Status of ε'/ε in the SM

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What is ε'/ε and why is it interesting?

- ε'/ε constitutes a fundamental test of CP violation!

$$\text{Re}\,(\varepsilon'/\varepsilon)_{\text{SM}} \propto \text{Im}(V_{td} V_{ts}^*) \;, \qquad V_{\text{CKM}} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- ε and ε' parametrize different sources of CP violation in K decays:

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon + \varepsilon', \qquad \eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - 2\,\varepsilon'.$$

- CP violation from K mixing is contained in ε:

$$|\varepsilon| = \tfrac{1}{3}|\eta_{00} + 2\eta_{+-}| = (2.228 \pm 0.011) \cdot 10^{-3}.$$

- ε' is a tinier effect and accounts for direct CP violation in K decays:

$$\text{Re}\,(\varepsilon'/\varepsilon) = \tfrac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.6 \pm 2.3) \cdot 10^{-4}$$

  demonstrates the existence of direct CP violation in K decays.

- Small size of ε' makes it sensitive to new sources of CP violation.

Dynamical features of $K \to \pi\pi$ decay

- **Isospin decomposition:**

$$A[K^0 \to \pi^+\pi^-] = A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}}\, A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \frac{1}{\sqrt{2}}\, (\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^0 \to \pi^0\pi^0] = A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} = \mathcal{A}_{1/2} + \sqrt{2}\, (\mathcal{A}_{3/2} + \mathcal{A}_{5/2})$$

$$A[K^+ \to \pi^+\pi^0] = \frac{3}{2}\, A_2^+\, e^{i\chi_2^+} = \frac{3}{2}\left(\mathcal{A}_{3/2} - \frac{2}{3}\, \mathcal{A}_{5/2}\right).$$

- **Measured $K \to \pi\pi$ branching ratios:**

$$A_0 = (2.704 \pm 0.001) \cdot 10^{-7}\,\text{GeV},\ A_2 = (1.210 \pm 0.002) \cdot 10^{-8}\,\text{GeV},$$

$$\chi_0 - \chi_2 = (47.5 \pm 0.9)^\circ\ .$$

- **What do they tell us?**

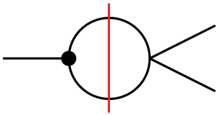
$$\varepsilon' = -\frac{i}{\sqrt{2}}\, e^{i(\chi_2 - \chi_0)}\, \omega\, \frac{\text{Im}A_0}{\text{Re}A_0} \left[1 - \frac{1}{\omega}\, \frac{\text{Im}A_2}{\text{Im}A_0}\right]$$

1. **$\Delta I = 1/2$ rule: $\omega = \frac{\text{Re}A_2}{\text{Re}A_0} \approx 1/22$**
   - ε' is suppressed by ω.
   - Small IB corrections to the ratio $\frac{\text{Im}A_2}{\text{Im}A_0}$ get amplified by $\omega^{-1} \approx 22$.
2. **Strong FSI: $\frac{\text{Abs}(\mathcal{A}_{1/2}/\mathcal{A}_{3/2})}{\text{Dis}(\mathcal{A}_{1/2}/\mathcal{A}_{3/2})} \approx 1 \to$ Absorptive $\sim$ Dispersive.**

Unitarity and Analyticity

1. **Unitarity:** $\delta_0(M_K) = (39.2 \pm 1.5)^\circ \longrightarrow$ $\boxed{A_0 \approx 1.3 \times \mathrm{Dis}(A_0)}$

(Colangelo-Gasser-Leutwyler '01)



$$A_I\, e^{i\,\delta_I} = \mathrm{Dis}(A_I) + i\mathrm{Abs}(A_I)$$

$$\tan\delta_I = \frac{\mathrm{Abs}(A_I)}{\mathrm{Dis}(A_I)}$$

$$A_I = \mathrm{Dis}(A_I)\,\sqrt{1+\tan^2\delta_I}$$

2. **Analyticity:** $\boxed{\mathrm{Dis}(A_I) \propto \mathrm{Abs}(A_I)}$

3. $\boxed{\text{Large } \delta_0 \longrightarrow \text{Large Abs}(A_0) \longrightarrow \text{Large Dis}(A_0)}$

4. **Message:**

   Unitarity and analyticity have to be contained in the theoretical framework, to get reliable predictions of the amplitudes!

Multi-scale framework

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	↓ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\rm QCD}^{(n_f=3)}$, $\mathcal{L}_{\rm eff}^{\Delta S=1,2}$
	↓ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

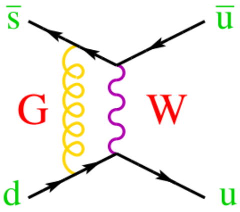
Short-distance description

$\Delta S = 1$ transitions for $K \to \pi\pi$

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} C_i(\mu)\, Q_i(\mu)$$

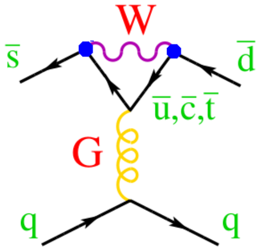
$$C_i(\mu) = z_i(\mu) + \tau\, y_i(\mu)$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$



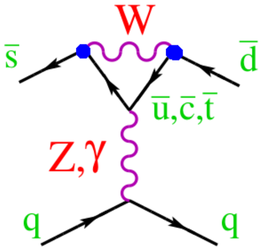
Current - Current operators

$Q_{1,2}$



QCD - Penguins operators

$Q_{3,4,5,6}$



Electroweak Penguins operators

$Q_{7,8,9,10}$

Long-distance description

- χPT is the QFT of QCD at low energies.
- χPT formulation is a consistent theoretical framework to describe pseudoscalar-octet dynamics.
- Perturbative expansion in powers of p^2/Λ_χ^2 where $\Lambda_\chi \sim 1$GeV.
- Chiral symmetry fixes all allowed operators, at given order in p.
- $\mathcal{O}(G_F\, p^2)$: Goldstone Interactions (π, K, η)

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda\, L_\mu\, L^\mu) + G_{27}\, F^4 \left(L_{\mu\,23}\, L_{11}^\mu + \frac{2}{3}\, L_{\mu\,21}\, L_{13}^\mu\right)$$

$$G_{8,27} = -\frac{G_F}{\sqrt{2}}\, V_{ud}\, V_{us}^*\, g_{8,27} \quad ; \quad L_\mu = -i\, U^\dagger D_\mu U \quad ; \quad \lambda_{ij} \equiv \delta_{i3}\delta_{j2} \quad ; \quad U \equiv \exp\{i\sqrt{2}\phi/F\}$$

- Short-distance dynamics encoded in Low-Energy Couplings.

Estimation of LECs

- From phenomenological data or with additional input from theory.
- Principle of calculation LECs: perform a matching between two EFTs.
- Large N_C–limit: T-product of two colour singlet currents factorizes,

$$\langle J \cdot J \rangle = \langle J \rangle \langle J \rangle \left\{ 1 + \mathcal{O}\left(\frac{1}{N_C}\right) \right\}$$

  - $\langle J \rangle$ has a well-known representation in χPT.
  - Matching between EFTs can be done at leading order in $1/N_C$.
- Weak couplings of $\mathcal{O}(G_F\, p^2)$ and $\mathcal{O}(e^2\, G_8\, p^0)$:

$$g_8^\infty = -\frac{2}{5}\, C_1(\mu_{SD}) + \frac{3}{5}\, C_2(\mu_{SD}) + C_4(\mu_{SD}) - 16\, L_5\, B(\mu_{SD})\, C_6(\mu_{SD})$$

$$g_{27}^\infty = \frac{3}{5}\left[C_1(\mu_{SD}) + C_2(\mu_{SD}) \right]$$

$$(e^2\, g_8\, g_{ewk})^\infty = -3\, B(\mu_{SD})\, C_8(\mu_{SD}) - \frac{16}{3}\, B(\mu_{SD})\, C_6(\mu_{SD})\, e^2\, (K_9 - 2\, K_{10})$$

$$B(\mu_{SD}) \equiv \frac{\langle \bar{q} q \rangle}{F_\pi^3} = \left[\frac{M_K^2}{(m_s + m_d)(\mu_{SD}) F_\pi} \right]^2 \left[1 - \frac{16 M_K^2}{F_\pi^2}\, (2 L_8 - L_5) + \frac{8 M_\pi^2}{F_\pi^2}\, L_5 \right]$$

Strong cancellation in simplified analysis

Good numerical approximation to consider only $Q_{6,8}$ operators:

$$\mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.2 \cdot 10^{-3} \left\{ B_6^{(1/2)} \left(1 - \underbrace{\Omega_{\text{eff}}}_{\text{IB}}\right) - 0.48 \, B_8^{(3/2)} \right\}$$

$$\text{IB} \equiv \mathrm{O}[(m_u - m_d)p^2, e^2 p^2]$$

- **Large–N_C limit:**

  $B_6^{(1/2)} = B_8^{(3/2)} = 1 \longrightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 9.0 \cdot 10^{-4} \sim \mathcal{O}(10^{-3})$

- **Buras-Gorbahn-Jäger-Jamin '15:**

  $B_6^{(1/2)} = 0.57, \; B_8^{(3/2)} = 0.76 \rightarrow \mathrm{Re}(\varepsilon'/\varepsilon) \approx 2.4 \cdot 10^{-4} \sim \mathcal{O}(10^{-4})$

Strong cancellation between Q_6 and Q_8!

Anatomy of ε'/ε calculation in χPT

(All operators, not only Q_6 and Q_8)

$$\mathrm{Re}(\varepsilon'/\varepsilon) = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon|}\left[\frac{\mathrm{Im}A_0^{(0)}}{\mathrm{Re}A_0^{(0)}}\,(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2^{\mathrm{emp}}}{\mathrm{Re}A_2^{(0)}}\right]$$

~~Strong cancelation:~~ $Q_6 \uparrow \quad - \quad Q_8 \downarrow \quad \neq 0$

1. $O(p^4)$ χPT Loops: Large correction $\Delta_L \mathcal{A}_n^{(X)} \rightarrow \frac{1}{N_C}\log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3}\times 2$

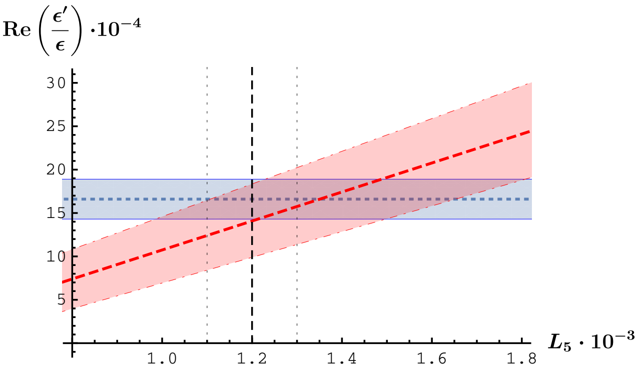
$$\mathcal{A}_n^{(X)} = a_n^{(X)}\left[1 + \Delta_L\mathcal{A}_n^{(X)} + \Delta_C\mathcal{A}_n^{(X)}\right] \quad ; \quad X = 27,\ 8,\ \varepsilon,\ \gamma,\ Z,\ g.$$

$$\Delta_L\mathcal{A}_{1/2}^{(8)} = 0.27 \pm 0.05 + i\,0.47\,, \quad \Delta_L\mathcal{A}_{3/2}^{(g)} = -0.50 \pm 0.20 - i\,0.21\,, \ldots$$

2. $O(p^4)$ LECs fixed at $N_C \rightarrow \infty$: Small correction $\Delta_C \mathcal{A}_n^{(X)}$
3. Isospin Breaking $O[(m_u - m_d)p^2, e^2p^2]$: Sizeable correction

   $\Omega_{\mathrm{eff}} = (11 \pm 9)\cdot 10^{-2}$ (V.Cirigliano-HG-A.Pich-A.Rodríguez-Sánchez '19)
4. $\mathrm{Re}\,g_8$ and $\mathrm{Re}\,g_{27}$ fixed from phenomenological fit.

SM prediction of Re $(\varepsilon'/\varepsilon)$ V.Cirigliano-HG-A.Pich-A.Rodríguez-Sánchez '19



$$\mathrm{Re}\,(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (14 \pm 5)\cdot 10^{-4}$$

in good agreement with the experimental value!

$$\mathrm{Re}\,(\varepsilon'/\varepsilon)|_{\mathrm{exp}} = (16.6 \pm 2.3)\cdot 10^{-4}$$

Recent lattice prediction of ε'/ε arXiv:2004.09440

Excellent complementarity between Lattice and χPT!

Lattice arXiv:2004.09440

- Unitarity & analyticity ✓
- All $1/N_C$ corr. included ✓
- IB corr. not included ✗
- Lattice prediction:

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{Lattice}} = (21.7 \pm 8.4) \cdot 10^{-4}$$

χPT arXiv:1911.01359

- Unitarity & analyticity ✓
- All $1/N_C$ corr. included, except in matching ✗
- IB corr. included ✓
- χPT prediction:

$$\text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\chi\text{PT}} = (13.8 \pm 4.5) \cdot 10^{-4}$$

What happens if we include IB corrections from χPT in Lattice?

$$\Omega_{\text{eff}} = 0.11 \to \text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{SM}} = 16.7 \cdot 10^{-4} \quad \text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

Future directions and improvements

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{SM}} = \left(13.8\,^{+0.5}_{-0.4}\,{}_{m_s}\,^{+1.7}_{-1.3}\,{}_{\mu_{\mathrm{SD}}}\,^{+3.1}_{-3.2}\,{}_{\nu_\chi} \pm 1.3_{\gamma_5} \pm 2.1_{L_{5,8}} \pm 1.3_{L_7} \pm 0.2_{K_i} \pm 0.3_{X_i}\right) \cdot 10^{-4}$$

Summary: $$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)_{\mathrm{SM}} = \left(13.8 \pm 1.3_{\gamma_5} \pm 2.5_{\mathrm{LECs}} \pm 3.5_{\frac{1}{N_C}}\right) \cdot 10^{-4}$$

- **Close to be completed:**
  - **Wilson coefficients at NNLO** $(\pm 1.2_{\gamma_5})$ (Cerdà et al)
- **Future directions:**
  - **Improved lattice input** $(\pm 2.5_{\mathrm{LECs}})$ Expected
  - **g_8 and higher-order LECs at NLO** $(\pm 3.5_{\frac{1}{N_C}})$ New ideas

Best strategy:

χPT (amplitudes) + Lattice (LECs)

Thank you for your attention!