OPE of Green functions and (some of) its phenomenological applications

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Green Functions of Chiral Currents

- The vacuum expectation values of the T-ordered product of local operators
  - Vector and axial-vector currents:

$$V_\mu^a(x) = \overline{q}(x)\gamma_\mu T^a q(x) \qquad A_\mu^a(x) = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)$$

  - Scalar and pseudoscalar densities:

$$S^a(x) = \overline{q}(x) T^a q(x) \qquad P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x)$$

- Connection to phenomenological processes via the LSZ reduction formula
- Nontrivial three-point Green functions in QCD:
  - Set I: $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$
    - Order parameters of χSB in the chiral limit
  - Set II: $\langle ASP\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle AAV\rangle$, $\langle VVV\rangle$

Operator Product Expansion

- Framework to study behaviour of the GFs at high energies
- At short distances (i.e. for large external momenta), the Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$
\begin{aligned}
\Pi_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(\lambda p, \lambda q; \lambda r) \xrightarrow{\lambda \to \infty} \quad & \lambda\, C^{\mathbb{1}}_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(p, q; r) \\
& + \frac{1}{\lambda^2} C^{\langle \overline{q} q \rangle}_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(p, q; r) \langle \overline{q} q \rangle \\
& + \frac{1}{\lambda^3} C^{\langle G^2 \rangle}_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(p, q; r) \langle G^a_{\mu\nu} G^{\mu\nu,a} \rangle \\
& + \frac{1}{\lambda^4} C^{\langle \overline{q} \sigma \cdot G q \rangle}_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(p, q; r) \langle \overline{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle \\
& + \frac{1}{\lambda^5} C^{\langle \overline{q} q \rangle^2}_{\mathcal{O}_1^a \mathcal{O}_2^b \mathcal{O}_3^c}(p, q; r) \langle (\overline{q} X q)(\overline{q} X q) \rangle + \ldots
\end{aligned}
$$

- Our simplifications:
  - Chiral limit
  - Only the lowest-order contribution of such a condensate

OPE: Effective Propagation of QCD Condensates

- QCD condensates are obtained through the effective propagation of non-local condensates [TK, K. Kampf and J. Novotný '20]:

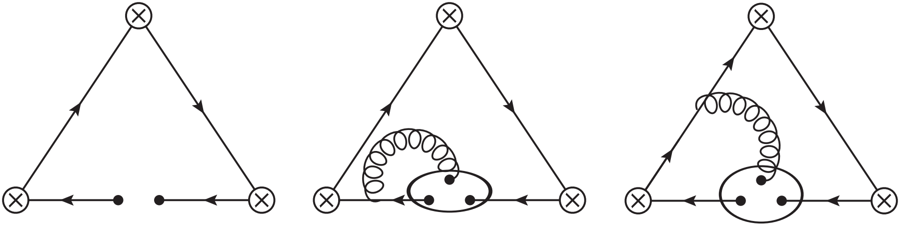
$$\langle \overline{q}^A_{i,\alpha}(x) q^B_{k,\beta}(y) \rangle = \Bigg(\frac{\langle \overline{q}q \rangle}{2^2 \cdot 3^2} \delta_{ik} - \frac{g_s \langle \overline{q}\sigma \cdot G q \rangle}{2^5 \cdot 3^2} \left[F^{\langle \overline{q}q \rangle}(x,y) \right]_{ki} + \\ + \frac{i\pi\alpha_s \langle \overline{q}q \rangle^2}{2^3 \cdot 3^7} \left[G^{\langle \overline{q}q \rangle}(x,y) \right]_{ki} + \dots \Bigg) \delta_{\alpha\beta} \delta^{AB}$$

$$g_s \langle \overline{q}^A_{i,\alpha}(x) \mathcal{A}^a_\mu(u) q^B_{k,\beta}(y) \rangle = \Bigg(\frac{g_s \langle \overline{q}\sigma \cdot G q \rangle}{2^7 \cdot 3^2} \left[F^{\langle \overline{q}\mathcal{A}q \rangle}_\mu(x,u,y) \right]_{ki} + \\ + \frac{\pi\alpha_s \langle \overline{q}q \rangle^2}{2^3 \cdot 3^5} \left[G^{\langle \overline{q}\mathcal{A}q \rangle}_\mu(x,u,y) \right]_{ki} + \dots \Bigg) (T^a)_{\beta\alpha} \delta^{AB}$$

  - Omitting the effective propagation *"has been one of the main source of errors in the existing QSSR literature"* [S. Narison '04]
- Computation of the contributions to the GFs
  - Straightforward, but lengthy — easiest in the Fock–Schwinger gauge
  - Fourier transform needed to convert the result into momentum representation
  - Cancelation of logarithmic terms at the lowest-order

OPE: Quark and Quark-gluon Condensates

- Contributions only to the order parameters of the χSB — example: $\langle VVP \rangle$
- Quark condensate: $\langle \overline{q}q \rangle$
  - Implicates the chiral symmetry breaking
  - LO: studied a long time ago [B. Moussallam '94]
  - NLO: gluonic corrections at $\mathcal{O}(\alpha_s)$ [M. Jamin and V. Mateu '08], an opportunity to explore the renormalisation dependence in full QCD
- Quark-gluon condensate: $\langle \overline{q}\sigma \cdot Gq \rangle$
  - Studied a long time ago, even outside of chiral limit [V. Elias, T. G. Steele and M. Scadron '88]



Odd-intrinsic Parity Sector of QCD

- Special interest of ours: $\langle VVP \rangle$, $\langle AAP \rangle$, $\langle VAS \rangle$, $\langle VVA \rangle$, $\langle AAA \rangle$
- NLO resonance Lagrangian [K. Kampf and J. Novotný '11]:

$$\mathcal{L}_{\mathrm{R}\chi\mathrm{T}}^{(6,\,\mathrm{odd})} = \varepsilon^{\mu\nu\alpha\beta} \sum_X \sum_i \kappa_i^X \big(\widehat{\mathcal{O}}_i^X\big)_{\mu\nu\alpha\beta}$$

  - Dynamics assumed to be saturated with the lightest resonances only
  - 67 operators and 67 corresponding unknown couplings κ_i^X
  - X: the single-resonance (V, A, S, P), double-resonance (VV, AA, SA, SV, VA, PA, PV) and triple-resonance fields (VVP, VAS, AAP)

- How to obtain the unknown coupling constants?
  - Phenomenological (decay widths) or theoretical (OPE, B–L etc.) constraints

$\langle VVP \rangle$ Green Function: Matching RχT-OPE

- OPE for all momenta large [TK, K. Kampf and J. Novotný '20]:

$$\mathcal{F}^{\mathrm{OPE}}_{VVP}\left((\lambda p)^2, (\lambda q)^2, (\lambda r)^2\right) = \frac{\langle \overline{q} q \rangle}{6\lambda^4} \frac{p^2+q^2+r^2}{p^2 q^2 r^2} - \frac{g_s \langle \overline{q} \sigma \cdot G q \rangle}{72\lambda^6} \frac{r^2(p^4+q^4)+3(p^2-q^2)^2(p^2+q^2)+4r^6}{p^4 q^4 r^4} + \mathcal{O}\left(\frac{1}{\lambda^8}\right)$$

- RχT contribution [K. Kampf and J. Novotný '11]:

$$\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{VVP}(p^2,q^2,r^2) = \frac{B_0 N_c}{16\pi^2 r^2} - \frac{4B_0 F_V^2 \kappa_3^{VV} p^2}{(p^2-M_V^2)(q^2-M_V^2)r^2} + \frac{16\sqrt{2}B_0 d_m F_V \kappa_3^{PV}}{(p^2-M_V^2)(r^2-M_P^2)}$$
$$+ \frac{32 B_0 d_m \kappa_5^P}{r^2-M_P^2} + \frac{8B_0 d_m F_V^2 \kappa^{VVP}}{(p^2-M_V^2)(q^2-M_V^2)(r^2-M_P^2)} - \frac{2B_0 F_V^2(8\kappa_2^{VV}-\kappa_3^{VV})}{(p^2-M_V^2)(q^2-M_V^2)}$$
$$+ \frac{2\sqrt{2}B_0 F_V}{(p^2-M_V^2)r^2}\left[p^2(\kappa_{16}^V+2\kappa_{12}^V) - q^2(\kappa_{16}^V-2\kappa_{17}^V+2\kappa_{12}^V) - r^2(8\kappa_{14}^V+\kappa_{16}^V+2\kappa_{12}^V)\right]$$
$$+ (p \leftrightarrow q)$$

- Matching the RχT contribution onto OPE with quark-gluon condensate fails!
- Another resonance multiplets needed!

$\langle VVP \rangle$ Green Function: Parametrization

- A suitable parametrization $\mathcal{F}_{VVP}(p^2, q^2, r^2)$ should satisfy:
  - OPE for all momenta large (quark-gluon condensate included)
  - OPE for two momenta large [M. Knecht and A. Nyffeler '01]:

$$\mathcal{F}_{VVP}^{\mathrm{OPE}}\left((\lambda p)^2, (q-\lambda p)^2, q^2\right) = \frac{\langle \bar{q} q \rangle}{3\lambda^2} \frac{1}{p^2 q^2} + \mathcal{O}\left(\frac{1}{\lambda^3}\right)$$

  - Brodsky–Lepage behaviour [G. P. Lepage and S. J. Brodsky '80, '81]:

$$\lim_{Q^2 \to \infty} Q^2 \mathcal{F}_{\gamma^* \gamma^* \pi^0}(0, -Q^2) = 2F$$

    for the pion transition form factor

$$\mathcal{F}_{\gamma^* \gamma^* \pi^0}(p^2, q^2) = \frac{2}{3B_0 F} \lim_{r^2 \to 0} r^2 \mathcal{F}_{VVP}(p^2, q^2, r^2)$$

  - Anomaly:

$$\mathcal{F}_{\gamma^* \gamma^* \pi^0}(0, 0) = \frac{N_c}{12\pi^2 F}$$

$\langle VVP \rangle$ Green Function: Parametrization

- „LMD+V+P", i.e. THS [T. Husek and S. Leupold '15]
  - An addition of the quark-gluon condensate contribution to the OPE spoils the simultaneous fulfillment of the OPE and B–L
- „THS+P" [TK, K. Kampf and J. Novotný (in preparation)]
  - Taking into account another pseudoscalar multiplet saves the day!
  - At the Lagrangian level, the number of operators increases from 10 to 29
  - Example: monomials of the extended RχT Lagrangian at $\mathcal{O}(p^6)$ with three resonance fields

Operator	Coupling constant	Operator	Coupling constant
$\langle V_1^{\mu\nu} V_1^{\alpha\beta} P_1 \rangle$	κ^{VVP}	$\langle \{V_1^{\mu\nu}, V_2^{\alpha\beta}\} P_1 \rangle$	λ_1^{VVP}
		$\langle V_1^{\mu\nu} V_1^{\alpha\beta} P_2 \rangle$	λ_2^{VVP}
		$\langle V_2^{\mu\nu} V_2^{\alpha\beta} P_1 \rangle$	λ_3^{VVP}
		$\langle \{V_1^{\mu\nu}, V_2^{\alpha\beta}\} P_2 \rangle$	λ_4^{VVP}
		$\langle V_2^{\mu\nu} V_2^{\alpha\beta} P_2 \rangle$	λ_5^{VVP}

$\langle VVP \rangle$ Green Function: Parametrization

- „THS+P"
  - Pion transition form factor reproduces the one of THS!
  - In our notation:

$$\mathcal{F}_{\gamma^*\gamma^*\pi^0}(p^2,q^2) = \frac{N_c}{12\pi^2 F}\frac{M_{V_1}^4 M_{V_2}^4}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)}\times$$
$$\times\left[1-\frac{4\pi^2F^2}{N_c}\frac{p^2+q^2}{M_{V_1}^2 M_{V_2}^2}\left(6+\frac{p^2q^2}{M_{V_1}^2 M_{V_2}^2}\right)+\frac{8\pi^2F^2}{N_c}\frac{\alpha_5 p^2 q^2}{M_{V_1}^4 M_{V_2}^4 M_{P_1}^2 M_{P_2}^2}\right]$$

  - Connection with THS:

$$\alpha_5 = \frac{M_{V_1}^4 M_{V_2}^4 M_{P_1}^2 M_{P_2}^2}{(4\pi F)^6}\kappa$$

- Does an addition of other multiplets bring something new?

$\langle VVP \rangle$ Green Function: Parametrization

- „(THS+P)+V+P"
  - General parametrization:

$$\mathcal{F}_{VVP}(p^2,q^2,r^2) = \frac{B_0 F^2}{r^2(r^2-M_{P_1}^2)(r^2-M_{P_2}^2)(r^2-M_{P_3}^2)} \times$$

$$\times \frac{\mathcal{P}_{VVP}(p^2,q^2,r^2)}{(p^2-M_{V_1}^2)(p^2-M_{V_2}^2)(p^2-M_{V_3}^2)(q^2-M_{V_1}^2)(q^2-M_{V_2}^2)(q^2-M_{V_3}^2)}$$

  - $\mathcal{P}_{VVP}(p^2,q^2,r^2)$ is dimension–16 Bose-symmetrical polynomial
  - Satisfying the requirements: $165 \to 40$ parameters
  - Pion transition form factor: 7 parameters, only one for single off-shell

$$\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,-Q^2) =$$

$$= \frac{8\pi^2 F^2 Q^2 \beta_2 + M_{V_1}^2 M_{V_2}^2 M_{V_3}^2 M_{P_1}^2 M_{P_2}^2 M_{P_3}^2 (24\pi^2 F^2 Q^4 + N_c M_{V_1}^2 M_{V_2}^2 M_{V_3}^2)}{12\pi^2 F M_{V_1}^2 M_{V_2}^2 M_{V_3}^2 M_{P_1}^2 M_{P_2}^2 M_{P_3}^2 (Q^2+M_{V_1}^2)(Q^2+M_{V_2}^2)(Q^2+M_{V_3}^2)}$$

  - Parameter β_2 can be determined in several ways
    - Fit onto the BABAR/BELLE/CLEO data set
    - Determination from $\omega \to \pi\gamma$

$\langle VVP \rangle$ Green Function: Pion Transition Form Factor

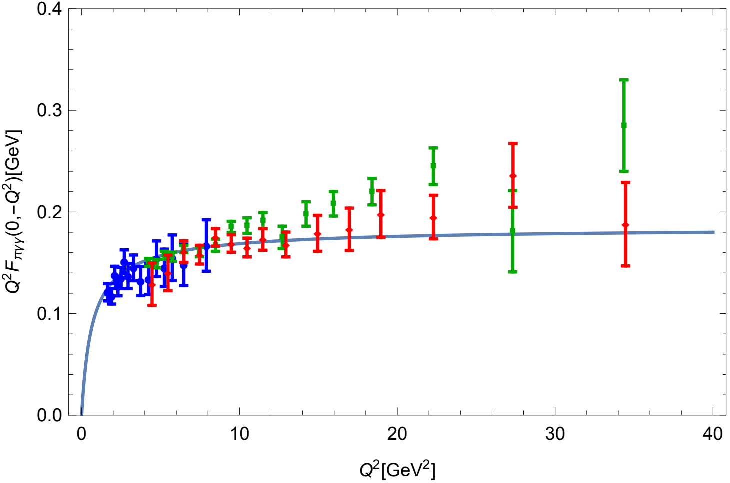


Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,-Q^2)$. Obtained value: $\beta_2 = (1536 \pm 41)\,\mathrm{GeV}^{14}$.

$\langle VVP \rangle$ Green Function: Pion Transition Form Factor

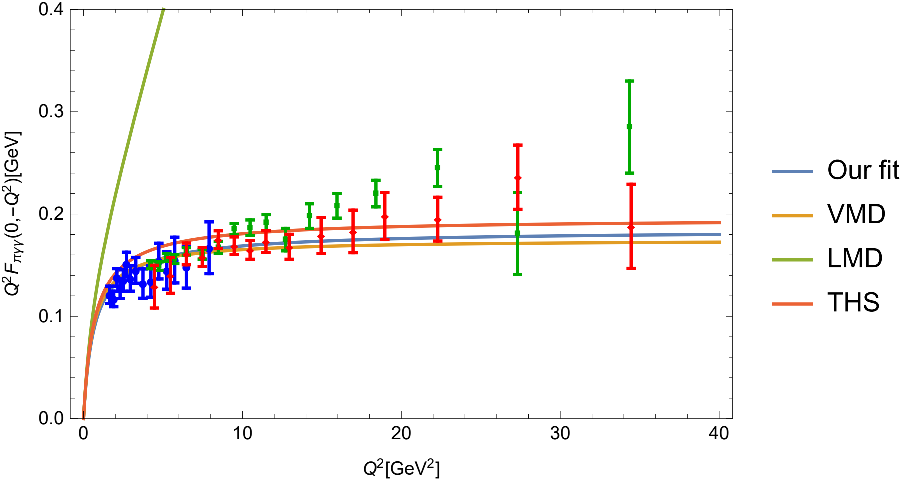


Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data with the formfactor $\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,-Q^2)$ for $\beta_2 = (1536 \pm 41)\,\mathrm{GeV}^{14}$, in comparison with VMD, LMD and THS.

$\langle VVP \rangle$ Green Function: Pion Transition Form Factor

- What about the remaining six parameters? All known! For example from...
  - „Subleading" B–L [V. A. Novikov, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov '84]:

$$\frac{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(-Q^2,-Q^2)}{\mathcal{F}_{\gamma^*\gamma^*\pi^0}(0,0)} = \frac{8\pi^2 F^2}{3}\left(\frac{1}{Q^2} - \frac{8\delta^2}{9Q^4} + \ldots\right), \quad \delta^2 = (0.2 \pm 0.02)\,\mathrm{GeV}^2$$

gives

$$\beta_{24} = -\frac{1}{9} M_{P_1}^2 M_{P_2}^2 M_{P_3}^2 \left(9M_{V_1}^2 + 18M_{V_2}^2 + 18M_{V_3}^2 - 8\delta^2\right)$$

  - Decay $\omega \to \pi^0 \ell^+ \ell^-$ $(\ell = e, \mu)$ gives constraints on β_6 and β_{12}
    - Relevant form factor [T. Husek and S. Leupold '15]:

$$\mathcal{F}_{V\omega\pi^0}(q^2) = \frac{1}{M_\omega F_\omega} \lim_{p^2 \to M_{V_1}^2} \lim_{r^2 \to 0} (p^2 - M_{V_1}^2) r^2 \mathcal{F}_{VVP}(p^2, q^2, r^2)$$

$$\Gamma_{\omega \to \pi^0 \ell^+ \ell^-} = \frac{\alpha^2}{72\pi M_\omega^3} \int_{4m_\ell^2}^{(M_\omega - M_\pi)^2} \frac{|\mathcal{F}_{V\omega\pi^0}(q^2)|^2}{q^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} \left(1 + \frac{2m_\ell^2}{q^2}\right) \lambda^{\frac{3}{2}}(M_\omega^2, M_\pi^2, q^2)\,\mathrm{d}q^2$$

    - Sensitive to uncertainties of $\Gamma_{\omega \to \pi^0 \ell^+ \ell^-}$

Work in progress!

Conclusion

- Contributions of the QCD condensates up to dimension–6 evaluated in the chiral limit for all the Green functions of chiral currents
- Special emphasis on the odd-intrinsic parity sector of QCD
  - Resonance contributions to the $\langle VVP \rangle$, $\langle VAS \rangle$, $\langle AAP \rangle$, $\langle VVA \rangle$, $\langle AAA \rangle$ evaluated with two resonance multiplets of each kind
  - Matching onto „extended“ OPE fulfilled for order parameters of χSB
    - $\langle VVP \rangle$ seems to be the most promising one
    - Several phenomenological applications studied
  - Matching for anomalous GFs is complicated due to the perturbative contribution
    - Logarithmic terms are needed to be taken care of

Thank you for your attention!

See arXiv:2006.13006 [hep-ph] for details, another paper is being finished