The N/D study on the singularity structure of πN scattering amplitudes

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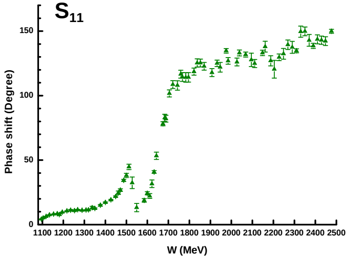
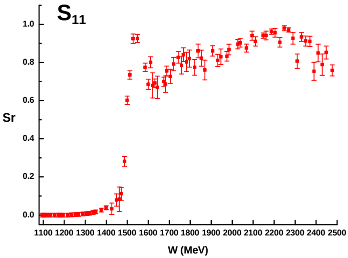
Contents

- The πN scattering $\rightarrow$ one of the most fundamental and important processes in nuclear or hadron physics
- Decades of researching
- Various experiments and phenomena
  $(L_{2I\ 2J}$ convention, $W = \sqrt{s}$, $S_r = 1 - \eta^2)$ [SAID: WI 08]





- Problems to study
  - Low energy properties:
    $\pi N \sigma$-term, subthreshold expansions
    [C. Ditsche et. al. 2012 JHEP][Hoferichter et. al. 2016 Phys.Rept.]
  - Intermediate resonances: $\Delta(1232)$, $N^*(890)$ $N^*(1440)$, $N^*(1535)\cdots$
- Methods
  - Perturbative calculation
  - Lattice QCD
  - Dispersion relations

The Production (PKU) Representation

The factorized S matrix and the separable singularities:

$$S^{phy.} = \prod_i S^{R_i} \cdot S^{cut} \; . \qquad (1)$$

S^{cut}: no longer contains any pole:

$$\begin{aligned} S^{cut} &= e^{2i\rho f(s)} \; , \\ \rho(s) &= \frac{\sqrt{(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)}}{s} \\ f(s) &= \frac{s}{\pi}\int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} + \frac{s}{\pi}\int_{R'} \frac{\mathrm{Im}_R f(s')}{s'(s'-s)} \; . \end{aligned} \qquad (2)$$

Subtraction constant can be determined!

Mandelstam Analyticity(Polynomial boundedness of scattering amplitudes)

[Z. Y. Zhou and H.Z., NPA, 2006]

$$f(0) = 0 \; . \qquad (3)$$

Phase shift components

$$\text{Im}_{L,R} f(s) = -\frac{1}{2\rho(s)} \log|S^{phy}(s)| \ , \quad S^{phy} = 1 + 2i\rho T \ . \qquad (4)$$

The eq.(4) may be modified in πN scatterings . [QuZhi Li et al., 2021, ArXive:2102.00977]

The phase is additive, $\delta(s) = \sum_i \delta_{R_i} + \delta_{b.g.}$.

$$\delta_{b.g.}(s) = \rho(s) f(s) \ . \qquad (5)$$

The left hand cut $\rightarrow$ (empirically) negative phase shift (proved in quantum mechanical potential scatterings)

[T. Regge 1958 Nuovo Cimento]

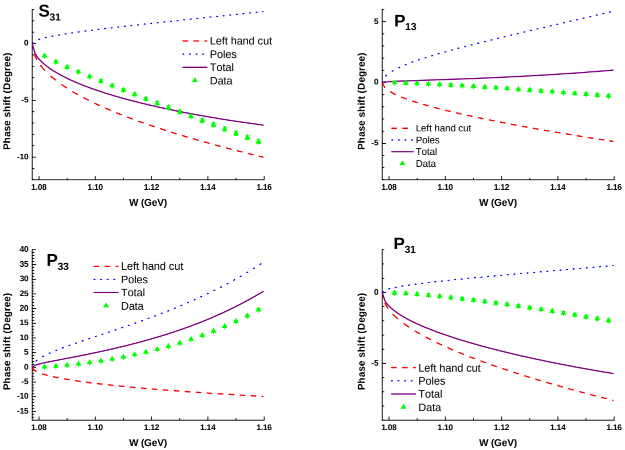
- contributions of poles
  - bound states $\rightarrow$ negative phase shift
  - virtual states (usually hidden !) $\rightarrow$ positive phase shift
  - resonances $\rightarrow$ positive phase shift

Tree level phase shift results

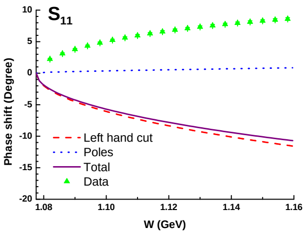
[Y. F. Wang, D. L. Yao, H.Q. Zheng, EPJC 2018]

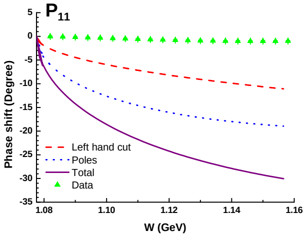
$L_{2I\ 2J}$ convention, $W = \sqrt{s}$, data: green triangles [SAID: WI 08]

Except S11 and P11 channel, other channels agree with data qualitatively:



FINDING S_{11} HIDDEN POLE





P_{11}

Adding a shadow pole, due to nucleon pole,gives positive phase shifts.

S_{11}

Assuming exist a resonance, fitting data to give the position

Up to $O(p^3)$, position of the resonance:

$$\sqrt{s} = (0.895 - 0.164i)\,GeV \tag{6}$$

$N^*(890)$ pole in N/D method

QuZhi Li et al., 2021, ArXive:2102.00977

$$T(s) = N(s)/D(s)\ . \tag{7}$$

where :

- D(s) only contains right hand cut:

  $\mathrm{Im}_R[D(s)] = -\rho(s)N(s)$;

- N(s) contains left hand cut and poles(bound states):
  $\mathrm{Im}_L[N(s)] = \mathrm{Im}_L[T(s)]D(s)$.

According to dispersion relation:

$$D(s) = 1 - \frac{s-s_0}{\pi}\int_R \frac{\rho(s')N(s')}{(s'-s)(s'-s_0)}ds' \ ,$$
$$N(s) = N(s_0) + \frac{s-s_0}{\pi}\int_L \frac{D(s')\mathrm{Im}_L[T(s')]}{(s'-s)(s'-s_0)}ds' \ . \qquad (8)$$

$\mathrm{Im}_L T$ as an input.

$$N(s) = N(s_0) + \tilde{B}(s,s_0) + \frac{s-s_0}{\pi}\int_R \frac{\tilde{B}(s,s')\rho(s')N(s')}{(s'-s_0)(s-s')}ds' \qquad (9)$$

$$\tilde{B}(s,s') = \frac{s-s'}{\pi}\int_L \frac{\mathrm{Im}_L T(\tilde{s})}{(\tilde{s}-s)(\tilde{s}-s')}d\tilde{s}$$

Analytic continuation:

$$D^{\mathrm{II}}(s) = D(s) + 2i\rho N(s) \ , \quad N^{\mathrm{II}}(s) = N(s) \ , \qquad (10)$$

A toy model calculation

$$N(s) = \sum_i \frac{\gamma_i}{s - s_i} \ , \tag{11}$$

$$D(s) = 1 - \frac{s - s_0}{\pi} \int_R \frac{\rho(s')N(s')}{(s' - s)(s' - s_0)} ds' \ . \tag{12}$$

	$Case$ I	$Case$ II
s_1	0	$-m_N^2$
γ_1 (GeV^2)	0.79	1.34
$\sqrt{s_{pole}}$(GeV)	0.810 - 0.125i	0.788 - 0.185i

Table: Subthreshold pole locations using input Eq. (11).

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A toy model calculation

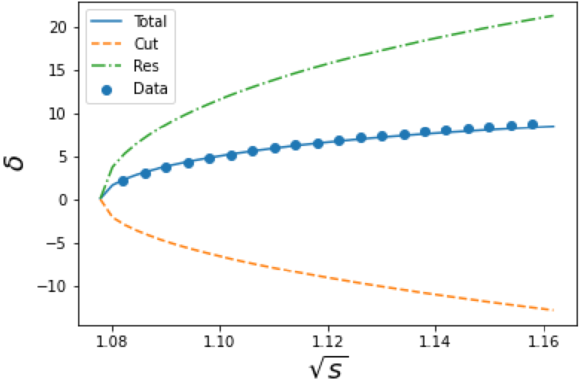

Figure: fit to the S_{11} channel phase shift data, taking $Case$ II as an example

$\mathcal{O}(p^2)$ calculation

The cut structure of πN partial wave amplitudes:

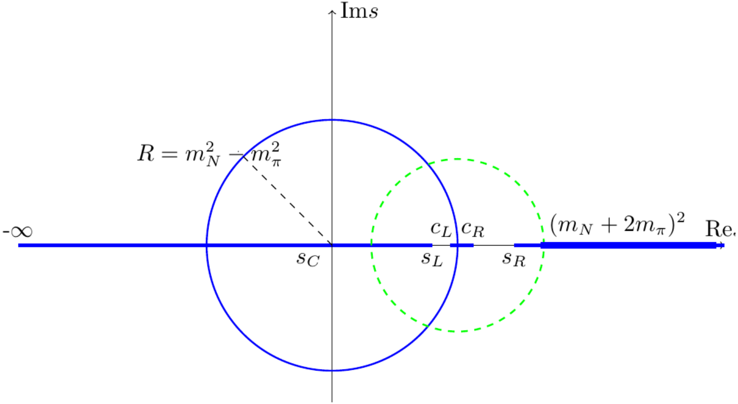

Figure: green dashed circle:χPT range of application. circular cut: t-channel box diagram;sigmet cut (c_L, c_R):u-channel nucleon exchange.

$\mathcal{O}(p^2)$ calculation

Partial wave projection of χPT amplitudes encounter a severe problem at $s = 0$,

$$T\big[\mathcal{O}(p^n)\big](s \to 0) \sim Cs^{-n-1/2} \ , \tag{13}$$

Violating Froissart bound: $T(s) \sim O(s^{-1})$(up to some logarithmic corrections).
General argument gives instead

$$T \sim s^{-\alpha_{\Delta(0)}} \tag{14}$$

$\alpha_\Delta(0)(\simeq 0)$: the intercept parameter of the Regge trajectory of $\Delta(1232)$.
An N/D calculation is nevertheless still doable and gives position:

$$\sqrt{s} = 1.01 \pm 0.19i \ \mathrm{GeV} \ , \tag{15}$$

of $\mathcal{O}(p^2)$ chiral lagrangian with reasonable range of LECs.

A dynamic model calculation

$$\text{disc}\ T(s) = \text{disc}\ T^{(1)}(s) + \text{disc}\ T^{\rho}(s) + \text{disc}\,[\frac{a+bs}{\sqrt{s}}]\ . \tag{16}$$

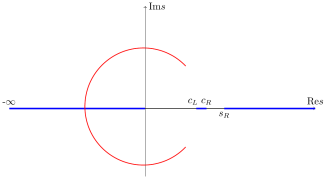

Figure: The $l.h.c.$ by t-channel ρ exchange(circular arc)and u-channel N exchange(line segment from c_L to c_R).

$$\sqrt{s} = 0.90 - 0.20i\text{GeV}\ . \tag{17}$$

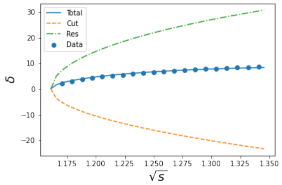

Figure: Phase shift decomposition

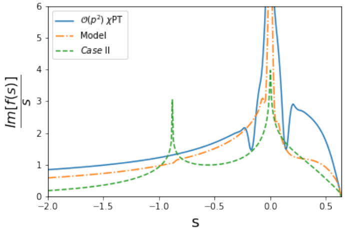

Figure: Comparison among different "spectral" functions. The singular behaviours of $T(s)$ at $s=0$ are $\mathcal{O}(s^{-5/2})$, $\mathcal{O}(s^{-1/2})$ and $\mathcal{O}(s^0)$ for $\mathcal{O}(p^2)$ χPT , model Eq. (16) and $Case$ II, respectively.

$$f(s) = \frac{s}{\pi} \int_L \frac{\mathrm{Im}_L f(s')}{s'(s'-s)} ds', \quad \delta_{b.g.} = \rho(s) f(s) \tag{18}$$

- $\mathrm{Im_L f(s)/s} > 0$ for $s \in (-\infty, s_L) \Rightarrow \delta_{b.g.} < 0$.

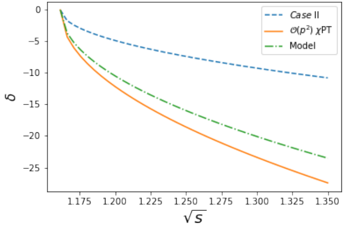

Figure: Different inputs give cuts' contributions.

Three different calculations tell us the existence of $N^*(890)$ doesn't depend on the details of model .

Virtual poles

[Li QZ, HZ, arXiv:2108.03734]

The virtual states was found firstly in S_{11} channel.
u channel nucleon pole exchange contributes a cut $\in [c_L, c_R]$, with $c_L = \frac{(m_N^2 - m_\pi^2)^2}{m_N^2}$ and $c_R = m_N^2 + 2m_\pi^2$.

$$\begin{aligned} s \to c_L: &\qquad T(s) \to -\frac{g^2 m_N^4}{16\pi F^2 \left(4m_N^2 - m_\pi^2\right)} \ln \frac{s - c_L}{c_L - c_R} \ , \\ s \to c_R: &\qquad T(s) \to \frac{g^2 m_N^2 (m_N^2 + 2m_\pi^2)}{\pi F^2 (4m_N^2 - m_\pi^2)} \ln \frac{c_R - c_L}{s - c_R} \ , \end{aligned} \tag{19}$$

$$\begin{aligned} s \to c_L, &\quad S \simeq A_{c_L} + B_{c_L} \ln \frac{s - c_L}{c_L - c_R} \ , \\ s \to c_R, &\quad S \simeq A_{c_R} + B_{c_R} \ln \frac{s - c_R}{c_R - c_L} \ , \end{aligned} \tag{20}$$

Virtual poles

- $S(c_L), S(c_R) \to -\infty$ which are exact (correct to any order of chiral expansions)
- $S = +1$ at s_L and s_R by definition.
- $S(s)$ is real when $s \in (s_L, c_L) \cup (c_R, s_R)$

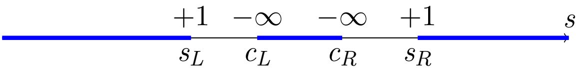

Figure: the value of S-matrix at: s_L, c_L, c_R and s_R.

⇒ There have to be two S matrix zeros:

$$
\begin{aligned}
v_L &= c_L - (c_R - c_L)e^{-A_{c_L}/B_{c_L}} \ , \\
v_R &= c_R + (c_R - c_L)e^{-A_{c_R}/B_{c_R}} \ .
\end{aligned}
\tag{21}
$$

Virtual poles

The u channel nucleon pole term is contained in invariant amplitude $B^I(s,u)$(I: total isospin):

$$-\frac{m_N^2 g^2}{F^2}\frac{1}{u-m_N^2} \in B^{1/2}(s,u)\ , \quad \frac{2m_N^2 g^2}{F^2}\frac{1}{u-m_N^2} \in B^{3/2}(s,u)\ . \tag{22}$$

- m_N, g and F denote the nucleon mass,axial vector coupling constant,and pion decay constant.
- the sign and even the value of these parameters are immune of chiral corrections.

Virtual poles

The helicity partial wave amplitudes are read:

$$T_{++}^{I,J} = 2m_N A_C^{I,J}(s) + (s - m_\pi^2 - m_N^2) B_C^{I,J}(s) \ ,$$
$$T_{+-}^{I,J} = -\frac{1}{\sqrt{s}}[(s - m_\pi^2 + m_N^2) A_S^{I,J}(s) + m_N(s + m_\pi^2 - m_N^2) B_S^{I,J}(s)].$$

The parity eigenstates can be obtained by the linear combinations:

$$T_{\pm}^{I,J} = T_{++}^{I,J} \pm T_{+-}^{I,J} \ . \tag{23}$$

Amplitudes $T_{\pm}^{I,J}$ are corresponding to orbital angular momentum $L = J \mp 1/2$ with $P = (-1)^{J \pm 1/2}$.

$$S_{\pm}^{I,J} = 1 + 2i\rho T_{\pm}^{I,J} \tag{24}$$

Virtual poles

After partial wave projection,the pole term leads partial wave amplitudes to behave in the neighbourhood of c_R like :

$$
\begin{aligned}
s \to c_R, \quad & T_{\pm}^{1/2,J} \to \frac{g^2 m_N^2 (m_N^2 + 2m_\pi^2)}{16\pi F^2 (4m_N^2 - m_\pi^2)} \ln \frac{c_R - c_L}{s - c_R} \to \infty \ , \\
s \to c_R, \quad & T_{\pm}^{3/2,J} \to -\frac{g^2 m_N^2 (m_N^2 + 2m_\pi^2)}{8\pi F^2 (4m_N^2 - m_\pi^2)} \ln \frac{c_R - c_L}{s - c_R} \to -\infty \ .
\end{aligned}
\tag{25}
$$

As to $s \to c_L$:

- $T_{\pm}^{1/2,J} \to \mp(-1)^{J+1/2}\infty.$
- $T_{\pm}^{3/2,J} \to \pm(-1)^{J+1/2}\infty.$

Essential Singularities for πN scatterings

Following the same reasons mentioned above ,we can conclude that:

- $s \in (c_R, s_R)$, the $S_{\pm}^{1/2,J}$ must contain a zero.
- $s \in (s_L, c_L)$, $S_{+}^{1/2,J}$and $S_{-}^{3/2,J}$ contain a zero for J = 1/2, 5/2, 9/2, ..., while $S_{-}^{1/2,J}$ and $S_{+}^{3/2,J}$ contain a zero for J = 3/2, 7/2, 11/2, ...

$$T^{\mathrm{II}}_{+\pm}(s,t) = 8\pi \sum_{J=1/2} (2J+1) \left[\frac{T^J_+(s)}{S^J_+(s)} \pm \frac{T^J_-(s)}{S^J_-(s)} \right] d^J_{1/2,\pm 1/2}(\cos\theta) \quad (26)$$

$s = c_L, c_R$, accumulation of poles on sheet II. So essential singularities [A. Martin,1970] of $T(s,t)$ on sheet II of s. That is valid to all orders of perturbation chiral expansions.

Summary

- $N^*(890)$ can be found in N/D calculation.
- Due to nucleon exchange of u-channel, there exist virtual poles in partial wave amplitudes.
- The virtual poles accumulate and form the essential singularities of $T^{II}(s,t)$.

S_{11} channel: lowest potential-nature resonance?

- S_{11} channel $\to$ no s-channel intermediate states $\to$ potential nature interaction
- Square-well potential (μ: reduced mass)

$$U(r) = 2\mu V(r) = \begin{cases} -2\mu V_0 & (r \leq L), \\ 0 & (r > L), \end{cases}$$

- Phase shift ($k' = (k^2 + 2\mu V_0)^{1/2}$)

$$\delta_{\mathsf{sw}}(k) = \arctan\left[\frac{k \tan k'L - k' \tan kL}{k' + k \tan(kL) \tan(k'L)}\right]$$

- Fit result (20 data): $L = 0.829$ fm and $V_0 = 144$ MeV, $\chi^2_{\text{sw}}/\text{d.o.f} = 0.740$
- Pole position: $k = -346i$ MeV $\rightarrow 0.872 - 0.316i$ GeV. Hidden pole fit $(0.861 \pm 0.053) - (0.130 \pm 0.075)i$ GeV