

10th International Workshop on Chiral Dynamics
16 November 2021



Extracting few-body matrix elements from lattice QCD

Marc Illa



UNIVERSITAT DE
BARCELONA



ICCUB



EXCELENCIA
MARÍA
DE MAEZTU

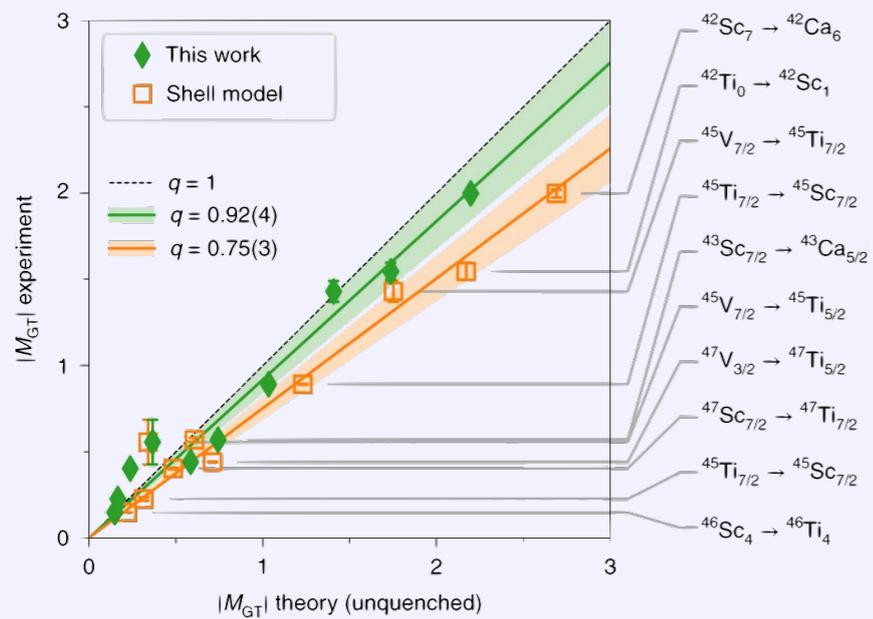


QUANTUM
SCIENCE
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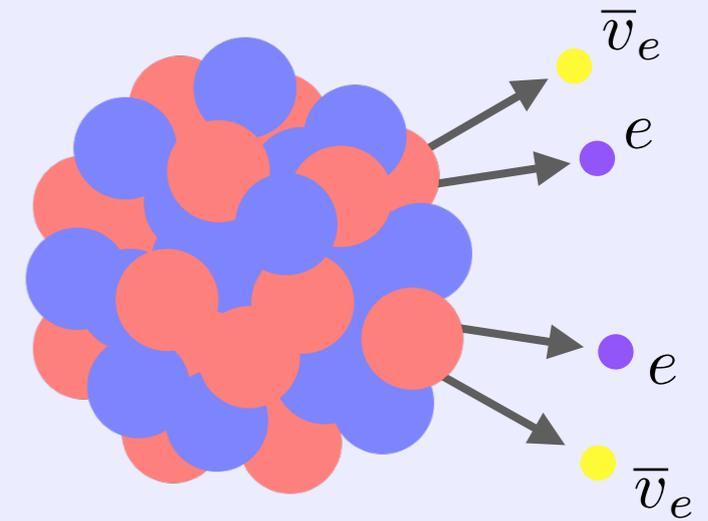
UNIVERSITY of
WASHINGTON

Quenching of axial charge



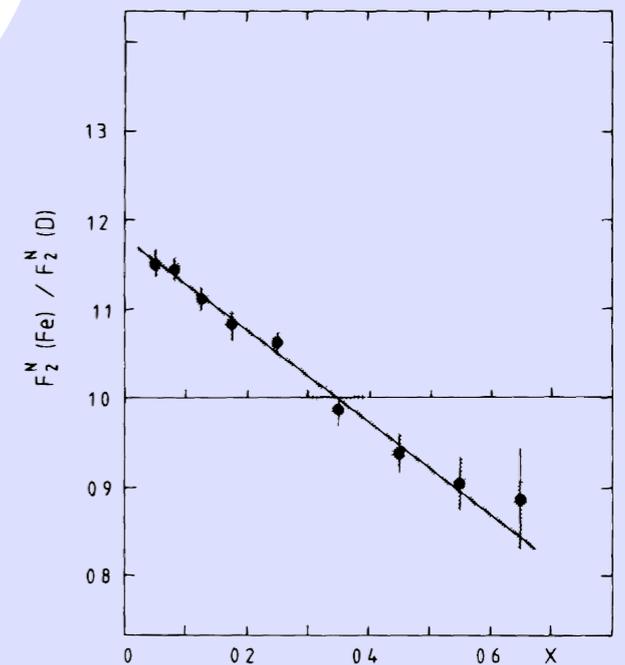
Gysbers et al., *Nature Phys.* 15 (2019)

Double- β decay

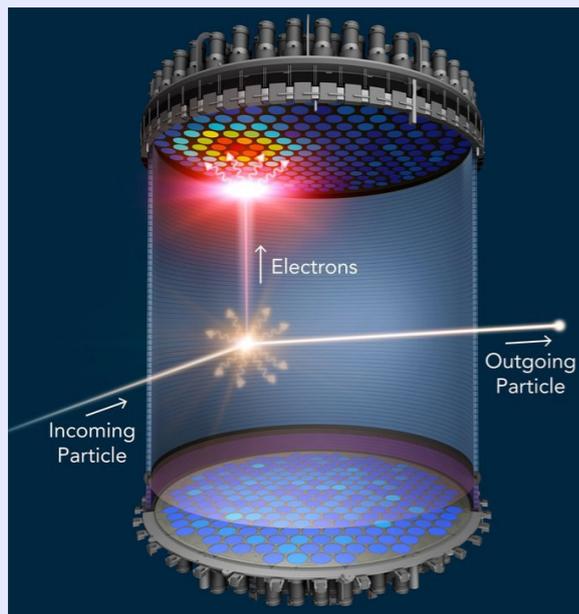


Nuclear matrix elements

EMC effect



EMC Collaboration, *PLB* 123 (1983)



Interaction with dark matter

SLAC National Accelerator Laboratory

Nuclear physics with lattice QCD



Spectroscopy

Σ^- in dense nuclear matter

[PRL 109 \(2012\), 172001](#)

Light-nuclei spectrum

[PRD 87 \(2013\), 034506](#)

Charged multi-hadron systems with lattice QCD+QED

[PRD 103 \(2021\), 054504](#) + QCDSF Collaboration

Baryon-baryon interactions

[PRD 96 \(2017\), 114510](#), [PRD 103 \(2021\), 054508](#)

Variational study of two nucleons

[arXiv:2108.10835](#)

See recent review: Davoudi et al, [Phys. Rept. 900 \(2021\), 1](#)

Matrix elements

$np \rightarrow d\gamma$ radiative capture process

[PRL 115 \(2015\), 132001](#), [PRD 92 \(2015\), 114502](#)

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Gluon structure

[PRD 96 \(2017\), 094512](#)

Scalar, axial and tensor matrix elements

[PRL 120 \(2018\), 152002](#)

Momentum fraction of ^3He

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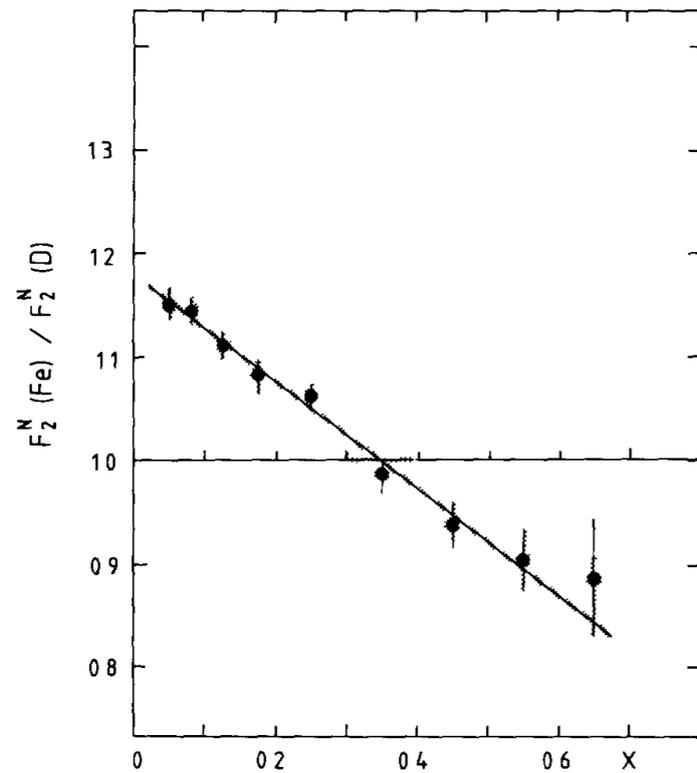
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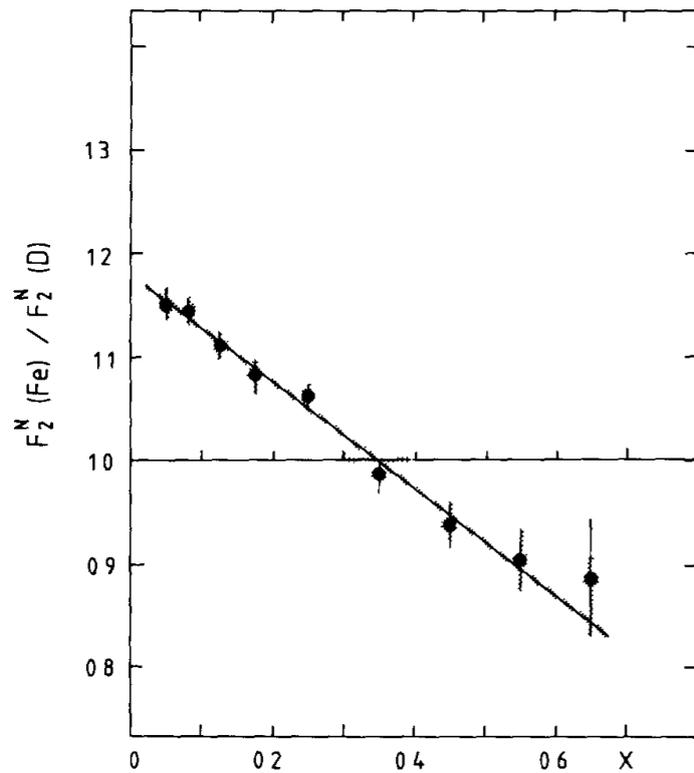


EMC Collaboration, [PLB 123 \(1983\)](#)

$$F_2(x, Q^2) = \sum_q x e_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$$

parton distribution function

Momentum fraction of ^3He



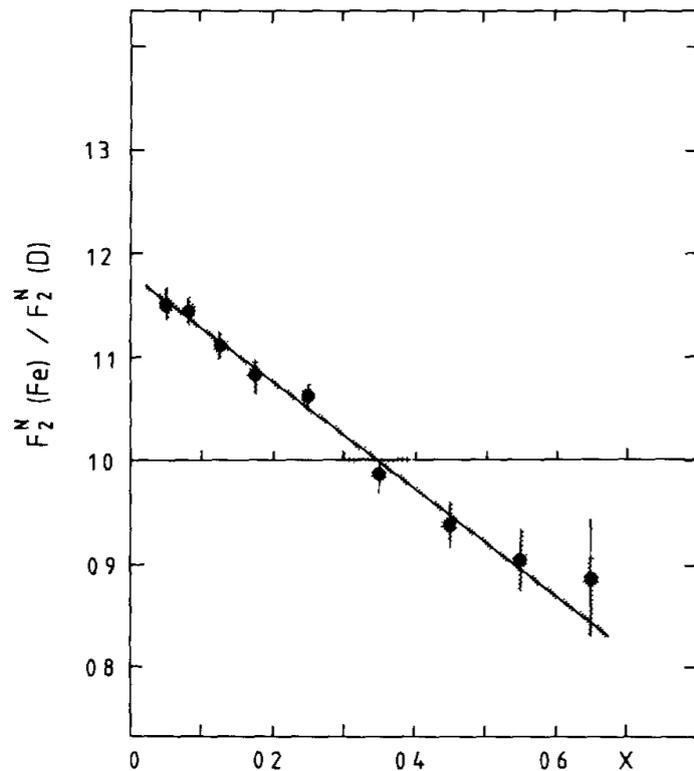
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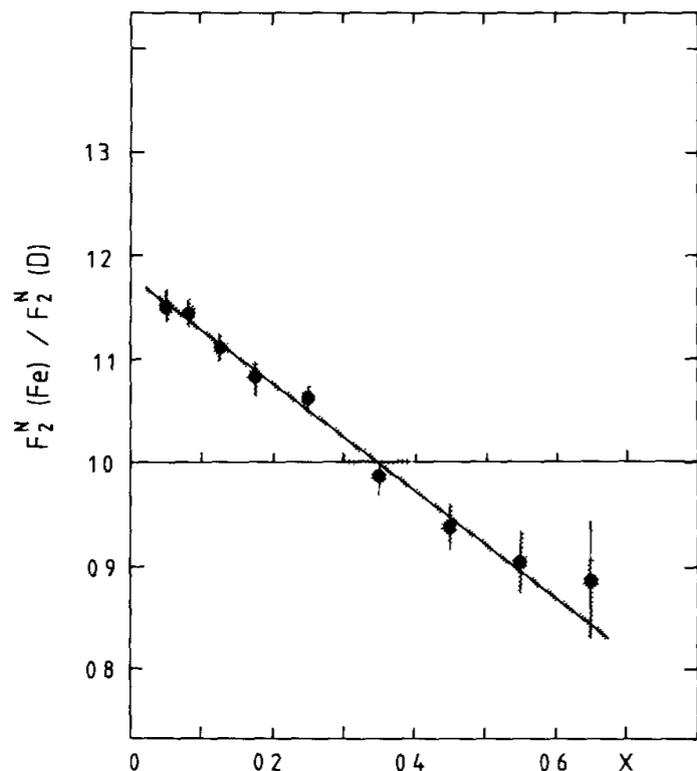
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Much easier to compute the moments of PDFs:

$$\langle x^n \rangle_q = \int_{-1}^1 dx x^n q(x, Q^2)$$

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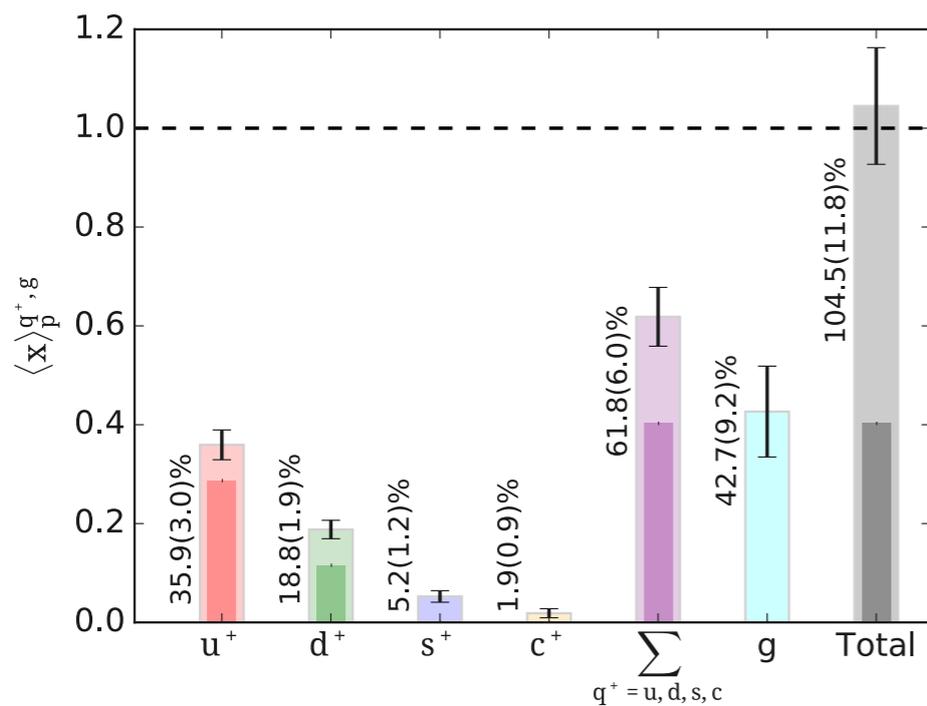
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Full flavor decomposition of the proton momentum fraction at the physical point

Alexandrou et al. [ETMC], [PRD 101 \(2020\)](#)

Observing EMC effects on the moments for light nuclei

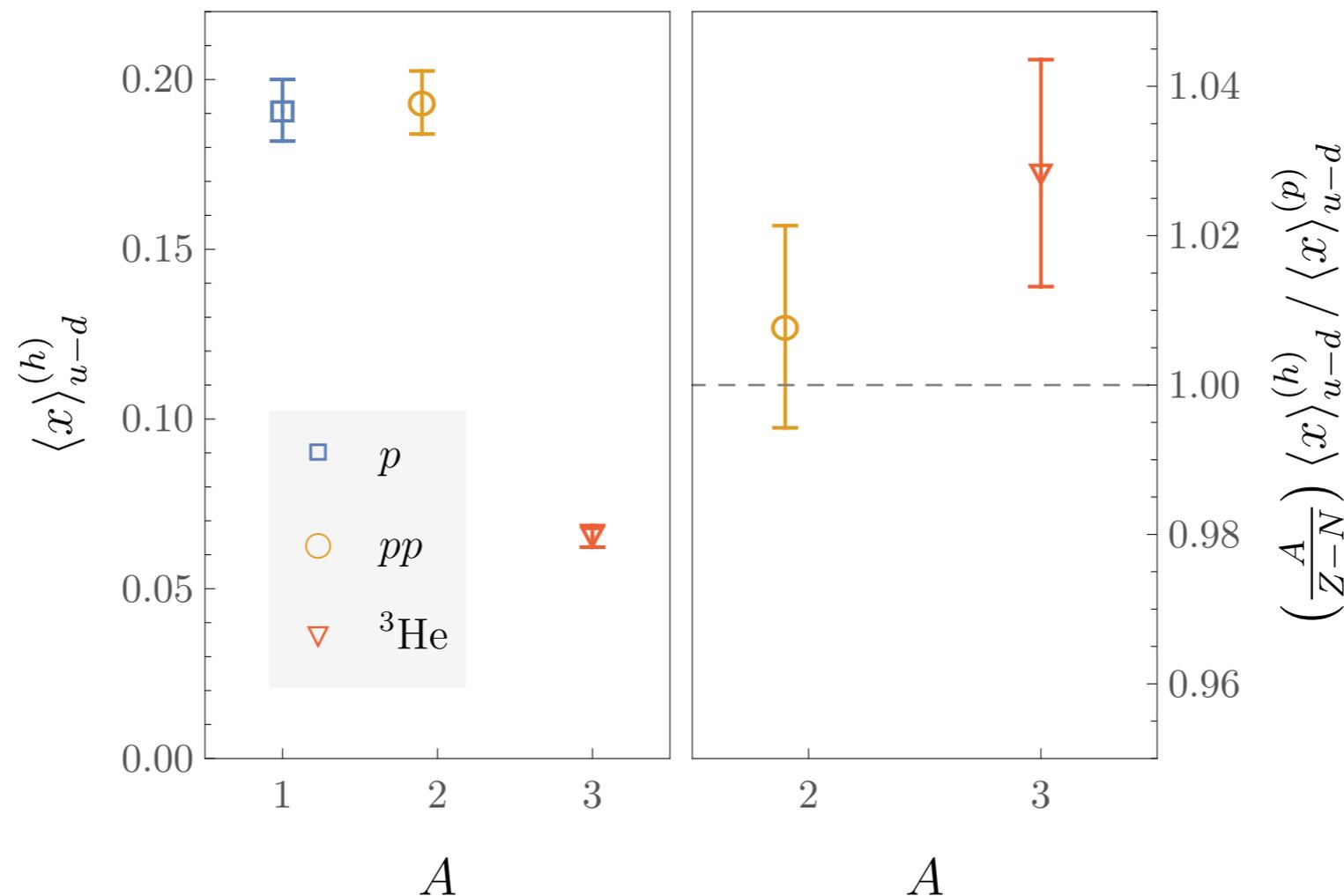
Ensemble parameters: $m_u = m_d = m_s$ $m_\pi \sim 806 \text{ MeV}$
 $a \sim 0.145 \text{ fm}$ $L = 32 \sim 4.6 \text{ fm}$

Momentum fraction of ${}^3\text{He}$

Detmold et al. [NPLQCD]
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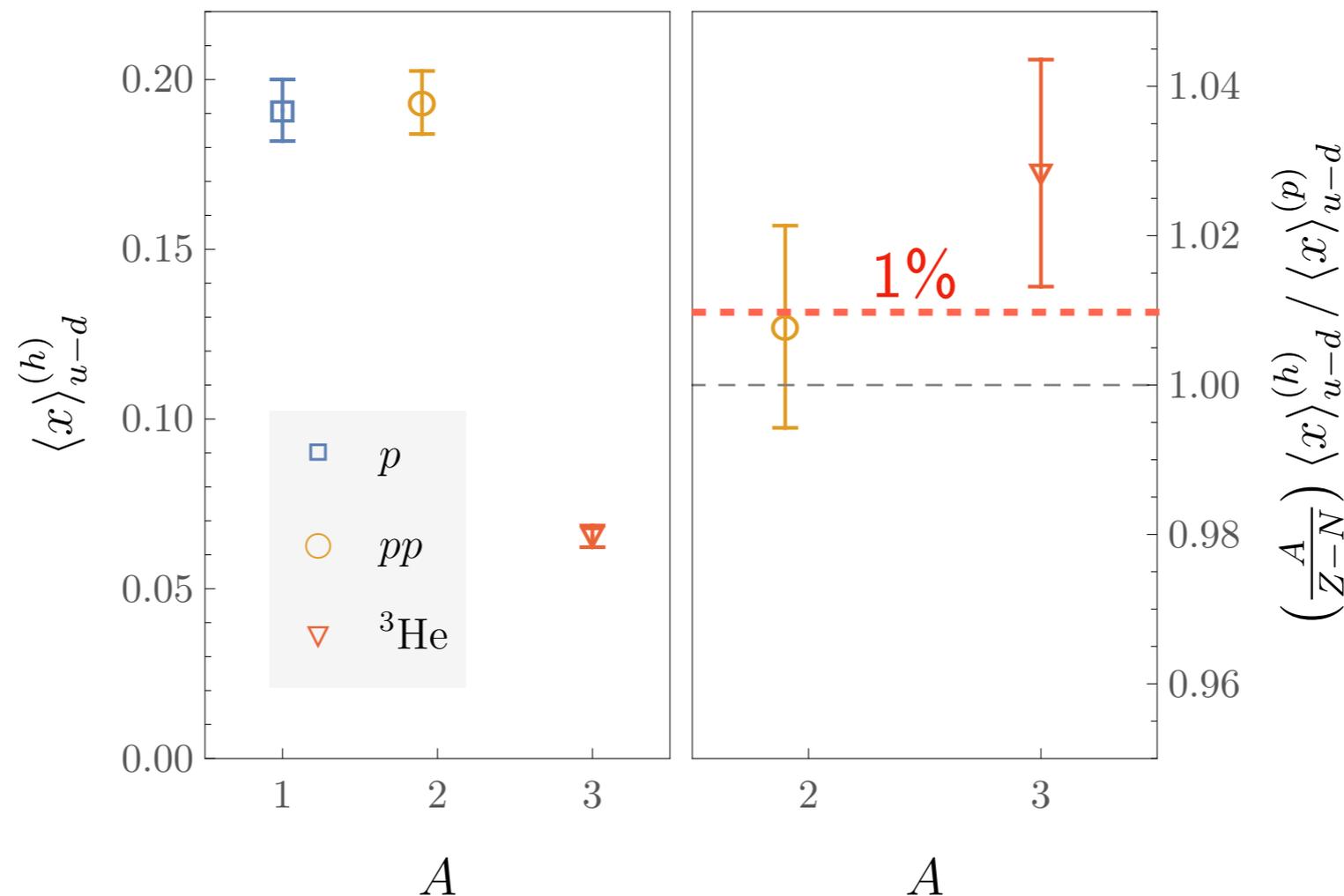


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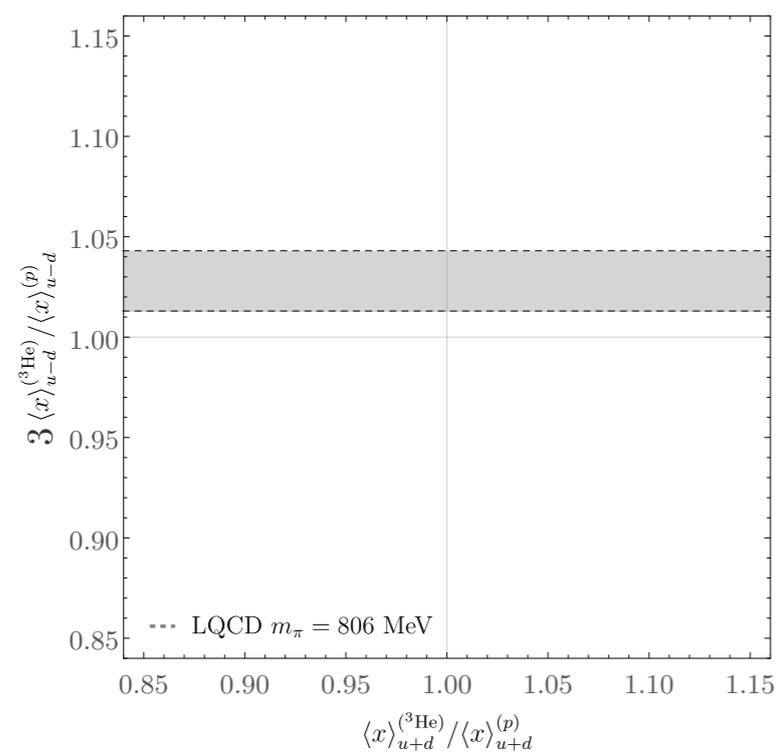


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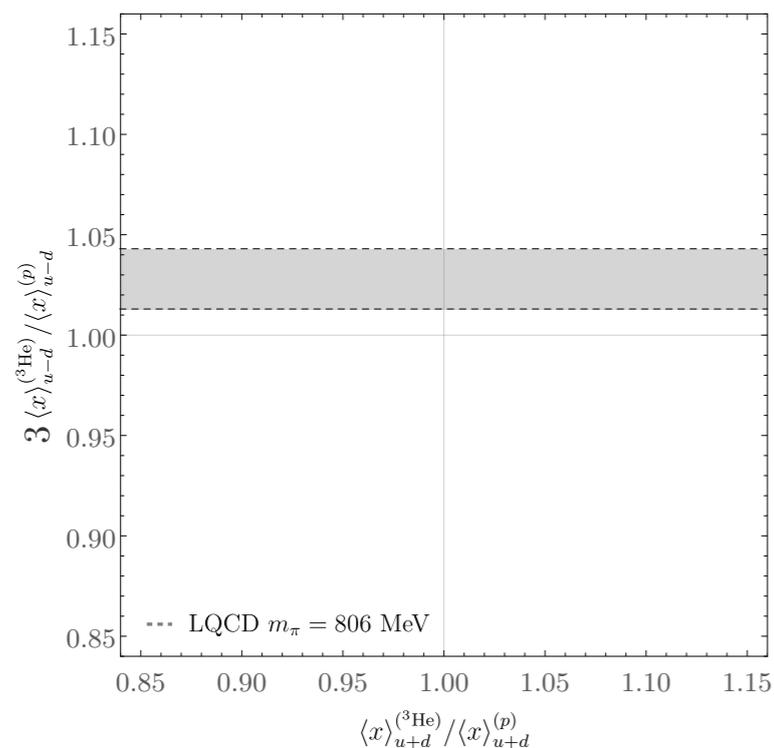


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Chen and Detmold, [PLB 625 \(2005\)](#)



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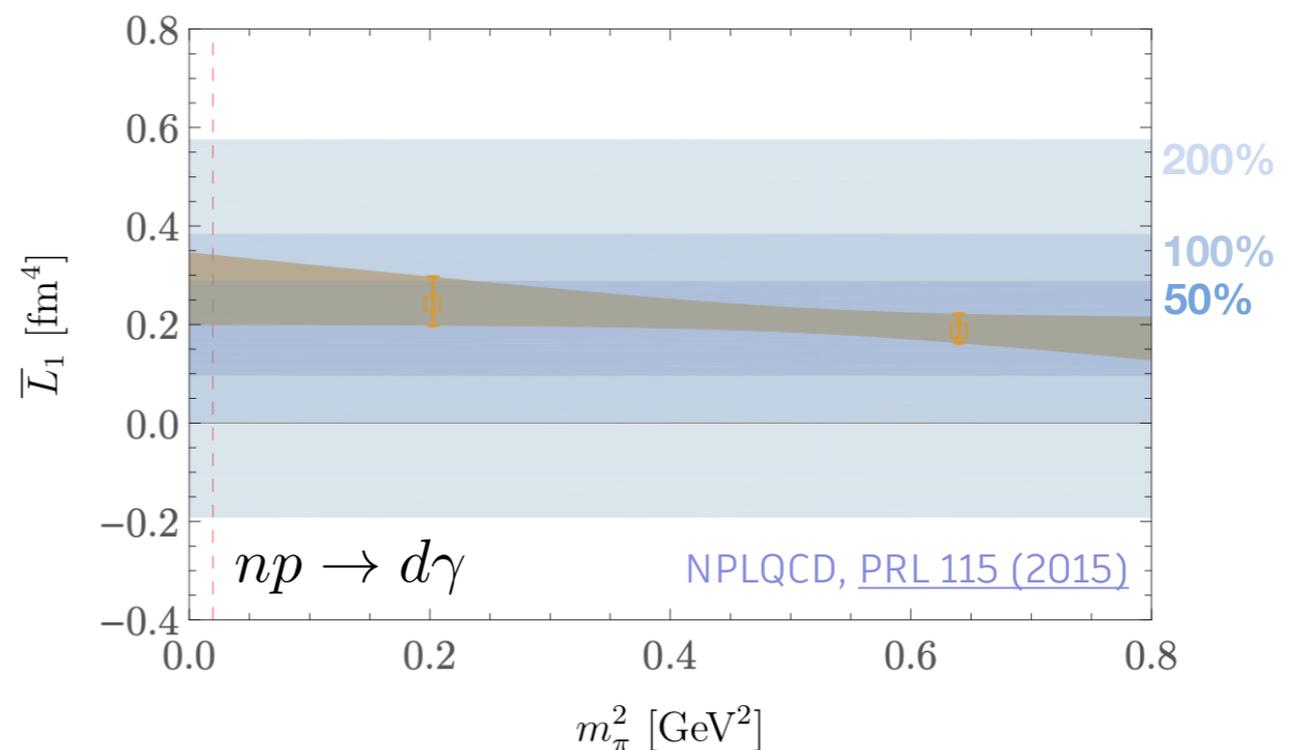
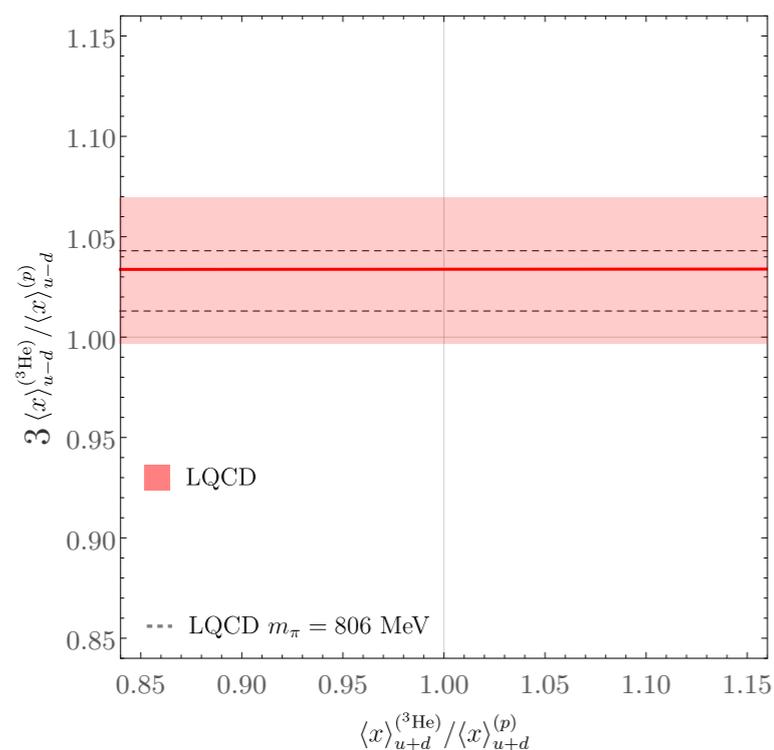
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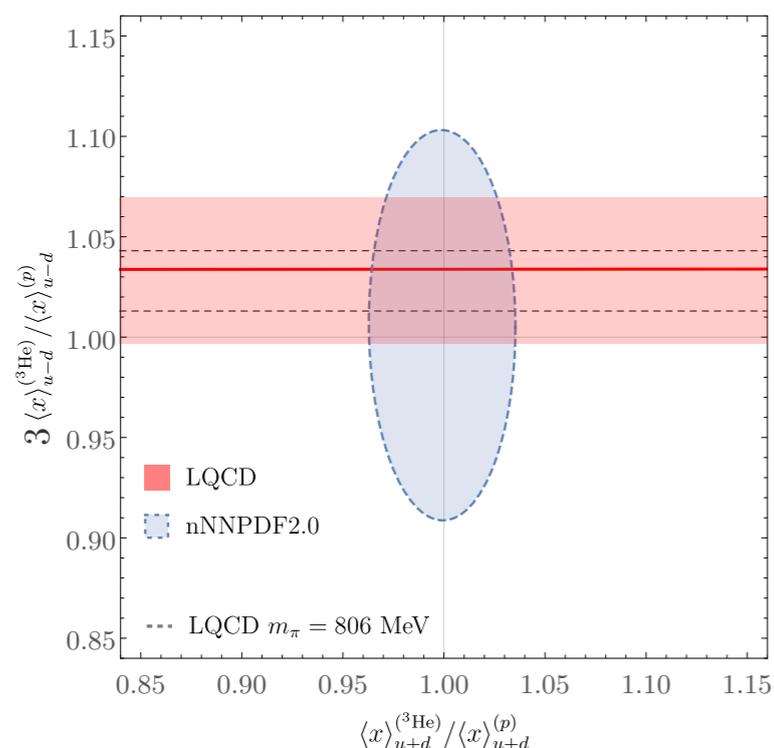
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Combine with experimental data from global fits provided by nNNPDF2.0

Ball et al. [NNPDF], [NPB 855 \(2012\)](#)

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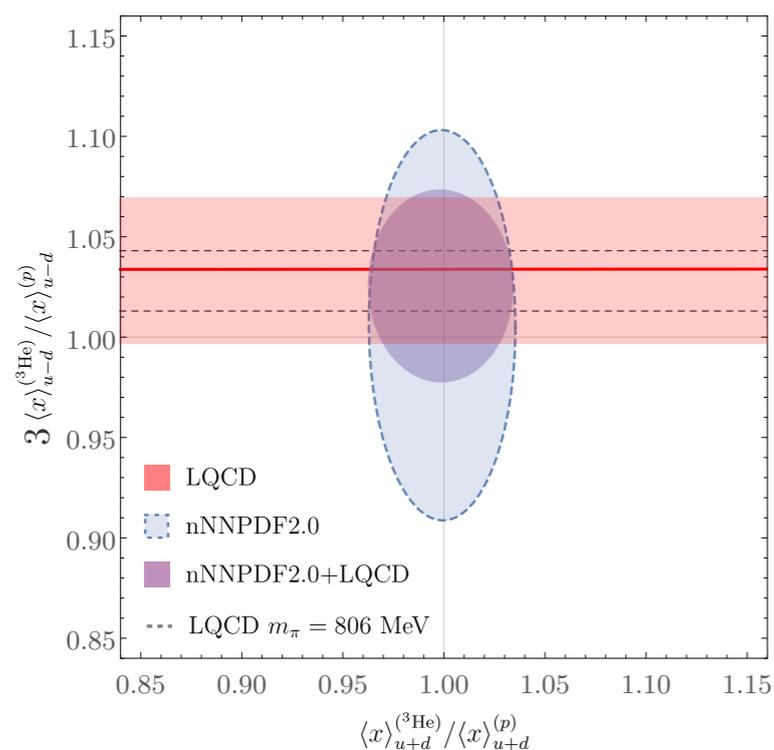
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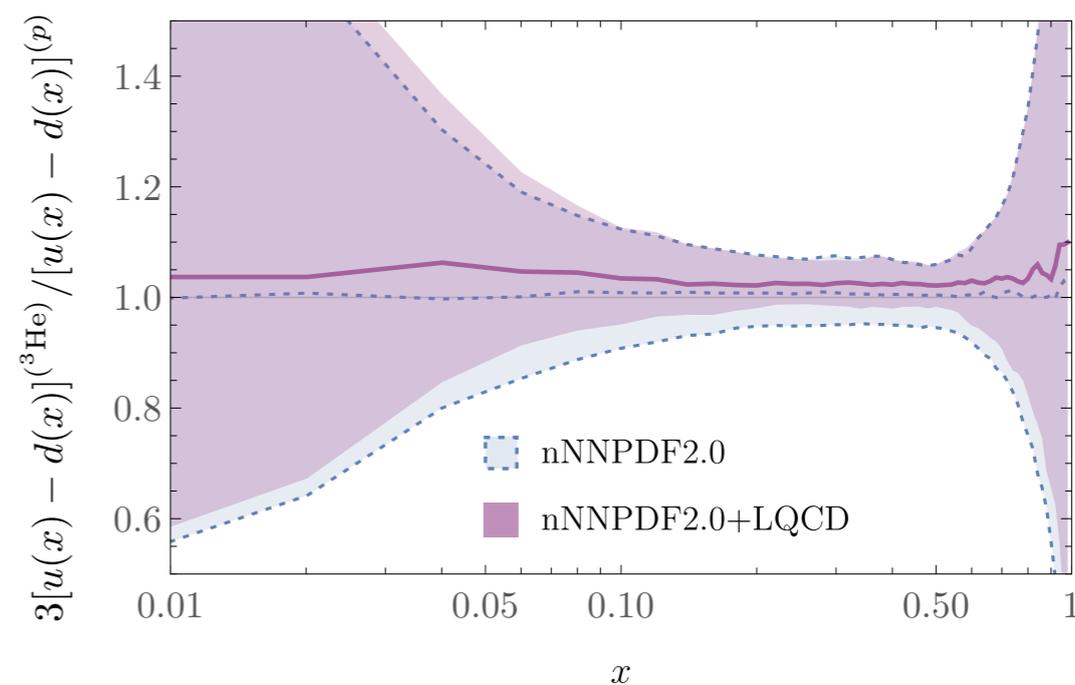
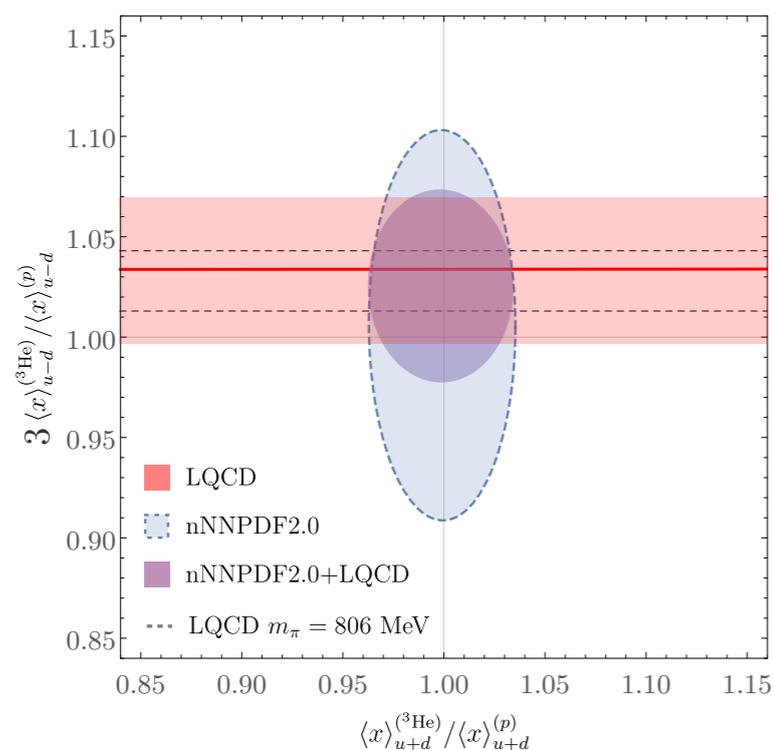
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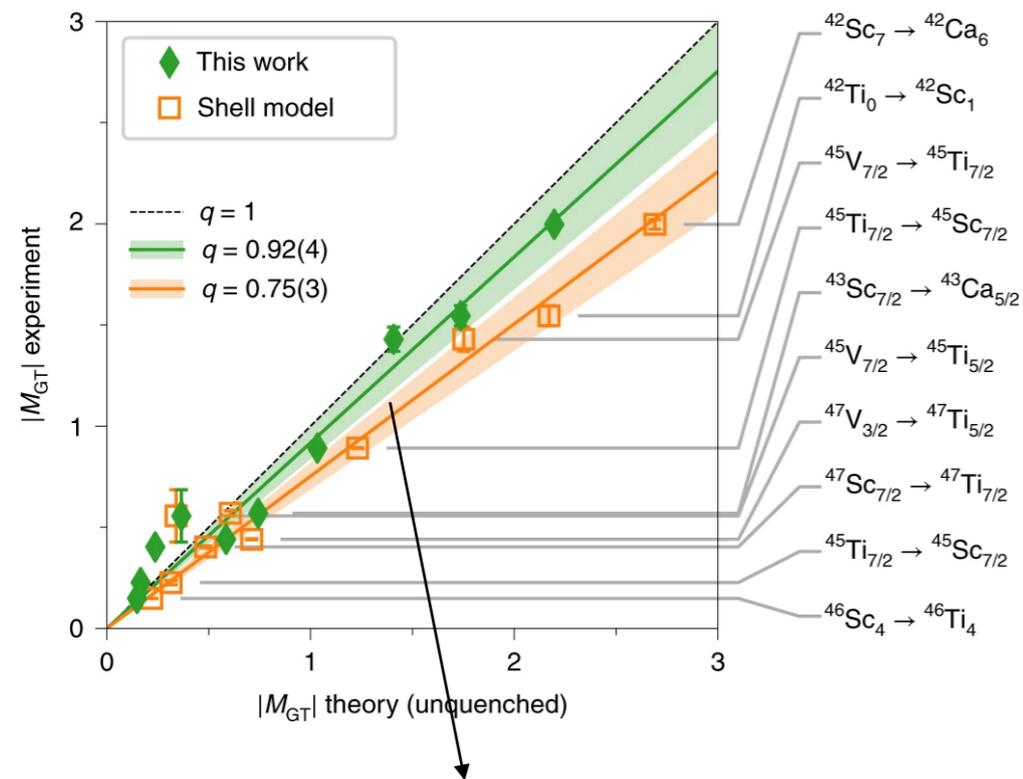
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Triton axial charge

P. Gysbers et al., *Nature Phys.* 15 (2019)

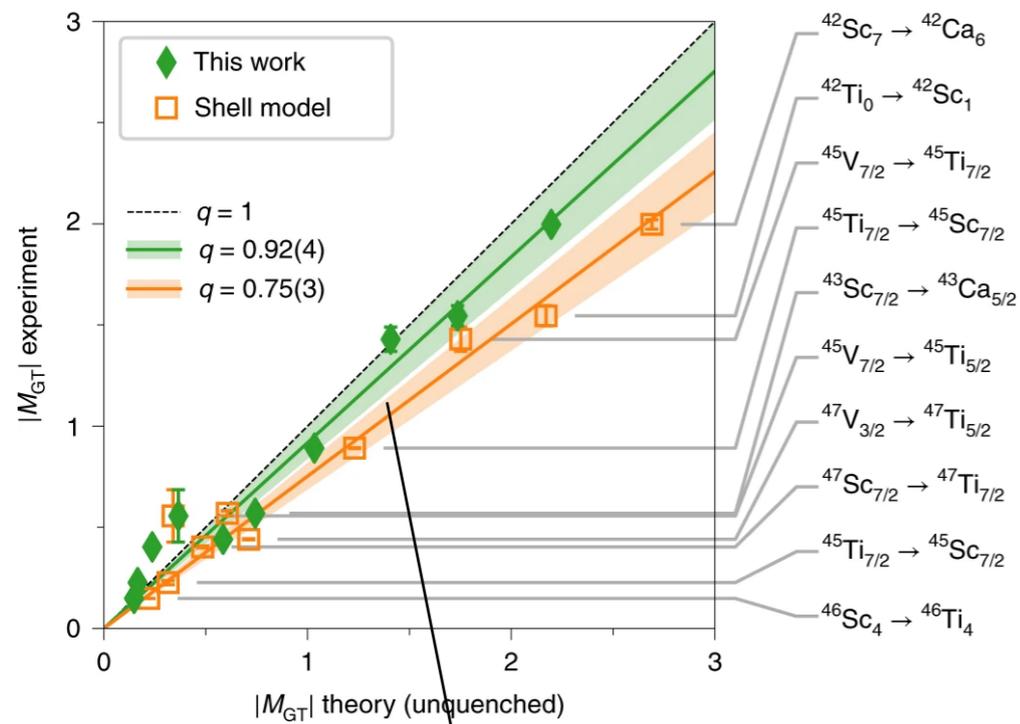


Including nuclear correlations and two-body weak currents ($\square \rightarrow \blacklozenge$)

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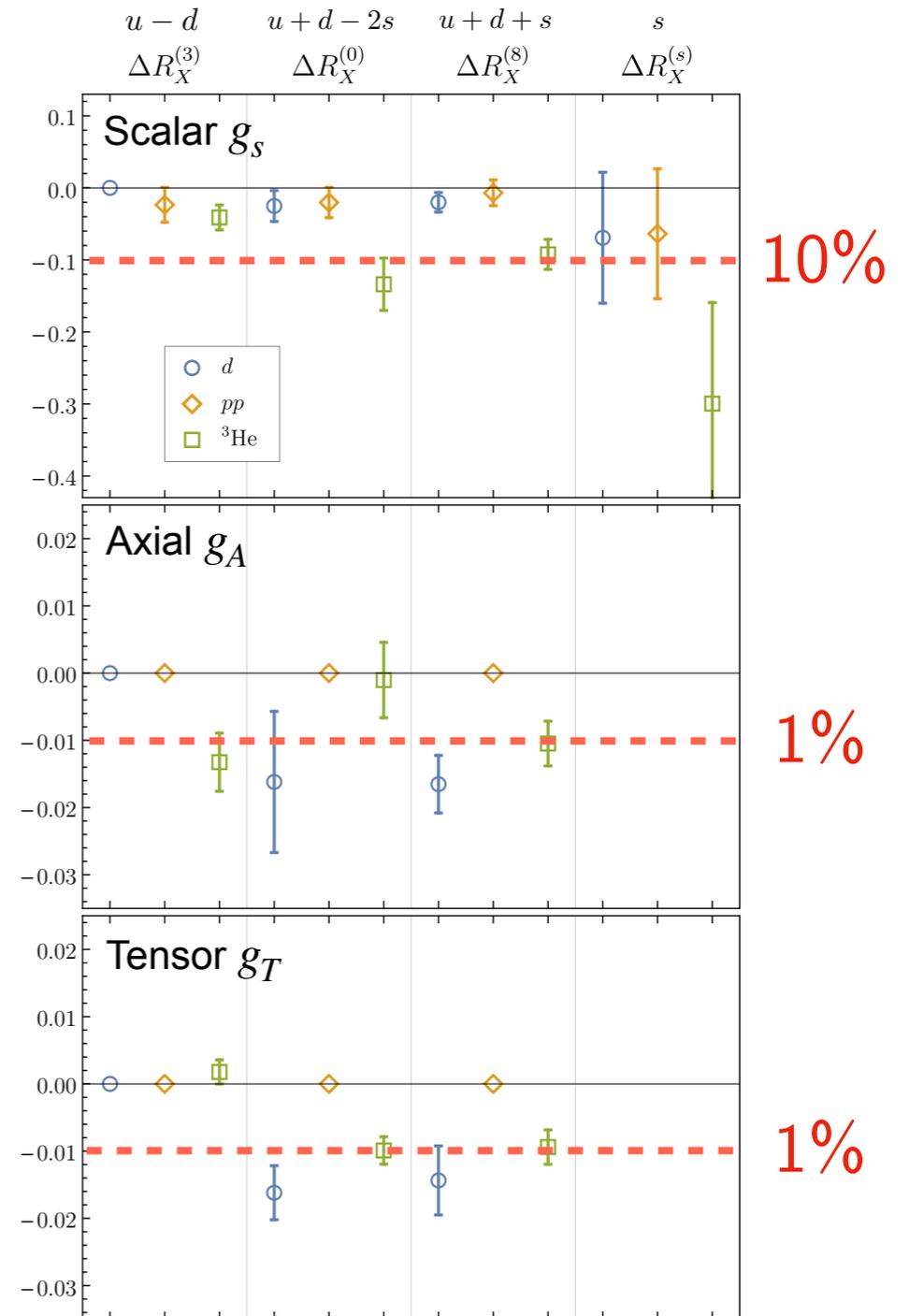
Nuclear effects computed from first principles

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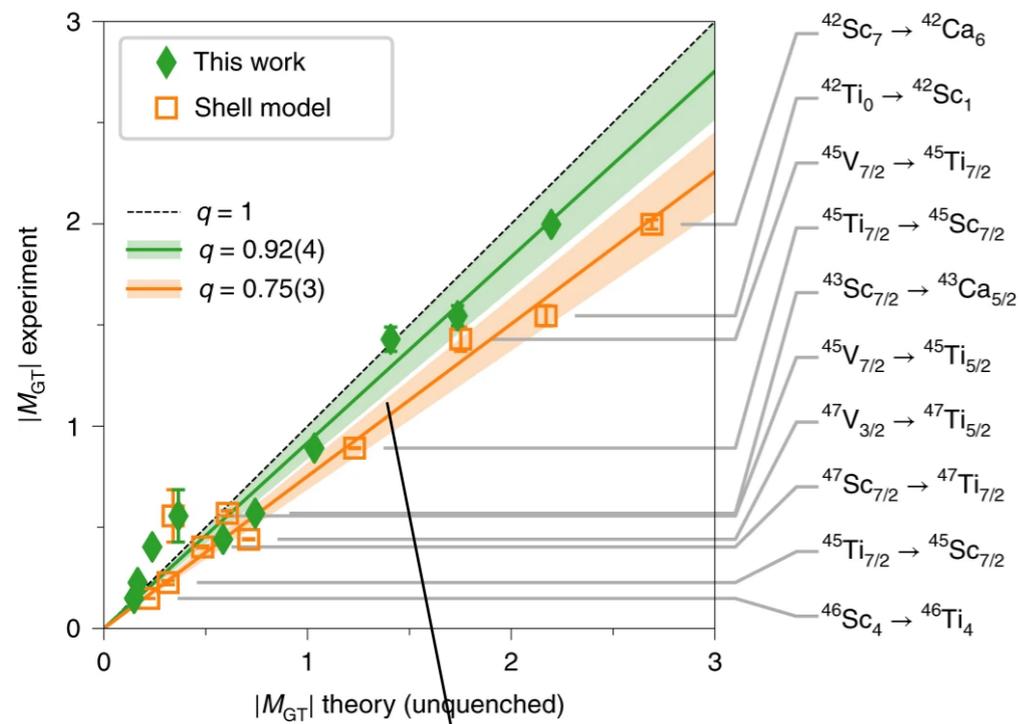


Chang et al. [NPLQCD], *PRL* 120 (2018)

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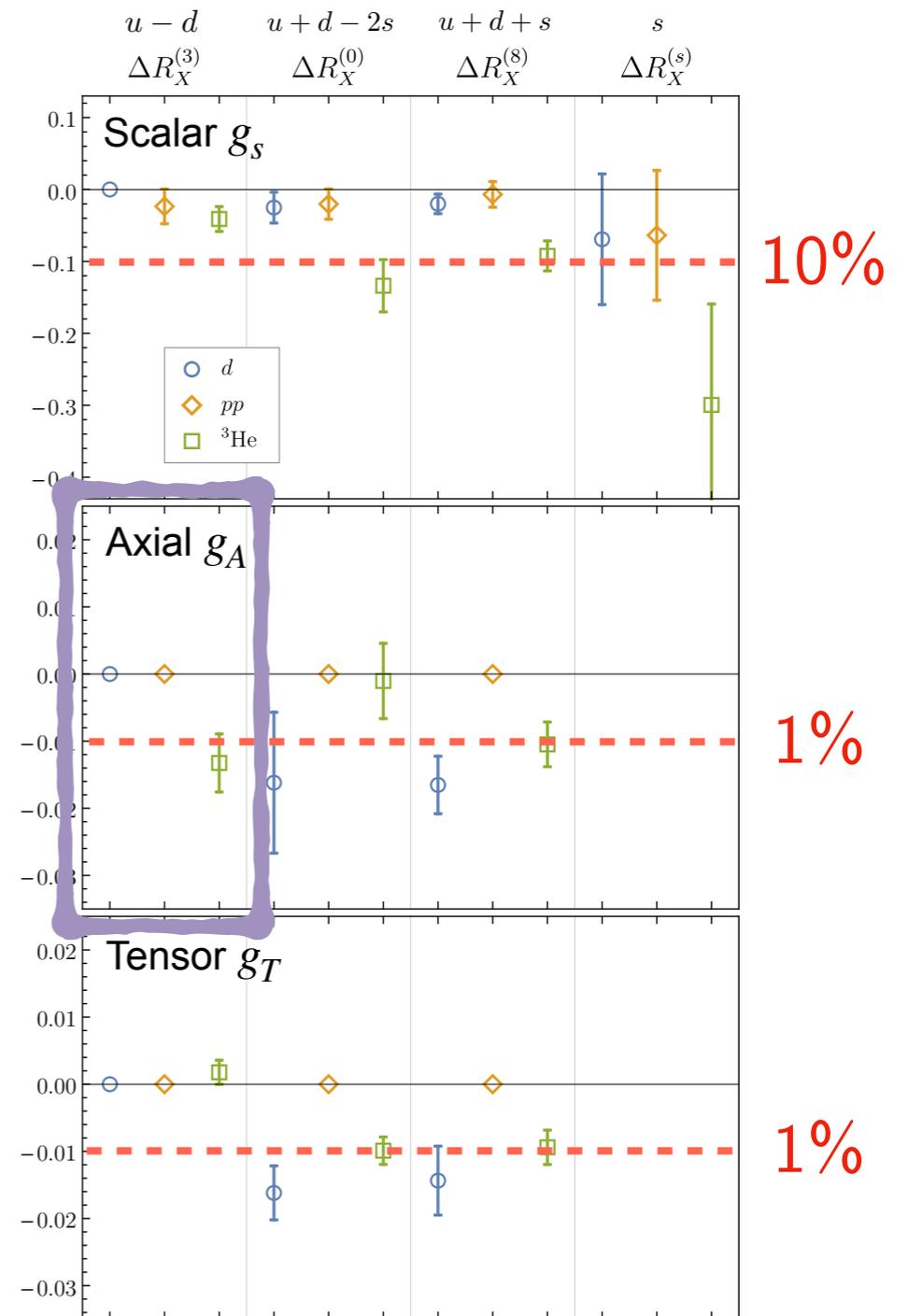
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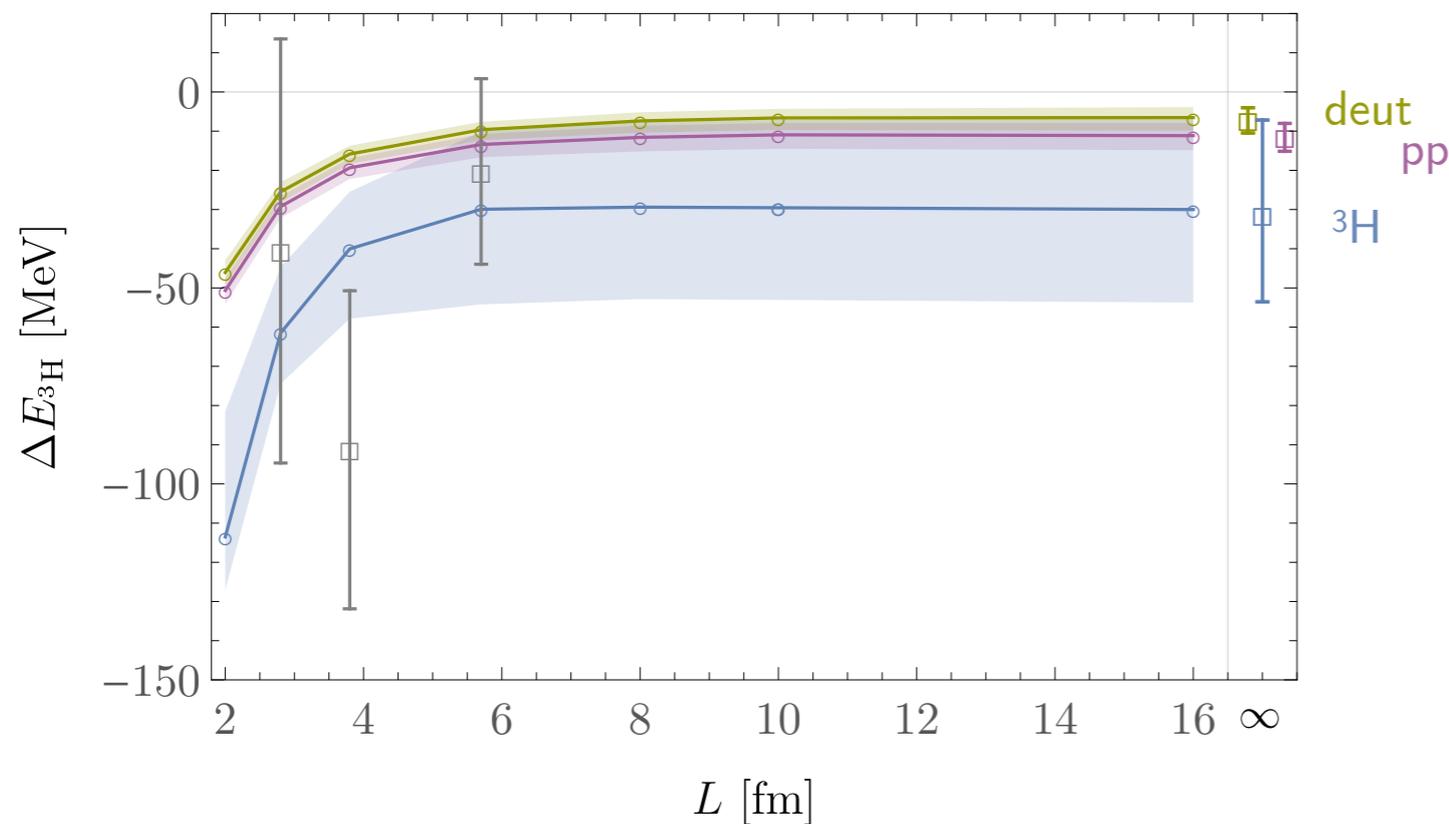
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Parreño et al. [NPLQCD]

PRD 103 (2021), 074511

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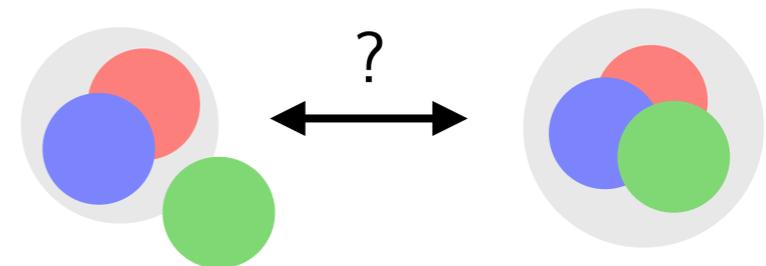
First, we have to determine the ground-state energy of ${}^3\text{H}$



Use π EFT to extrapolate to infinite-volume

Eliyahu, Bazak and Barnea, [PRC 102 \(2020\)](#)

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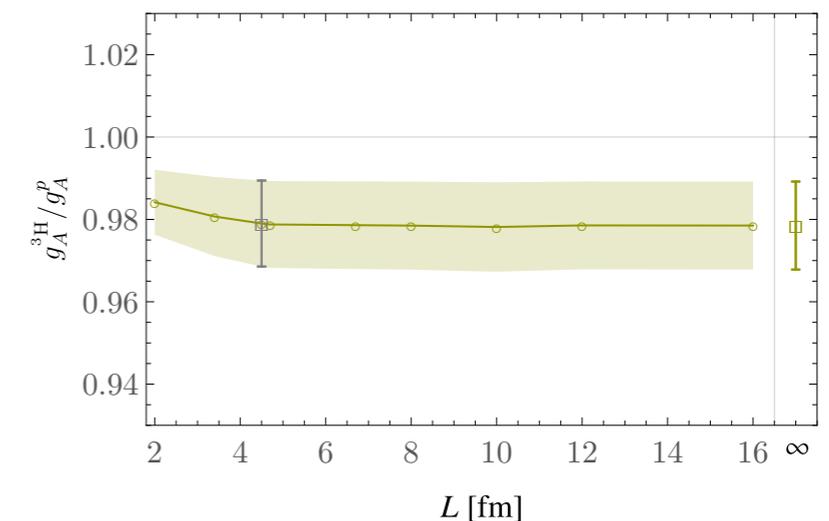
Then we can compute the Gamow-Teller matrix element $\langle \mathbf{GT} \rangle = g_A^{3\text{H}} / g_A^p$

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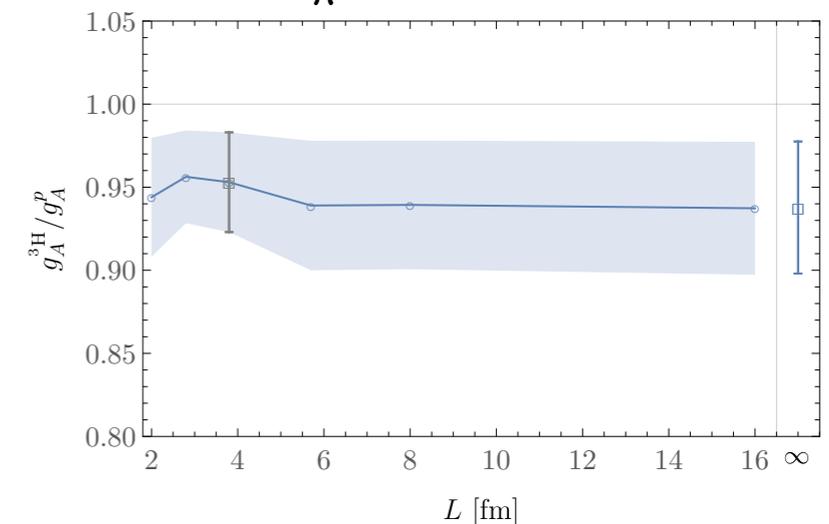
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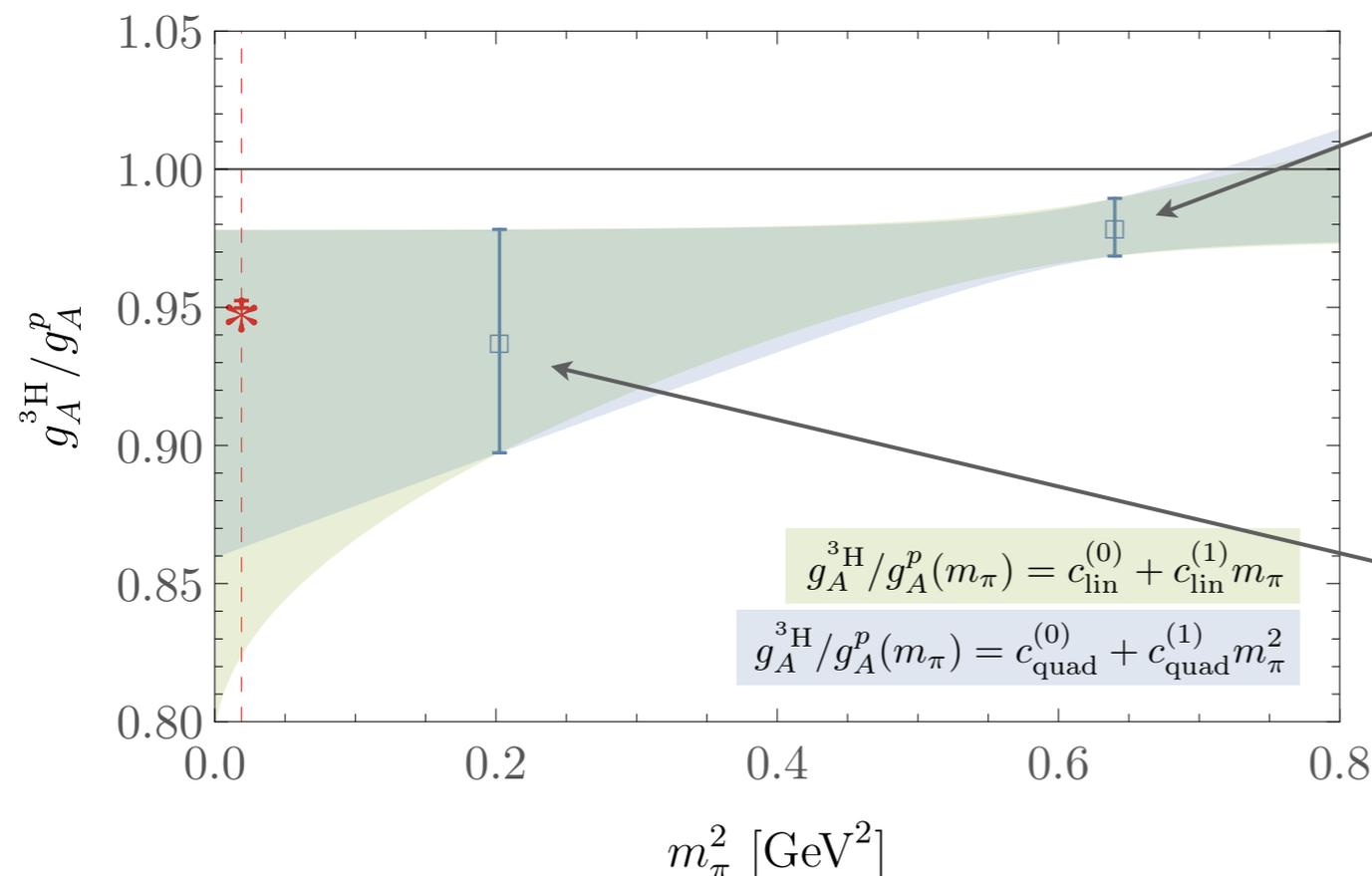
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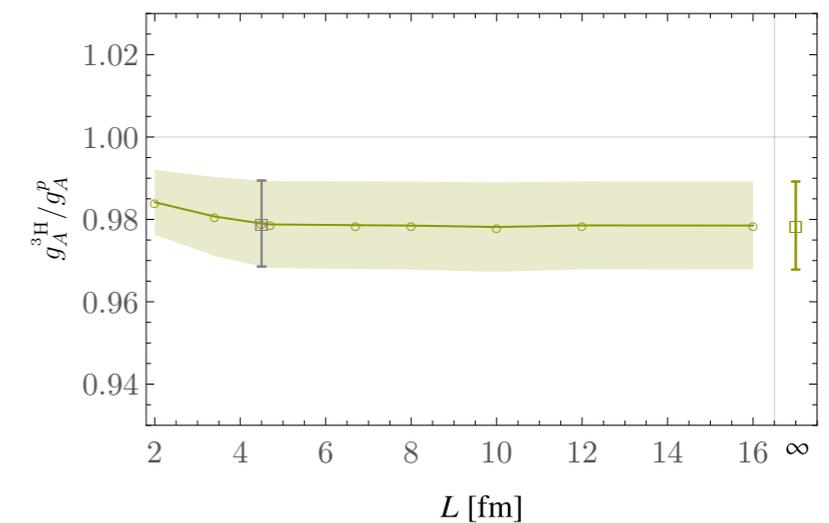
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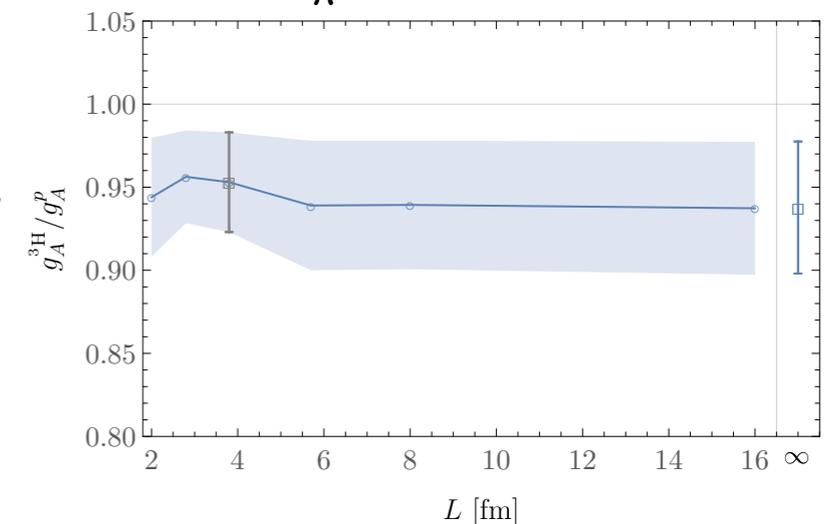
[Detmold and Shanahan, PRD 103 \(2021\)](#)



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Conclusions

- ✿ LQCD is able to reproduce the triton axial charge as well as the helium momentum fraction, indicating that first-principles calculations are possible, but still need closer-to-physical values for the quark masses
- ✿ While lattice calculations are only able to reach light nuclei ($A \leq 4$), they can be used to reach larger systems with EFTs and many-body techniques
[Barnea et al., PRL 114 \(2015\)](#)
[Contessi et al., PLB 772 \(2017\)](#)
[Bansal et al., PRC 98 \(2018\)](#)

Thank you!

