The Pole Counting Rule and X, Y, Z States

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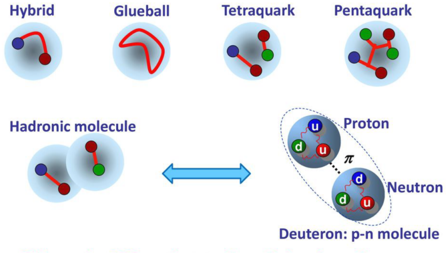
Contents

X, Y, Z States

- Many X, Y, Z states have been observed since 2003, when $X(3872)$ was detected by Belle[Choi et al., 2003].
- Exploring their properties is one of most active topics in hadron physics.
- An important topic is distinguishing hadronic molecules from compact states.



Pole counting rule, PCR

A resonance is a compact state, if there are two poles near the threshold found in S-wave scattering amplitude. Otherwise, it is a molecular state with only one pole near the threshold[Morgan, 1992].

- S-wave scattering amplitude:

$$T = (M - ik)^{-1}, \quad S = \frac{M + ik}{M - ik}, \text{ pole position: } M - ik = 0. \tag{1}$$

  - Potential scattering: $M = -\frac{1}{a_0} + \frac{r_0}{2}k^2 + O\left(k^4\right)$. $|r_0| \sim (200\text{MeV})^{-1}$, two roots satisfy:

$$|k_1| + |k_2| \geq \frac{2}{|r_0|}. \tag{2}$$

    Only one pole is near the threshold $\rightarrow$ molecular state.
  - CDD poles with weak coupling: $M = \frac{k^2 - k_0^2}{g^2}$, $g^2 \rightarrow$ small $\Rightarrow |k_1| + |k_2| \geq g^2 \Rightarrow$Two poles are near the threshold $\rightarrow$ compact state.

Applications in previous works

- $X(3872)$: Two poles are near the threshold of $D^*\bar{D} \Rightarrow 2^3P_1\ c\bar{c}$ state[Zhang et al., 2009].
- $Z_c(3900)$: One pole exits near $D^*\bar{D}$ threshold $\Rightarrow$ $D^*\bar{D}$ molecule[Gong et al., 2016].
- $X(4660)$: There exists a pole as a molecule of $\Lambda_c\bar{\Lambda}_c$. Meanwhile, $X(4660)$ is a compact state[Cao et al., 2019].
- ...

Application on $X(6900)$

In 2020, LHCb Collaboration observed a structure around 6.9 GeV, named as $X(6900)$, in the di-J/ψ invariant mass spectrum. [Aaij et al., 2020a]。

- Valence quarks： $c\bar{c}c\bar{c}$.
- Mass and width：

$$M = 6886 \pm 11 \pm 11 \quad \text{MeV},$$
$$\Gamma = 168 \pm 33 \pm 69 \quad \text{MeV}.$$

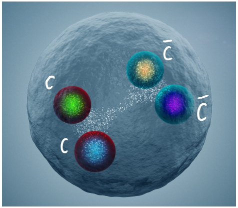
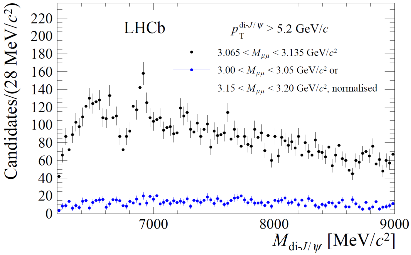


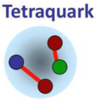
Figure: Schematic diagram of $X(6900)$. Image credit: CERN

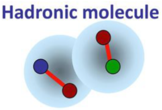


Related researches

- Compact state:
  With methods of QCD sum rule[Chen et al., 2020b], quark model[liu et al., 2020, Lü et al., 2020] regard $X(6900)$ as a compact state.

- Hadronic molecular state:
  Dynamically generated resonance.[Gong et al., 2020, Liang et al., 2021]





- Other opinions:
  There does exist a structure called $X(6200)$.
  $X(6900)$ may not to be a resonance but generated by the interference of $J/\psi\psi(2S)$, $J/\psi\psi(3770)$ with background.[Dong et al., 2021]。

Motivations & Approaches

Q.-F. Cao and H. Chen et al., Chin.Phys.C 45 (2021) 103102

- Motivations:
  - A general parametrization.
  - Data driven.
  - Respect the analycity of scattering amplitudes.

- Approaches:
  - Flatté-like parametrization $\Rightarrow$ fit data.
  - Analyse nature of resonances:

    $$\begin{cases} \text{Pole counting rule} \\ \text{Spectral density function sum rule} \end{cases}$$

  - A simple study on $X(7200)$.

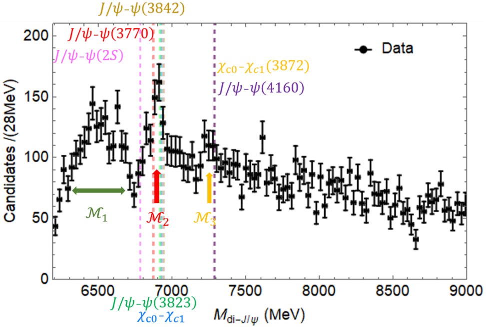
Flatté-like parametrization

$$\mathcal{M}_1 = \frac{g_1 n_{11}(s) e^{i\phi_1}}{s - M_1^2 + iM_1\Gamma_{11}(s)},\ \text{the structure around 6.5 GeV},$$

$$\mathcal{M}_i = \frac{g_i n_{i1}(s) e^{i\phi_i}}{s - M_i^2 + iM_i \sum_{j=1}^{2} \Gamma_{ij}(s)},\ \text{i=2, for } X(6900),\ \text{i=3 for } X(7200), \tag{3}$$

$$\mathcal{M}_{\rm NoR} = c_0\, e^{c_1(\sqrt{s}-2m)} \sqrt{\frac{s-4m^2}{s}},\ \text{coherent background.}$$

- $n_{ij}(s) = (p_{ij}/p_0)^l F_l(s)$, $\Gamma_{ij}(s) = g_{ij}\rho_{ij}(s) n_{ij}^2(s)$
- ρ_{ij}: two-body phase space of couple channel, l: orbital angular momentum number.
- F_l: form factor. $F_0^2 = 1$, $F_1^2 = 1/(1+z)$, $z = (p_{ij}/p_0)^2$[Chung et al., 1995]。



Spectral density function sum rule, SDFSR

- S. Weinberg: Proposed the renormalization constant of deutron: Z [Weinberg, 1965]:
  - $Z = 0 \to$ molecular state, $Z = 1 \to$ compact state.
  - Deutron is a molecular state of p and n.
  - Only suitable for S-wave stable particles which are near the thresholds.
- Expansion[Baru et al., 2004, Kalashnikova and Nefediev, 2009]:

$$|\Psi\rangle = \begin{pmatrix} \sum_a c_a |\psi_a\rangle \\ \sum_i \chi_i |M_1(i) M_2(i)\rangle \end{pmatrix},$$

$|\psi_a\rangle$: Bare elementary state.
$|M_1 M_2\rangle$: Two-hadron continuum.
$\langle\psi_0|\Psi\rangle = c_0(E)$: Possibility of finding an elementary state in a physical hadronic state.

- S-wave:

$$\int |c_0(E)|^2 d\alpha \Rightarrow \int \omega(E)\, dE \equiv \mathcal{Z}$$

- $\omega(E)$: Spectral density:

$$\omega(E) = \frac{1}{2\pi} \frac{\Gamma_{\mathrm{I}} + \Gamma_{\mathrm{II}}}{\left|E - E_f + \frac{i}{2}\Gamma_{\mathrm{I}} + \frac{i}{2}\Gamma_{\mathrm{II}}\right|^2}.$$

- $E = \sqrt{s} - m_{th}$, $E_f = M - m_{th}$, $\Gamma = \tilde{g}\sqrt{2\mu E}\theta(E)$. $\tilde{g} = 2g/m_{th}$.
  g: coupling constant in Eq. (3).
- $\mathcal{Z} \sim Z$, but suitable for S-wave resonances.

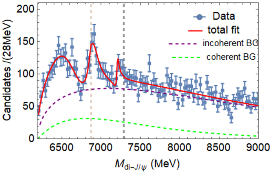
Fit results

- A: $X(6900), X(7200)$ both couple to S-wave di-J/ψ (S-S couplings).

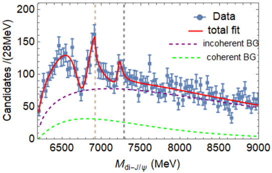
$$|\mathcal{M}|^2 = \left|\sum_{i=1}^{3} \mathcal{M}_i + \mathcal{M}_{\mathrm{NoR}}\right|^2 + \mathcal{B}.\mathcal{G}.,$$

  where $\mathcal{B}.\mathcal{G}.$ is the incoherent background.
  Three coupling cases for $X(6900)$:
  - Case I: di-J/ψ and $J/\psi, \psi(3770)$,
  - Case II: di-J/ψ and $J/\psi, \psi(3823)$,
  - Case III: di-J/ψ and $J/\psi, \psi(3842)$.

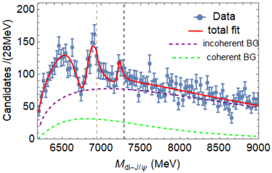
  One coupling case for $X(7200)$: di-J/ψ + $J/\psi, \psi(4160)$.
- Results: Both compact and molecular states are possible.



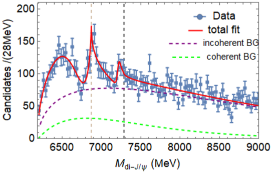
(a) Case I: elementary



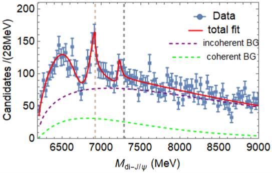
(b) Case II: elementary



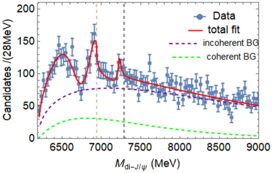
(c) Case III: elementary



(d) Case I: molecular



(e) Case II: molecular



(f) Case III: molecular

Figure: Fit projections for S-S coupling

- Pole positions of S-S couplings in the complex s plane:

Sol.	Case	State	Sheet II	Sheet III
Sol. I	I	$X(6900)$	$6885.4-68.0i$	$6874.4-80.0i$
		$X(7200)$	$7202.2-16.6i$	$7187.1-18.0i$
	II	$X(6900)$	$6947.6-172.0i$	$6810.4-274.0i$
		$X(7200)$	$7220.8-31.0i$	$7220.8-31.0i$
	III	$X(6900)$	$6845.2-117.0i$	$6789.2-138.0i$
		$X(7200)$	$7221.9-28.0i$	$7221.9-28.0i$
Sol. II	I	$X(6900)$	$6937.9-97.0i$	$6527.3-323.0i$
		$X(7200)$	$7210.7-27.5i$	$7037.3-47.5i$
	II	$X(6900)$	$6933.9-111.0i$	$6443.8-275.0i$
		$X(7200)$	$7218.9-24.0i$	$7067.9-41.5i$
	III	$X(6900)$	$6933.3-113.0i$	$6452.3-275.0i$
		$X(7200)$	$7221.9-23.0i$	$7073.7-41.0i$

- Definition of Riemann sheets (i=2,3),

	I	II	III	IV
ρ_{i1}	+	-	-	+
ρ_{i2}	+	+	-	-

- Sol.I: Compact states,

  Sol.II: Molecular states.

- Faraway poles: more than 3 times of lineshape width away from the threshold.

- SDFSR for S-S couplings: $\mathcal{Z}=\int_{E_{\min}}^{E_{\max}}\omega(E)\,dE$,

	Case	$[E_f-\Gamma,E_f+\Gamma]$	$[E_f-2\Gamma,E_f+2\Gamma]$
Sol. I	I	0.459	0.671
	II	0.379	0.592
	III	0.468	0.681
Sol. II	I	0.184	0.344
	II	0.243	0.418
	III	0.259	0.438

- B: $X(6900), X(7200)$ both couple to P-wave di-J/ψ (P-P couplings):

$$|\mathcal{M}|^2 = \left|\sum_{i=1}^{3} \mathcal{M}_i + \mathcal{M}_{\mathrm{NoR}}\right|^2 + \mathcal{B}.\mathcal{G}.,$$

Couple channels:

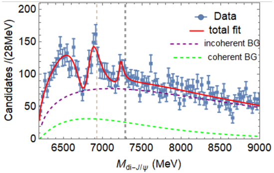
  - Case I: $X(6900) \to J/\psi J/\psi, \chi_{c0}\chi_{c1}$; $X(7200) \to J/\psi J/\psi, \chi_{c0}\chi_{c1}(3872)$;
  - Case II: $X(6900) \to J/\psi J/\psi, J/\psi\psi(3770)$; $X(7200) \to J/\psi J/\psi, J/\psi\psi(4160)$.

- Pole positions:

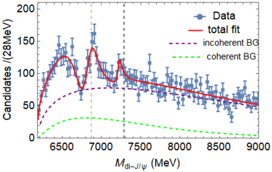
	State	Sheet II	Sheet III
Case I	$X(6900)$	$6838.7 - 119.0i$	$6840.9 - 113.0i$
	$X(7200)$	$7220.8 - 31.0i$	$7232.5 - 23.0i$
Caes II	$X(6900)$	$6844.1 - 122.0i$	$6841.2 - 110.5i$
	$X(7200)$	$7221.4 - 32.0i$	$7234.4 - 22.0i$

- But it is neither suitable for PCR nor SDFSR.

- Fit results:



(a) Case I



(b) Case II

- C: One of $X(6900), X(7200)$ couples to S-wave di-J/ψ, another couples to P-wave di-J/ψ.
  - Case I: S-wave for $X(6900) \to J/\psi J/\psi, J/\psi\psi(3770)$; P-wave for $X(7200) \to J/\psi J/\psi, J/\psi\psi(4160)$, (S-P couplings).
  - Case II: P-wave for $X(6900) \to J/\psi J/\psi, J/\psi\psi(3770)$; S-wave for $X(7200) \to J/\psi J/\psi, J/\psi\psi(4160)$, (P-S couplings).
  - $$|\mathcal{M}|^2 = \left|\sum_{i=1}^{2} \mathcal{M}_i + \mathcal{M}_{\text{NoR}}\right|^2 + |\mathcal{M}_3|^2 + \mathcal{B}\cdot\mathcal{G}., \tag{4}$$

    $X(6500)$ always interfer with $X(6900)$ to fit the dip around 6.8 GeV.
  - Pole positions:

	Sol.	State	Sheet II	Sheet III
Case I	I	$X(6900)$	$6901.0 - 32.6i$	$6884.4 - 61.7i$
		$X(7200)$	$7196.2 - 19.5i$	$7200.8 - 17.4i$
	II	$X(6900)$	$6894.8 - 65.3i$	–
		$X(7200)$	$7097.8 - 17.6i$	$7128.1 - 14.0i$
Case II		$X(6900)$	$6900.5 - 14.5i$	$6900.3 - 15.2i$
		$X(7200)$	$7362.2 - 67.9i$	–

$X(2900)$

In year 2020, LHCb Collaboration declared two structures $X_0(2900)$, $X_1(2900)$ found in $D^- K^+$ invariant mass spectrum via $B^\pm \to D^+ D^- K^\pm$[Aaij et al., 2020b].
Masses and widths are:

$$m_{X_0} = 2.866 \pm 0.007 \pm 0.002 \text{ GeV},$$
$$\Gamma_{X_0} = 57 \pm 12 \pm 4 \text{ MeV},$$

and

$$m_{X_1} = 2.904 \pm 0.005 \pm 0.001 \text{ GeV},$$
$$\Gamma_{X_1} = 110 \pm 11 \pm 4 \text{ MeV}.$$

Valence quarks: $ud\bar{s}\bar{c}$.

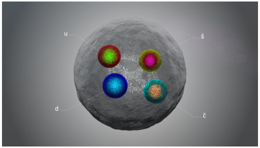
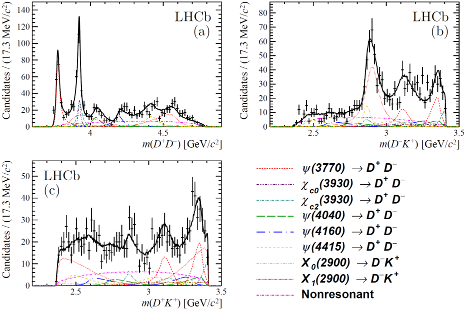


Figure: Image credit: CERN



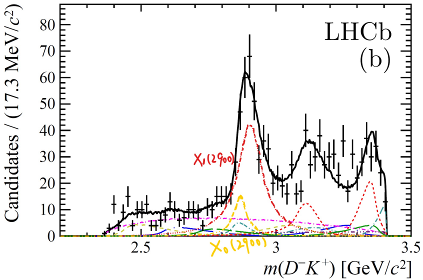
Background

- Methods in related works& Conclusions
  - Quark model[He et al., 2020]$\Rightarrow$ compact state.
  - Triangle diagram+final state interaction[Burns and Swanson, 2020, Liu et al., 2020]
    $\Rightarrow$ threshold singularity of $\bar{D}_1 K$, $\bar{D}^* K^*$
  - QCD sum rule[Chen et al., 2020a], L-S equation[Hu et al., 2020], qBSE[He and Chen, 2020, Qi et al., 2021], quark model[Xue et al., 2021]
    $\Rightarrow$ hadronic molecule of $\bar{D}^* K^*$, $\bar{D}_1 K$
- Our motivation:
  - QCD sum rule, quark model, qBSE, L-S equation $\Rightarrow$ Pole positions $\surd$, Fit data $\times$.
  - We want to fit the data and give the pole positions.

Our approaches

H. Chen, H.-R. Qi and H.-Q. Zheng, Eur.Phys.J.C 81 (2021) 9, 812.

- Consider $X_1(2900)$ couples to $\bar{D}_1 K$ and $\bar{D}K$.
- Dynamically generated $X_1(2900)$: heavy meson chiral perturbation theory, HMχPT$\Rightarrow$ perturbative scattering amplitudes of $\bar{D}K$, $\bar{D}_1 K$ $\Rightarrow$ couple channel K-matrix $\Rightarrow$ fit data $\Rightarrow$ pole positions.
- Explicitly introduced $X_1(2900)$: Flatté parametrization $\Rightarrow$ fit data $\Rightarrow$ pole positions.



Only $X_1(2900)$ is under consideration.

Dynamically generated $X_1(2900)$

- Heavy meson fields:

$$H_a^{(\bar{Q})} = \left[P_a^{*(\bar{Q})\mu}\gamma_\mu - P_a^{(\bar{Q})}\gamma_5\right]\frac{1-\not{v}}{2}, \quad T_a^{(\bar{Q})\mu} = \left[P_{2a}^{(\bar{Q})\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}P_{1a\nu}^{(\bar{Q})}\gamma_5\left(g^{\mu\nu} - \frac{1}{3}\left(\gamma^\mu - v^\mu\right)\gamma^\nu\right)\right]\frac{1-\not{v}}{2},$$

$$\bar{H}_a^{(\bar{Q})} = \frac{1-\not{v}}{2}\left[P_a^{*(\bar{Q})\mu\dagger}\gamma_\mu + P_a^{(\bar{Q})\dagger}\gamma_5\right]. \quad \bar{T}_{a\mu}^{(\bar{Q})} = \frac{1-\not{v}}{2}\left[P_{2a\mu\nu}^{(\bar{Q})\dagger}\gamma^\nu + \sqrt{\frac{3}{2}}P_{1a}^{(\bar{Q})\nu\dagger}\left(g_{\mu\nu} - \frac{1}{3}\gamma_\nu\left(\gamma_\mu - v_\mu\right)\right)\gamma_5\right]$$

- $Q \to c$, $(P^{(\bar{Q})},\ P^{*(\bar{Q})}) \to (\bar{D}, \bar{D}^*)$.
- $Q \to c$, $(P_1^{(\bar{Q})},\ P_2^{*(\bar{Q})}) \to (\bar{D}_1, \bar{D}_2^*)$.

- Interaction lagrangian[Ding, 2009]:

$$\begin{aligned}
\mathcal{L}_{\bar{D}\bar{D}\phi\phi} &= -\, i\beta\left\langle \bar{H}_a^{(\bar{Q})} v^\mu \left(\mathcal{V}_\mu\right)_{ab} H_b^{(\bar{Q})}\right\rangle, \\
\mathcal{L}_{\bar{D}_1\bar{D}_1\phi\phi} &= -\, i\beta_2\left\langle \bar{T}_{a\lambda}^{(\bar{Q})} v^\mu \left(\mathcal{V}_\mu\right)_{ab} T_b^{(\bar{Q})\lambda}\right\rangle, \\
\mathcal{L}_{\bar{D}\bar{D}_1\phi\phi} &= -\, i\zeta_1\left\langle \bar{H}_a^{(\bar{Q})} \left(\mathcal{V}_\mu\right)_{ab} T_b^{(\bar{Q})\mu}\right\rangle + h.c. \,.
\end{aligned}$$

- $\mathcal{V}_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)$, $\xi = \exp\left(i\phi/f_\pi\right)$.

- Isospin symmetry:

$$\left|K\bar{D}\right\rangle_{I=0} = \frac{1}{\sqrt{2}}\left(\left|K^+D^-\right\rangle - \left|K^0\bar{D}^0\right\rangle\right), \left|K\bar{D}\right\rangle_{I=1} = \frac{1}{\sqrt{2}}\left(\left|K^+D^-\right\rangle + \left|K^0\bar{D}^0\right\rangle\right),$$

$$\left|K\bar{D}_1\right\rangle_{I=0} = \frac{1}{\sqrt{2}}\left(\left|K^+D_1^-\right\rangle - \left|K^0\bar{D}_1^0\right\rangle\right), \left|K\bar{D}_1\right\rangle_{I=1} = \frac{1}{\sqrt{2}}\left(\left|K^+D_1^-\right\rangle + \left|K^0\bar{D}_1^0\right\rangle\right).$$

- Partial wave amplitudes: helicity basis $\rightarrow$ LSJ basis.[Chung, 1971]

$$\left\langle \lambda_3\lambda_4 \left|\mathcal{M}^{IJ}(s)\right| \lambda_1\lambda_2 \right\rangle = \frac{1}{2\pi}\int_{-1}^{1} \left\langle \Omega'\lambda_3\lambda_4|\mathcal{M}^{I}(s,t)|00\lambda_1\lambda_2 \right\rangle d^J_{\lambda,\lambda'}(z_s)dz_s.$$

$$\mathcal{T}^{IJ}_{L,L'} \equiv \left\langle JML'S' \left|\mathcal{M}^{I}(s)\right| JMLS \right\rangle = \sum_{\lambda_i}\sqrt{\frac{2L+1}{2J+1}}\sqrt{\frac{2L'+1}{2J+1}}\left\langle \lambda_3\lambda_4 \left|\mathcal{M}^{IJ}(s)\right| \lambda_1\lambda_2 \right\rangle \times \left\langle L'0S'\lambda' \mid J\lambda'\right\rangle \left\langle s_3,\lambda_3,s_4,-\lambda_4 \mid S'\lambda'\right\rangle \times \left\langle L0S\lambda \mid J\lambda\right\rangle \left\langle s_1,\lambda_1,s_2,-\lambda_2 \mid S\lambda\right\rangle.$$

- z_s : $\cos\theta_s$.
- $\lambda = \lambda_1 - \lambda_2$, $\lambda' = \lambda_3 - \lambda_4$.
- S: total spin, L: orbital angular momentum.

Unitarization

- K-matrix:

$$\begin{pmatrix} \mathcal{T}_{\bar{D}K\to\bar{D}K} & \mathcal{T}_{\bar{D}K\to\bar{D}_1K} + \mathcal{P}_{12} \\ \mathcal{T}_{\bar{D}_1K\to\bar{D}K} + \mathcal{P}_{12} & \mathcal{T}_{\bar{D}_1K\to\bar{D}_1K} \end{pmatrix}^{IJ} \equiv \begin{pmatrix} \mathcal{K}_{11} & \mathcal{K}_{12} \\ \mathcal{K}_{21} & \mathcal{K}_{22} \end{pmatrix}^{IJ}$$

  - $\mathcal{P}_{12} \to c_{012} + c_{112}\,(\sqrt{s} - m_{\text{th}2})$.

- Unitarization:

$$T = \mathcal{K} \cdot [1 - i\rho(s)\mathcal{K}]^{-1},$$

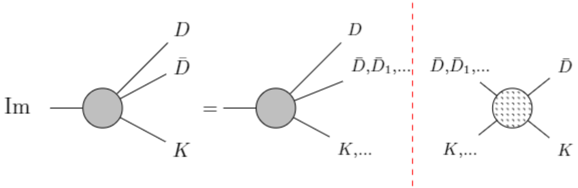
$$\rho = \text{diag}\,\{\rho_1, \rho_2\}\,.$$

- Full amplitude:

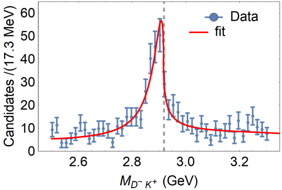
$$\mathcal{A} = \alpha_1(s)\,T_{11}(s) + \alpha_2(s)\,T_{21}(s),$$

$\alpha_{1,2}$ are polynomials of s.

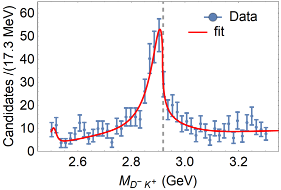
$\mathcal{A}$ satisfies the theorem of FSI:



Results& Discussions



(a) $IJ^P = 01^-$, pole position: (2.928-0.034i) GeV in sheet II.



(b) $IJ^P = 11^-$, pole position: (2.925-0.039i) GeV in sheet II.

Only one pole near the threshold $\rightarrow$ hadronic molecule.

	$IJ^P = 01^-$	$IJ^P = 11^-$
parameters	values	
β_2	-0.21 $\pm$ 0.03	-0.26$\pm$ 0.07
ζ_1	-0.90 $\pm$ 0.48	5.85 $\pm$ 2.00
c_{012}	25.25$\pm$2.48	70.00 $\pm$ 27.87
c_{112}	35.04 $\pm$ 8.46	71.37 $\pm$ 35.89
a_{10}	35.42$\pm$ 5.58	14.41$\pm$ 3.00
a'_{20}	-6.30$\pm$ 1.72	10.00$\pm$ 2.13
a'_{11}	fixed=0	4.09$\pm$ 1.63
b_0	10.15 $\pm$ 2.92	13.70 $\pm$ 1.29

- $\zeta_1 \approx \pm 0.16$ estimated from $K_1 \rightarrow K\rho$.[Dong et al., 2020]
- $X_1(2900)$ is more like an iso-singlet.

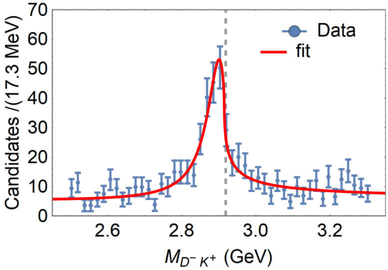
Explicitly introduced $X_1(2900)$

- Couple channels:

$$X_1(2900) \to J^P = 1^- \begin{cases} S\text{-wave } \bar{D}_1 K \\ P\text{-wave } \bar{D} K \end{cases}$$

- Invariant mass spectrum:

$$\frac{d\sigma}{d\sqrt{s}} = p_{\bar{D}K} \cdot \left| \frac{g \cdot n_{\bar{D}K}(s)}{s - M_X^2 + iM_X \left(g_1 \rho_{\bar{D}K}(s) n^2_{\bar{D}K}(s) + g_2 \rho_{\bar{D}_1 K}(s) \right)} \right|^2 + b.g.$$

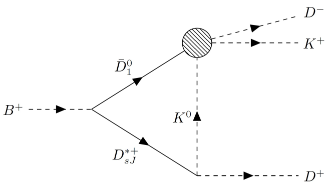


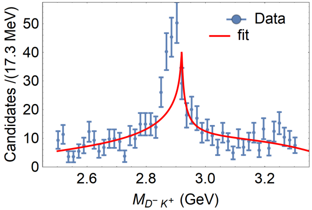
RS	pole position (GeV)
II	$2.910 - 0.039i$
III	$2.435 - 0.012i$

Only one pole near the threshold $\to$ hadronic molecule.

Triangle cusp?

Discussions about the possibility of $X_1(2900)$ to be a triangle cusp.





- D_{sJ}^* mass: $m^* - i\Gamma \to$ free parameter.
- $m^* = (2.544 \pm 0.097)$ GeV, $\Gamma = (0.048 \pm 0.059)$ GeV
- The mass and width are not consistent with the suggestions that D_{sJ}^* is $D_{s1}^*(2860)$ or $D_{s1}^*(2700)$ in Refs. [Burns and Swanson, 2020, Liu et al., 2020].

Conclusions

- Pole counting rule is a powerful method to distinguish a hadronic molecule from a compact state.
- For $X(6900)$, both a compact state and a molecular state are possible based on current data. More data $\Rightarrow$ understand its nature better.
- For $X_1(2900)$, it should be a $\bar{D}_1 K$ molecule.
- Almost all X, Y, Z states with exotic quantum numbers $\Rightarrow$ hadronic molecules.
- $X(3872) \Rightarrow c\bar{c}$ quantum number $\Rightarrow$ large portion of charmonium.

Thanks for Listening!

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Remarks

- Couplings of $X(6900)[X(7200)]$ with nearby channels are studied.
- Flatté-like parametrization is used to fit the data and PCR, SDFSR are used to analyse the nature of resonances.
- Both compact and molecular states are possible. PCR and SDFSR give consistent conclusions.
- We suggest that experiments measure processes, such as $X(6900) \to J/\psi\psi(3770), J/\psi\psi(3823), J/\psi\psi(3842), \chi_{c0}\chi_{c1}$; $X(7200) \to J/\psi\psi(4160), \chi_{c0}\chi_{c1}(3872)$, to clarify their nature.

Remarks

- $X_1(2900)$ can be produced dynamically via the couple channel interactions between $\bar{D}K$ and $\bar{D}_1K$.
- $X_1(2900)$ is a S-wave hadronic molecule of $\bar{D}_1K$ with $J^P = 1^-$ and is leaning toward an iso-singlet.
- Results of explicitly introduced $X_1(2900)$ also support this point of view.
- $X_1(2900)$ can hardly be produced by a triangle cusp.
- We suggest experiments to measure processes, e.g. $B^+ \to D^0X^+, X^+ \to \bar{D}^0K^+$ and $B^0 \to D^+X^-, X^- \to D^-K^0$ to determine the isospin of $X_1(2900)$.

Couple channels involved:

$$X(6900)\begin{cases} J^{PC}=0^{++},\ S\text{—wave}\begin{cases} J/\psi\ J/\psi + J/\psi\psi(2S) \\ J/\psi\ J/\psi + J/\psi\psi(3770)\end{cases} \\ J^{PC}=2^{++},\ S\text{—wave}\begin{cases} J/\psi\ J/\psi + J/\psi\psi(2S) \\ J/\psi\ J/\psi + J/\psi\psi(3770) \\ J/\psi\ J/\psi + J/\psi\psi(3823) \\ J/\psi\ J/\psi + J/\psi\psi(3842)\end{cases} \\ J^{PC}=1^{-+},\ P\text{—wave}\rightarrow J/\psi\ J\psi + \chi_{c0}\chi_{c1} \\ J^{PC}=(0,1,2)^{-+},\ P\text{—wave}\rightarrow J/\psi\ J/\psi + J/\psi\psi(3770)\end{cases}$$

$$X(7200)\begin{cases} J^{PC}=(0,2)^{++},\ S\text{—wave}\rightarrow J/\psi\ J/\psi + J/\psi\psi(4160) \\ J^{PC}=(0,1,2)^{-+},\ P\text{—wave}\rightarrow J/\psi\ J/\psi + J/\psi\psi(4160) \\ J^{PC}=1^{-+},\ P\text{—wave}\rightarrow J/\psi\ J\psi + \chi_{c0}\chi_{c1}(3872)\end{cases}$$

Three-point loop integral function:

$$C_0(p_1^2, p_2^2, p_3^2; m_2^2, m_1^2, m_3^2) =$$

$$\frac{\mu^{4-D}}{i}\int \frac{d^D k}{(2\pi)^D}\left\{\frac{1}{[k^2 - m_1^2 + i\varepsilon][(k-p_1)^2 - m_2^2 + i\varepsilon]} \times \frac{1}{[(k-p_2)^2 - m_3^2 + i\varepsilon]}\right\}.$$

- Analyse the singularity of C_0 with different m^*.
- $X_1(2900)$ can hardly be produced by a triangle cusp.