Dispersive analysis of low energy pion photo- and electroproduction

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November 17, 2021

Talk given at the 10th International Workshop on Chiral Dynamics
IHEP, Beijing (online)

Contexts

Theoretical studies on π photo- and electroproduction

- In 1957, CGLN decomposition was established to study π production by using fixed-t DR[Chew et al., 1957];
- Current algebra results[Gaffney, 1967; Bhatia and Narayanaswamy, 1968];
- Effective Lagrangian description[Peccei, 1969];
- Phenomenological models: unitarity isobar model[Drechsel et al., 1999], dynamic model[Kamalov et al., 2001] and JBW model[Mai et al., 2021], et. al.
- χPT: one-loop calculations $\Longrightarrow$ HBχPT [Bernard et al., 1994], EOMS[Hilt et al., 2013; Guerrero et al., 2019; Guerrero and Vicente, 2020];
- Unitarity+χPT: couple channel N/D description of πN and γN scatterings[Gasparyan and Lutz, 2010], et. al.
  See also slides by Ulf-G. Meissner.

Method

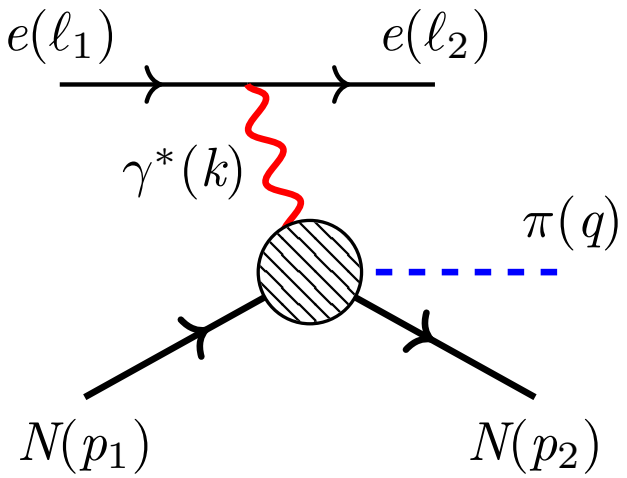
- Inhomogeneous Omnès formalism:
  - Left-hand cut (LHC) contributions $\Longleftarrow$ χPT;
  - Right-hand cut (RHC) contributions $\Longleftarrow$ Omnès solution.
- Omnès like formalisms have been successfully applied to many other processes:
  - $\gamma\gamma \to \pi\pi$[Mao et al., 2009; Garcia-Martin and Moussallam, 2010; Dai and Pennington, 2014];
  - $\eta \to 3\pi$[Albaladejo and Moussallam, 2017; Colangelo et al., 2018];
  - Heavy meson decay[Albaladejo et al., 2017; Yao et al., 2018];
  - π form factors[Ropertz et al., 2018];
  - XYZ states[Chen et al., 2019];
  - ...

Amplitude decomposition

- Electroproduction: $e(\ell_1) + N(p_1) \to e(\ell_2) + N(p_2) + \pi(q)$
  $\xrightarrow[\text{approximation}]{\text{OPE}} \gamma^*(k) + N(p_1) \to N(p_2) + \pi(q) \quad Q^2 = -k^2$

- Lorentz decomposition: $\mathcal{M} = \epsilon_\mu \mathcal{M}^\mu$ [Bernard et al., 1994],

$$\mathcal{M}^\mu = -ie \langle N\pi | J^\mu(0) | N \rangle = \bar{u}(p_2) \left(\sum_{i=1}^{6} A_i M_i^\mu \right) u(p_1) .$$



ϵ_μ: polarization vector of photon (photoproduction)
or $e\frac{\overline{u}(\ell_1)\gamma_\mu u(\ell_2)}{k^2}$ (electroproduction).

- Isospin decomposition:

$$\mathcal{M}(\gamma^* + N \to \pi^a + N) = \chi_2^\dagger \left\{ \delta^{a3} \mathcal{M}^{(+)} + i\epsilon^{a3b} \tau^b \mathcal{M}^{(-)} + \tau^a \mathcal{M}^{(0)} \right\} \chi_1 ,$$

$$\mathcal{M}^{\frac{1}{2},\frac{1}{2}} = -\sqrt{\frac{1}{3}} \left(\mathcal{M}^{(+)} + 2\mathcal{M}^{(-)} + 3\mathcal{M}^{(0)} \right), \quad \mathcal{M}^{\frac{1}{2},-\frac{1}{2}} = \sqrt{\frac{1}{3}} \left(\mathcal{M}^{(+)} + 2\mathcal{M}^{(-)} - 3\mathcal{M}^{(0)} \right) .$$

CGLN decomposition and PWA

- CGLN amplitude $\rightleftharpoons$ Helicity amplitude
- CGLN decomposition[Chew et al., 1957]: $\mathcal{M}^{\mu} = \frac{4\pi\sqrt{s}}{m_N}\chi_2^{\dagger}\mathbf{F}\chi_1$,

$$\mathbf{F} = i\boldsymbol{\sigma}\cdot\boldsymbol{b}\mathcal{F}_1 + \boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}\boldsymbol{\sigma}\cdot(\hat{\boldsymbol{k}}\times\boldsymbol{b})\mathcal{F}_2 + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\hat{\boldsymbol{q}}\cdot\boldsymbol{b}\mathcal{F}_3 + i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}\hat{\boldsymbol{q}}\cdot\boldsymbol{b}\mathcal{F}_4$$
$$- i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{q}}b_0\mathcal{F}_7 - i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}b_0\mathcal{F}_8 \quad (b_\mu = \epsilon_\mu - \frac{\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{k}}}{|\boldsymbol{k}|}k_\mu)\ .$$

- CGLN amplitudes $\xrightarrow[\text{expansion}]{\text{PW}}$ $E_{l\pm}$, $M_{l\pm}$, $S_{l\pm}$[Berends et al., 1967],

$$\mathbb{F}(s,t) = \sum_l \begin{bmatrix} \mathbb{G}_l(x)_{4\times4} & \mathbb{O} \\ \mathbb{O} & \mathbb{H}_l(x)_{2\times2} \end{bmatrix} \mathbb{M}_l(s) \quad (x = \hat{\boldsymbol{q}}\cdot\hat{\boldsymbol{k}})\ ,$$

$\mathbb{G}_l(x), \mathbb{H}_l(x)$ both are linear combination of legendre function $P_l(x)$ and its derivative.

Chiral perturbation theory

- Lorentz-invariant chiral Lagrangian up to $\mathcal{O}(p^2)$[Fettes et al., 2000]:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \sum_{i=1}^{2} \mathcal{L}^{(i)}_{\pi N} \,,$$

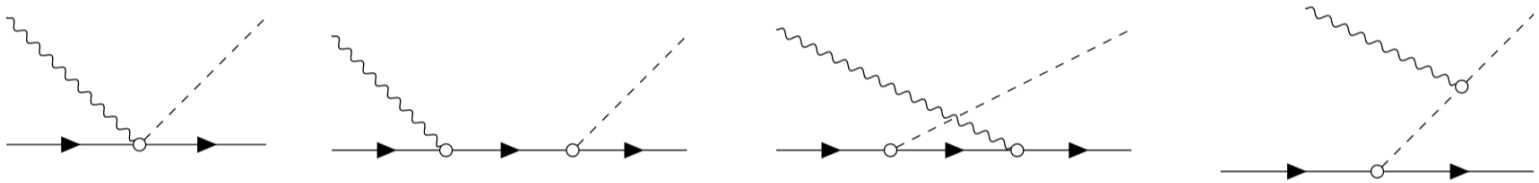
$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{F^2}{4} \operatorname{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] + \frac{F^2}{4} \operatorname{Tr}\left(\chi U^\dagger + U \chi^\dagger\right) \,,$$

$$\mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} = \bar{\Psi}\left(i \not{D} - m_N + \frac{g_A}{2} \not{u} \gamma_5\right) \Psi + \bar{\Psi} \sigma^{\mu\nu} \left[\frac{c_6}{2} f^+_{\mu\nu} + \frac{c_7}{2} v^{(s)}_{\mu\nu}\right] \Psi + \cdots \,,$$

pion's parameterization:

$$U(x) = \exp\left(i \frac{\Phi}{F}\right), \quad \Phi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} .$$

- Tree level diagrams[Guerrero et al., 2019]:



Dispersive representation

- PW optical theorem($\mathcal{S} = 1 + 2i\rho\mathcal{T}$):

$$\mathrm{Im}\,\mathcal{M}(s+i\epsilon) = \mathcal{T}^*(s+i\epsilon)\rho(s+i\epsilon)\mathcal{M}(s+i\epsilon)\ ;$$

$\Longrightarrow$ Unitarity relation: $\mathcal{M}^+ = \mathcal{S}^+\mathcal{M}^-$; $(\mathcal{M}^\pm(s) = \lim_{\epsilon\to 0}\mathcal{M}(s\pm i\epsilon))$

- **If the PWA can be divided into two parts:** $\mathcal{M} = \mathcal{M}_R + \mathcal{M}_L$;
- Define an auxiliary function, $\Omega(s)$, that only contains RHC and manifests the same unitarity relation as $\mathcal{M}(s)$: $\Omega^+ = \mathcal{S}^+\Omega^-$;
  $\Longrightarrow$ $\mathrm{Im}\left(\Omega^{-1}\mathcal{M}_R\right) = -\left(\mathrm{Im}\Omega^{-1}\right)\mathcal{M}_L$;

★ n-th subtracted DR for $\mathrm{Im}\left(\Omega^{-1}\mathcal{M}_R\right)$ [Babelon et al., 1976]:

$$\boxed{\mathcal{M}^I(s) = \mathcal{M}_L^I(s) + \Omega^I(s)\left(\frac{s^n}{\pi}\int_{s_R}^{\infty}\frac{\sin\delta^I(s')\mathcal{M}_L^I(s')}{|\Omega(s')|s'^n\left(s'-s\right)}\mathrm{d}s' + \mathcal{P}_{n-1}^I(s)\right)}\ .$$

Omnès solution[Omnès, 1958]: $\Omega^I(s) = \mathcal{P}^I(s)\exp\left[\frac{s}{\pi}\int_{s_R}^{\infty}\frac{\delta^I(s')}{s'(s'-s)}\mathrm{d}s'\right]$.

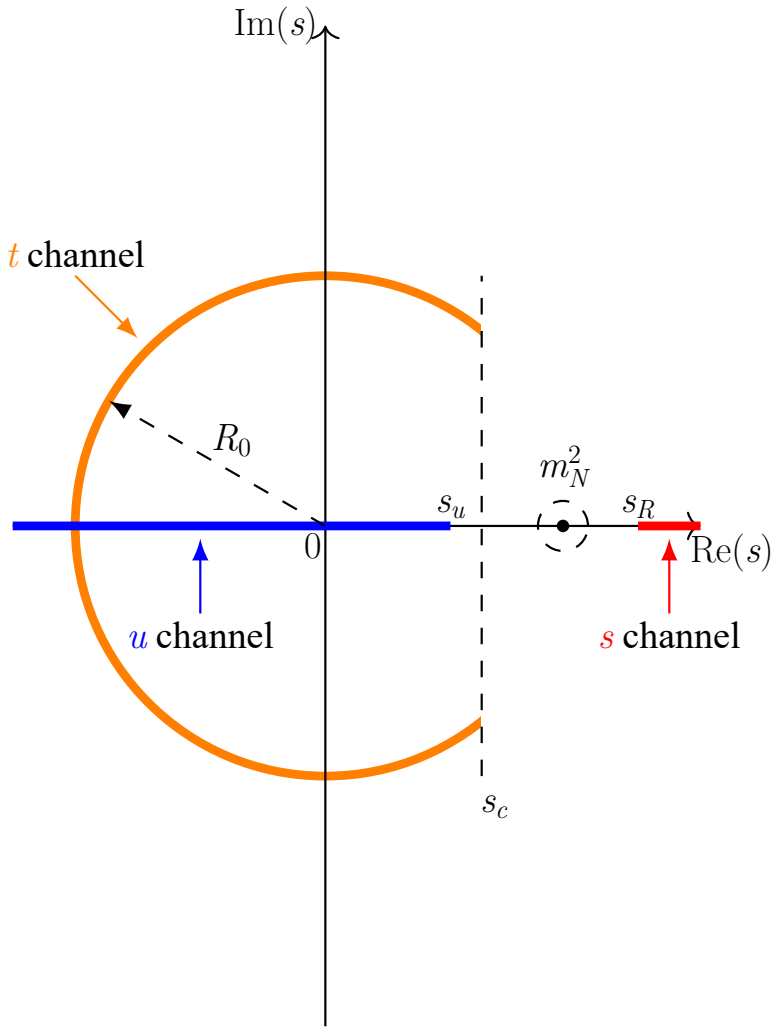
Singularity structure of PWA ($Q^2 = 0$)[Kennedy and Spearman, 1962]

I Unitarity cut, $s \in [s_R, \infty)$ on account of s-channel continuous spectrum;

II t-channel cut, 1. arc stems from $4m_\pi^2 \leq t \leq 4m_N^2$; 2. $s \in (-\infty, 0]$ corresponding to $t \geq 4m_N^2$;

III u-channel cut, $s \in \left(-\infty, s_u = \frac{m_N\left(m_N^2 - m_\pi^2 - m_\pi m_N\right)}{m_\pi + m_N}\right]$ due to $u \geq (m_\pi + m_N)^2$;

IV Pole, $s = m_N^2$, due to t-channel pion exchange and u-channel nucleon exchange.



Singularity structure of PWA ($Q^2 \neq 0$)

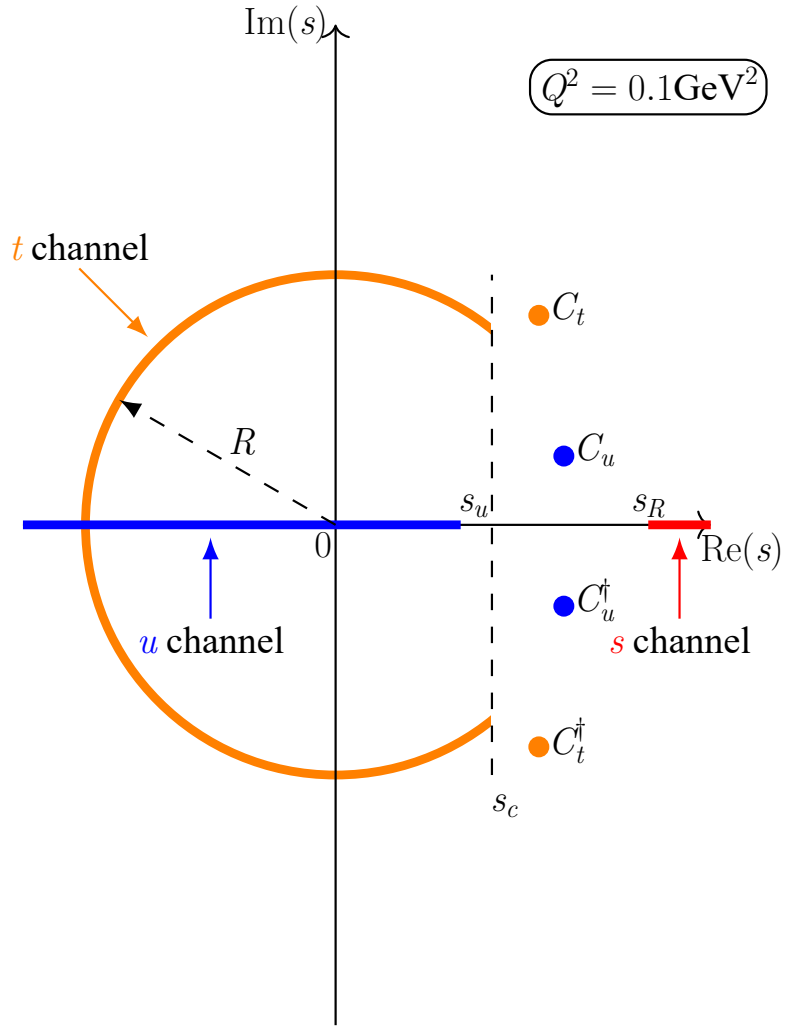
I Unitarity cut, $s \in [s_R, \infty)$ on account of s-channel continuous spectrum;

II t-channel cut, 1. arc stems from $4m_\pi^2 \leq t \leq 4m_N^2$; 2. $s \in (-\infty, 0]$ corresponding to $t \geq 4m_N^2$;

III u-channel cut, $s \in \left(-\infty, s_u = \frac{m_N^3 - m_\pi^2 m_N - m_\pi\left(m_N^2+Q^2\right)}{m_\pi + m_N}\right]$ due to $u \geq (m_\pi + m_N)^2$;

IV Cut due to t-channel pion exchange, the branch points locate at 0, C_t, $C_t^\dagger$;

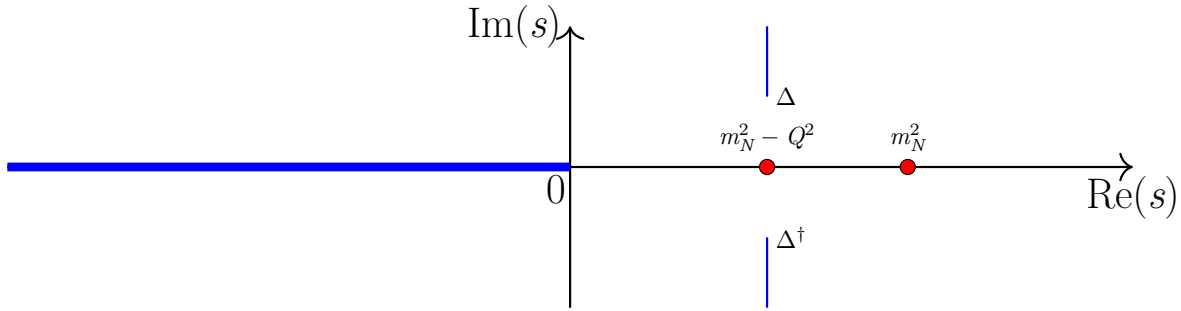
V Cut due to u-channel nucleon exchange, the branch points locate at 0, C_u, $C_u^\dagger$.



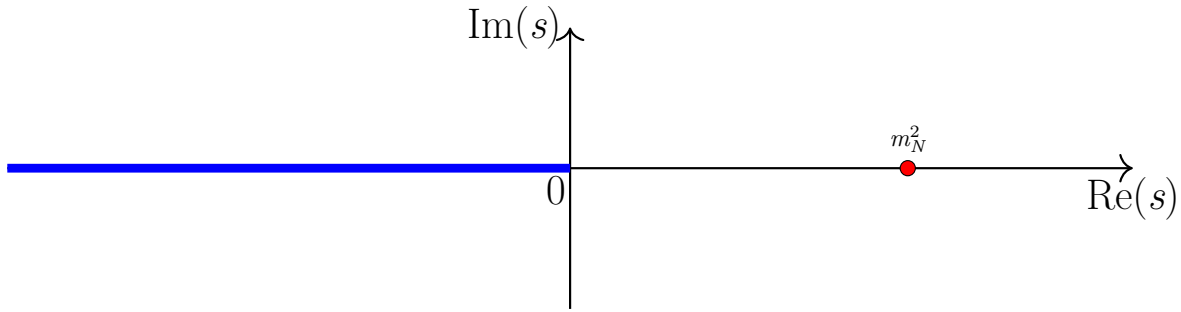
★ **In real photon point,** C_t, $C_u \xrightarrow{Q^2 \to 0} m_N^2$

Kinematic singularities

- Kinematic singularities: in our consideration, stems from relativistic kinematics such that $\sqrt{s-s_L}\sqrt{s-s_R}$, etc.



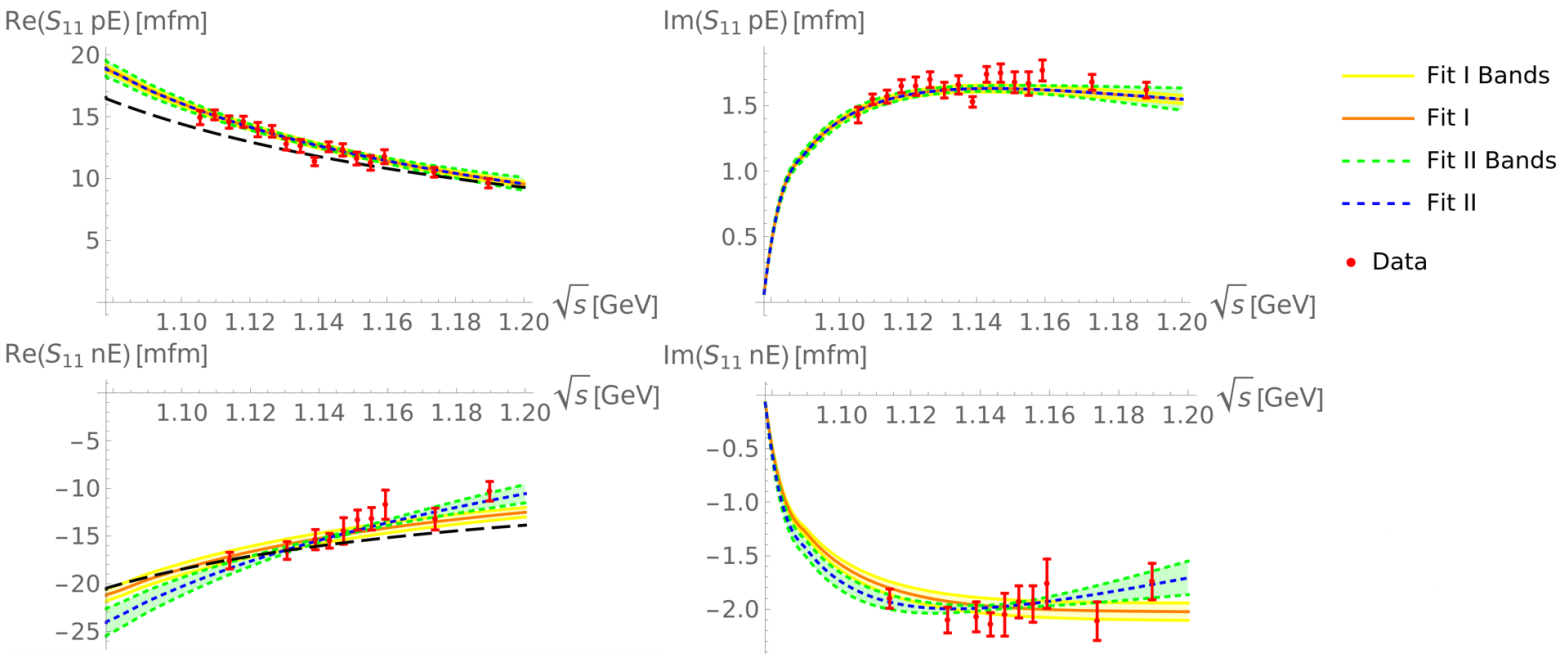
$$\Downarrow \left(Q^2 \to 0\right)$$



- **All cuts do not cover the unitarity cut, so the whole singularities can be divided into RHC and LHC.**

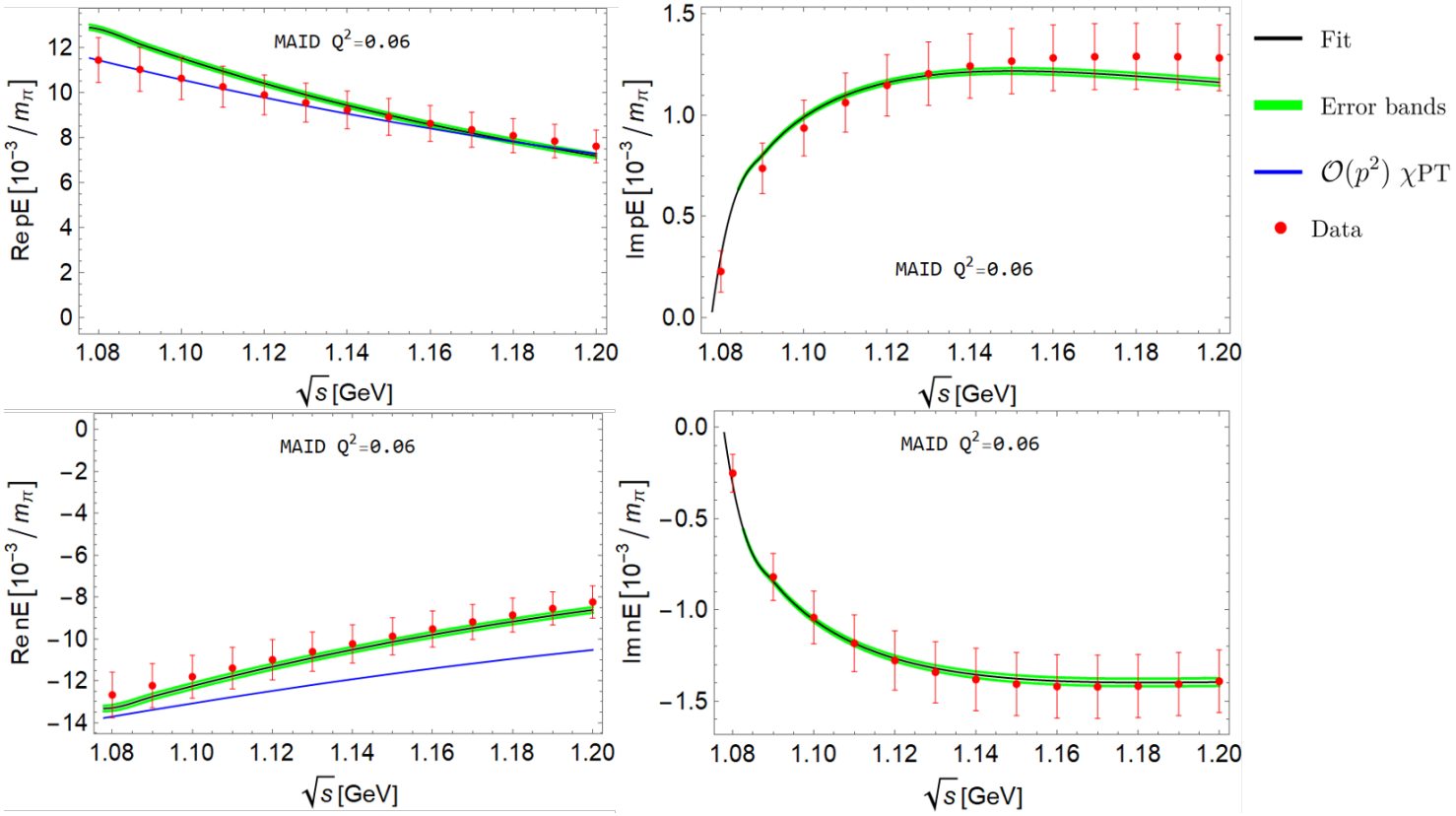
Fit results of photoproduction[Ma et al., 2021]

- Data:
  - SAID[Workman et al., 2012] $\Longrightarrow$ photoproduction;
  - DMT2001[Kamalov and Yang, 1999] and MAID2007[Drechsel et al., 2007] $\Longrightarrow$ electroproduction.
- Fitting S_{11} PW $\longrightarrow E_{0+}, S_{0+}$, with p and n targets;
  Combination fit: $Q^2 \sim 0 - 0.1\text{GeV}^2$& $\sqrt{s} \sim 1.08 - 1.2\text{GeV}$;
  Phase shifts $\Longleftarrow$ Roy-Steiner equation[Hoferichter et al., 2016].



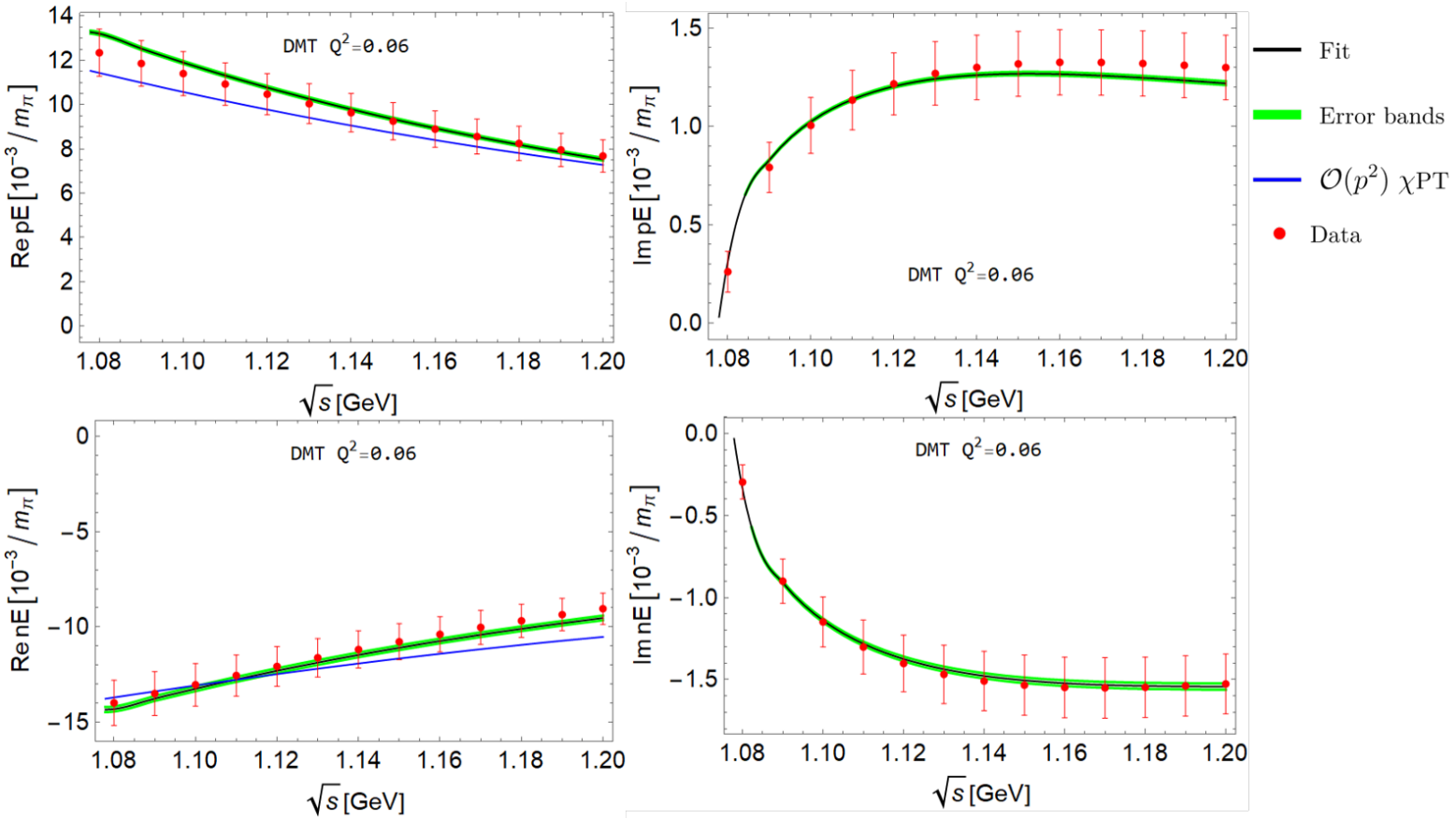
Fit results of electroproduction[Cao et al., 2021]—E_{0+}

- E_{0+} ($Q^2 = 0.06\mathrm{GeV}^2$)
- Subtraction constant: independent of Q^2 when neglecting the contribution of vector meson.



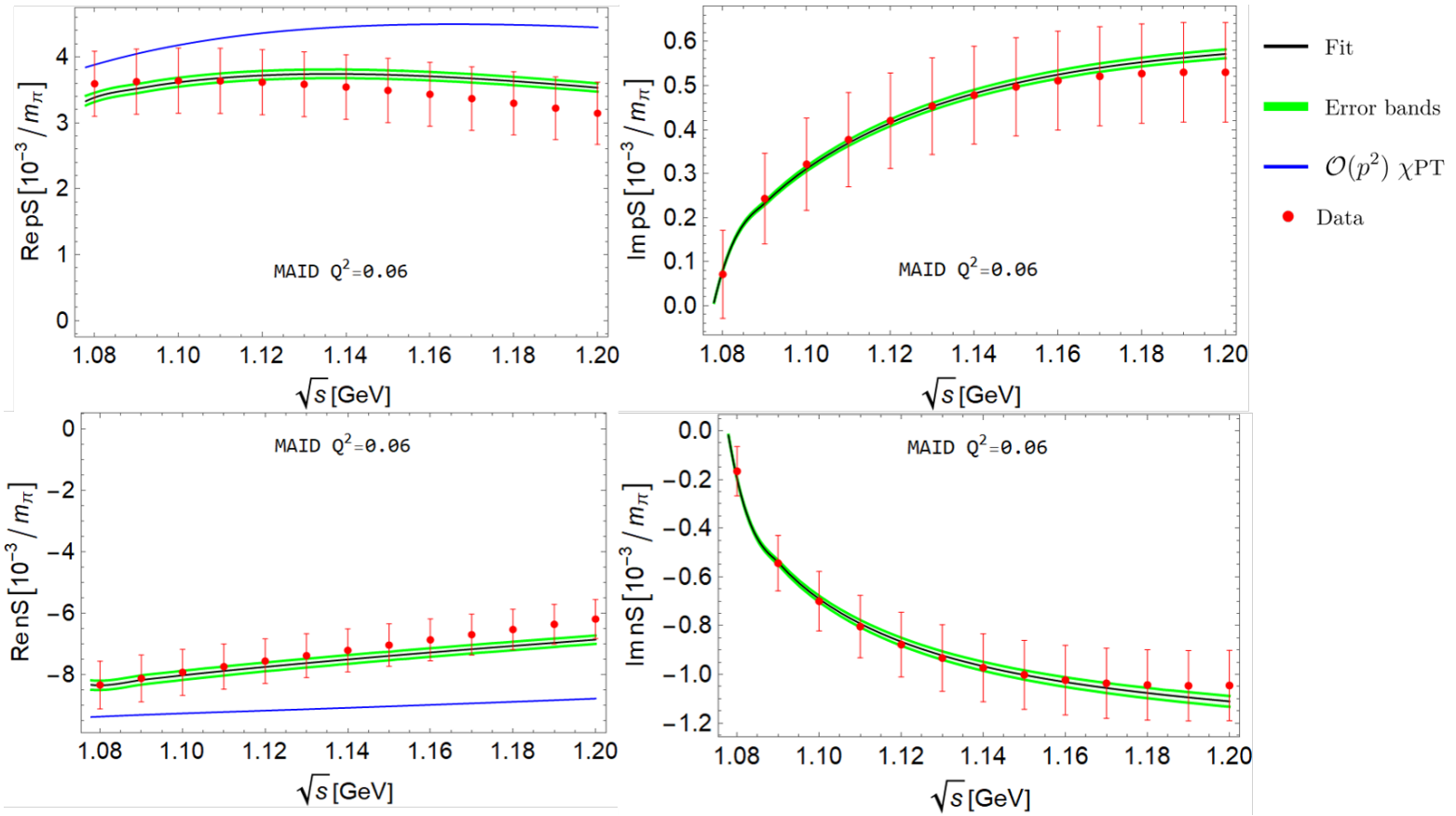
Fit results of electroproduction—E_{0+}

- E_{0+} $(Q^2 = 0.06\text{GeV}^2)$



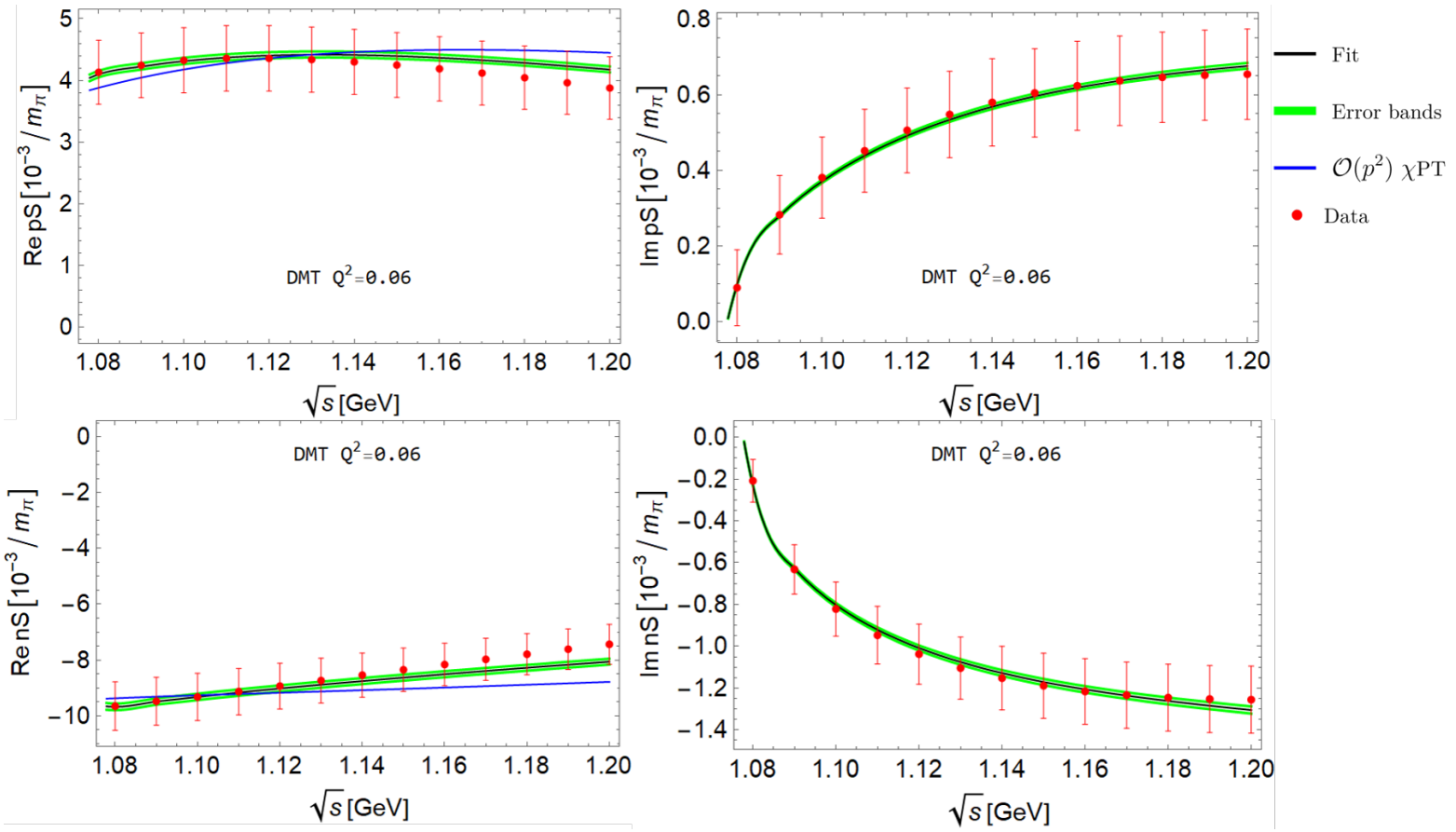
Fit results of electroproduction—S_{0+}

- S_{0+} $(Q^2 = 0.06\text{GeV}^2)$



Fit results of electroproduction—S_{0+}

- S_{0+} $(Q^2 = 0.06\text{GeV}^2)$



Fit discussion

- Estimating error bars acording to $\mathrm{err}\left(\mathcal{M}\right)=\sqrt{\left(e_s\right)^2+\left(e_r\right)^2\left(\mathcal{M}\right)^2}$, $e_s^{R,I}=0.4,0.1\left[10^{-3}/m_\pi\right]$, $e_r=10\%$;
- Truncation of the DR: Λ dependence $\rightleftharpoons$ subtraction constant a;

$$\frac{s}{\pi}\int_{s_R}^{\Lambda}\frac{\sin\delta^I\left(s'\right)\mathcal{M}_L^I\left(s'\right)}{\left|\Omega\left(s'\right)\right|s'\left(s'-s\right)}\mathrm{d}s'+a$$

- $\mathcal{O}(p^4)$[Hilt et al., 2013], $\mathcal{O}(p^3)$ with Δ[Guerrero et al., 2019; Guerrero and Vicente, 2020] calculations are advantageous compared with an $\mathcal{O}(p^2)$ calculation;
- Unitarization effects are automatically fulfilled in our scheme.

Analytic continuation

- Possible hidden pole $N^*(890)$ in S_{11} channel[Wang et al., 2018; Li et al., 2021].
- Pion photo- and electroproduction $\longrightarrow$ Electromagnetic couplings
- Analytical continuation to second sheet:

$$\mathcal{M}^{\mathrm{II}}(s) = \frac{\mathcal{M}(s)}{\mathcal{S}(s)} \xrightarrow{\mathcal{S}(s)\simeq\mathcal{S}'(s_\mathrm{p})(s-s_\mathrm{p})} \mathcal{M}^{\mathrm{II}}(s) \simeq \frac{\mathcal{M}(s)}{\mathcal{S}'(s_\mathrm{p})(s-s_\mathrm{p})} .$$

- The couplings are defined as the residue via:

$$\mathcal{M}^{\mathrm{II}}(s \to s_\mathrm{p}) \simeq \frac{g_{\gamma N} g_{\pi N}}{s - s_\mathrm{p}}, \quad \mathcal{T}^{\mathrm{II}}(s \to s_\mathrm{p}) \simeq \frac{{g_{\pi N}}^2}{s - s_\mathrm{p}} ,$$

$$\Longrightarrow \boxed{g_{\gamma N} g_{\pi N} \simeq \frac{\mathcal{M}(s_\mathrm{p})}{\mathcal{S}'(s_\mathrm{p})}, \quad {g_{\pi N}}^2 \simeq \frac{\mathcal{T}(s_\mathrm{p})}{\mathcal{S}'(s_\mathrm{p})}} .$$

Residues photon decay amplitudes

- Residue $\Longrightarrow$ Decay amplitude[Workman et al., 2013a; Tiator et al., 2016]

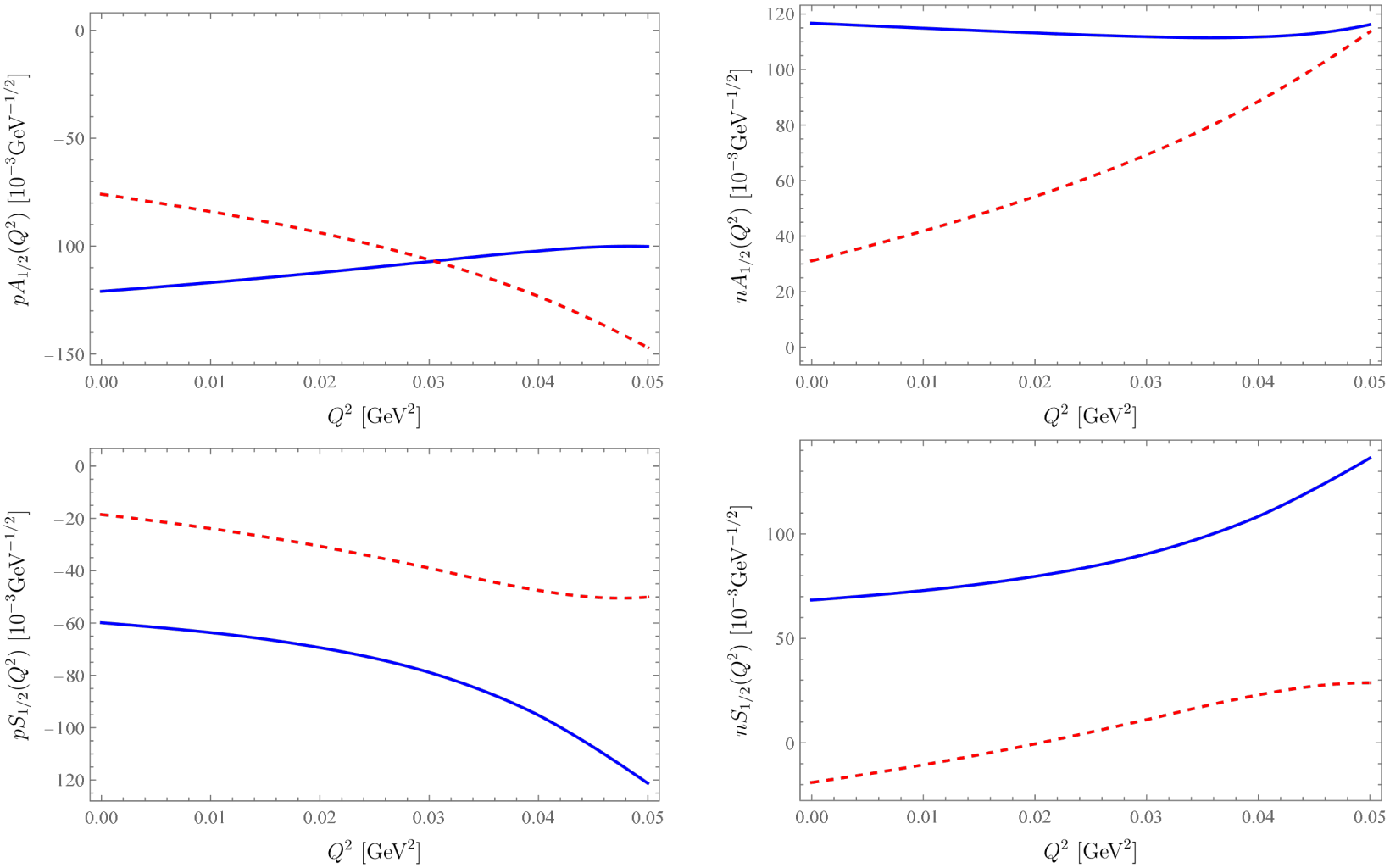
$$A_{1/2}^{\text{pole}}\left(Q^2\right) = g_{\gamma N}^{E}\sqrt{\frac{3\pi\, W_{\text{p}}}{m_N k_{\text{p}}^{\text{cm}}}}, \quad S_{1/2}^{\text{pole}}\left(Q^2\right) = g_{\gamma N}^{S}\sqrt{\frac{3\pi\, W_{\text{p}}}{2m_N k_{\text{p}}^{\text{cm}}}}$$

		$g_{\pi N}\cdot g_{\gamma N}(10^{-2}\text{GeV}^2)$		$A_{1/2}(\text{GeV}^{-1/2})$	
Target	Pole Position	Moduli	Phase (degree)	Moduli	Phase (degree)
p	$0.882+0.190i$	1.212	-79	0.17	-129
	$0.960+0.192i$	1.467	-71	0.19	-43
n	$0.882+0.190i$	0.642	111	0.09	62
	$0.960+0.192i$	1.111	103	0.14	130

- $N^*(890)$ vs $N^*(1535)$:

	$N^*(890)$	$N^*(1535)$
$\lvert g_{\pi N}\rvert^2$	0.2GeV^2[Wang et al., 2018]	0.08GeV^2[Arndt et al., 2006]
$\lvert g_{\gamma N}\rvert$	0.032GeV[Ma et al., 2021]	0.024GeV[Workman et al., 2013b; Švarc et al., 2014]
$\lvert pA_{1/2}\rvert$	$0.17\text{GeV}^{-1/2}$[Ma et al., 2021]	$0.007\text{GeV}^{-1/2}$[Workman et al., 2013b; Švarc et al., 2014]

Virtual-photon decay amplitudes



Pole position: $\sqrt{s} = 0.89 - 0.19i$ in the unit of GeV.

Summary

I Single pion photo- and electroproduction are analyzed at low energies using a dispersive representation;

II The analytic structure of PWA;

III χPT+Omnès solution with a few parameters $\Longrightarrow$ describing data at $Q^2 \leq 0.1\mathrm{GeV}^2$;

IV The residues $|g_{\pi N}|$ of $N^*(890)$ is large than $N^*(1535)$. However, $|g_{\gamma N}|$ is comparable with $N^*(1535)$.

Thanks for Listening!