O}_2 | N_1 N_2 \rangle \xrightarrow{N_c \to \infty} \langle N_3 | \mathcal{O}_1 | N_1 \rangle \langle N_4 | \mathcal{O}_2 | N_2 \rangle$$

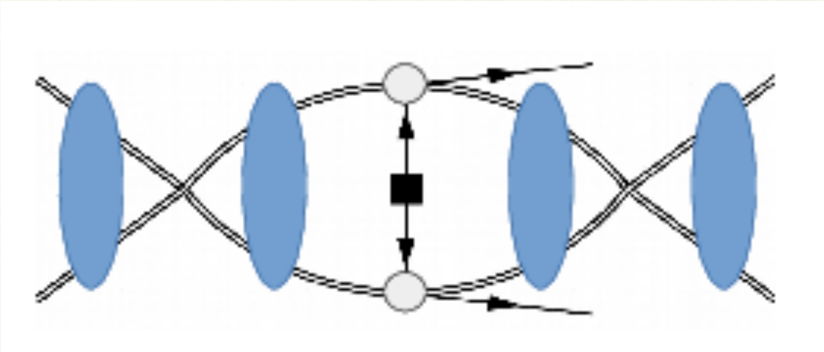
Dashen and Manohar, 1993; Dashen et al. 1995 ; Kaplan and Savage, 1996
Kaplan and Manohar, 1997

Light Majorana Exchange Mechanism

Cirigliano et al., 2018a, 2018b, 2019

- Neutrino exchange potential





$$V_{\nu L}^{{}^1S_0}(q) = \frac{\tau^{(1)+}\tau^{(2)+}}{q^2}\left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(q^2+m_\pi^2)^2}\right]$$

- Contact term required for renormalization

$$\mathcal{L}_{\Delta L=2}^{NN} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C\bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(N^\dagger u\tau^+ u^\dagger N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tau^+\tau^+\right)\left(N^\dagger\tau^a N\right)^2\right] + \mathrm{H.c}$$

$$u = \exp\left(\frac{i}{F_0}\phi_a\tau^a\right)$$

Relation to Charge Independence Breaking

- CIB isotensor Lagrangian

$$\tilde{Q}_\pm = \frac{1}{4}\left[u^\dagger \tau^3 u \pm u \tau^3 u^\dagger\right]$$

$$\mathcal{L}_{CIB}^{NN} = \frac{e^2}{2}\left\{(\mathcal{C}_1 + \mathcal{C}_2)\left[\left(N^\dagger \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \tau^a N\right)^2\right]\right.$$
$$\left.(\mathcal{C}_1 - \mathcal{C}_2)\left[\left(N^\dagger \tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger \tau^a N\right)^2\right]\right\}$$

Epelbaum and Meißner 1999, Walzl et al. 2001

- Chiral symmetry dictates

$$g_\nu^{NN} = \mathcal{C}_1$$

- Approximate

$$g_\nu^{NN} = \frac{1}{2}(\mathcal{C}_1 + \mathcal{C}_2)$$

Cirigliano et al. 2018a, 2018b, 2019, 2021

Results: LEC Constraints

- LNV contact term

Richardson et al. 2021

$$\left(N^\dagger \sigma^i \tau^+ N\right)\left(N^\dagger \sigma^i \tau^+ N\right) = -3\left(N^\dagger \tau^+ N\right)\left(N^\dagger \tau^+ N\right)$$

$$g_\nu^{NN} \sim O(N_c)$$

- CIB contact terms

$$\mathcal{L}^{\Delta I=2}_{\text{LO-in-}N_c} = \bar{\mathcal{C}}_3 \left[\left(N^\dagger \sigma^i \tilde{Q}_+ N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_+^2\right)\left(N^\dagger \sigma^i \tau^a N\right)^2\right]$$

$$\mathcal{L}^{\Delta I=2}_{\text{NLO-in-}N_c} = \bar{\mathcal{C}}_6 \left[\left(N^\dagger \sigma^i \tilde{Q}_- N\right)^2 - \frac{1}{6}\mathrm{Tr}\left(\tilde{Q}_-^2\right)\left(N^\dagger \sigma^i \tau^a N\right)^2\right]$$

$$\bar{\mathcal{C}}_3 \sim O(N_c) \qquad \mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

$$\bar{\mathcal{C}}_6 \sim O(1) \qquad \mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

Results: Isospin Breaking Two-Nucleon Interactions

- Four classes of charge symmetry of nuclear forces

$$V_I \propto \hat{1}_1 \hat{1}_2, \ \tau_1^a \tau_2^a \lesssim O(N_c)$$

$$V_{II} \propto \tau_1^3 \tau_2^3 - \frac{1}{3}\tau_1^a - \frac{1}{3}\tau_2^a \lesssim O(N_c)$$

$$V_{III} \propto \tau_1^3 + \tau_2^3 \lesssim O(1)$$

$$V_{IV} \propto \tau_1^3 - \tau_2^3, \ (\vec{\tau}_1 \times \vec{\tau}_2)^3 \lesssim O(1)$$

- Class (IV) occurs for P-wave or higher
  - suppressed in EFT power counting

van Kolck 1993, Miller 1994, Miller et al. 2006

WIMP-Light Nucleus Scattering

- Recent proposals for light nuclei as direct detection targets

Guo and McKinsey, 2013; Ito and Seidel, 2013; Gerbier, et al. 2014; Profumo 2016; Hertel, et al. 2017; Maris, et al. 2019

- Momentum transfer bounded from above by a few MeV

$$q \ll m_\pi$$

Körber, et al. 2017

- (Almost) model-independent framework—minimal assumptions about dark matter
  - Spin 0 or 1/2
  - Heavy WIMP $m_\chi \gtrsim O(1)$ GeV
- Novel theoretical constraints can be used to simplify many-body calculations

One-Nucleon Currents

- Zero derivatives

Fitzpatrick et al., 2013;
Hill and Solon, 2015

$$\mathcal{L}_{\chi N}^{(PT)} = \left(\chi^\dagger\chi\right)\left[C_{1,\chi N}^{(PT)}\left(N^\dagger N\right) + C_{4,\chi N}^{(PT)}\left(N^\dagger\tau^3 N\right)\right] + \left(\chi^\dagger\sigma^i\chi\right)\left[C_{3,\chi N}^{(PT)}\left(N^\dagger\sigma^i N\right) + C_{2,\chi N}^{(PT)}\left(N^\dagger\sigma^i\tau^3 N\right)\right]$$

$$C_{1,\chi N}^{(PT)},\, C_{2,\chi N}^{(PT)} \sim O(N_c)$$

$$C_{3,\chi N}^{(PT)},\, C_{4,\chi N}^{(PT)} \sim O(1)$$

- One derivative—16 terms, but only 6 are $O(N_c)$
  - 3 suppressed by WIMP mass

$$\mathcal{L}_{\chi N}^{\mathrm{LO-in-}N_c} \supset \left(\chi^\dagger\chi\right)\left[C_5^{(\not{P}T)}\epsilon^{ijk}\nabla^j\left(N^\dagger\sigma^k\tau^3 N\right) + C_{11}^{(\not{P}\not{T})}\nabla^i\left(N^\dagger\sigma^i\tau^3 N\right)\right] + C_{12}^{(\not{P}\not{T})}\left(\chi^\dagger\sigma^i\chi\right)\nabla^i\left(N^\dagger N\right)$$

Two-Nucleon Currents

- Seven operators, only three $O(N_c)$ LECs

$$\mathcal{L}_{\chi NN}^{\text{LO-in-}N_c} = C_{1,\chi NN}^{(\text{SI},\,s)} \left(\chi^\dagger \chi\right)\left(N^\dagger N\right)\left(N^\dagger N\right)$$
$$+ \epsilon^{ijk}\epsilon^{3ab} C_{1,\chi NN}^{(\text{SD},\,v)} \left(\chi^\dagger \sigma^i \chi\right)\left(N^\dagger \sigma^j \tau^a N\right)\left(N^\dagger \sigma^k \tau^b N\right)$$
$$+ C_{1,\chi NN}^{(\text{SI},\,t)} \left(\chi^\dagger \chi\right)\left[\left(N^\dagger \sigma^i \tau^3 N\right)\left(N^\dagger \sigma^i \tau^3 N\right) - \frac{1}{2}\left(N^\dagger \sigma^i \tau^a N\right)\left(N^\dagger \sigma^i \tau^a N\right)\right]$$

- For WIMP-deuteron scattering need $C_{1,\chi NN}^{(\text{SI},\,s)}$ and

$$C_{2,\chi NN}^{(\text{SI},\,s)} \left(\chi^\dagger \chi\right)\left(N^\dagger \sigma^i N\right)\left(N^\dagger \sigma^i N\right) \qquad C_{2,\chi NN}^{(\text{SI},\,s)} \sim O(1/N_c)$$

$$C_{1,\chi NN}^{(\text{SD},\,s)} \left(\chi^\dagger \sigma^i \chi\right)\left(N^\dagger \sigma^i N\right)\left(N^\dagger N\right) \qquad C_{1,\chi NN}^{(\text{SD},\,s)} \sim O(1)$$

Single-Nucleon Cross Section

- Unpolarized scattering cross section at LO in EFT

$$\sigma^{\rm SI}_{0,p/n} = \frac{m^2_{\chi N}}{\pi}\left(C^{(PT)}_{1,\chi N} \pm C^{(PT)}_{4,\chi N}\right)^2$$

$$\sigma^{\rm SD}_{0,p/n} = \frac{3m^2_{\chi N}}{\pi}\left(C^{(PT)}_{2,\chi N} \pm C^{(PT)}_{3,\chi N}\right)^2$$

$$\left.\frac{\sigma^{\rm (SI)}_{0,N}}{\sigma^{\rm (SD)}_{0,N}}\right|_{\text{LO-in-}N_c} = \frac{C^{(PT)2}_{1,\chi N}}{3C^{(PT)2}_{2,\chi N}} \sim \frac{1}{3}$$

Deuteron Cross Section

- Symmetry conserving unpolarized cross section

$$\sigma_{0,d}^{\mathrm{SI}} = \frac{m_{\chi d}^2}{\pi}\left[2C_{1,\chi N}^{(PT)} - \frac{\gamma}{\pi}\left(\mu-\gamma\right)^2\left(C_{1,\chi NN}^{(\mathrm{SI},\,s)} + C_{2,\chi NN}^{(\mathrm{SI},\,s)}\right)\right]^2$$

$$\sigma_{0,d}^{\mathrm{SD}} = \frac{2m_{\chi d}^2}{\pi}\left[2C_{3,\chi N}^{(PT)} - \frac{\gamma}{\pi}\left(\mu-\gamma\right)^2 C_{1,\chi NN}^{(\mathrm{SD},\,s)}\right]^2$$

$$\frac{\sigma_{0,d}^{\mathrm{SD}}}{\sigma_{0,d}^{\mathrm{SI}}} = \frac{2C_{3,\chi N}^{(PT)2}}{C_{1,\chi N}^{PT)2}} \sim \frac{2}{N_c^2} \qquad \frac{\sigma_{0,d}^{\mathrm{SI}}}{\sigma_{0,N}^{\mathrm{SI}}} = \frac{4m_{\chi d}^2}{m_{\chi N}^2} \sim O(1)$$

$$\frac{\sigma_{0,d}^{SD}}{\sigma_{0,N}^{SD}} = \frac{8m_{\chi d}^2}{3m_{\chi N}^2}\frac{1}{\left(1 \pm \frac{C_{2,\chi N}^{(PT)}}{C_{3,\chi N}^{(PT)}}\right)^2} \sim O(1/N_c^2)$$

WIMP-Three body Scattering

- For triton or helium-3

$$\frac{\sigma_{0,H}^{\mathrm{SI}}}{\sigma_{0,H}^{\mathrm{SD}}} = \frac{9C_{1,\chi N}^{(PT)2}\left(1 \mp \frac{C_{4,\chi N}^{(PT)}}{C_{1,\chi N}}\right)^2}{3C_{2,\chi N}^{(PT)2}\left(1 \pm \frac{C_{3,\chi N}^{(PT)}}{C_{2,\chi N}^{(PT)}}\right)^2} \approx \frac{3C_{1,\chi N}^{(PT)2}}{C_{2,\chi N}^{(PT)2}} \sim O(1)$$

- SI and SD cross sections are of the same size as corresponding nucleon cross sections

Conclusions

- Large-N_c spin-flavor symmetry constrains relative sizes of LECs without data
- Constraints should guide the interpretation of data
- Constraints can reduce required input in *ab initio* studies that use EFT interactions
- Leading order LECs should be priority for lattice QCD calculations