Explicit renormalization of the nucleon-nucleon interaction in chiral EFT and non-perturbative effects

A. M. Gasparyan, Ruhr-Universität Bochum

in collaboration with E. Epelbaum

November 16, 2021, Chiral Dynamics 2021, Beijing

Chiral EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Consistent with symmetries

Chiral EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Consistent with symmetries

↓

“Perturbative” calculation of the S-matrix, spectrum, etc.

Chiral EFT

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)} + \ldots$$

Consistent with symmetries

↓

"Perturbative" calculation of the S-matrix, spectrum, etc.

expansion parameter: $Q = \dfrac{q}{\Lambda_b}$

$$q \in \{|\vec{p}|\,, M_\pi\}\,, \qquad \Lambda_b \sim M_\rho$$

Weinberg power counting for NN-interaction

Weinberg, S., **NPB363**, 3 (1991)

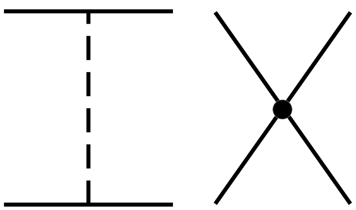
For potential (2N-irreducible) contributions:

$$D = 2L + \sum_{i=\text{vertices}} \left(d_i + \frac{n_i}{2} - 2 \right)$$

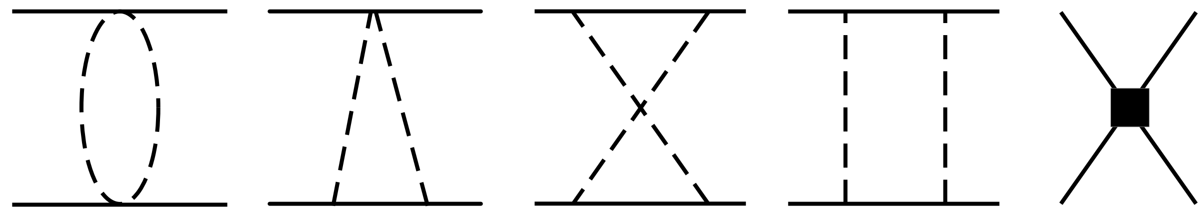
d_i – number of derivatives and quark masses

n_i – number of nucleon fields, L – number of loops

$\mathcal{O}(Q^0)$



$\mathcal{O}(Q^2)$



Weinberg power counting for NN-interaction

Weinberg, S., **NPB363**, 3 (1991)

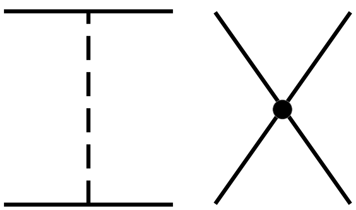
For potential (2N-irreducible) contributions:

$$D = 2L + \sum_{i=\text{vertices}} \left(d_i + \frac{n_i}{2} - 2 \right)$$

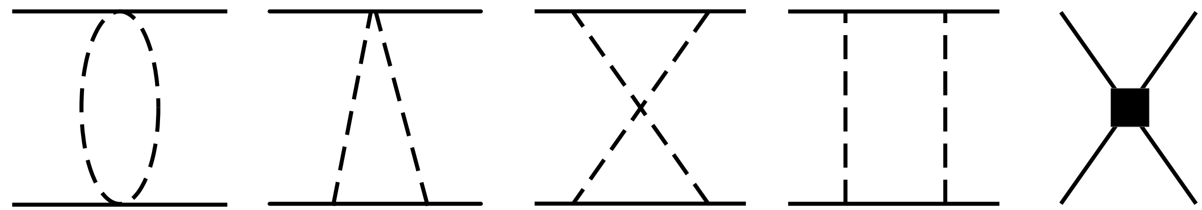
d_i – number of derivatives and quark masses

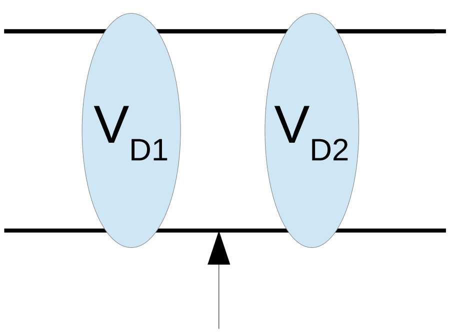
n_i – number of nucleon fields , L – number of loops

$\mathcal{O}(Q^0)$



$\mathcal{O}(Q^2)$





$$\sim \frac{m_N q}{\Lambda_b^2} \sim 1$$

Enhancement due to the infrared singularity: V_0 must be iterated

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \ldots$$

$$T_2 = V_2 + V_2 G V_0 + V_0 G V_2 + V_2 G V_0 G V_0 + \ldots$$

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} \left(V_0 G\right)^m V_2 \left(G V_0\right)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m\, V_2\, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

→ Regulator: cutoff Λ

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m \, V_2 \, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

→ Regulator: cutoff Λ

Finite cutoff: $\Lambda \approx \Lambda_b$

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]} , \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m \, V_2 \, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]} , \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

→ Regulator: cutoff Λ

Finite cutoff: $\Lambda \approx \Lambda_b$

Phenomenological success (NN)

P. Reinert, H. Krebs, and E. Epelbaum, **EPJA54**, 86 (2018), 171
D. R. Entem, R. Machleidt, and Y. Nosyk, **PRC96**, 024004 (2017)

Regularization

Divergent:

$$T_0 = V_0 + V_0 G V_0 + V_0 G V_0 G V_0 + \cdots = \sum_{n=0}^{\infty} T_0^{[n]}\,, \qquad T_0^{[n]} \sim p^n$$

$$T_2 = \sum_{m,n=0}^{\infty} (V_0 G)^m\, V_2\, (G V_0)^n = \sum_{m,n=0}^{\infty} T_2^{[m,n]}\,, \qquad T_2^{[m,n]} \sim p^{m+n+2}$$

→ Regulator: cutoff Λ

Finite cutoff: $\Lambda \approx \Lambda_b$

Phenomenological success (NN)

P. Reinert, H. Krebs, and E. Epelbaum, **EPJA54**, 86 (2018), 171
D. R. Entem, R. Machleidt, and Y. Nosyk, **PRC96**, 024004 (2017)

Consistent with power counting?

Renormalization

Renormalization: power counting in terms of renormalized quantities

$$T_0^{[n]} = V_0(GV_0)^n \sim \mathcal{O}(Q^0)$$

Renormalization

Renormalization: power counting in terms of renormalized quantities

$$T_0^{[n]} = V_0 (G V_0)^n \sim \mathcal{O}(Q^0)$$

$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1} dp}{(\Lambda_{\rm V})^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$$

G. P. Lepage, nucl-th/9706029
J. Gegelia, **JPG25**, 1681 (1999)

Renormalization

Renormalization: power counting in terms of renormalized quantities

$$T_0^{[n]} = V_0(GV_0)^n \sim \mathcal{O}(Q^0)$$

$$\Lambda \approx \Lambda_b : \int \frac{p^{n-1}dp}{(\Lambda_{\rm V})^n} \sim \left(\frac{\Lambda}{\Lambda_V}\right)^n \sim \left(\frac{\Lambda_b}{\Lambda_b}\right)^n \sim \mathcal{O}(Q^0)$$

G. P. Lepage, nucl-th/9706029
J. Gegelia, **JPG25**, 1681 (1999)

Rigorously proven
under rather general conditions
If T_0 converges:

$$\mathrm{T}_0 = \sum_{n=0}^{\infty} T_0^{[n]}$$

AG, E.Epelbaum,
(2021), 2110.15302

Renormalization. NLO

$$T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from momenta: $p \sim \Lambda \,, p' \sim \Lambda$ in $V_2(p', p)$

Renormalization. NLO

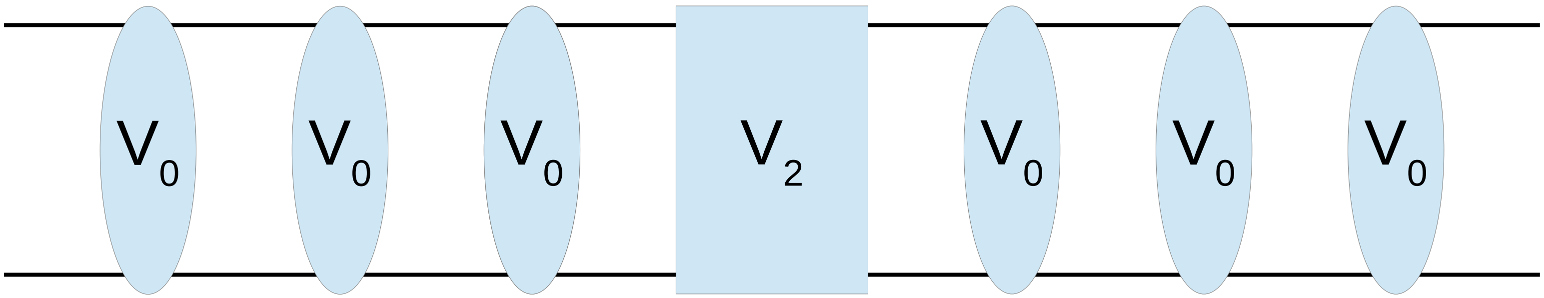
$$T_2^{[m,n]} = (V_0 G)^m \, V_2 \, (G V_0)^n \sim \mathcal{O}(Q^0) \neq \mathcal{O}(Q^2)$$

Power-counting violating contributions from momenta: $p \sim \Lambda \, , p' \sim \Lambda \text{ in } V_2(p', p)$

Can be absorbed by LO contact interactions? $\mathbb{R}\left(T_2^{[m,n]}\right) \sim \frac{q^2}{\Lambda_b^2} \left(\frac{\Lambda}{\Lambda_V}\right)^{m+n}$?

BPHZ subtraction scheme

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



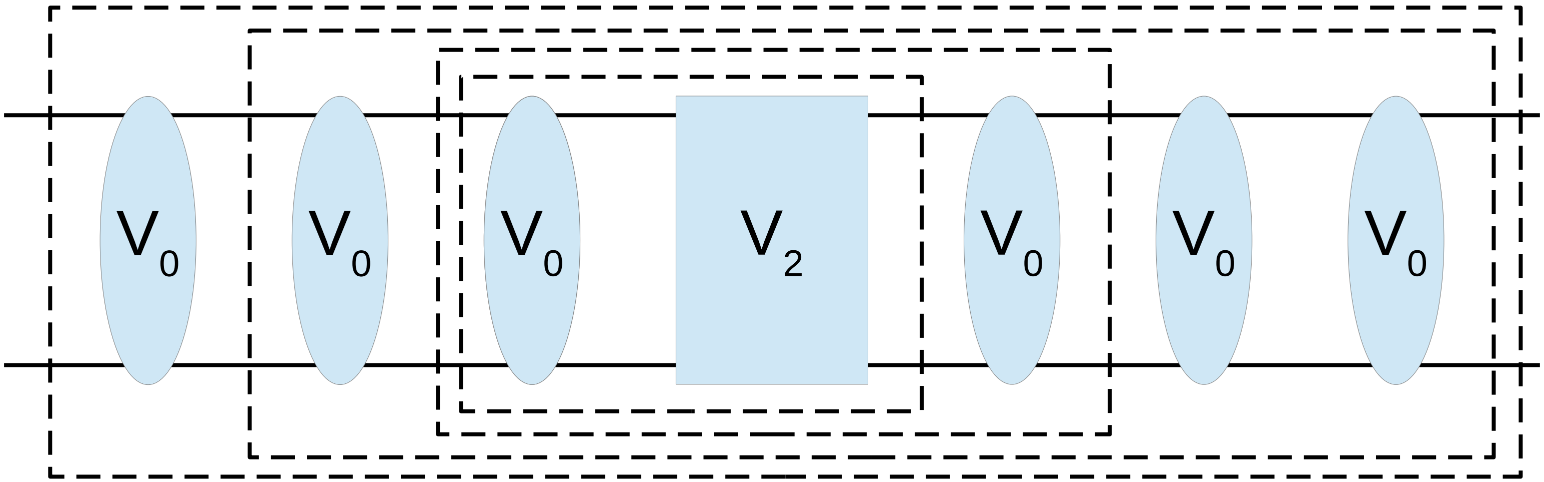
Subtraction operation: $\mathbb{T}(X)(p', p, p_{\text{on}}) = X(p' = 0, p = 0, p_{\text{on}} = 0)$

Renormallized amplitude: $\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}$

$$U_k = ((m_{k,1}, n_{k,1}), (m_{k,2}, n_{k,2}), \ldots), \quad m \geq m_{k,i+1} \geq m_{k,i} \geq 0, \; n \geq n_{k,i+1} \geq n_{k,i} \geq 0 .$$

BPHZ subtraction scheme

N. N. Bogoliubov, O. S. Parasiuk, **AM97**, 227 (1957); K. Hepp, **CMP2**, 301 (1966); W. Zimmermann, **CMP15**, 208 (1969)



Subtraction operation:

$$\mathbb{T}(X)(p', p, p_{\rm on}) = X(p'=0, p=0, p_{\rm on}=0)$$

Renormallized amplitude:

$$\mathbb{R}(T_2^{[m,n]}) = T_2^{[m,n]} + \sum_{U_k \in \mathcal{F}^{m,n}} \prod_{(m_i,n_i)\in U_k} (-\mathbb{T}^{m_i,n_i}) T_2^{[m,n]}$$

$$U_k = ((m_{k,1}, n_{k,1}), (m_{k,2}, n_{k,2}), \dots), \quad m \geq m_{k,i+1} \geq m_{k,i} \geq 0, \; n \geq n_{k,i+1} \geq n_{k,i} \geq 0.$$

Power counting in the perturbative case

AG, E.Epelbaum, (2021), 2110.15302

Convergent series in V_0:

$$\mathbb{R}\left(T_2\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$$

Power counting in the perturbative case

AG, E.Epelbaum, (2021), 2110.15302

Convergent series in V_0 :

$$\mathbb{R}\left(T_2\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)$$

$$\left|\mathbb{R}(T_2^{[m,n]})(p_{\rm on})\right| \leq \frac{8\pi^2 \mathcal{M}_1}{m_N \Lambda_V}\left(\mathcal{M}_2 \frac{\Lambda}{\Lambda_V}\right)^{m+n} \frac{p_{\rm on}^2}{\Lambda_b^2} \log \Lambda / M_\pi$$

$$\mathcal{M}_1\,,\mathcal{M}_2 \sim 1$$

S-waves. Non-perturbative LO. Fredholm formula

$$T_0 = V_0 R = \bar{R} V_0 \qquad R = \frac{1}{\mathbb{1} - G V_0} = \frac{N}{D}\,, \quad \bar{R} = \frac{1}{\mathbb{1} - V_0 G} = \frac{\bar{N}}{D}$$

Convergent series in V_0:

$$N = \sum_{i=0}^{\infty} N_i\,, \quad D = \sum_{i=0}^{\infty} D_i$$

Fredholm determinant contains the non-perturbative dynamics:

$$D = D(p_{\rm on})$$

(Quasi-) bound state:

$$D(p_{\rm on}) \sim \frac{p_{\rm on}}{M_\pi}$$

$$T_0(p_{\rm on}) = \frac{N_0(p_{\rm on})}{D(p_{\rm on})} \sim \mathcal{O}(M_\pi / p_{\rm on})$$

Enhanced at threshold

$$\text{For } p_{\rm on} > M_\pi : \; T_0(p_{\rm on}) \sim \mathcal{O}(Q^0)$$

NLO. Using Fredholm formula.

$$T_2(p_{\rm on}) = (1 + T_0 G) V_2 (1 + G T_0) = \frac{N_2(p_{\rm on})}{D(p_{\rm on})^2}$$

Convergent series in V_0 :

$$N = \sum_{i=0}^{\infty} N_i \,, \; D = \sum_{i=0}^{\infty} D_i$$

The same for the counter terms:

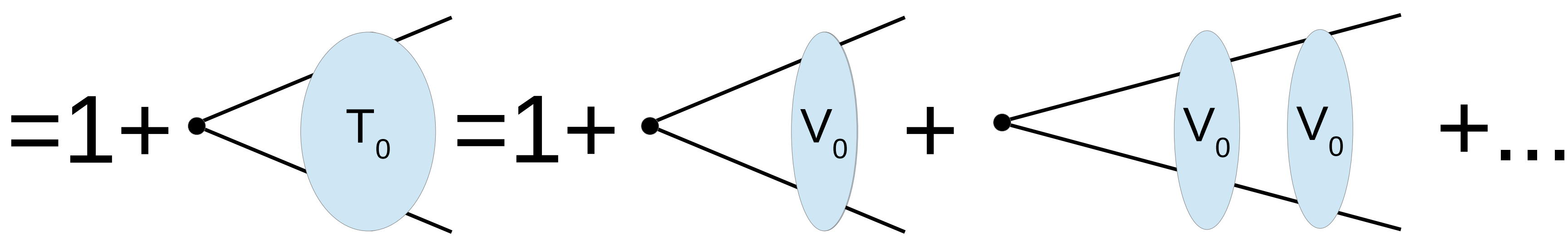
$$\delta T_2 = (1 + T_0 G) \delta V_0^{ct} (1 + G T_0)$$

S-waves. NLO.
Subtractions in the non-perturbative case

The series for ***R($T_2^{[m,n]}$)*** can be summed explicitly

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}\right) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)\left(p_{\rm on}\right) = T_2(p_{\rm on}) - T_2(p_{\rm on}=0)\left[\frac{\psi_{p_{\rm on}}(0)}{\psi_{p_{\rm on}=0}(0)}\right]^2$$

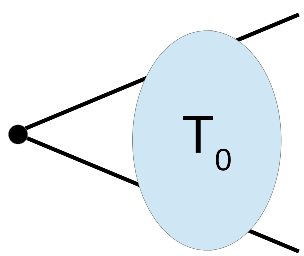
$\psi_{p_{\rm on}}(0)$

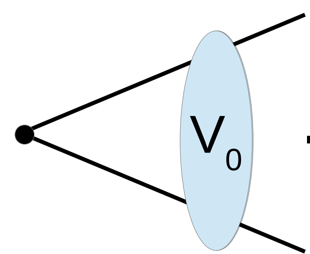


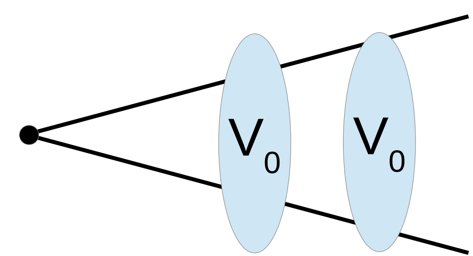
S-waves. NLO.
Subtractions in the non-perturbative case

The series for $R(T_2^{[m,n]})$ can be summed explicitly

$$\mathbb{R}\left(T_2\right)(p_{\rm on}) = \sum_{m,n=0}^{\infty} \mathbb{R}\left(T_2^{[m,n]}\right)(p_{\rm on}) = T_2(p_{\rm on}) - T_2(p_{\rm on}=0)\left[\frac{\psi_{p_{\rm on}}(0)}{\psi_{p_{\rm on}=0}(0)}\right]^2$$

$\psi_{p_{\rm on}}(0)$ =1+ 

=1+ 

\+ 

+...

$$\mathbb{R}\left(T_2\right)(p_{\rm on}=0) = 0$$

Non-perturbative NLO. Renormalizability. Constraint on V_0

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}\right) = \frac{1}{D(p_{\rm on})^2}\left[N_2(p_{\rm on}) - N_2(0)\frac{N_\psi(p_{\rm on})}{N_\psi(0)}\right]$$

$$\psi_{p_{\rm on}}(0) = N_\psi(p_{\rm on})/D(p_{\rm on})$$

Convergent series in V_0:

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\,,\ N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

Non-perturbative NLO. Renormalizability. Constraint on V_0

$$\mathbb{R}\left(T_2\right)(p_{\text{on}}) = \frac{1}{D(p_{\text{on}})^2}\left[N_2(p_{\text{on}}) - N_2(0)\frac{N_\psi(p_{\text{on}})}{N_\psi(0)}\right]$$

$$\psi_{p_{\text{on}}}(0) = N_\psi(p_{\text{on}})/D(p_{\text{on}})$$

Convergent series in V_0:

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\,,\;\; N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

Non-perturbative NLO. Renormalizability. Constraint on V_0

$$\mathbb{R}\left(T_2\right)\left(p_{\rm on}\right) = \frac{1}{D(p_{\rm on})^2}\left[N_2(p_{\rm on}) - N_2(0)\frac{N_\psi(p_{\rm on})}{N_\psi(0)}\right]$$

$$\psi_{p_{\rm on}}(0) = N_\psi(p_{\rm on})/D(p_{\rm on})$$

Convergent series in V_0:

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\,,\ N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

Power counting is restored if

$$N_\psi(0) \neq 0$$

Inside the region: $V_0 \to \lambda V_0\,,\ |\lambda| < 1$

Non-perturbative NLO. Renormalizability. Constraint on V_0

$$\mathbb{R}\,(T_2)\,(p_{\rm on}) = \frac{1}{D(p_{\rm on})^2}\left[N_2(p_{\rm on}) - N_2(0)\frac{N_\psi(p_{\rm on})}{N_\psi(0)}\right]$$

$$\psi_{p_{\rm on}}(0) = N_\psi(p_{\rm on})/D(p_{\rm on})$$

Convergent series in V_0:

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\,,\ N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

Power counting is restored if

$$N_\psi(0) \neq 0$$

Inside the region: $V_0 \to \lambda V_0\,,\ |\lambda| < 1$

Subtraction at p_{on}=0:

$$\mathbb{R}\,(T_2)\,(p_{\rm on}) \sim \frac{p_{\rm on}^2/\Lambda_b^2}{D(p_{\rm on})^2} \sim \mathcal{O}(Q^2)$$

even if

$$D(p_{\rm on}) \sim \frac{p_{\rm on}}{M_\pi}$$

Non-perturbative NLO. Renormalizability. Constraint on V_0

$$\mathbb{R}\left(T_2\right)(p_{\rm on}) = \frac{1}{D(p_{\rm on})^2}\left[N_2(p_{\rm on}) - N_2(0)\frac{N_\psi(p_{\rm on})}{N_\psi(0)}\right]$$

$$\psi_{p_{\rm on}}(0) = N_\psi(p_{\rm on})/D(p_{\rm on})$$

Convergent series in V_0 :

$$N_2 = \sum_{i=0}^{\infty} N_{2;i}\,,\ N_\psi = \sum_{i=0}^{\infty} N_{\psi;i}$$

Power counting is restored if

$$N_\psi(0) \neq 0$$

Inside the region: $V_0 \to \lambda V_0\,,\ |\lambda| < 1$

Subtraction at p_{on}=0: $\mathbb{R}\left(T_2\right)(p_{\rm on}) \sim \frac{p_{\rm on}^2/\Lambda_b^2}{D(p_{\rm on})^2} \sim \mathcal{O}(Q^2)$

even if $D(p_{\rm on}) \sim \frac{p_{\rm on}}{M_\pi}$

More constraints at higher orders!

Cutoff dependence

$$V_0^\Lambda = V_0 F_\Lambda \,, \qquad F_\Lambda = \frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}} \,,\; e^{-\vec{q}^{\,2}/\Lambda^2} \,,\; e^{-(p^2+p'^2)/\Lambda^2}$$

Cutoff dependence

$$V_0^\Lambda = V_0 F_\Lambda \,, \qquad F_\Lambda = \frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}} \,, \; e^{-\vec{q}^{\,2}/\Lambda^2} \,, \; e^{-(p^2+p'^2)/\Lambda^2}$$

Cutoff dependence

$$V_0^\Lambda = V_0 F_\Lambda \,, \qquad F_\Lambda = \frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}} \,, \; e^{-\vec{q}^{\,2}/\Lambda^2} \,, \; e^{-(p^2+p'^2)/\Lambda^2}$$

Perturbation:

$$\delta V_0^\Lambda = V_0 - V_0^\Lambda \sim \mathcal{O}(Q^2)$$

For small momenta
~ like a contact interaction

$$\delta_\Lambda V_0 = V_0^{\Lambda+\delta\Lambda} - V_0^\Lambda \sim \frac{\delta\Lambda}{\Lambda}\mathcal{O}(Q^2)$$

Cutoff dependence

$$V_0^\Lambda = V_0 F_\Lambda \,, \qquad F_\Lambda = \frac{\Lambda^2}{\Lambda^2 + \vec{q}^{\,2}} \,, \; e^{-\vec{q}^{\,2}/\Lambda^2} \,, \; e^{-(p^2+p'^2)/\Lambda^2}$$

Perturbation:

$$\delta V_0^\Lambda = V_0 - V_0^\Lambda \sim \mathcal{O}(Q^2)$$

For small momenta
~ like a contact interaction

$$\delta_\Lambda V_0 = V_0^{\Lambda+\delta\Lambda} - V_0^\Lambda \sim \frac{\delta\Lambda}{\Lambda}\mathcal{O}(Q^2)$$

$$\delta T_2^\Lambda = (1 + T_0 G)\delta V_0^\Lambda (1 + G T_0) \sim \mathcal{O}(Q^0)$$

After renormalization:

$$\mathbb{R}\left(\delta T_2^\Lambda\right) \sim \mathcal{O}(Q^2)$$

Cutoff dependence

$$V_0^\Lambda = V_0 F_\Lambda\,, \qquad F_\Lambda = \frac{\Lambda^2}{\Lambda^2+\vec{q}^{\,2}}\,,\; e^{-\vec{q}^{\,2}/\Lambda^2}\,,\; e^{-(p^2+p'^2)/\Lambda^2}$$

Perturbation:

$$\delta V_0^\Lambda = V_0 - V_0^\Lambda \sim \mathcal{O}(Q^2)$$

For small momenta
~ like a contact interaction

$$\delta_\Lambda V_0 = V_0^{\Lambda+\delta\Lambda} - V_0^\Lambda \sim \frac{\delta\Lambda}{\Lambda}\mathcal{O}(Q^2)$$

$$\delta T_2^\Lambda = (1+T_0 G)\delta V_0^\Lambda(1+GT_0) \sim \mathcal{O}(Q^0)$$

After renormalization:

$$\mathbb{R}\left(\delta T_2^\Lambda\right) \sim \mathcal{O}(Q^2)$$

In the non-perturbative case guaranteed only when:

$$N_\psi(0) \neq 0$$

Summary

- ✔ NN Chiral EFT with a finite cutoff:
  NLO interaction is treated perturbatively.
  LO interaction is summed (perturbatively) up to an arbitrary order
  or treated non-perturbatively

- ✔ Power-counting breaking contributions at NLO can be absorbed
  by the renormalization of the LO contact interactions
  for perturbative LO under rather general conditions

- ✔ In the case of non-perturbative LO,
  the requirement of renormalizability
  imposes certain constraints on the LO potential

Summary

- ✔ NN Chiral EFT with a finite cutoff:
  NLO interaction is treated perturbatively.
  LO interaction is summed (perturbatively) up to an arbitrary order
  or treated non-perturbatively

- ✔ Power-counting breaking contributions at NLO can be absorbed
  by the renormalization of the LO contact interactions
  for perturbative LO under rather general conditions

- ✔ In the case of non-perturbative LO,
  the requirement of renormalizability
  imposes certain constraints on the LO potential

Outlook

- Generalization: N^2LO, N^3LO,…
- Other applications: currents, few body,…