HVP contribution to $g - 2$

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MH, Hoid, Kubis JHEP 08 (2019) 137, EPJC 80 (2020) 988

Colangelo, MH, Stoffer JHEP 02 (2019) 006, PLB 814 (2021) 136073

Colangelo, MH, Kubis, Niehus, Ruiz de Elvira 2110.05493

Hadronic vacuum polarization from e^+e^- data

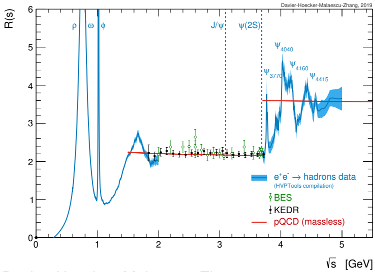
HVP from e^+e^- data

$$a_\mu^{\text{HVP, LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{s_{\text{thr}}}^{\infty} ds \frac{\hat{K}(s)}{s^2} R_{\text{had}}(s) \qquad R_{\text{had}}(s) = \frac{3s}{4\pi\alpha^2}\sigma(e^+e^- \to \text{hadrons}(+\gamma))(s)$$

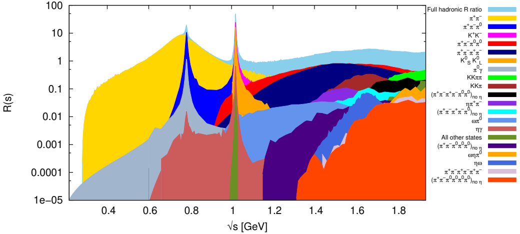
$$= 6931(40) \times 10^{-11}$$

- The "theory" prediction a_μ^{SM} is actually **based on experiments** (ISR, direct scan)
  $\hookrightarrow$ propagation of experimental uncertainties
- Uncertainty estimate includes Aoyama et al. 2020:
  - different methodologies for the combination of data sets Davier et al. 2019, Keshavarzi et al. 2020
  - conservative estimate of systematic errors from tensions in the data
  - cross checks from analyticity/unitarity constraints Colangelo et al. 2018, Ananthanarayan et al. 2018, Davier et al. 2019, MH et al. 2019
  - full NLO radiative corrections Campanario et al. 2019

Hadronic vacuum polarization from e^+e^- data



Davier, Hoecker, Malaescu, Zhang 2019



Keshavarzi, Nomura, Teubner 2018

- Decades-long effort to measure e^+e^- cross sections
  - up to about 2 GeV: sum of exclusive channels
  - above: inclusive data + narrow resonances + pQCD
- What does this have to do with **chiral dynamics**?
  - Unitarization of **$\pi\pi$ scattering** gives the ρ resonance Talk by M. Niehus, We 22:50
  - 3π, $\pi^0\gamma$ channels constrained by **chiral anomaly** Talk by B.-L. Hoid on $\pi^0 \to e^+e^-$, Tu 23:50

- $\sigma(e^+e^- \to 3\pi)$ determined by $\gamma^* \to 3\pi$ matrix element

$$\langle 0|j_\mu(0)|\pi^+(p_+)\pi^-(p_-)\pi^0(p_0)\rangle = -\epsilon_{\mu\nu\rho\sigma}\, p_+^\nu p_-^\rho p_0^\sigma \mathcal{F}(s,t,u;q^2)$$

- Constrained by **chiral anomaly**

$$\mathcal{F}(0,0,0;0) = F_{3\pi} = \frac{1}{4\pi^2 F_\pi^3} = 32.23(10)\,\text{GeV}^{-3}$$

$\hookrightarrow$ absorbs dominant chiral corrections into F_π (same as for $\pi^0 \to \gamma\gamma$)

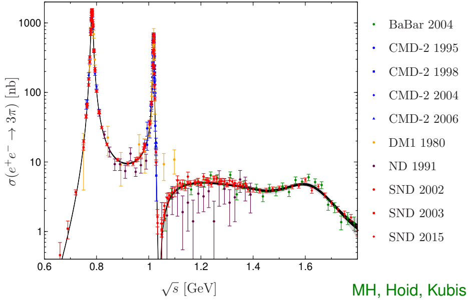
- For physical M_π: further quark-mass renormalization $\simeq 7\%$ Bijnens et al. 1989
- Can implement **unitarity/analyticity** constraints by Khuri–Treiman methods

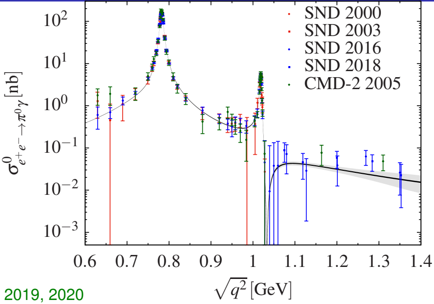
$\hookrightarrow$ Talks by T. Isken, H. Akdag, E. Passemar, M. Niehus, D. Stamen

- Accounts for $\pi\pi$ rescattering, but not for 3π unitarity
- $\sigma(e^+e^- \to \pi^0\gamma)$ determined by $\gamma^* \to \pi^0\gamma$ matrix element

$\hookrightarrow$ also $\pi^0 \to \gamma\gamma$ anomaly plays a role

Hadronic vacuum polarization: 3π, $\pi^0\gamma$ channels





MH, Hoid, Kubis 2019, 2020

- **Normalization** $a(q^2)$ parameterized as

$$a(q^2) = \underbrace{\alpha_A}_{\text{chiral anomaly}} + \underbrace{\frac{q^2}{\pi}\int_{s_{\text{thr}}}^{\infty} ds' \frac{\text{Im}\,\mathcal{A}(s')}{s'(s'-q^2)}}_{\omega,\,\phi\text{ resonances}} + \underbrace{C_p(q^2)}_{\text{conformal polynomial}}$$

$\hookrightarrow \alpha_A = \frac{F_{3\pi}}{3}(1 + \text{quark-mass corrections})$ determines low-energy cross section

- Fills the gap in the **threshold region** where data are scarce
- Similarly for $e^+e^- \to \pi^0\gamma$

How well do we understand the anomalies?

- **$\pi^0\gamma\gamma$ anomaly** Talks by K. Kampf, I. Larin

$$F_{\pi\gamma\gamma}\big|_{\text{LET}} = \frac{1}{4\pi^2 F_\pi} = 0.2745(3)\,\text{GeV}^{-1} \qquad F_{\pi\gamma\gamma}\big|_{\text{PrimEx}} = 0.2754(21)\,\text{GeV}^{-1}$$

$\hookrightarrow$ tested at 0.8% experimentally

- **$3\pi\gamma$ anomaly** Talk by M. Niehus

$$F_{3\pi}\big|_{\text{LET}} = 32.23(10)\,\text{GeV}^{-3}$$

$$F_{3\pi}\big|_{\gamma\pi^-\to\pi^-\pi^0} = 35.3(4.0)\,\text{GeV}^{-3} \qquad F_{3\pi}\big|_{\pi^- e^-\to\pi^- e^-\pi^0} = 31.7(3.6)\,\text{GeV}^{-3}$$

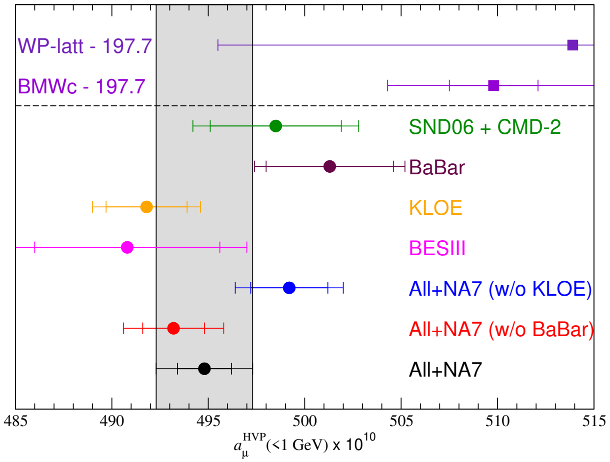
$\hookrightarrow$ only tested at 10% level!

- Ways to improve
  - Analysis of $\gamma\pi^- \to \pi^-\pi^0$ with $\rho(770)$ as lever MH, Kubis, Sakkas 2012, MH, Kubis, Zanke 2017

    $\hookrightarrow$ **COMPASS Primakoff program**
  - **Lattice QCD** Briceño et al. 2016, Alexandrou et al. 2018

    $\hookrightarrow$ need chiral extrapolation

Hadronic vacuum polarization: 2π channel



Assumption: suppose all changes occur in 2π channel below 1 GeV

$\hookrightarrow a_\mu^{\text{total}}[\text{WP20}] - a_\mu^{2\pi,<1\text{ GeV}}[\text{WP20}] = 197.7 \times 10^{-10}$

- **Chiral extrapolation** part of systematic error budget

  $\hookrightarrow$ extrapolation to (or interpolation around) physical quark masses

- Biggest contribution from ***I* = 1 *ud* isospin-symmetric correlator**

  $\hookrightarrow$ phenomenologically dominated by 2π channel, first correction from 4π

- ChPT not enough Golterman, Maltman, Peris 2017

$$a_\mu^{I=1} = \frac{\alpha^2}{24\pi^2}\bigg(-\log\frac{M_\pi^2}{m_\mu^2} - \frac{31}{6} + 3\pi^2\sqrt{\frac{M_\pi^2}{m_\mu^2}} + \mathcal{O}\bigg(\frac{M_\pi^2}{m_\mu^2}\log^2\frac{M_\pi^2}{m_\mu^2}\bigg)\bigg)$$

  $\hookrightarrow$ "convergence" in M_π/m_μ

- Need to provide information on the $\rho(770)$ resonance

  $\hookrightarrow$ **inverse-amplitude method at two-loop order** Talk by M. Niehus

- Decomposition of **pion form factor**

$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{elastic } \pi\pi \text{ scattering}} \times \underbrace{G_\omega(s)}_{\text{isospin-breaking } 3\pi \text{ cut}} \times \underbrace{G_{\text{in}}(s)}_{\text{inelastic effects: } 4\pi, \ldots}$$

- Omnès factor

$$\Omega_1^1(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty \mathrm{d}s' \frac{\delta_1^1(s')}{s'(s'-s)}\right\}$$

$\hookrightarrow$ can get **pion-mass dependence from IAM** Guo et al. 2009

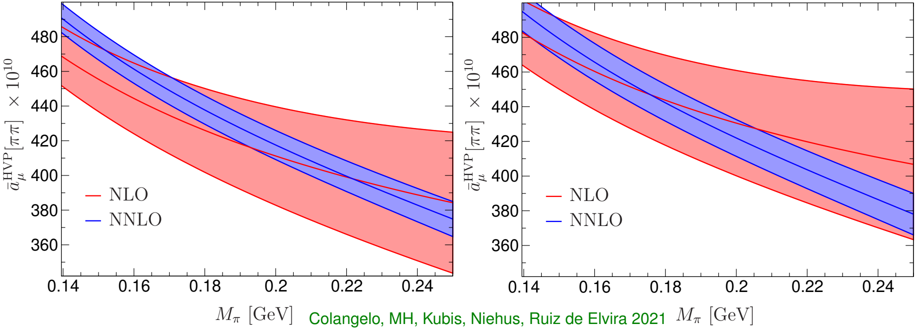
- $G_\omega(s)$ does not contribute to $I=1$ correlator
- $G_{\text{in}}(s)$ parameterized as normal or conformal polynomial

$\hookrightarrow$ free parameters can be matched to $\langle r_\pi^2\rangle$ (and c_π)

- Pion-mass dependence of $\langle r_\pi^2\rangle$ at two loops known Bijnens, Colangelo, Talavera 1998

$\hookrightarrow$ new LEC r_{V1}^r

Predicting the pion-mass dependence from the IAM



Colangelo, MH, Kubis, Niehus, Ruiz de Elvira 2021

- Free parameters:
  - LECs in $\delta_1^1(s)$: combined fit to data Colangelo, MH, Stoffer 2019 and lattice Andersen et al. 2019
  - r^r_{V1}: resonance saturation $r^r_{V1} = 2.0 \times 10^{-5}$ in concord with lattice Feng, Fu, Jin 2020
- Check physical point:

$$\bar{a}_\mu^{\rm HVP}[\pi\pi, \leq 1\,\text{GeV}] \equiv a_\mu^{\rm HVP}[\pi\pi, \leq 1\,\text{GeV}]\big|_{\text{no FSR},\, G_\omega = 1} = 486.3(1.4)(2.1) \times 10^{-10}$$

$$\bar{a}_\mu^{\rm HVP}[\pi\pi, \leq 1\,\text{GeV}]\big|^{\rm NLO} = 460.4(0.3)(14.9)(7.2) \times 10^{-10}$$

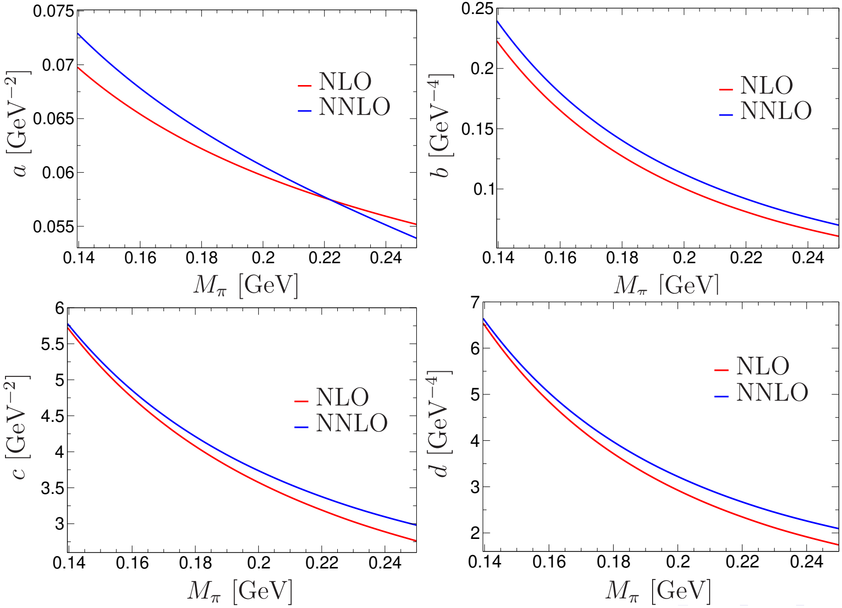
$$\bar{a}_\mu^{\rm HVP}[\pi\pi, \leq 1\,\text{GeV}]\big|^{\rm NNLO} = 482.4(0.1)(0.7)(8.0) \times 10^{-10}$$

Possible application to lattice QCD

1. **Chiral LECs as fit parameters**:
   - Describes $\pi\pi$ physics
   - Need to add $a_\mu^{\text{HVP}}[ud, I=1, \text{non-}\pi\pi] = \zeta + M_\pi^2 \xi$

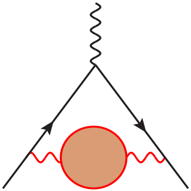
     $\hookrightarrow$ **infrared singularities** will be totally dominated by 2π
   - Can provide **independent constraints from other lattice calculations**: δ_1^1, F_π, $\langle r_\pi^2 \rangle$
2. **Simple parameterizations**:
   - Only possible for integrated HVP or space-like integrand $\frac{\bar{\Pi}(-Q^2)}{Q^2} = \frac{a+bQ^2}{1+cQ^2+dQ^4}$
   - Test infrared singularities Golterman, Maltman, Peris 2017, e.g., M_π^{-2}, $\log M_\pi^2$
   - Fits to $\{a, b, c, d\}$ indicate singularity as strong as M_π^{-2} in $[0.14, 0.25]$ GeV
   - **Purely empirical finding**, no analytic approximation to full IAM nor true chiral behavior
   - Could help inform lattice fits

Possible application to lattice QCD

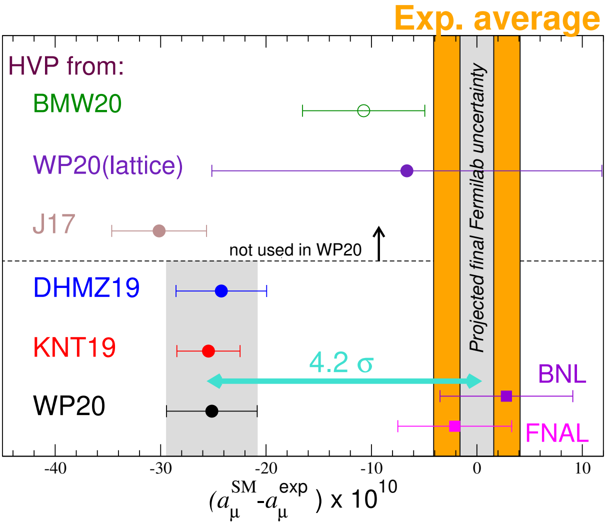


Conclusions

- Control over **HVP contribution** to $g-2$ critical for interpretation of Fermilab E989 experiment
- What can we say from **chiral symmetry**?
  - Chiral anomaly constrains threshold region in $e^+e^- \to 3\pi, \pi^0\gamma$
  - Chiral extrapolation via IAM
  - Finite-volume corrections Aubin, Blum, Golterman, Peris 2020



The situation after the Fermilab announcement



Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	Sec. 8	Eq. (8.14)	279(76)	

Table 1: Summary of the contributions to a_μ^{SM}. After the experimental number from E821, the first block gives the main results for the hadronic contributions from Secs. 2 to 5 as well as the combined result for HLbL scattering from phenomenology and lattice QCD constructed in Sec. 8. The second block summarizes the quantities entering our recommended SM value, in particular, the total HVP contribution, evaluated from e^+e^- data, and the total HLbL number. The construction of the total HVP and HLbL contributions takes into account correlations among the terms at different orders, and the final rounding includes subleading digits at intermediate stages. The HVP evaluation is mainly based on the experimental Refs. [37–89]. In addition, the HLbL evaluation uses experimental input from Refs. [90–109]. The lattice QCD calculation of the HLbL contribution builds on crucial methodological advances from Refs. [110–116]. Finally, the QED value uses the fine-structure constant obtained from atom-interferometry measurements of the Cs atom [117].

Breakdown of the HVP error

HVP from e^+e^- data

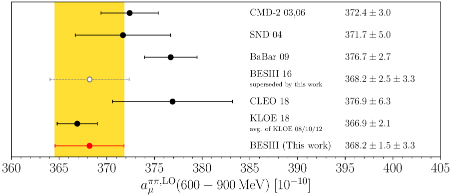
$$a_\mu^{\text{HVP, LO}} = 6931(28)_{\text{exp}}(28)_{\text{sys}}(7)_{\text{DV+QCD}} \times 10^{-11}$$

- DV+QCD: comparison of inclusive data and pQCD in transition region
- Sensitivity of the data is better than the quoted error

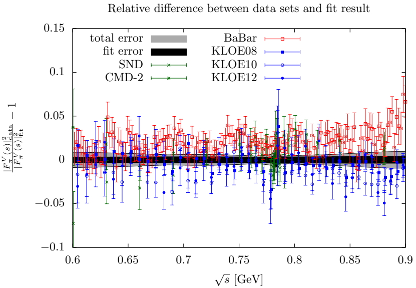
  $\hookrightarrow$ would get $4.2\sigma \to 4.8\sigma$ when ignoring additional systematic error
- There was broad consensus to adopt **conservative error estimates**

  $\hookrightarrow$ **merging procedure** in WP20 covers tensions in the data and different methodologies for the combination of data sets
- Systematic effect dominated by [fit w/o KLOE - fit w/o BaBar]/2

Cross checks from analyticity and unitarity



BESIII 2009.05011



Colangelo, MH, Stoffer 2018

- For "simple" channels $e^+e^- \to 2\pi, 3\pi$ can derive form of the cross section from **general principles of QCD** (analyticity, unitarity, crossing symmetry)
  $\hookrightarrow$ strong cross check on the data sets (covering about 80% of HVP)
- Uncovered an error in the covariance matrix of BESIII 16 (now corrected), all other data sets passed the tests