$A = 4 - 7$ Ξ Hypernuclei Based on Interactions from Chiral EFT

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HL, J. Haidenbauer, U.-G. Meißner, A. Nogga (arXiv:2109.06648 [nucl-th] (2021))

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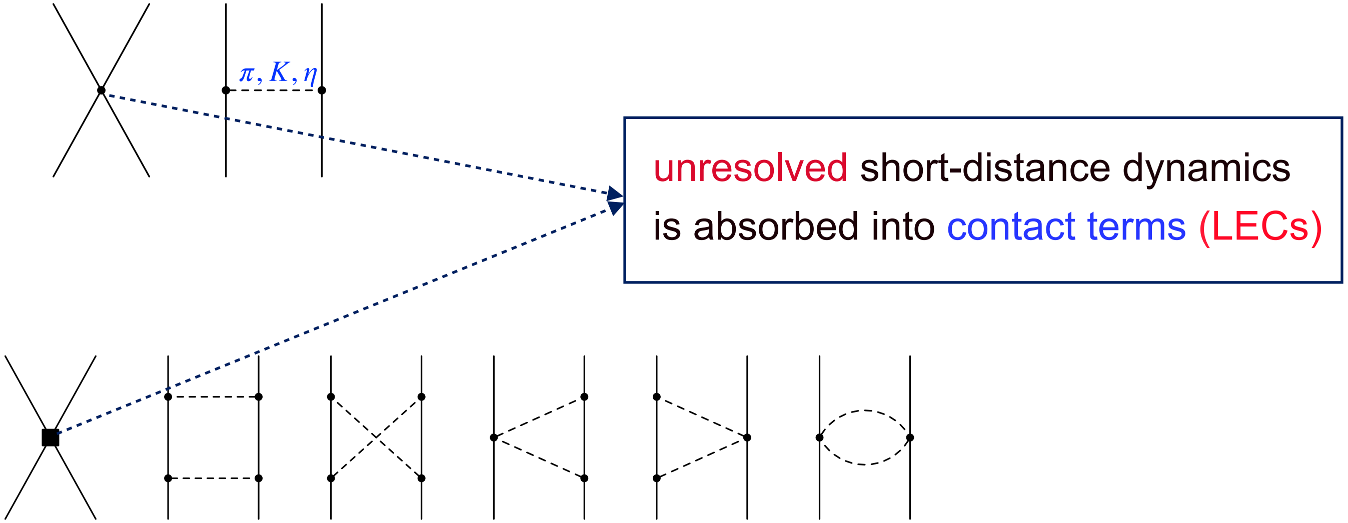
Baryon-Baryon interactions in χEFT

LO: H. Polinder et al., NPA 779 (2006). NLO: J. Haidenbauer et al., NPA 915 (2013)

- degrees of freedom: octet baryons $(N, \Lambda, \Sigma, \Xi)$, pseudoscalar mesons (π, K, η)
- based on Weinberg power counting as in the NN case



LO:

unresolved short-distance dynamics is absorbed into contact terms (LECs)

NLO:

- exploit $SU(3)_f$ to fix BBM couplings and relate various LECs, allow $SU(3)_f$ breaking where it seems appropriate

→ number of contact terms:

NN: 2 (LO) 7 (NLO)
YN: + 3 (LO) + 11 (NLO)
YY: + 1 (LO) + 4 (NLO) ⇒ 28 LECs for S=-2 at NLO

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Constraints from S=-2 sector: ΞN

- some data/ limits for ΞN (in)elastic cross sections (J.K. Ahn et al, PLB 633 (2006) 214)

$\sigma_{\Xi^- p\to\Lambda\Lambda} = 4.3^{+6.3}_{-2.7}$ mb, at $p_\Xi = 500$ MeV/c

$\sigma_{\Xi^- p\to\Xi^- p}$ < 24 mb; $\sigma_{\Xi^- p\to\Lambda\Lambda}$ < 12 mb (200 < p_Ξ < 800 MeV/c)

$\sigma_{\Xi^- N} = 12.7^{+3.5}_{-3.1}$ mb (400 < p_Ξ < 600 MeV/c) (S. Aoki et al., NPA 644 (1998))

→ ΞN interaction can not be very strong

- events support Ξ-bound states:

$\Xi^- - {}^{14}N\,({}^{15}_{\Xi^-}C)$: $B_\Xi = 3.87(1.03) \pm 0.21(0.18)$ MeV (KISO) K. Nakazawa et al., PTEP 033D02 (2015)

$= 4.96 \pm 0.77$ MeV (KINKA) M. Yoshimoto et al., PTEP 073D02 (2021)

$= 1.27 \pm 0.21$ MeV (IBUKI) S.H. Hayakawa et al., PRL 126 (2021)

→ Ξ^- is captured in nuclear 1p state

$B_\Xi = 8.00 \pm 0.77$ (KINKA) , 6.27 ± 0.27 (IRRAWADDY) (M.Yoshimoto et al., PTEP (2021))

→ s-shell Ξ– nuclear state

${}^{12}C(K^-, K^+)\,{}^{12}_{\Xi^-}Be$: T. Fukuda et al. PRC 58 (1998), P. Khaustov PRC 61 (2000)

T. Nagae et al., PoS (INPC2016) 038, AIP Conf. Proc 2130 (2019), $B_\Xi \approx 9$ MeV (and 2 MeV)

→ Ξ single particle potential $U_\Xi \approx -14$ MeV ($U_\Lambda \approx -30$ MeV)

Constraints from S=-2 sector: $\Lambda\Lambda$

- $\Lambda\Lambda$ hypernuclei: ${}^{6}_{\Lambda\Lambda}$He (Nagara), ${}^{10}_{\Lambda\Lambda}$Be (Demachi-Yanagi), ${}^{11}_{\Lambda\Lambda}$Be (Hida)

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) - 2B_{\Lambda}({}^{5}_{\Lambda}\text{He}) = 0.67 \pm 0.17\,\text{MeV}$$

(K. Nakazawa et al., NPA 835 (2010))

→ $\Lambda\Lambda$ interaction is weakly attractive

YY interaction at NLO

J. Haidenbauer et al., NPA 954 (2016) 273, EPJA 55 (2019) 23

- fulfil all the constraints from S=-2 sector (see J. Haidenbauer talk)

- study ${}^{6}_{\Lambda\Lambda}$He, ${}^{5}_{\Lambda\Lambda}$He and ${}^{4}_{\Lambda\Lambda}$H using LO and NLO

(HL et al., EPJA 57 (2021) 217)

→ ${}^{6}_{\Lambda\Lambda}$He is fairly well described with NLO; ${}^{5}_{\Lambda\Lambda}$He is predicted to be bound

	$a_{\Lambda\Lambda}$	$a_{\Xi^0 n}({}^1S_0)$	$U_{\Xi}(p_{\Xi}=0)$
NLO(500)	-0.62	-1.30 - i0.07	-5.5
NLO(550)	-0.61	-2.05 - i0.27	-3.8
NLO(600)	-0.66	-1.95 - i0.25	-4.3
NLO(650)	-0.70	-1.41 - i0.09	-4.1

$a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm (A. Gasparyan PRC 85 (2012))

$U^{c}_{\Xi} \approx -14$ MeV

- Kohno et al., use Ξ s.p. potential obtained with NLO to study finite systems: $\Xi^{-}-{}^{14}N$, $\Xi^{-}-{}^{12}C$

(M. Kohno PRC 100 (2019), M. Kohno, K. Miyagawa., arXiv:2107.03784)

→ reasonable agreement with experiment can be obtained

Possible lightest Ξ hypernucleus?

variational Gaussian expansion method using ESC08c and HAL QCD

E. Hiyama et al., PRL 124 (2020) 092501

- $\Xi N - \Lambda\Sigma - \Sigma\Sigma$, $\Lambda\Lambda - \Xi N$ coupling effects are renormalized into $V_{\Xi N - \Xi N}$
- → Nijmegen ESC08c leads to $NN\Xi(J^\pi, T) = (3/2^+, 1/2)$; $NNN\Xi(1^+,0), (0^+,1)(1^+,1)$ bound states
- HAL QCD predicts a loosely bound $(1^+,0)$ state in $NNN\Xi$

Faddeev equation for $NN\Xi$ with modern ΞN potentials

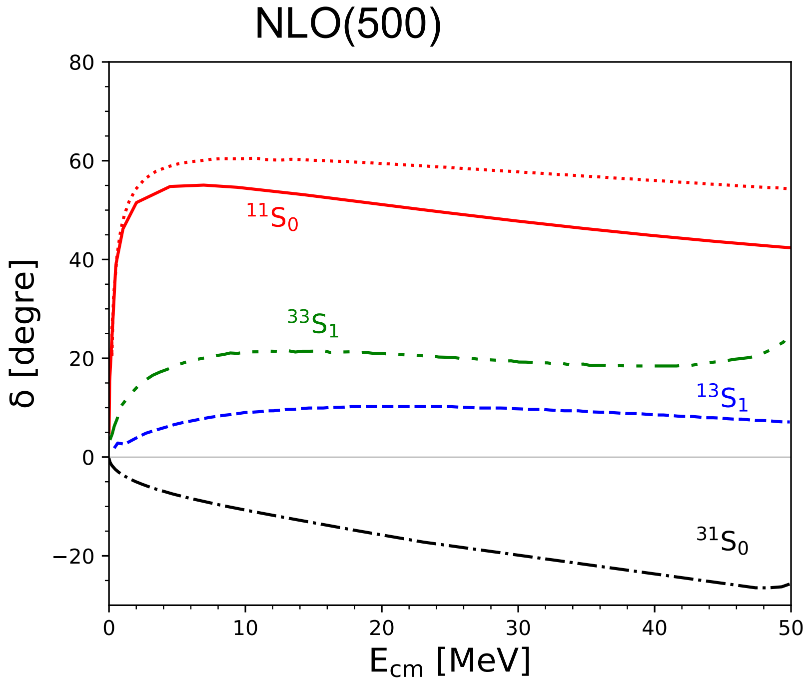
K. Miyagawa and M. Kohno FBS 65 (2021)

- original ΞN potentials are employed to obtain $T_{\Xi N, \Xi N}$
- → no bound states are obtained with chiral NLO or HAL QCD

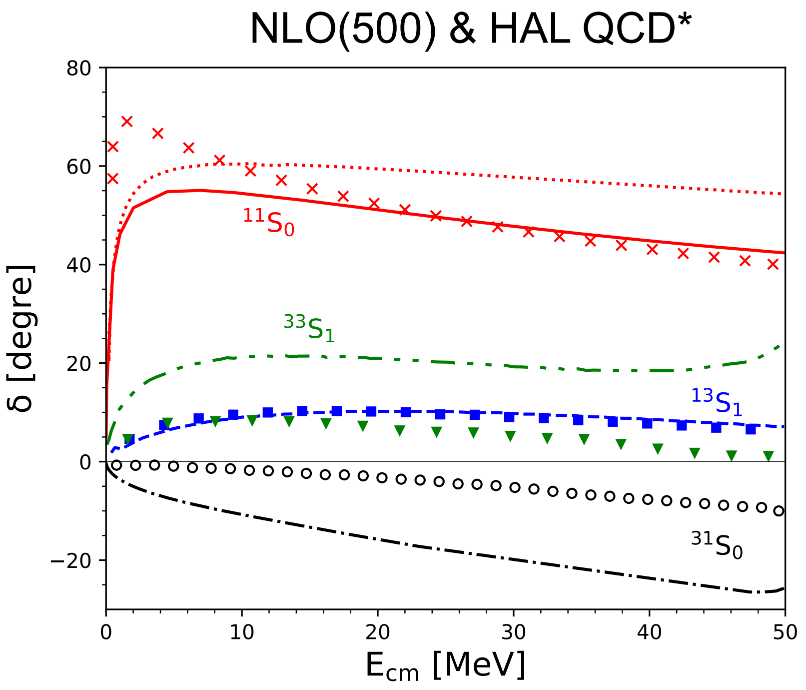
Our aim:

- **study predictions of chiral interactions for $A = 4 - 7$ Ξ systems using Jacobi NCSM**
  - input Hamiltonian: chiral NN ($\mathrm{N}^4\mathrm{LO}^+(450)$) and S=-2 NLO potentials
  - $\Xi N - \Lambda\Sigma - \Sigma\Sigma$ transition in S=-2 is explicitly considered
  - $\Lambda\Lambda - \Xi N\,(^{11}S_0)$ coupling is effectively incorporated by re-adjusting the strength of $V_{\Xi N - \Xi N}$

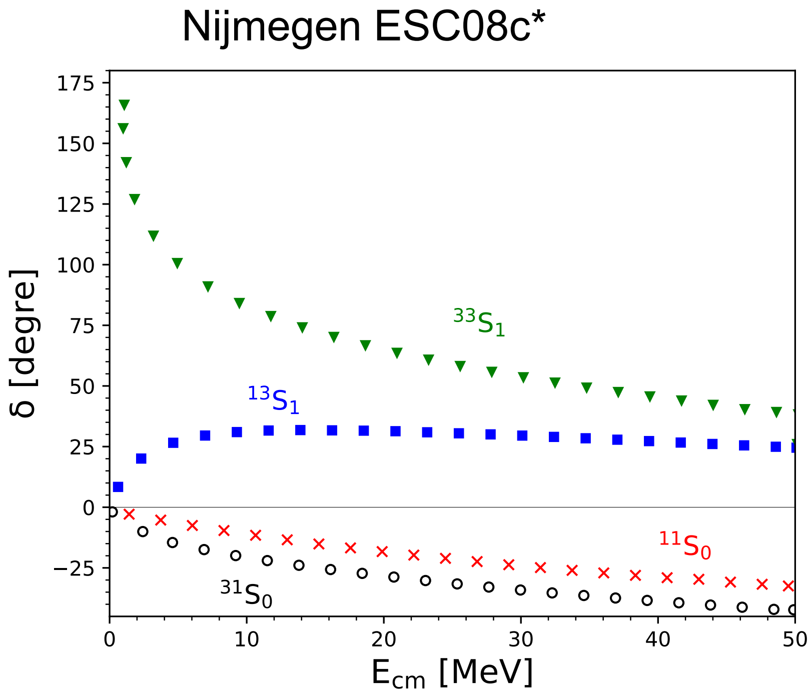
ΞN phase shifts predicted by modern interactions



ΞN phase shifts predicted by modern interactions



(* E. Hiyama et al. PRL 124, 092501 (2020))



(* E. Hiyama et al PRL 124, 092501 (2020))

→
- $^{11}S_0$ is rather attractive in NLO and HAL QCD, but repulsive in ESC08c
- $^{33}S_1$ is strongly attractive in ESC08c (lead to a ΞN bound state), it is only moderately (weakly) attractive in NLO (HAL QCD)

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Jacobi No-Core Shell Model (J-NCSM)

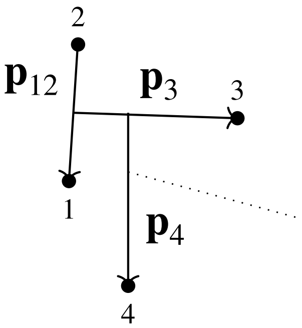
- an expansion of the wavefunction in a many-body HO basis depending on Jacobi coordinates

$$\mathbf{p}_{12} = \frac{m_2}{m_1+m_2}\mathbf{k}_1 - \frac{m_1}{m_1+m_2}\mathbf{k}_2$$

$$\mathbf{p}_3 = \frac{m_2+m_1}{m_1+m_2+m_3}\mathbf{k}_3 - \frac{m_3}{m_1+m_2+m_3}(\mathbf{k}_2+\mathbf{k}_1)$$

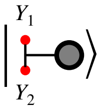
$$\vdots$$

$$\mathbf{p}_A = \frac{\sum_{i=1}^{A-1} m_i}{\sum_{i=1}^{A} m_i}\mathbf{k}_A - \frac{m_A}{\sum_{i=1}^{A} m_i}\sum_{i=1}^{A-1}\mathbf{k}_i$$



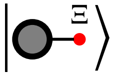
  - explicit removal of c.m. motion
  - all particles are active (no inert core) → employ microscopic BB interactions
  - converge slowly → require soft interactions (use techniques e.g., Vlow_k, SRG)

- basis functions are split into two orthogonal sets:



$$= |\mathcal{N}JT, \alpha_{A-2}\,\underbrace{\alpha_{Y_1Y_2}}_{|\Lambda\Sigma\rangle,|\Sigma\Sigma\rangle}\,n_\lambda\lambda; ((l_{Y_1Y_2}S_{Y_1Y_2})J_{Y_1Y_2}(\lambda J_{A-2})I_\lambda)J, ((t_{Y_1}t_{Y_2})T_{Y_1Y_2}T_{A-2})T\rangle \equiv |\alpha^{*(Y_1Y_2)}\rangle$$

(JJ-coupling)



$$= |\mathcal{N}JT, \alpha_{A-1}\,n_\Xi\,l_\Xi\,t_\Xi; (J_{A-1}(l_\Xi\,s)I_\Xi)\,J, (T_{A-1}t_\Xi)T\rangle \equiv |\alpha^{*(\Xi)}\rangle$$

- basis truncation:

$$\mathcal{N} = \mathcal{N}_{A-2} + N_{Y_1Y_2} + 2n_\lambda + \lambda = \mathcal{N}_{A-1} + 2n_\Xi + \lambda_\Xi \le \mathcal{N}_{max} \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$$

→ require $\mathcal{N}_{max} \to \infty$ extrapolation

(HL et al. arXiv:2008.11565, arXiv:2103.08395)

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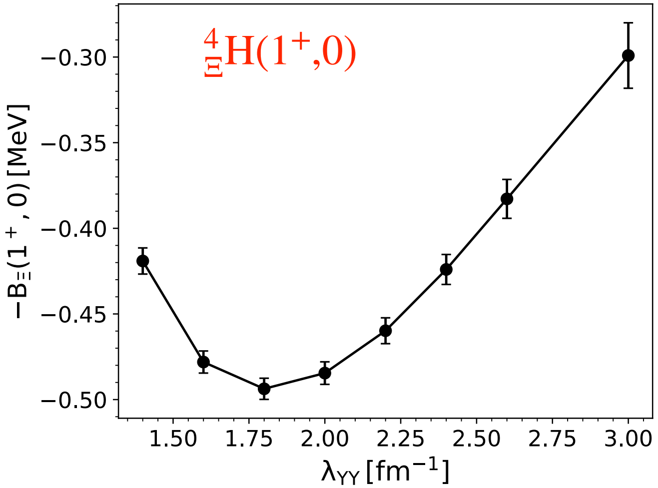
Dependence of energies on SRG

- apply SRG evolution to $\mathrm{N^4LO^+(450)}$ and S=-2 NLO(500) to speed up convergence:

$$\lambda_{NN} = 1.6\,\mathrm{fm}^{-1},\ \ 1.4 \leq \lambda_{YY} \leq 3.0\,\mathrm{fm}^{-1}$$

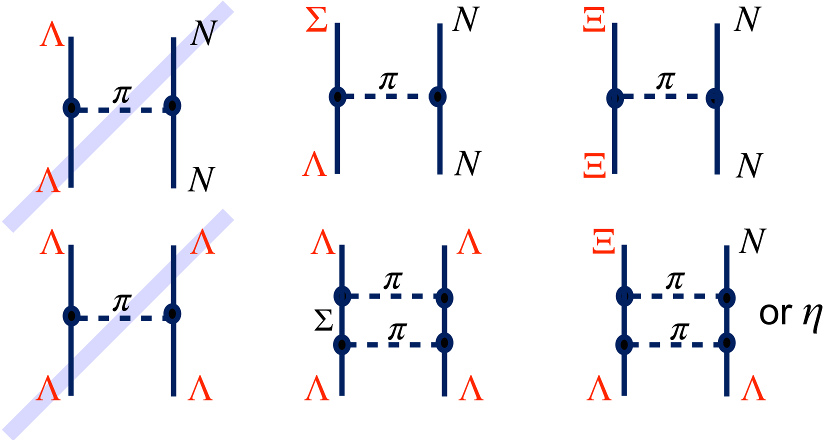
→ estimate effect of the omitted SRG-induced 3BFs



	ΔB^{SRG}_{Ξ} [keV]	$\Delta B^{SRG}_{\Lambda\Lambda}$ [keV]	ΔB^{SRG}_{Λ} [keV]
A=4	190 ± 30	-	740 ± 2
A=5	460 ± 140	80 ± 40	1501 ± 20
A=6	-	100 ± 17	-

- effect of SRG evolution on B_Ξ is moderately large, however, $\Delta B^{SRG}_{\Xi} \ll B_\Xi$

→ SRG dependence will not affect conclusions on the existence of bound states



suppressed by isospin symmetry

Results for A=4-7

	NLO(500)		others	
	B_Ξ [MeV]	Γ [MeV]	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74	$0.36(16)(26)^{(1)}$	$0.06^{(1)}$
			$10.20^{(2)}$	$0.89^{(2)}$
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2	$3.55^{(2)}$	$0.43^{(2)}$
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01	$10.11^{(2)}$	$0.03^{(2)}$
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	-	-	-	-
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19	$1.7^{(3)}$	$0.2^{(3)}$
			$2.0^{(4)}$	$0.45^{(4)}$
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2	$3.15^{(5)}$	$0.02^{(5)}$
			$1.8^{(6)}$	$2.64^{(6)}$

(1) HAL QCD (t/a=12)

(2) Nijmegen ESC08c
E.Hiyama et al., PRL 124 (2020)

(3) K. Myint, Y. Akaishi PTPS 117 (1994)

(4) E. Friedman, A. Gal PLB 820(2021)

(5) HAL QCD (t/a=11)

(6) Nijmegen ESC04d
H. Fujioko APFB2021, March (2021)

$$\Gamma \simeq -2\,\mathrm{Im}\,\langle \Psi_{B_\Xi} | T_{\Xi N-\Xi N} | \Psi_{B_\Xi} \rangle$$

- $\Xi^- p$ Coulomb interaction contributes roughly 200, 600 and 400 keV to $NNN\Xi$, ${}^{5}_{\Xi}\mathrm{H}$ and ${}^{7}_{\Xi}\mathrm{H}$

Results for A=4-7

	$\langle V^{S=-2}\rangle$ [MeV]					E [MeV]
	$^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$	total	
$^{4}_{\Xi}$H$(1^+,0)$	-1.95	0.02	-0.7	-2.31	-5.21	-8.97
$^{4}_{\Xi}$n$(0^+,1)$	-0.6	0.25	-0.004	-0.74	-1.37	-9.07
$^{4}_{\Xi}$n$(1^+,1)$	-0.02	0.16	-0.13	-1.14	-1.30	-9.0
$^{4}_{\Xi}$H$(0^+,0)$	-0.002	0.08	-0.01	-0.006	-0.11	-6.94
$^{5}_{\Xi}$H$(1/2^+,1/2)$	-0.96	0.94	-0.58	-3.63	-4.88	-31.43
$^{7}_{\Xi}$H$(1/2^+,3/2)$	-1.23	1.79	-0.79	-6.74	-8.04	-33.22

→ the attraction in $^{33}S_1$ is essential for the binding of A=4-7 Ξ-hypernuclei

Estimate partial-wave contributions

- **Assumption:**
  - no particle conversion contributing
  - both core nucleons and Ξ are in s-wave states
- **A=3 system:**

$$^{3}_{\Xi}\mathrm{H}(\frac{1}{2}^{+},\frac{1}{2}):\ \tilde{V}_{\Xi N}\approx\frac{3}{16}V_{\Xi N}^{^{11}S_0}+\frac{9}{16}V_{\Xi N}^{^{31}S_0}+\frac{1}{16}V_{\Xi N}^{^{13}S_1}+\frac{3}{16}V_{\Xi N}^{^{33}S_1}$$

$$^{3}_{\Xi}\mathrm{H}(\frac{3}{2}^{+},\frac{1}{2})\ :\tilde{V}_{\Xi N}\approx\frac{1}{4}V_{\Xi N}^{^{13}S_1}+\frac{3}{4}V_{\Xi N}^{^{33}S_1}$$

- **A=4 system:**

$$^{4}_{\Xi}\mathrm{H}(1^{+},0):\ \tilde{V}_{\Xi N}\approx\frac{1}{6}V_{\Xi N}^{^{11}S_0}+\frac{1}{3}V_{\Xi N}^{^{13}S_1}+\frac{1}{2}V_{\Xi N}^{^{33}S_1}$$

$$^{4}_{\Xi}\mathrm{H}(0^{+},1):\ \tilde{V}_{\Xi N}\approx\frac{1}{6}V_{\Xi N}^{^{11}S_0}+\frac{1}{3}V_{\Xi N}^{^{31}S_0}+\frac{1}{2}V_{\Xi N}^{^{33}S_1}$$

$$^{4}_{\Xi}\mathrm{H}(1^{+},1):\ \tilde{V}_{\Xi N}\approx\frac{1}{6}V_{\Xi N}^{^{31}S_0}+\frac{1}{6}V_{\Xi N}^{^{13}S_1}+\frac{2}{3}V_{\Xi N}^{^{33}S_1}$$

$$^{4}_{\Xi}\mathrm{H}(0^{+},0):\ \tilde{V}_{\Xi N}\approx\frac{1}{2}V_{\Xi N}^{^{31}S_0}+\frac{1}{2}V_{\Xi N}^{^{13}S_1}$$

- **A=5 system:**

$$^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^{+},\frac{1}{2}):\ \tilde{V}_{\Xi N}\approx\frac{1}{16}V_{\Xi N}^{^{11}S_0}+\frac{3}{16}V_{\Xi N}^{^{31}S_0}+\frac{3}{16}V_{\Xi N}^{^{13}S_1}+\frac{9}{16}V_{\Xi N}^{^{33}S_1}$$

Estimate partial-wave contributions

- **Assumption:**
  - no particle conversion contributing
  - both core nucleons and Ξ are in s-wave states

$(J_{core}, T_{core}, m^t_{\Xi})$	${}^4_{\Xi}H(1^+,0)$	${}^4_{\Xi}n(0^+,1)$	${}^4_{\Xi}n(1^+,1)$	${}^4_{\Xi}\mathrm{H}(0^+,0)$
$(\frac{1}{2},\frac{1}{2},-\frac{1}{2})$	49.66	97.48	97.44	49.98
$(\frac{1}{2},\frac{1}{2},\frac{1}{2})$	49.66	–	–	49.98
$(\frac{1}{2},\frac{3}{2},\frac{1}{2})$	–	0.54	0.55	–
$(\frac{1}{2},\frac{3}{2},-\frac{1}{2})$	–	1.6	1.6	–
others	0.16	0.17	0.22	0.02

$(J_{core}, T_{core}, m^t_{\Xi})$	${}^5_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$
$(0,0,-\frac{1}{2})$	96.03
$(0,0,\frac{1}{2})$	1.1
$(0,1,\frac{1}{2})$	2.1
others	0.3

$(J_{core}, T_{core}, m^t_{\Xi})$	${}^7_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$
$(0,1,-\frac{1}{2})$	94.44
$(0,2,-\frac{1}{2})$	0.7
$(0,2,\frac{1}{2})$	2.8
others	1.13

Probabilities (%) of finding nucleons with certain (J_{core}, T_{core}) and a Ξ (m^t_{Ξ}) in the A=4-7 wavefunctions

Summary

we studied A=4-7 Ξ hypernuclei using J-NCSM and NLO(500) ΞN potential

- found 3 loosely bound states $(1^+,0)$, $(0^+,1)$, $(1^+,1)$ in $NNN\Xi$

  ${}^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$, ${}^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$ are more tightly bound

  - these bindings are predominantly due to the attraction of ΞN potential in ${}^{33}S_1$ channel
  - all bound states are predicted to have very small decay widths

future works: study dependence of B_Ξ on chiral cutoff. Include SRG-induced 3BF

experimental confirmation?

- $NNN\Xi$ could be produced in heavy ion collisions or in ${}^4\mathrm{He}(K^-, K^+)$ reaction
- ${}^{7}_{\Xi}\mathrm{H}$ is expected to be produced and studied in ${}^7\mathrm{Li}(K^-, K^+)$ reaction at J-PARC **(H. Fujioka et al., FBS 69(2021))**
- production of ${}^{5}_{\Xi}\mathrm{H}$?

$\bar{p}\, p \to \Xi^- \, \bar{\Xi}^+$ PANDA **(J.I. Pütz PhD Thesis (2020))**

$\hookrightarrow {}^4\mathrm{He} \to {}^{5}_{\Xi}\mathrm{H}$

$\to \bar{\Xi}^+ \; \Xi^*(1530)$, then $\Xi^*(1530) \to \Xi^- + \pi^+$

$\hookrightarrow {}^4\mathrm{He} \to {}^{5}_{\Xi}\mathrm{H}$

Thank you for the attention!

Results for A=4-7

	$\|\Xi N\rangle$					
	$\|^{11}S_0\rangle$	$\|^{31}S_0\rangle$	$\|^{13}S_1\rangle$	$\|^{33}S_1\rangle$	$J \geq 2$	total
$^{4}_{\Xi}\mathrm{H}(1^+,0)$	12.88	0.18	25.91	35.72	24.80	99.49
$^{4}_{\Xi}\mathrm{n}(0^+,1)$	8.24	13.32	0.23	23.29	54.73	99.81
$^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.14	9.22	9.83	33.08	47.56	99.83
$^{4}_{\Xi}\mathrm{H}(0^+,0)$	0.02	11.87	14.65	0.11	73.33	99.98
$^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$	4.82	12.18	14.37	35.53	32.59	99.49
$^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$	3.71	12.92	11.11	38.36	32.94	99.04

Probabilities (%) of finding a pair $|\Xi N\rangle$ in different partial-wave states

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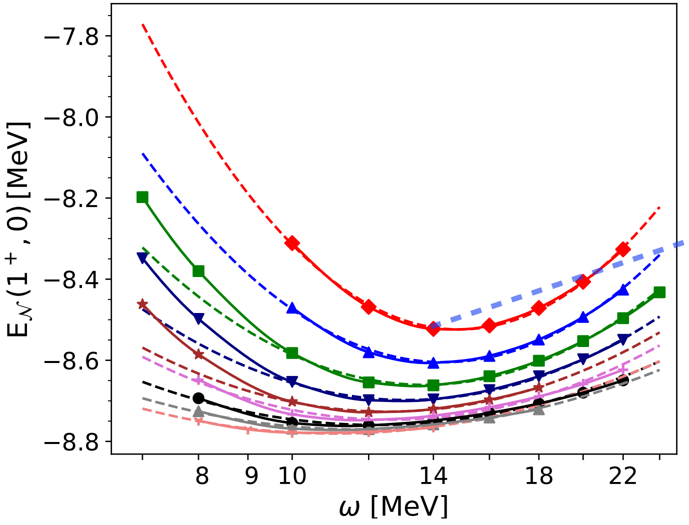
Extrapolation of the results

- extract converged binding energy

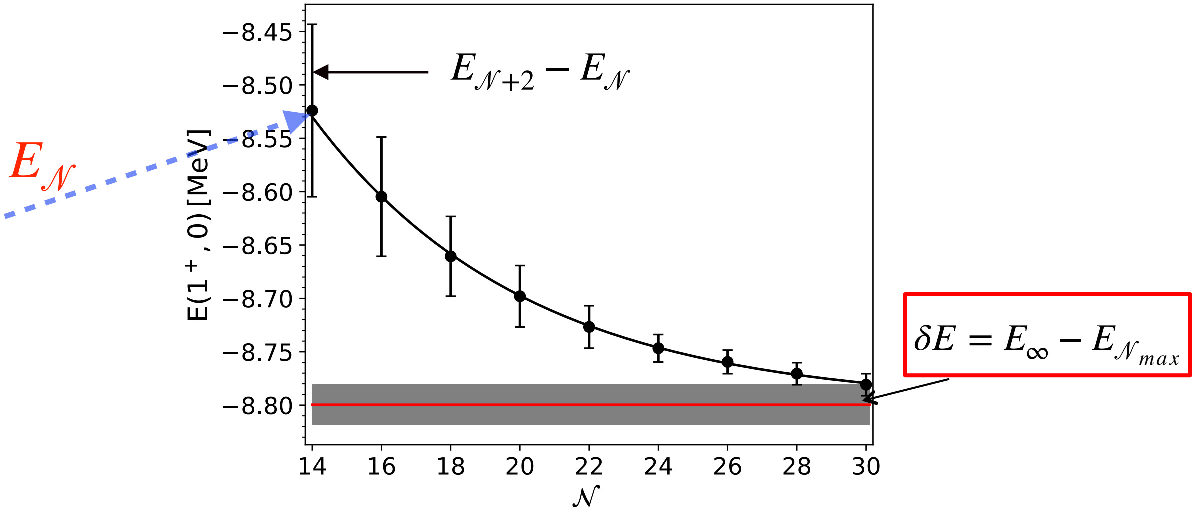
$$\mathcal{N} = \mathcal{N}_{A-2} + N_{Y_1Y_2} + 2n_\lambda + \lambda = \mathcal{N}_{A-1} + 2n_\Xi + \lambda_\Xi \le \mathcal{N}_{max} \quad \Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$$

$$E_b(\omega, \mathcal{N}) = E_{\mathcal{N}} + \kappa(\log(\omega) - \log(\omega_{opt}))^2$$



(a) $E_{\mathcal{N}}({}^4_\Xi\mathrm{H}(1^+,0))$ as a function of ω.

$$E_{\mathcal{N}} = E_\infty + Ae^{-b\mathcal{N}}$$



- extract converged separation energy B_Ξ:

$$B_{\Xi,\mathcal{N}} = E_{\mathcal{N}}({}^3\mathrm{H}) - E_{\mathcal{N}}({}^4_\Xi\mathrm{H}), \quad \mathcal{N} = 14,\cdots,30$$

- $\mathcal{N}_{max}({}^5_\Xi\mathrm{H}) = 16, \quad \mathcal{N}_{max}({}^7_\Xi\mathrm{H}) = 12$

$$B_{\Xi,\mathcal{N}} = B_{\Xi,\infty} + Ae^{-b\mathcal{N}}$$