Strangeness $S = -3$ and -4 baryon-baryon interactions in chiral effective field theory

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(Hoai Le, Ulf-G. Meißner, Andreas Nogga)

BB interaction in chiral effective field theory

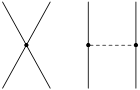
Baryon-baryon interaction in SU(3) χEFT à la Weinberg (1990) [up to NLO]
(in complete analogy to the study of *NN* in χEFT by E. Epelbaum et al.)

Advantages:

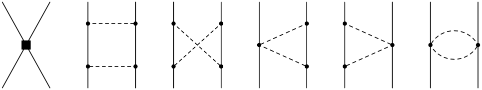
- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three-baryon forces and external current operators in a consistent way

- degrees of freedom: octet baryons (N, Λ, Σ, Ξ), pseudoscalar mesons (π, K, η)
- pseudoscalar-meson exchanges
- contact terms – represent unresolved short-distance dynamics)

LO :



NLO :



LO: H. Polinder, J.H., U.-G. Meißner, NPA 779 (2006) 244

NLO: J.H., N. Kaiser, U.-G. Meißner, A. Nogga, S. Petschauer, W. Weise, NPA 915 (2013) 24

Contact terms for BB

LO: $\mathcal{L}^{(0)} = \tilde{C}_1 \langle \bar{B}_a (\Gamma_i B)_a \bar{B}_b (\Gamma_i B)_b \rangle + \tilde{C}_2 \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \rangle + \ldots$

NLO: $\mathcal{L}^{(2)} = C_1 \left(\left\langle (\partial^\mu \bar{B})_a (\Gamma_i B)_a (\partial_\mu \bar{B})_b (\Gamma_i B)_b \right\rangle + \langle \bar{B}_a (\Gamma_i \partial^\mu B)_a \bar{B}_b (\Gamma_i \partial_\mu B)_b \rangle \right) + \ldots$

$+ C_1^\chi \langle \bar{B}_a \bar{B}_b (\Gamma_i B)_b (\Gamma_i B)_a \chi \rangle + \ldots$

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

$$\chi = 2B_0 \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \approx \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}$$

a, b ... Dirac indices of the particles

$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu \gamma_5\,,\ \Gamma_5 = \gamma_5$

$\tilde{C}_i$, C_i, C_i^χ ... low-energy constants (LECs)
(need to be fixed by a fit to NN, YN, ... data)

(S. Petschauer and N. Kaiser, NPA 916 (2013) 1)

$SU(3)$ structure for scattering of two octet baryons $\rightarrow$
$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

BB interaction can be given in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8a}, C^{8s}, C^{10^*}, C^{10}, C^{27}

	Channel	Isospin	$V_{{}^3S_1,{}^1P_1,\ldots}$	Isospin	$V_{{}^1S_0,{}^3P_0,{}^3P_1,{}^3P_2,\ldots}$
$S=\ \ 0$	$NN \rightarrow NN$	0	C^{10^*}	1	C^{27}
$S=-1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a}+C^{10^*}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}+C^{8_s}\right)$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(-C^{8_a}+C^{10^*}\right)$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}+C^{8_s}\right)$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a}+C^{10^*}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}+9C^{8_s}\right)$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	C^{10}	$\frac{3}{2}$	C^{27}
$S=-3$	$\Xi\Lambda \rightarrow \Xi\Lambda$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a}+C^{10}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}+C^{8_s}\right)$
	$\Xi\Lambda \rightarrow \Xi\Sigma$	$\frac{1}{2}$	$\frac{1}{2}\left(-C^{8_a}+C^{10}\right)$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}+C^{8_s}\right)$
	$\Xi\Sigma \rightarrow \Xi\Sigma$	$\frac{1}{2}$	$\frac{1}{2}\left(C^{8_a}+C^{10}\right)$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}+9C^{8_s}\right)$
	$\Xi\Sigma \rightarrow \Xi\Sigma$	$\frac{3}{2}$	C^{10^*}	$\frac{3}{2}$	C^{27}
$S=-4$	$\Xi\Xi \rightarrow \Xi\Xi$	0	C^{10}	1	C^{27}

10 and 10^* representations interchange their roles when going from the $S=0,-1$ to the $S=-3,-4$ channels

BB contact interaction up to NLO

SU(3) structure + breaking of SU(3) symmetry

(S. Petschauer, N. Kaiser, NPA 916 (2013) 1)

Contact interaction - partial-wave projected

$$
\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) + C^{\chi}_{^1S_0}(m_K^2 - m_\pi^2) \\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) + C^{\chi}_{^3S_1}(m_K^2 - m_\pi^2) \\
V(\alpha) &= C_\alpha \, p\, p' \qquad \alpha \,\hat{=}\, {}^1P_1,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2 \\
V(^3D_1 \leftrightarrow {}^3S_1) &= C_{^3S_1 - ^3D_1}\, p'^2,\ C_{^3S_1 - ^3D_1}\, p^2 \\
V(^1P_1 \leftrightarrow {}^3P_1) &= C_{^1P_1 - ^3P_1}\, p\, p'
\end{aligned}
$$

SU(3) symmetry $\Rightarrow$ number of contact terms:

LO: 2 (NN) + 3 (YN) + 1 (YY) NLO: 7 (NN) + 11 (YN) + 4 (YY)

(6 + 6 for $^1S_0$ + $^3S_1$ partial waves)

SU(3) symmetry breaking contact terms C_i^χ:

for $S = 0$ to $S = -4$: 6 LECs for $^1S_0$ and 6 LECs for $^3S_1$

$\rightarrow$ cannot be determined from presently available data

(we assume $C_i^\chi = 0$, unless data require the opposite)

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T^{\nu'\nu,J}_{\rho'\rho}(p',p) &= V^{\nu'\nu,J}_{\rho'\rho}(p',p) \\
&+ \sum_{\rho'',\nu''}\int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p)
\end{aligned}
$$

$\rho',\ \rho = \Lambda N,\ \Sigma N\quad (\Lambda\Lambda,\ \Xi N,\ \Lambda\Sigma,\ \Sigma\Sigma)\quad (\Xi\Lambda,\ \Xi\Sigma)$

LS equation is solved for particle channels (in momentum space)
(SU(3) symmetry is broken by mass difference of baryons: $\mu_\rho = M_{B_1}M_{B_2}/(M_{B_1}+M_{B_2})$)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$
V^{\nu'\nu,J}_{\rho'\rho}(p',p) \to f^\Lambda(p')\,V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^\Lambda(p);\quad f^\Lambda(p) = e^{-(p/\Lambda)^4}
$$

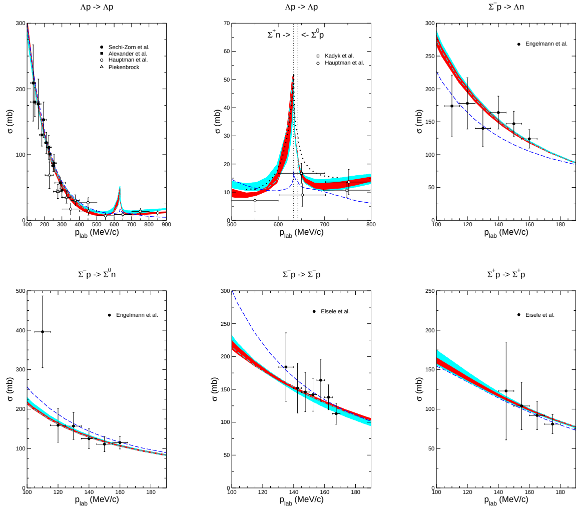
consider values $\Lambda = 500$ - 650 MeV [guided by NN, achieved χ^2]

ideally the regulator (Λ) dependence should be absorbed completely by the LECs

in practice there is a residual regulator dependence (shown by bands below)

- tells us something about the convergence
- tells us something about the size of higher-order contributions

YN integrated cross sections



NLO13 ... all S-wave LECs are fixed from a fit directly to available YN data

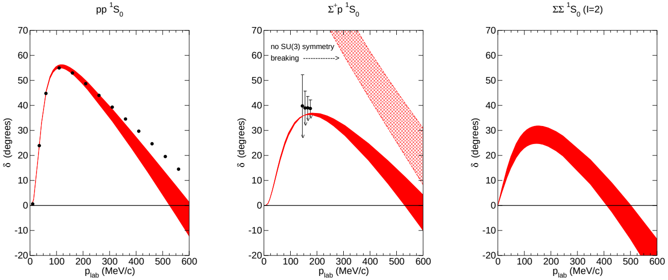
NLO19 ... consider constraints from the NN interaction within (broken) SU(3) symmetry

NLO13: J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

NLO19: J.H., U.-G. Meißner, A. Nogga, EPJA 56 (2020) 91

Jülich '04: J.H., U.-G. Meißner, PRC 72 (2005) 044005

Breaking of SU(3) symmetry



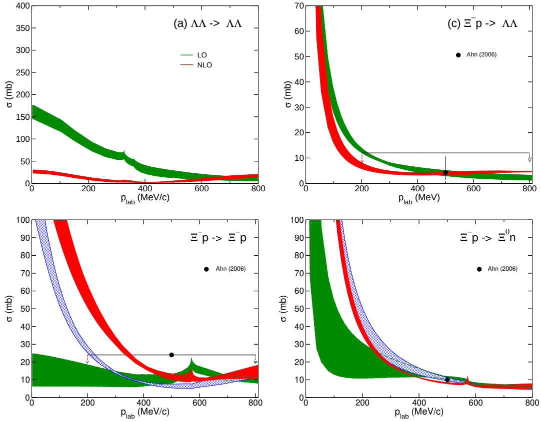
$$
\begin{aligned}
V_{pp} &= \tilde{C}^{27} + C^{27}(p^2 + p'^2) + \frac{1}{2}C_1^{\chi}(m_K^2 - m_\pi^2) + V^{OBE} + V^{TBE} \\
V_{\Sigma^+ p} &= \tilde{C}^{27} + C^{27}(p^2 + p'^2) + \frac{1}{4}C_1^{\chi}(m_K^2 - m_\pi^2) + V^{OBE} + V^{TBE} \\
V_{\Sigma^+\Sigma^+} &= \tilde{C}^{27} + C^{27}(p^2 + p'^2) + V^{OBE} + V^{TBE} \\
V_{\Xi^0\Sigma^+} &= \tilde{C}^{27} + C^{27}(p^2 + p'^2) + \frac{1}{4}C_2^{\chi}(m_K^2 - m_\pi^2) + V^{OBE} + V^{TBE} \\
V_{\Xi^0\Xi^0} &= \tilde{C}^{27} + C^{27}(p^2 + p'^2) + \frac{1}{2}C_2^{\chi}(m_K^2 - m_\pi^2) + V^{OBE} + V^{TBE}
\end{aligned}
$$

J.H., U.-G. Meißner, S. Petschauer, EPJA 51 (2015) 17:

one can determine $\tilde{C}^{27}$, C^{27}, C_1^{χ} from a combined fit to pp and $\Sigma^+ p$

$C_1^{\chi} < 0 \Rightarrow$ increasing repulsion for $S = 0 \rightarrow S = -1 \rightarrow S = -2$

Selected results for $S = -2$: ΞN scattering



LO : H. Polinder, J.H., U.-G. Meißner, PLB 653 (2007) 29

NLO16: J.H., U.-G. Meißner, S. Petschauer, NPA 954 (2016) 273

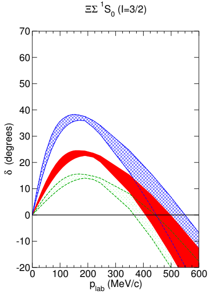
NLO19: J.H., U.-G. Meißner, EPJA 55 (2019) 23

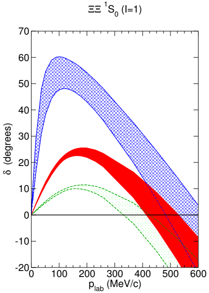
NLO16 versus NLO19: differences in the SU(3) breaking in the ${}^3S_1$-${}^3D_1$ partial wave

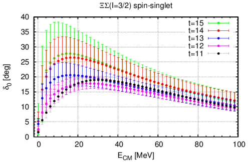
in-medium properties: $U_\Xi \approx +20$ MeV versus $U_\Xi \approx -5$ MeV (empirical: $U_\Xi \approx -15$ MeV)

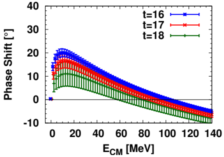
Breaking of SU(3) symmetry: S=-3,-4

(i) blue: $C_2^\chi = 0$; (ii) red: $C_2^\chi = -C_1^\chi/2$; (iii) green: $C_2^\chi = -C_1^\chi$



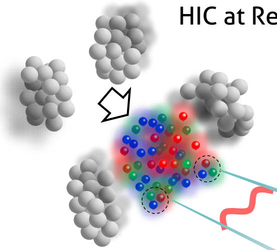






HAL QCD lattice results ($m_\pi = 146$ MeV): N. Ishii (ΞΣ), T. Doi (ΞΞ) Lattice2017 (Granada, June 2017)

How HIC Can Tell Us Interaction?



HIC at Relativistic Energies@RHIC, LHC

Production of Quark-Gluon Plasma

Crossover Transition Into Hadron
Particle Abundance – Thermal Eq.

Incl. **Rare Particles**

Measuring Pair Correlation
→Constrain Pairwise Interaction

$$Q = \sqrt{-\left(\frac{p_1 - p_2}{2} - \frac{(p_1 - p_2)\cdot P}{P^2} P\right)^2}$$

$$C_{AB}(Q) = \frac{N_{AB}^{\mathrm{pair}}(Q)}{N_A N_B(Q)} = \begin{cases} 1 & \textbf{No Correlation} \\ \text{others} & \textbf{Interaction, Interference etc} \end{cases}$$

Two-particle correlation function

Koonin-Pratt formalism (consider only correlations in S-waves)

Correlation function for identical particles ($\Lambda\Lambda$, $\Xi^-\Xi^-$, ...)

$$C(k) \simeq 1 - \frac{1}{2}\exp(-4k^2R^2) + \frac{1}{2}\int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2\right]$$

Correlation function for non-identical particles (Λp, $\Xi^- p$, $\Lambda\Xi^-$, ...)

$$C(k) \simeq 1 + \int_0^\infty 4\pi r^2\, dr\, S_{12}(\mathbf{r}) \left[|\psi(k,r)|^2 - |j_0(kr)|^2\right]$$

Boundary condition for wave function:

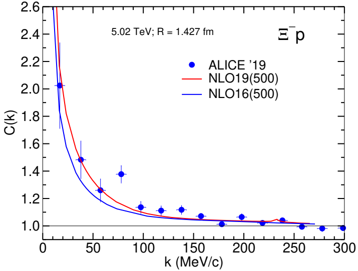
$$\psi(k,r) \to \frac{e^{-i\delta}}{kr}\sin(kr+\delta) = \frac{1}{2ikr}\left[e^{ikr} - e^{-2i\delta}e^{-ikr}\right] \quad (r \to \infty)$$

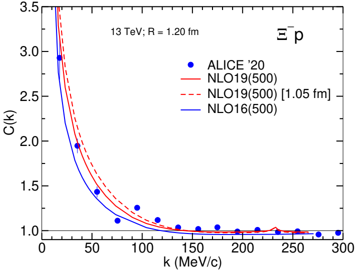
S_{12} ... source function, which describes the space-time distribution of emitted particles
$\rightarrow$ assume a spherical Gaussian shape for S_{12} with source radius R:

$S_{12}(\mathbf{r}) = \exp(-r^2/4R^2)/(2\sqrt{\pi}R)^3$

$\Psi^{(-)}(\mathbf{r},\mathbf{k})$ ... wave function in the outgoing state; k ... center-of-mass momentum

Results for $S = -2$: ΞN





ALICE Collaboration: *p*-Pb at 5.02 TeV (PRL 123 (2019) 112002) *pp* at 13 TeV (Nature 588 (2020) 232)

$R = 1.427$ fm; $\lambda = 0.513$ $R = 1.02$ fm; $\lambda = 1$

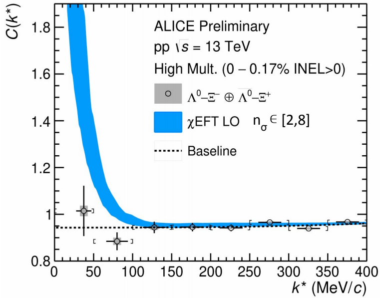
source radius: basically determined from fit to measured *pp* correlation function

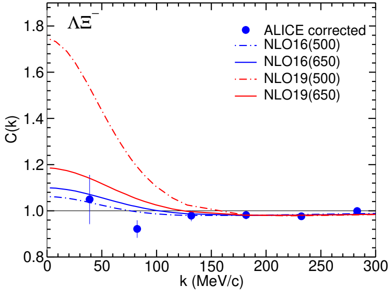
($R = 1.427$ fm for data from *p*-Pb collisions and $R = 1.18$ fm for data from *pp*)

$$C_{\rm th}(k) = \frac{1}{4} C_{^1S_0}(k) + \frac{3}{4} C_{^3S_1}(k)$$
$$C(k) = (a + b\,k)\,(1 + \lambda\,(C_{\rm th}(k) - 1))$$

a, b, λ ... additional parameters that need to be determined

Results for $S = -3$: $\Lambda\Xi^-$





Emma Chizzali (ALICE Collaboration) at SQM 2021 (May 2021): *pp* at 13 TeV

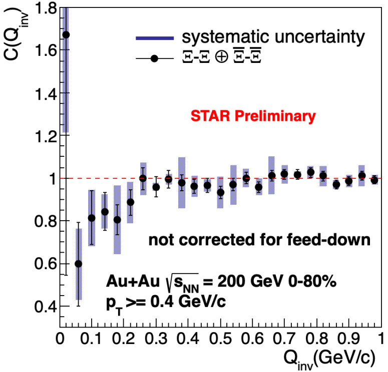
$R = 1.03$ fm; $\lambda = 0.36$

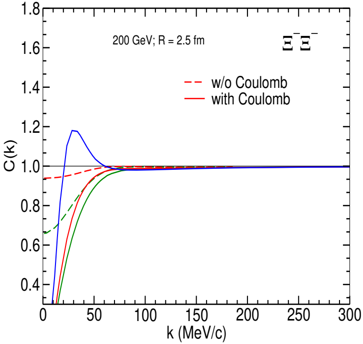
LO potential (J.H., U.-G. Meißner, PLB 684 (2010) 275):
produces a bound state $\rightarrow$ not supported by measurement

LO rel. chiral EFT potential (Z.-W. Liu et al., PRC 103 (2021) 025201): likewise too attractive

NLO19:
$a_s = -0.99 \cdots -0.89$ fm, $r_s = 4.63 \cdots 5.77$ fm; $a_t = -0.42 \cdots -1.66$ fm, $r_t = 6.33 \cdots 1.49$ fm

NLO16:
$a_s = -0.99 \cdots -0.89$ fm, $r_s = 4.63 \cdots 5.77$ fm; $a_t = 0.026 \cdots -0.12$ fm, $r_t = 32.0 \cdots 702$ fm





Moe Isshiki (STAR Collaboration) at SQM 2021 (arXiv:2109.10953): Au+Au at 200 GeV

only preliminary results: $R = 2.5 - 5$ fm; $\lambda = ??$

use for calculation: $R = 2.5$ fm; $\lambda = 1$

$a_s = -7.04$ fm (no SU(3) breaking) -1.71 fm (moderate SU(3) breaking) -0.71 fm (strong SU(3) breaking)

Summary

Baryon-baryon interaction constructed within chiral EFT

- Approach is based on a modified Weinberg power counting, analogous to applications for NN scattering
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints + $SU(3)_f$ breaking
- $S=-1$: Excellent results at next-to-leading order (NLO)
  Λp, ΣN low-energy data are reproduced with a quality comparable to phenomenological models
- $S=-2$: $\Lambda\Lambda$, ΞN results in agreement with empirical constraints
- moderately attractive Ξ-nuclear interaction (U_Ξ) can be achieved
- predictions for Ξ hypernuclei in talk by Hoai Le

Baryon-baryon two-body momentum correlation functions

- $S=-2$ (ΞN): predictions of our ΞN NLO interactions are in agreement with data from the ALICE Collaboration
- $S=-3$ and -4: measurements (ALICE, STAR) are just becoming available
  so far only qualitative conclusions can be drawn

Backup slides

	Channel	Isospin	V
$^1S_0$	$\Xi N \to \Xi N$	0	C^{8_a}
	$\Xi N \to \Xi N$	1	$\frac{1}{3}\left(C^{10}+C^{10^*}+C^{8_a}\right)$
	$\Xi N \to \Sigma\Lambda$	1	$\frac{\sqrt{6}}{6}\left(C^{10}-C^{10^*}\right)$
	$\Xi N \to \Sigma\Sigma$	1	$\frac{\sqrt{2}}{6}\left(C^{10}+C^{10^*}-2C^{8_a}\right)$
	$\Sigma\Lambda \to \Sigma\Lambda$	1	$\frac{1}{2}\left(C^{10}+C^{10^*}\right)$
	$\Sigma\Lambda \to \Sigma\Sigma$	1	$\frac{\sqrt{3}}{6}\left(C^{10}-C^{10^*}\right)$
	$\Sigma\Sigma \to \Sigma\Sigma$	1	$\frac{1}{6}\left(C^{10}+C^{10^*}+4C^{8_a}\right)$
$^3S_1$	$\Lambda\Lambda \to \Lambda\Lambda$	0	$\frac{1}{40}\left(27C^{27}+8C^{8_s}+5C^{1}\right)$
	$\Lambda\Lambda \to \Xi N$	0	$\frac{-1}{40}\left(18C^{27}-8C^{8_s}-10C^{1}\right)$
	$\Lambda\Lambda \to \Sigma\Sigma$	0	$\frac{\sqrt{3}}{40}\left(-3C^{27}+8C^{8_s}-5C^{1}\right)$
	$\Xi N \to \Xi N$	0	$\frac{1}{40}\left(12C^{27}+8C^{8_s}+20C^{1}\right)$
	$\Xi N \to \Sigma\Sigma$	0	$\frac{\sqrt{3}}{40}\left(2C^{27}+8C^{8_s}-10C^{1}\right)$
	$\Sigma\Sigma \to \Sigma\Sigma$	0	$\frac{1}{40}\left(C^{27}+24C^{8_s}+15C^{1}\right)$
	$\Xi N \to \Xi N$	1	$\frac{1}{5}\left(2C^{27}+3C^{8_s}\right)$
	$\Xi N \to \Sigma\Lambda$	1	$\frac{\sqrt{6}}{5}\left(C^{27}-C^{8_s}\right)$
	$\Sigma\Lambda \to \Sigma\Lambda$	1	$\frac{1}{5}\left(3C^{27}+2C^{8_s}\right)$
	$\Sigma\Sigma \to \Sigma\Sigma$	2	C^{27}

BB interaction up to NLO

Pseudoscalar-meson exchange

$$V^{OBE}_{B_1B_2\to B_1'B_2'} = -f_{B_1B_1'P}f_{B_2B_2'P}\frac{(\vec\sigma_1\cdot\vec q)\,(\vec\sigma_2\cdot\vec q)}{\vec q^{\,2}+m_P^2}, \quad \vec q=\vec p\,'-\vec p$$

$$V^{TBE}_{B_1B_2\to B_1'B_2'} = \dots$$

$f_{B_1B_1'P}$ ... coupling constants fulfil standard SU(3) relations
m_P ... mass of the exchanged pseudoscalar meson

$SU(3)$ symmetry breaking due to the mass splitting of the ps mesons
(m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)

Contact interaction - partial-wave projected

$$\begin{aligned}
V(^1S_0) &= \tilde C_{^1S_0} + C_{^1S_0}(p^2+p'^2) + C^\chi_{^1S_0}(m_K^2-m_\pi^2)\\
V(^3S_1) &= \tilde C_{^3S_1} + C_{^3S_1}(p^2+p'^2) + C^\chi_{^3S_1}(m_K^2-m_\pi^2)\\
V(\alpha) &= C_\alpha\, p\, p' \qquad \alpha \,\hat=\, {}^1P_1,\ {}^3P_0,\ {}^3P_1,\ {}^3P_2\\
V(^3D_1\leftrightarrow {}^3S_1) &= C_{^3S_1-{}^3D_1}\, p'^2,\ C_{^3S_1-{}^3D_1}\, p^2\\
V(^1P_1\leftrightarrow {}^3P_1) &= C_{^1P_1-{}^3P_1}\, p\, p'
\end{aligned}$$