Two-loop calculation of the nucleon self-energy

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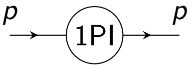
Nucleon self-energy

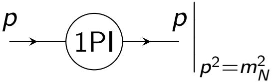
$\Sigma(p) =$ 

- Lattice Calculation (QCD) $\rightarrow$ unphysically high quark masses
- EFTs: study low-energy QCD and its dependence on the pion mass M^2 around the chiral limit
  - HBChPT
    $\rightarrow$ non-relativistic (convergence problems)
  - IR (ChPT)
    $\rightarrow$ taking only the IR part (unphysical cuts)
  - EOMS (ChPT)
    $\rightarrow$ preserves proper analytic structures of the Green's functions

Nucleon self-energy

The correction to the **nucleon mass** is given by the **self energy** Σ.

$$m_N = m^{\text{Bare}} + \Sigma(p^2 = m_N^2) = m^{\text{Bare}} +$$



Calculate $\Sigma(p)$ up to $\mathcal{O}(q^6)$ = including all two-loop diagrams
& solve for the physical nucleon mass m_N

- [Schindler et al., Nuc.Phys.A 803.1, 2008] with IR and $1/m$ expansion:

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\frac{M}{\mu} + k_4 M^4 + k_5 M^5 \ln\frac{M}{\mu} + k_6 M^5 + k_7 M^6 \ln\frac{M}{\mu} + k_8 M^6 \ln^2\frac{M}{\mu} + k_9 M^6$$

  with $k_i(m, g_A, F, l_3, l_4, c_{1-4}, d_{16}, d_{18}, e_i, \hat{g}_1)$

- With EOMS [Fuchs et al. Phys.Rev.D 68, 056005, 2003] we successfully renormalized as $1/m$ expansion

$$0 = \Sigma(p^2 = m_N^2) + \delta m$$

Chiral Perturbation Theory

- Quantum Chromodynamics (QCD) $\hat{=}$ strong interaction
- Chiral Perturbation Theory (ChPT) = EFT for low energies

Chiral Lagrangian up to chiral order $\mathcal{O}(q^4)$ in SU(2)
[Fettes et al., APhy 283.2, 2000]:

$$\mathcal{L} = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)}$$

$$\mathcal{L}_\pi^{(2)} = -\frac{1}{2} M^2 \vec{\pi} \cdot \vec{\pi} + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{8\alpha - 1}{8F^2} M^2 \left(\vec{\pi} \cdot \vec{\pi}\right)^2 + \cdots + O(\pi^6)$$

$$\mathcal{L}_\pi^{(4)} = -\frac{(l_3 + l_4) M^4}{F^2} \left(\vec{\pi} \cdot \vec{\pi}\right) + \cdots + O(\pi^3)$$

$$\mathcal{L}_{\pi N}^{(1)} = -\bar{\Psi} m \Psi + i \bar{\Psi} \partial\!\!\!/ \Psi + \frac{g_A}{2F} \bar{\Psi} \gamma_5 \vec{\tau} \cdot \partial\!\!\!/ \vec{\pi} \Psi + \cdots + O(\pi^5)$$

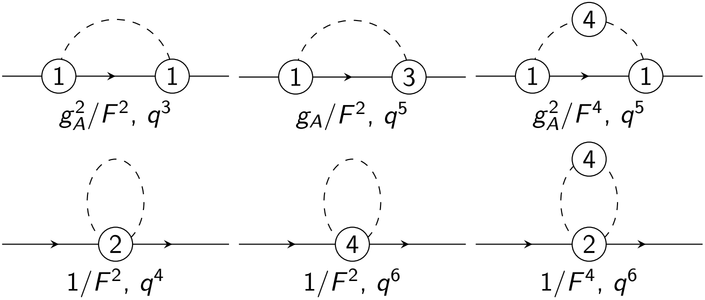
$$l_i \in \mathcal{L}_\pi^{(4)}, \quad c_i \in \mathcal{L}_{\pi N}^{(2)}, \quad d_i \in \mathcal{L}_{\pi N}^{(3)}, \quad e_i \in \mathcal{L}_{\pi N}^{(4)}$$

with $M/m < 1$ ($M = \mathcal{O}(q^1)$, $m = \mathcal{O}(\Lambda)$, $q/\Lambda < 1$)

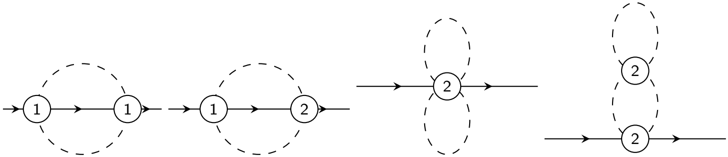
Diagrams - tree level and one-loop

Tree level

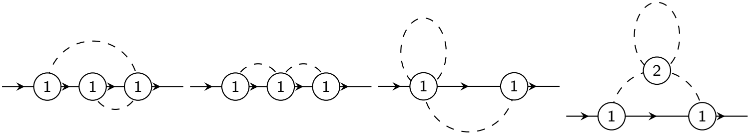
One-loop



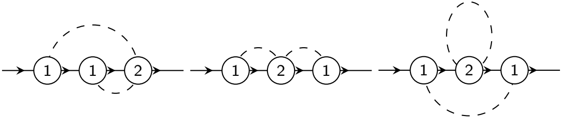
Diagrams - two-loop



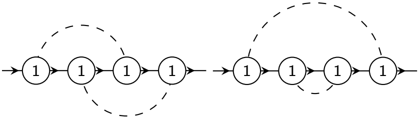
$\frac{c_i^{0-1}}{F^4}$, q^{5-6}



$\frac{g_A^2}{F^4}$, q^5



$c_i \frac{g_A^2}{F^4}$, q^6

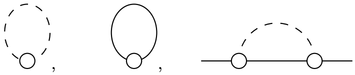


$\frac{g_A^4}{F^4}$, q^5

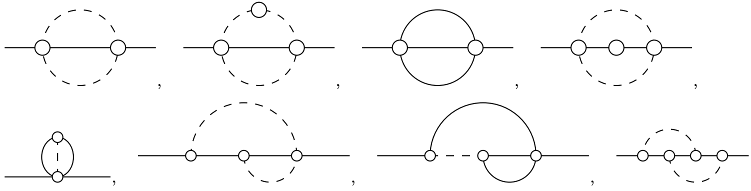
Reduction to master integrals

1. Feynman rules give mathematical expressions for the diagrams
2. Add suitable zeros to reduce the tensor rank of the integrals
3. Reduce to scalar integrals in $d + \ldots$ dimensions [Tarasov, NPhyB 502.1, 1996].
4. Express with a set of "Master Integrals" (TARCER [Mertig and Scharf, CPhyC 111.1, 1998])

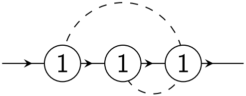
$T^{(1)}_{\pi}$, $T^{(1)}_{N}$, $T^{(1)}_{\pi N}$:



$T^{(2)}_{\text{I}}$ - $T^{(2)}_{\text{VIII}}$:



Reduction to master integrals: two-loop (d)+(e)



$$-i\left(\Sigma_d^{(2)}+\Sigma_e^{(2)}\right)=-\frac{ig_A^2(T_\pi^{(1)})^2\left((2d-3)m^2-(d-2)M^2\right)}{2(3d-4)F^4m}-\frac{3ig_A^2m(T_N^{(1)})^2}{F^4}$$

$$+\frac{ig_A^2T_N^{(1)}T_\pi^{(1)}\left(4(2d-3)m^2-(d-2)M^2\right)}{2(3d-4)F^4m}-\frac{3ig_A^2mM^2T_N^{(1)}T_{\pi N}^{(1)}}{F^4}$$

$$-\frac{ig_A^2T_{\text{I}}^{(2)}\left(\left(8d^2-32d+30\right)m^4+\left(-8d^2+33d-32\right)m^2M^2-\left(d^2-5d+6\right)M^4\right)}{(d-2)(3d-4)F^4m}$$

$$+\frac{4ig_A^2M^2T_{\text{II}}^{(2)}\left(m^2-M^2\right)\left((4d-6)m^2+(d-2)M^2\right)}{(d-2)(3d-4)F^4m}$$

$$-\frac{3ig_A^2mM^2T_{\text{V}}^{(2)}}{F^4}-\frac{3ig_A^2mM^4T_{\text{VI}}^{(2)}}{F^4}$$

Strategy of regions

fundamental papers:

[Gegelia et al., Theor.Math.Phys. 101, 1994]

[Beneke and Smirnov, Nucl.Phys.B 522 321-344, 1998]

- $l_i \sim q$ or $l_i \gg q$
- Taylor series in small scale $T_q \to$ massless propagators
- Solution as chiral expansion in d-dimension (integral products)

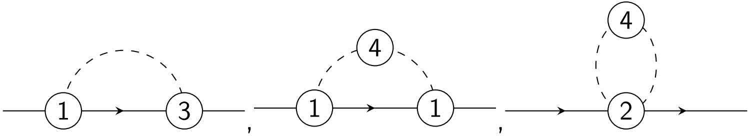
$$T_q\left[\int f(l_1,l_2,q)\mathrm{d}l_1\mathrm{d}l_2\right] = \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2$$
$$+\left(\int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\sim q\\ l_2\gg q}}\mathrm{d}l_1\mathrm{d}l_2 + \int T_q\left[f(l_1,l_2,q)\right]_{\substack{l_1\gg q\\ l_2\sim q}}\mathrm{d}l_1\mathrm{d}l_2\right) = S + R + M.$$

- For $l_i \sim q$ substituting $l_i^\nu \to Mq_i^\nu$

$$\frac{1}{(2\pi)^{2d}}M^{2d-2\alpha-2\beta-\gamma}\int\int \mathrm{d}^d q_1 \mathrm{d}^d q_2 \Big\{\left[q_1\cdot q_1 - 1\right]^\alpha\left[q_2\cdot q_2-1\right]^\beta$$
$$\times\left[2\,(p)\cdot(q_1+q_2)+(q_1+q_2)\cdot(q_1+q_2)\,M+\frac{p\cdot p-m^2}{M}\right]^\gamma\Big\}^{-1}$$

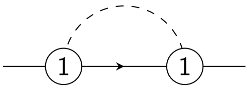
Renormalization - bare parameter shifts

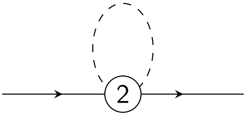
- Remove diagrams



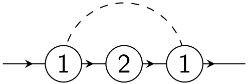
$$g_A^{\mathrm{B}} \to g_A^{\mathrm{B}} - 2(M^{\mathrm{B}})^2(2d_{16}^{\mathrm{B}} - d_{18}^{\mathrm{B}})$$

$$(M^{\mathrm{B}})^2 \to (M^{\mathrm{B}})^2 - \frac{2l_3^{\mathrm{B}}(M^{\mathrm{B}})^4}{(F^{\mathrm{B}})^2} \text{ and } F^{\mathrm{B}} \to F^{\mathrm{B}} - \frac{l_4^{\mathrm{B}}(M^{\mathrm{B}})^2}{F^{\mathrm{B}}}$$

in diagrams 

and 

- Remove nucleon contact interaction



$+ \dots$

Renormalization - PCB and divergent

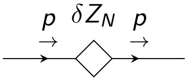
- EOMS shifts from [Siemens et al., PhyRevC 94, 014620, 2016]

$$m^{\mathrm{B}} = m_N + \hbar\delta m^{(1)} + \hbar^2\delta m^{(2)},$$

$$\delta m^{(1)} = \frac{3ig_A^2 m_N M^2 T_{\pi N}^{(1);\mathrm{div}+R}(m_N^2)}{2F^2} + \frac{3ig_A^2 m_N T_N^{(1)}}{2F^2} - \frac{3c_2}{128\pi^2 F^2}M^4 - \frac{iM^2 T_\pi^{(1);\mathrm{div}}(24c_1 - 3(c_2 + 4c_3))}{4F^2}$$

$\delta m^{(2)} =$ divergent and PCB terms that are analytic (in M^2)

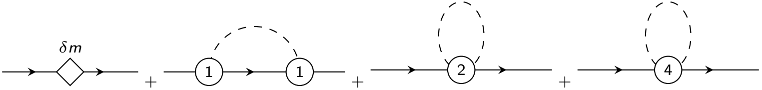
- Other corrections contributing: δM, δF, δg_A, δc_i, δe_i, δZ_N, no δZ_π
- Take only IRR and div part of the integrals
- Also no contribution as tree diagram:



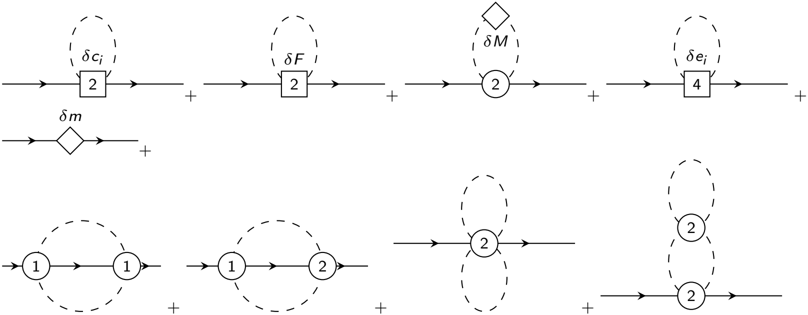
$$\hat{=} \quad i\delta Z_N \left(\not{p} - m^{\mathrm{R}}\right)$$

Renormalization - PCB and divergent

$1/F^2$ and g_A^2/F^2

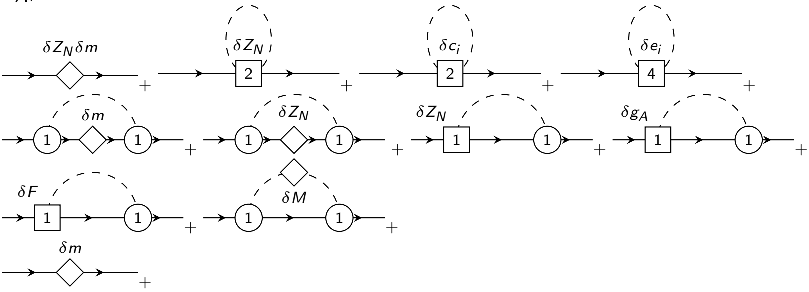


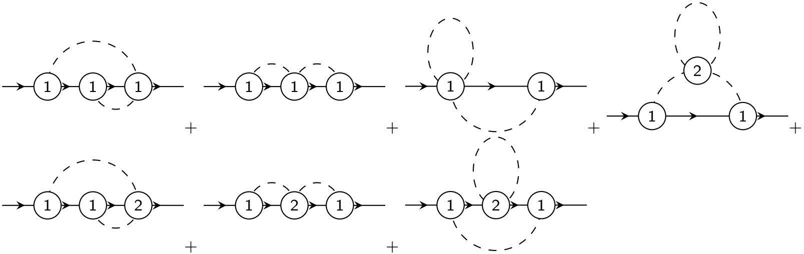
$1/F^4$



Renormalization - PCB and divergent

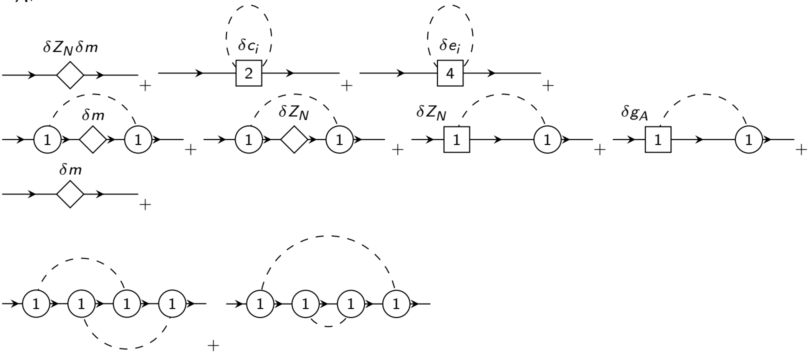
g_A^2/F^4





Renormalization - PCB and divergent

g_A^4/F^4



Renormalized nucleon self-energy - first result

$\Sigma(p^2 = m_N^2) + \delta m$ as $1/m$ expansion (in EOMS renormalization):

$$
\frac{3c_1 M^4 \ln(M/m_N)}{4\pi^2 F^2} - \frac{3c_2 M^4 \ln(M/m_N)}{32\pi^2 F^2} - \frac{3c_3 M^4 \ln(M/m_N)}{8\pi^2 F^2}
$$

$$
+ \frac{M^6}{32\pi^2 F^2}\Big\{ - 12 \ln(M/m_N) \text{ Sum of } e_i + \text{ Sum of } e_i \Big\}
$$

$$
- \frac{3g_A^2 M^3}{32\pi F^2} + \frac{3g_A^2 M^4}{32\pi^2 F^2 m_N} - \frac{3g_A^2 M^4 \ln(M/m_N)}{32\pi^2 F^2 m_N} + \frac{3g_A^2 M^5}{256\pi F^2 m_N^2} - \frac{g_A^2 M^6}{128\pi^2 F^2 m_N^3}
$$

$$
- \frac{M^6}{12288\pi^4 F^4 m_N}\Big\{ - 144 m_N (6c_1 - c_2 - 4c_3) \ln^2(M/m_N) - 144 m_N (2c_1 - c_3) \ln(M/m_N)
$$

$$
-24\left(6+\pi^2\right) c_1 m_N + 18 c_2 m_N + 3\pi^2 c_2 m_N + 72 c_3 m_N + 12\pi^2 c_3 m_N + 32 c_4 m_N + 60\pi^2 c_4 m_N - 30\Big\}
$$

$$
- \frac{9g_A^2 M^5 \ln(M/m_N)}{1024\pi^3 F^4} - \frac{g_A^2 M^6}{64\pi^2 F^4 m_N} + \frac{9g_A^2 M^6}{256\pi^4 F^4 m_N} - \frac{3g_A^2 M^6 \ln^2(M/m_N)}{256\pi^4 F^4 m_N} - \frac{5g_A^2 M^6 \ln(M/m_N)}{1024\pi^4 F^4 m_N}
$$

$$
+ \frac{g_A^2 M^6}{18432\pi^4 F^4}\Big\{3\pi^2(36c_1 + 5c_2 + 119c_3 - 226c_4)
$$

$$
+162 \ln(M/m_N)\,(3(8c_1 - c_2 - 4c_3)\ln(M/m_N)) + 2592c_1 - 438c_2 - 398c_3 + 2140c_4\Big\}
$$

$$
+ \frac{21 g_A^4 M^5 \ln(M/m_N)}{1024\pi^3 F^4} - \frac{g_A^4 M^5}{128\pi^3 F^4} - \frac{g_A^4 M^6\left(2592 \ln^2(M/m_N) + 864 \ln(M/m_N) + 239\left(8 + 3\pi^2\right)\right)}{98304\pi^4 F^4 m_N}
$$

Summary and Outlook

Summary

- Method for two-loop calculation
- EOMS applied (confirmation)
- Equation for m_N

Outlook:

- Solve $1/m$ result (numerically) for the nucleon mass
- Solve full EOMS result numerically
- Comparison with HB, IR and Lattice