Nucleon-level Effective Theory of $\mu \to e$ Conversion

Evan Rule | 10th International Workshop on Chiral Dynamics| November 17, 2021

Berkeley
UNIVERSITY OF CALIFORNIA

ER, Haxton, and McElvain, arXiv:2109.13503
Haxton, ER, McElvain, and Ramsey-Musolf, arXiv:21xx.xxxxx
Cirigliano, Fuyuto, Ramsey-Musolf, ER, arXiv:21xx.xxxxx

Background

$$B\left(\mu^- + (A,Z) \to e^- + (A,Z)\right) \equiv \frac{\Gamma\left(\mu^- + (A,Z) \to e^- + (A,Z)\right)}{\Gamma\left(\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right)}$$

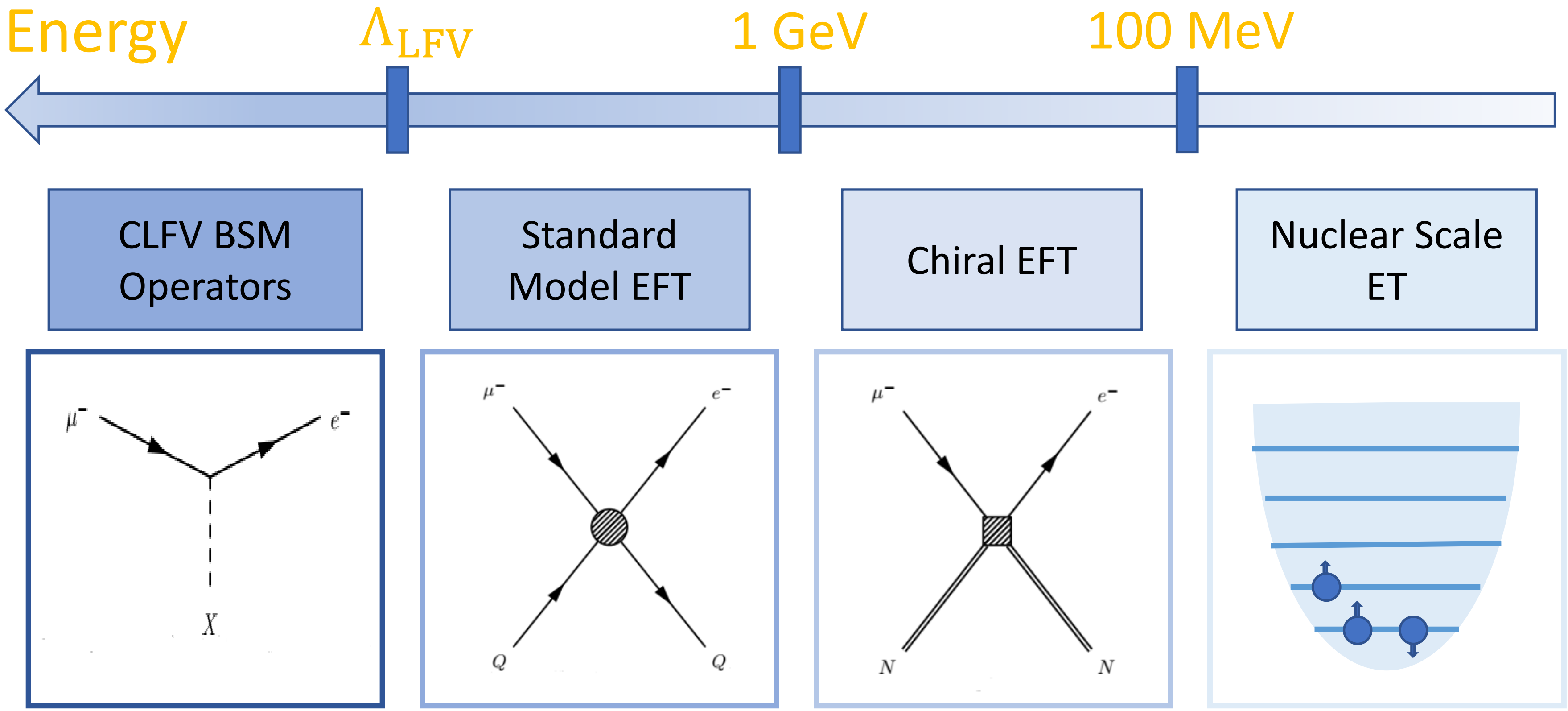
- Charged-lepton flavor violation can test BSM physics at scales beyond the reach of direct searches
- Next generation experiments Mu2e and COMET can improve current limits by four orders of magnitude†, probing scales $\lesssim 10^4$ TeV
- Experiments take place on atomic nucleus $^{27}\mathrm{Al}$
- Low-energy, highly-exclusive process

How much information about underlying CLFV operators can be extracted from observations of $\mu \to e$ conversion in nuclei?

†: Mu2e Collaboration, R. J. Abrams et al., arXiv:1211.7019
COMET Collaboration, Y. G. Cui et al

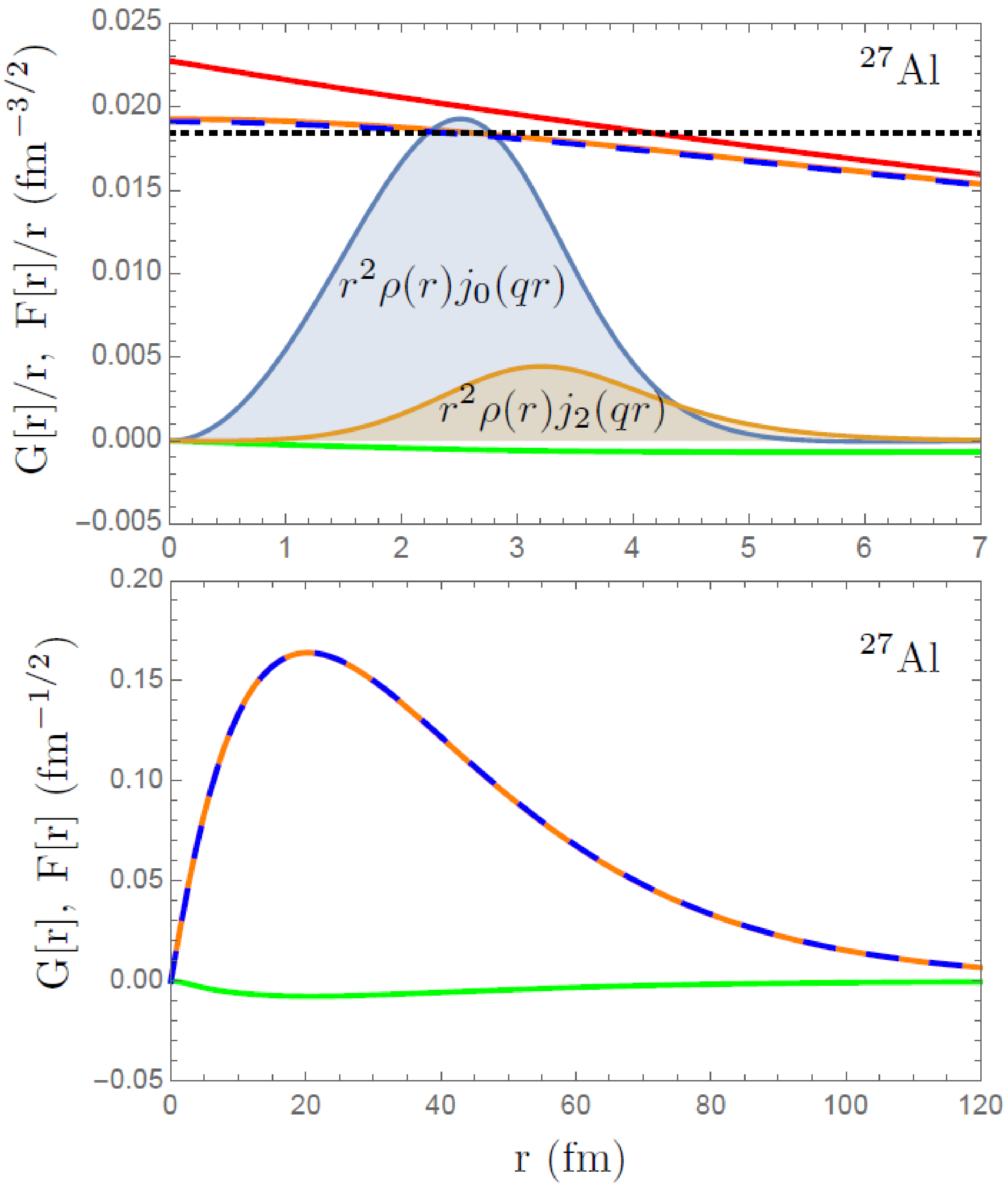
Why a Nucleon-level Effective Theory?

- CLFV operators must be reduced to the nucleon scale as these are the degrees of freedom employed in many-body nuclear methods
- Factors the CLFV leptonic physics from the nuclear physics
- Familiar form to other semileptonic processes: β decay and standard muon capture.
- Approximate forms for lepton wave functions are crucial



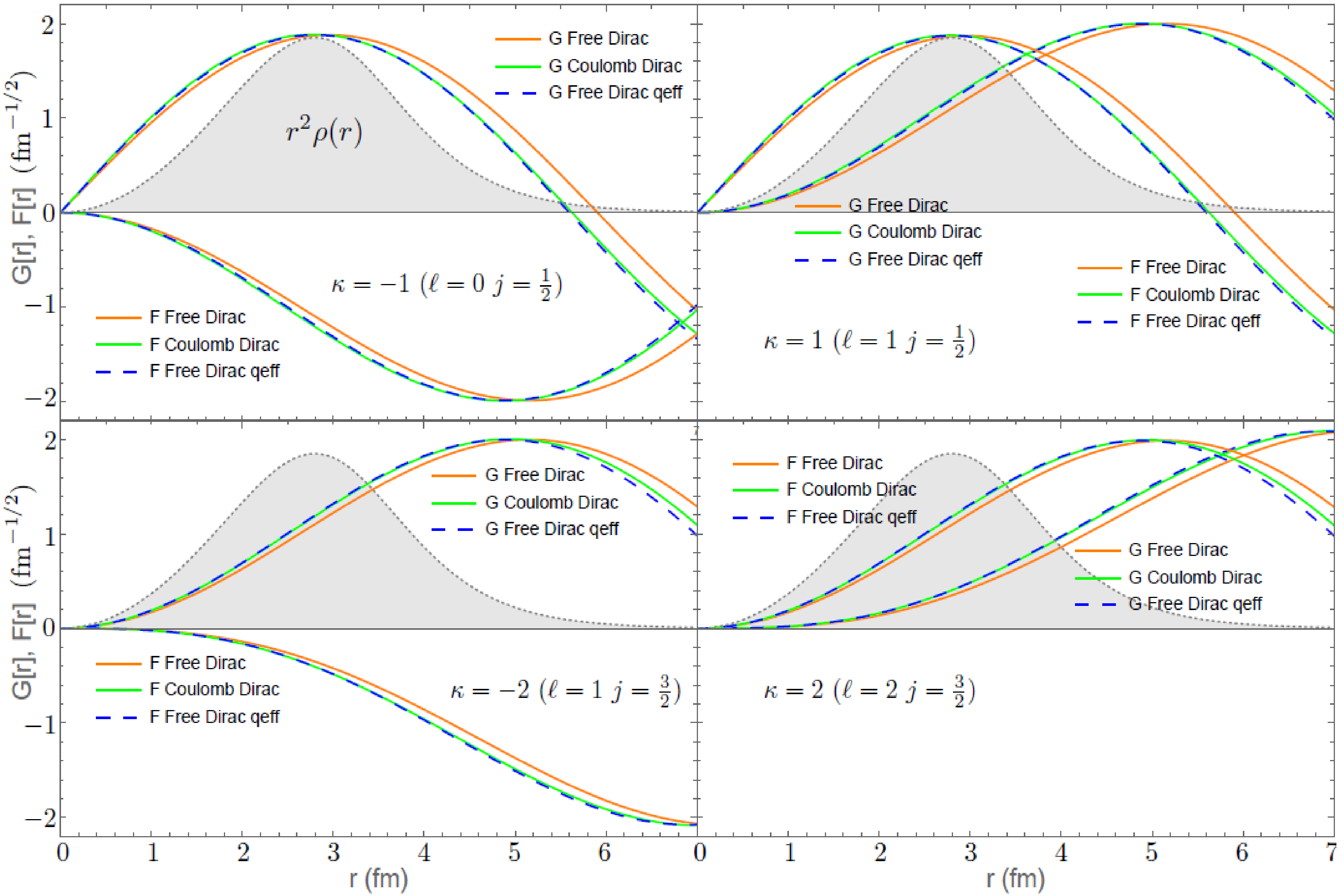
Muon Wave Functions

- Captured muons quickly cascade to $1s$ orbital of the nuclear Coulomb field
- Non-relativistic
- Muon wave function varies slowly over the scale of the nucleus: $a_0^\mu \approx 20$ fm, $r_N^{RMS} \approx 3.1$ fm
- We replace full muon wave function with an average value $\left|\phi_{1s}^{Z_{\rm eff}}(\vec{0})\right|$
- Resulting errors in decay rate are $\lesssim 1\%$ for ^{27}Al



Electron Wave Functions

- Outgoing electron has $E_e \approx m_\mu$ and is ultra-relativistic
- Electron wave function resembles a plane wave but is distorted by the Coulomb field of the nucleus
- We borrow from high-energy electron scattering studies and replace $e^{i\vec{q}\cdot\vec{x}} \rightarrow \frac{q_{\text{eff}}}{q} e^{i\vec{q}_{\text{eff}}\cdot\vec{x}}$
- $\vec{q}_{eff} = \vec{q} - V_C(0)\hat{q}$
- All Dirac spinor currents of electron and muon wave functions can be reduced in terms of Pauli spinors



Nucleon-level Operators

- Available Hermitian operators:
  $1_\mathrm{L},\ 1_\mathrm{N},\ i\hat{q},\ \vec{v}_N,\ \vec{\sigma}_L,\ \vec{\sigma}_N$

- We identify 16 independent operators through first order in $\vec{v}_N$

$$\mathcal{L}_\mathrm{eff} = \sqrt{2}G_F \sum_{\tau=0,1} \sum_{i=1}^{16} \tilde{c}_i^\tau\, \mathcal{O}_i t^\tau$$

$$\begin{aligned}
\mathcal{O}_1 &= 1_L\ 1_N \\
\mathcal{O}_2' &= 1_L\ i\hat{q}\cdot\vec{v}_N \\
\mathcal{O}_3 &= 1_L\ i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N] \\
\mathcal{O}_4 &= \vec{\sigma}_L\cdot\vec{\sigma}_N \\
\mathcal{O}_5 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N) \\
\mathcal{O}_6 &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{\sigma}_N \\
\mathcal{O}_7 &= 1_L\ \vec{v}_N\cdot\vec{\sigma}_N \\
\mathcal{O}_8 &= \vec{\sigma}_L\cdot\vec{v}_N \\
\mathcal{O}_9 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N) \\
\mathcal{O}_{10} &= 1_L\ i\hat{q}\cdot\vec{\sigma}_N \\
\mathcal{O}_{11} &= i\hat{q}\cdot\vec{\sigma}_L\ 1_N \\
\mathcal{O}_{12} &= \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N] \\
\mathcal{O}_{13}' &= \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]) \\
\mathcal{O}_{14} &= i\hat{q}\cdot\vec{\sigma}_L\ \vec{v}_N\cdot\vec{\sigma}_N \\
\mathcal{O}_{15} &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N] \\
\mathcal{O}_{16}' &= i\hat{q}\cdot\vec{\sigma}_L\ i\hat{q}\cdot\vec{v}_N
\end{aligned}$$

$$\tilde{R}_M^{\tau\tau'} = \tilde{c}_1^\tau \tilde{c}_1^{\tau'*} + \tilde{c}_{11}^\tau \tilde{c}_{11}^{\tau'*}$$
$$\tilde{R}_{\Phi''}^{\tau\tau'} = \tilde{c}_3^\tau \tilde{c}_3^{\tau'*} + (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\left(\tilde{c}_{12}^{\tau'*} - \tilde{c}_{15}^{\tau'*}\right)$$
$$\tilde{R}_{\Phi''M}^{\tau\tau'} = \mathrm{Re}\left[\tilde{c}_3^\tau \tilde{c}_1^{\tau'*} - (\tilde{c}_{12}^\tau - \tilde{c}_{15}^\tau)\,\tilde{c}_{11}^{\tau'*}\right]$$
$$\tilde{R}_{\tilde{\Phi}'}^{\tau\tau'} = \tilde{c}_{12}^\tau \tilde{c}_{12}^{\tau'*} + \tilde{c}_{13}^\tau \tilde{c}_{13}^{\tau'*}$$
$$\tilde{R}_{\Sigma''}^{\tau\tau'} = (\tilde{c}_4^\tau - \tilde{c}_6^\tau)(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^\tau \tilde{c}_{10}^{\tau'*}$$
$$\tilde{R}_{\Sigma'}^{\tau\tau'} = \tilde{c}_4^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_9^\tau \tilde{c}_9^{\tau'*}$$
$$\tilde{R}_{\Delta}^{\tau\tau'} = \tilde{c}_5^\tau \tilde{c}_5^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_8^{\tau'*}$$
$$\tilde{R}_{\Delta\Sigma'}^{\tau\tau'} = \mathrm{Re}\left[\tilde{c}_5^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_8^\tau \tilde{c}_9^{\tau'*}\right]$$

Charge or Current	Allowed Projection	Multipole Operator	LECs tested
1_N	Charge	$M_{J=0,2,\ldots}$	c_1, c_{11}
$\vec{v}_N \cdot \vec{\sigma}_N$	none	none	none
$\vec{\sigma}_N$	Longitudinal	$\Sigma''_{J=1,3,\ldots}$	c_4, c_6, c_{10}
$\vec{\sigma}_N$	Trans Electric	$\Sigma'_{J=1,3,\ldots}$	c_4, c_9
$\vec{v}_N$	Trans Magnetic	$\Delta_{J=1,3,\ldots}$	c_5, c_8
$\vec{v}_N \times \vec{\sigma}_N$	Longitudinal	$\Phi''_{J=0,2,\ldots}$	c_3, c_{12}, c_{15}
$\vec{v}_N \times \vec{\sigma}_N$	Trans Electric	$\tilde{\Phi}'_{J=2,4,\ldots}$	c_{12}, c_{13}

CLFV Decay Rate

$$\omega = \frac{G_F^2}{\pi}\,\frac{q_{\rm eff}^2}{1+\frac{q}{M_T}}\,|\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1}\sum_{\tau'=0,1}\Big\{\left[\tilde{R}_M^{\tau\tau'}\,W_M^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''}^{\tau\tau'}\,W_{\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'}^{\tau\tau'}\,W_{\Sigma'}^{\tau\tau'}(q_{\rm eff})\right]$$
$$+\frac{q_{\rm eff}^2}{m_N^2}\left[\tilde{R}_{\Phi''}^{\tau\tau'}\,W_{\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'}^{\tau\tau'}\,W_{\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta}^{\tau\tau'}\,W_{\Delta}^{\tau\tau'}(q_{\rm eff})\right]$$
$$-\frac{2q_{\rm eff}}{m_N}\left[\tilde{R}_{\Phi''M}^{\tau\tau'}\,W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'}\,W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff})\right]\Big\}$$

- Factorization of leptonic and nuclear physics
- Directly analogous to other semileptonic processes
- c_2, c_7, c_{14}, c_{16} not probed by elastic $\mu \to e$ conversion

$W_i^{\tau\tau'}(q_{\rm eff}) \leftrightarrow$ "Nuclear dials"

Limits on LECs

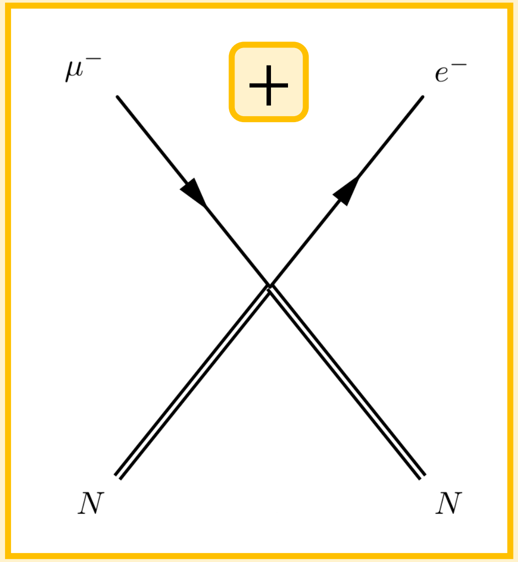
- Using our expression for the decay rate, we can constrain LECs using existing and future branching ratio limits for various nuclei
- We can also estimate the energy scale probed by each operator
- Assume only one operator is responsible for CLFV
- Coming soon: Mathematica script to compute $\mathrm{B}(\mu \rightarrow e)$ in terms of $\tilde{c}_i^\tau$ for a selection of nuclear targets

	Al $\left(B < 10^{-17}\right)$		Ti $\left(B < 6.1 \times 10^{-13}\right)^\dagger$	
Coupling	LEC Limit	~ Scale Probed	LEC Limit	~ Scale Probed
$\tilde{c}_1^0, \tilde{c}_{11}^0$	3.994E-10	10,000 TeV	7.380E-8	900 TeV
$\tilde{c}_1^1, \tilde{c}_{11}^1$	1.238E-8	2,000 TeV	1.316E-6	200 TeV
$\tilde{c}_3^0, \tilde{c}_{15}^0$	1.608E-8	2,000 TeV	3.801E-6	100 TeV
$\tilde{c}_3^1, \tilde{c}_{15}^1$	1.860E-7	1,000 TeV	7.344E-6	100 TeV
$\tilde{c}_4^0$	1.418E-8	2,000 TeV	1.504E-5	60 TeV
$\tilde{c}_4^1$	1.713E-8	2,000 TeV	1.718E-5	60 TeV
$\tilde{c}_5^0, \tilde{c}_8^0$	7.774E-8	1,000 TeV	5.802E-5	30 TeV
$\tilde{c}_5^1, \tilde{c}_8^1$	1.164E-7	1,000 TeV	6.521E-5	30 TeV
$\tilde{c}_6^0, \tilde{c}_{10}^0$	1.954E-8	2,000 TeV	1.794E-5	60 TeV
$\tilde{c}_6^1, \tilde{c}_{10}^1$	2.151E-8	2,000 TeV	1.999E-5	60 TeV
$\tilde{c}_9^0$	2.061E-8	2,000 TeV	2.758E-5	50 TeV
$\tilde{c}_9^1$	2.833E-8	1,000 TeV	3.360E-5	40 TeV
$\tilde{c}_{12}^0$	1.608E-8	2,000 TeV	3.797E-6	100 TeV
$\tilde{c}_{12}^1$	1.388E-7	700 TeV	7.342E-6	100 TeV
$\tilde{c}_{13}^0$	1.787E-6	200 TeV	8.422E-5	30 TeV
$\tilde{c}_{13}^1$	2.085E-7	500 TeV	3.718E-4	10 TeV

†: P. Wintz, Proc. 1st Int. Symp. on Lepton and Baryon Number Violation

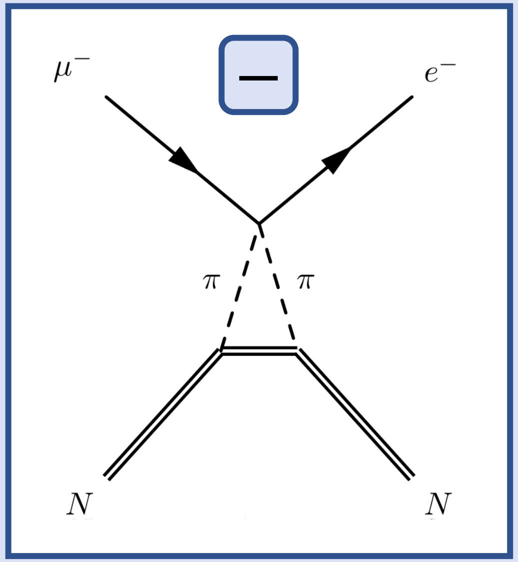
Connecting to ChPT

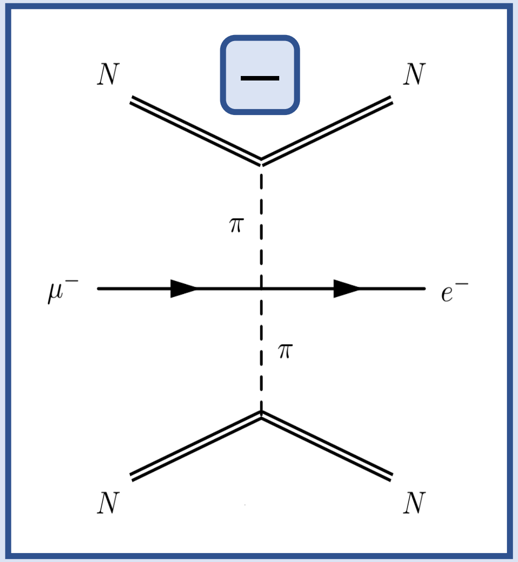
Leading Order



- Leading coherent contact interaction matches to c_1^0 and c_{11}^0

Next-to-Leading Order





- Form factors are absorbed in the LECs of our theory
- Two-nucleon diagram may be averaged to an effective one-body operator

- Match to HBChPT, then to LEFT and SMEFT
- Analogous matching has been carried out for WIMP dark matter effective theory

Bartolotta & Ramsey-Musolf, *Phys. Rev. C* **98**, 015208
Cirigliano, Fuyuto, Ramsey-Musolf, ER, arXiv:21xx.xxxxx

Bishara, F., Brod, J., Grinstein, B., and Zupan, J., arXiv:1708.02678

Backup Slides

Relation to $\mu \to e + \gamma$

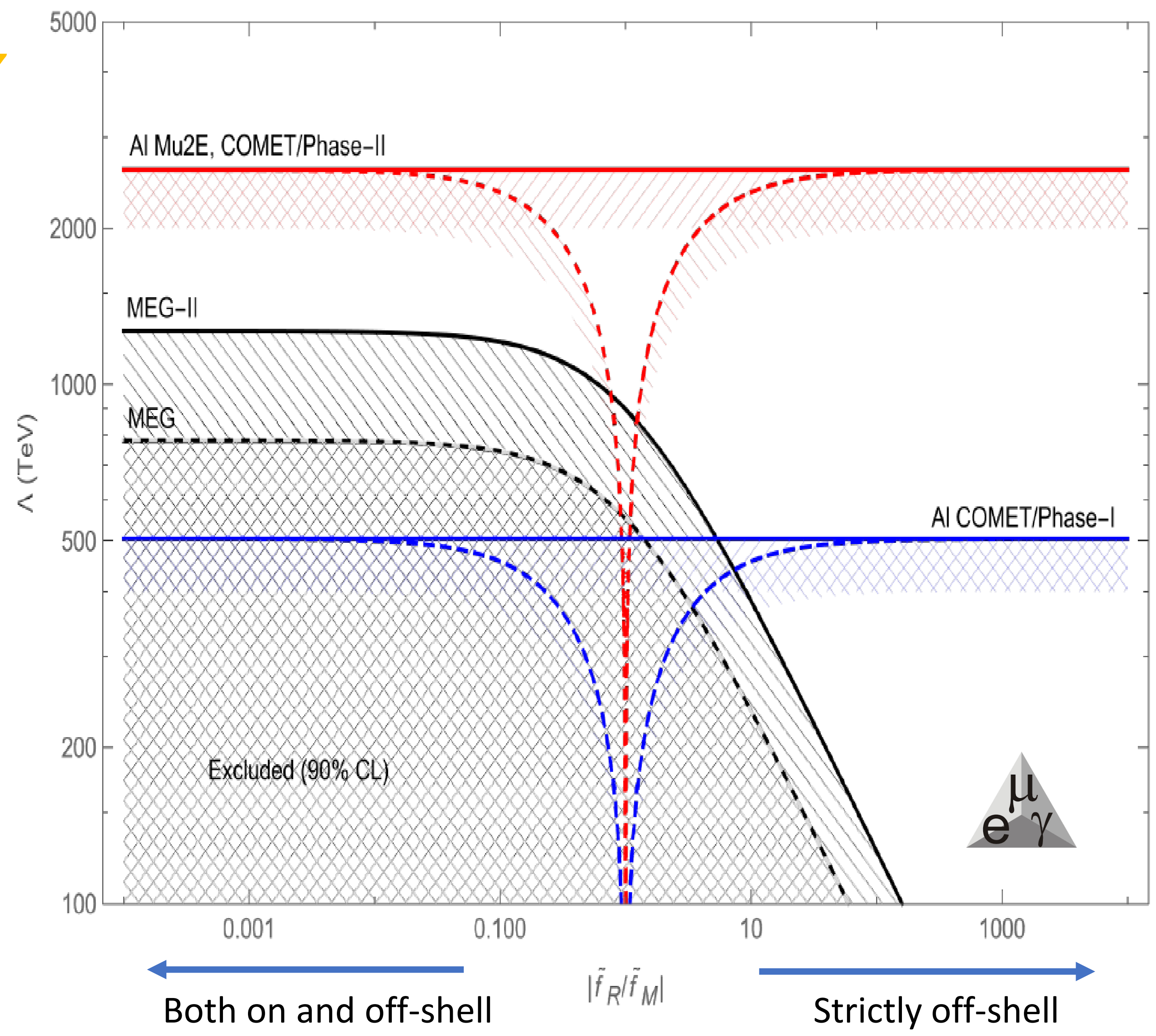
$$\Gamma^{\mu}_{\mu\to e} = \frac{1}{\Lambda^2}(q^2\gamma^\mu - q^\mu \not{q})\left[\tilde{f}_R^{\mu\to e}(q^2) + i\tilde{f}_A^{\mu\to e}(q^2)\gamma_5\right]$$
$$+ i\frac{m_\mu}{\Lambda^2}\sigma^{\mu\nu}q_\nu\left[\tilde{f}_M^{\mu\to e}(q^2) + i\tilde{f}_E^{\mu\to e}(q^2)\gamma_5\right]$$

On-shell photon $\mu \to e + \gamma$

- $\tilde{f}_R, \tilde{f}_A$ cannot contribute
- Constrained by MEG experiment
- MEG-II will improve limit by order of magnitude

Virtual photon-mediated $\mu \to e$ in Nuclei

- All four form factors can contribute
- Even more strict limits on CLFV EM couplings than MEG-II, assuming no cancellation between form factors



Nuclear Response Operators

- Two nuclear charges $1_N, \vec{v}_N \cdot \vec{\sigma}_N$ and three nuclear currents $\vec{v}_N, \vec{\sigma}_N, \vec{v}_N \times \vec{\sigma}_N$ should yield 11 multipole operators
- Nearly-exact P and CP of nuclear ground state permit only six of these operators to contribute to elastic $\mu \to e$ conversion

$$M_{JM;\tau}(q) \equiv \sum_{i=1}^{A} M_{JM}(q\vec{x}_i)\ t^\tau(i)$$

$$\Delta_{JM;\tau}(q) \equiv \sum_{i=1}^{A} \vec{M}_{JJ}^{M}(q\vec{x}_i) \cdot \frac{1}{q}\vec{\nabla}_i\ t^\tau(i)$$

$$\Sigma'_{JM;\tau}(q) \equiv -i\sum_{i=1}^{A} \left\{\frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^{M}(q\vec{x}_i)\right\} \cdot \vec{\sigma}(i)\ t^\tau(i)$$

$$\Sigma''_{JM;\tau}(q) \equiv \sum_{i=1}^{A} \left\{\frac{1}{q}\vec{\nabla}_i\ M_{JM}(q\vec{x}_i)\right\} \cdot \vec{\sigma}(i)\ t^\tau(i)$$

$$\tilde{\Phi}'_{JM;\tau}(q) \equiv \sum_{i=1}^{A} \left[\left(\frac{1}{q}\vec{\nabla}_i \times \vec{M}_{JJ}^{M}(q\vec{x}_i)\right) \cdot \left(\vec{\sigma}(i) \times \frac{1}{q}\vec{\nabla}_i\right) + \frac{1}{2}\vec{M}_{JJ}^{M}(q\vec{x}_i) \cdot \vec{\sigma}(i)\right]\ t^\tau(i)$$

$$\Phi''_{JM;\tau}(q) \equiv i\sum_{i=1}^{A} \left(\frac{1}{q}\vec{\nabla}_i M_{JM}(q\vec{x}_i)\right) \cdot \left(\vec{\sigma}(i) \times \frac{1}{q}\vec{\nabla}_i\right)\ t^\tau(i)$$

$$M_{JM}(q\vec{x}) = j_J(qx)Y_{JM}(\hat{x})$$

$$\vec{M}_{JL}^{M}(q\vec{x}) = j_J(qx)\vec{Y}_{JLM}(\hat{x})$$