# Patterns of $C$- and $CP$-violation in hadronic $\eta$ and $\eta'$ three-body decays

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presentation based on: Akdag, Isken, Kubis [arxiv:2111.02417 [hep-ph]] (2021)

Asymmetry between matter and antimatter

- Origin of matter from baryogenesis presumes $C$- and $CP$-violation [*Sakharov 1967*]
- In SM: violation from weak interaction is not sufficient to create observed asymmetry

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Search for new sources of $CP$-violation:

- Mostly neglected since 1960's: $T$-odd $P$-even (TOPE) operators in strong interactions
- Consider an eigenstate of $C$, we focus on the $\eta$ meson
  $\hookrightarrow$ Can investigate $CP$-violation in absence of weak interaction

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Ideal stage to investigate TOPE forces:

- $\eta \to \pi^0\pi^+\pi^-$
- $\eta' \to \eta\pi^+\pi^-$

} Dalitz plots sensitive to mirror symmetry breaking

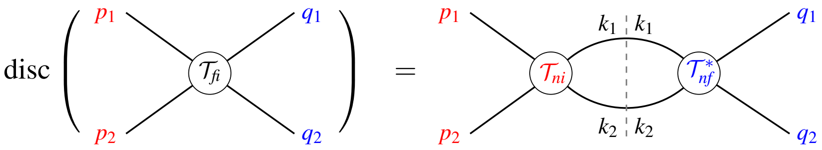
# Dispersive Framework

Evaluate three-particle decay in dispersive (*Khuri-Treiman*) framework:

Model independent and non-perturbative re-summation of final state interactions, based on

1. Unitarity ($\sim$ probability conservation) gives rise to *optical theorem*:

$$\mathcal{T}_{fi} - \mathcal{T}_{if}^{*} = i \sum_{n} \int \mathrm{d}\Pi_n (2\pi)^4 \delta^4 \Big( \sum_{i,n} p_i - k_n \Big) \mathcal{T}_{ni} \mathcal{T}_{nf}{}^{*}$$




2. Analyticity ($\sim$ causality)

   Dispersion relations: reconstruct whole amplitude with knowledge about discontinuity

Idea: derive $2 \to 2$ scattering amplitude and analytically continue to realm of $1 \to 3$ decay

$$\eta \to \pi^0\pi^+\pi^-$$

- $\eta \to \pi^+\pi^-\pi^0$ *breaks* G-parity; in the Standard Model consider transition with $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \xi \mathcal{M}_1^C(s,t,u)$$

[*Colangelo et al. 2018; Albaladejo et al. 2017; Guo et al. 2017; ...*]

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- For $C$-violating parts consider $C = -(-1)^{\Delta I}$, i.e. need even *total* isospin [*Gardner, Shi 2020*]

$$\mathcal{M}(s,t,u) = \xi \mathcal{M}_1^{C}(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

# Amplitude Decomposition

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- Bose symmetry: odd (even) $\pi\pi$-isospin must have odd (even) partial wave
- Reconstruction theorem: expand for fixed *two-body* isospin and partial wave

$$\mathcal{M}_1^{C}(s,t,u) = \mathcal{F}_0(s) + (s-u)\mathcal{F}_1(t) + (s-t)\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not C}(s,t,u) = (t-u)\mathcal{G}_1(s) + (u-s)\mathcal{G}_1(t) + (s-t)\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not C}(s,t,u) = 2(u-t)\mathcal{H}_1(s) + (u-s)\mathcal{H}_1(t) + (s-t)\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

- $C$-even terms are symmetric and $C$-odd ones antisymmetric under $t \leftrightarrow u$
- Note: $\mathcal{F}_I$, $\mathcal{G}_I$ and $\mathcal{H}_I$ are completely independent

- Single variable amplitudes $\mathcal{A} \in \{\mathcal{F}, \mathcal{G}, \mathcal{H}\}$ obey discontinuity relation

$$\text{disc}\mathcal{A}_I(s) = 2i\,\theta(s - 4M_\pi^2)\left[\mathcal{A}_I(s) + \hat{\mathcal{A}}_I(s)\right]\,\sin\delta_I(s)\,e^{-i\delta_I(s)}$$

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$$\mathcal{A}_I(s) = P(s)\,\Omega_I(s), \qquad \Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\delta_I(x)}{x(x-s)}\,\mathrm{d}x\right)$$

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- Inhomogeneities defined via partial wave (include left-hand cut contribution)

$$a_I(s) = \mathcal{A}_I(s) + \hat{\mathcal{A}}_I(s)$$

- Full solution

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{d}x}{x^n}\,\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(s)}{|\Omega_I(x)|\,(x-s)}\right)$$

- Subtraction polynomial $P_{n-1}$ fixed by asymptotics imposed on $\mathcal{A}_I$ and $\delta_I$ and by data

Regression to Dalitz plot [*KLOE-2, 2016*]

The SM amplitude $\mathcal{M}_1^C$:

- Minimal subtraction scheme 3 dof: $\chi^2_{\text{red}} \approx 1.054$
- Observables agree with current literature
  1. Taylor invariants [*Colangelo et al. 2018*]
  2. $\text{BR}(\eta \to 3\pi^0)/\text{BR}(\eta \to \pi^0\pi^+\pi^-)$ [*PDG 2020*]
  3. Dalitz plot parameters [*Colangelo et al. 2018, PDG 2020*]

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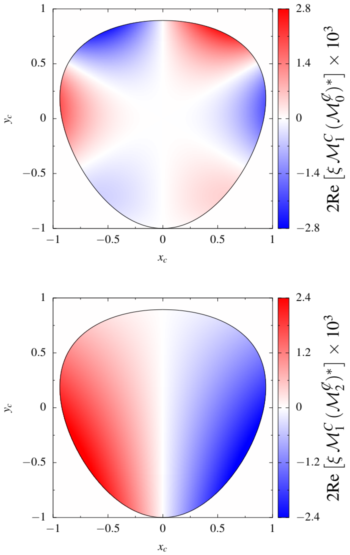
The BSM amplitude $\mathcal{M} = \xi\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$:

- Fix $\mathcal{M}_{0,2}^{\not C}$ by just one complex normalization each
- Full amplitude 7 dof: $\chi^2_{\text{red}} \approx 1.048$
- Upper limit on TOPE effects in per mille level
- All *C*- and *CP*-violating signals vanish within 1-2$\sigma$



- Effective BSM operators

$$X_0^{\not{C}} \sim g_0(s-t)(t-u)(u-s) + \mathcal{O}(p^8)$$
$$X_2^{\not{C}} \sim g_2(t-u) + \mathcal{O}(p^4)$$

- Obtain couplings by a Taylor expansion of $\mathcal{M}_0^{\not{C}}$, $\mathcal{M}_2^{\not{C}}$:

$$g_0 = (-3.3(3.6) + 6.7(12.6)\,i)\ \mathrm{GeV}^{-6}$$
$$g_2 = (0.001(15) - 0.006(42)\,i)\ \mathrm{GeV}^{-2}$$

- Relative deviation $|g_0/g_2| \approx 10^3\ \mathrm{GeV}^{-4} \Longrightarrow$ fit compensates kinematic suppression of $X_0^{\not{C}}$

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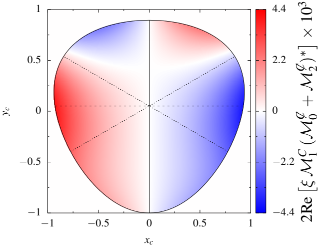


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- Dalitz-plot asymmetries in $10^{-4}$ ($g_0$ in $\text{GeV}^{-6}$, $g_2$ in $10^3\ \text{GeV}^{-2}$):

$$A_{LR} = -0.943\,\mathrm{Re}\,g_0 - 0.300\,\mathrm{Im}\,g_0 - 2.493\,\mathrm{Re}\,g_2 - 0.936\,\mathrm{Im}\,g_2 = -7.6(4.7)$$
$$A_Q = \ \ 0.479\,\mathrm{Re}\,g_0 + 0.443\,\mathrm{Im}\,g_0 + 0.536\,\mathrm{Re}\,g_2 + 0.336\,\mathrm{Im}\,g_2 = \ \ 4.1(4.3)$$
$$A_S = -2.971\,\mathrm{Re}\,g_0 - 0.850\,\mathrm{Im}\,g_0 - 0.057\,\mathrm{Re}\,g_2 - 0.043\,\mathrm{Im}\,g_2 = \ \ 3.7(4.3)$$

# *Generalization to* $\eta' \to \pi^0\pi^+\pi^-$

Can not repeating the high-precision analysis for $\eta' \to \pi^0\pi^+\pi^-$

- $\eta' \to \pi^0\pi^+\pi^-$ rare decay mode
- Precise investigation of Dalitz plot [*BESIII 2017*] not possible yet... [*Isken et al. 2021 (in preperation)*]

What happens for an increased phase space ($M_\eta \to M_{\eta'}$)?

- Both decays are driven by same $\Delta I = 0, 2$ operators
- Assume *same* couplings $g_0$ and $g_2$ to predict $C$-odd amplitudes
- $\mathcal{M}_0^{\not{C}}$ dominates $\mathcal{M}_2^{\not{C}}$ by $10^2$

$\Longrightarrow$ TOPE forces in $\eta' \to \pi^0\pi^+\pi^-$ are more sensitive to isoscalar transitions

$$\eta' \to \eta\pi^+\pi^-$$

# Amplitude Decomposition

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^C(s,t,u)$$

[*Isken et al. 2017, Isken et al. 2021 (in preparation)*]

# Amplitude Decomposition

- $\eta' \to \eta\pi^+\pi^-$ *conserves* G-parity; in the Standard Model consider transition with $\Delta I = 0$
- $C$-violating driven by isospin $\Delta I = 1$

$$\mathcal{M}(s,t,u) = \mathcal{M}_0^{C}(s,t,u) + \mathcal{M}_1^{\not{C}}(s,t,u)$$

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- Sensitive to a different class of BSM operators!
- Reconstruction theorem: two different intermediate states

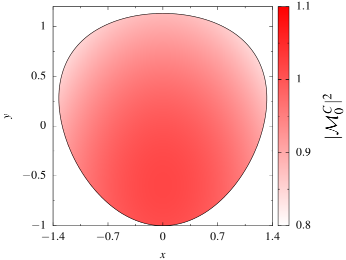
$$\mathcal{M}_0^{C}(s,t,u) = \mathcal{F}_{\pi\pi}(s) + \mathcal{F}_{\eta\pi}(t) + \mathcal{F}_{\eta\pi}(u)$$
$$\mathcal{M}_1^{\not{C}}(s,t,u) = (t-u)\,\mathcal{G}_{\pi\pi}(s) + \mathcal{G}_{\eta\pi}(t) - \mathcal{G}_{\eta\pi}(u)$$

- $\mathcal{F}_{\pi\pi}$, $\mathcal{F}_{\eta\pi}$ and $\mathcal{G}_{\eta\pi}$ in $S$-waves, $\mathcal{G}_{\pi\pi}$ in $P$-wave
- Solution analogous to $\eta \to 3\pi$

Regression to Dalitz plot [*BESIII, 2018*]

The SM amplitude $\mathcal{M}_0^C$:

- Minimal subtraction scheme fails to describe data accurately
- Need at least 4 dof: $\chi^2_{\text{red}} \approx 0.994$



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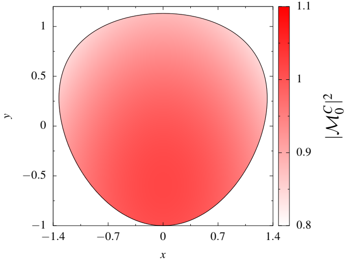
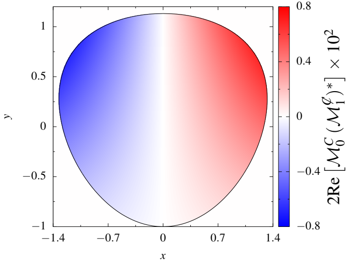
Apply same subtraction scheme for BSM amplitude:

- Fix $\mathcal{M}_1^{\not{C}}$ by two complex coefficients with the same phase
- Full amplitude 7 dof: $\chi^2_{\text{red}} \approx 0.994$
- Upper limit on TOPE effects in per cent level
- All $C$- and $CP$-violating signals vanish within $< 1.5\sigma$



# BSM Coupling

- Effective BSM operator

$$X_1^{\not{C}} \sim g_1\,(t-u)\,(1+s\,\delta g_1) + \mathcal{O}(p^6)$$

- Obtain coupling and $s$-dependent correction by a Taylor expansion of $\mathcal{M}_1^{\not{C}}$:

$$g_1 = (0.17(27) - 0.3(5.7)\,i)\ \mathrm{GeV}^{-2}$$

$$\delta g_1 \approx -4(99)\ \mathrm{GeV}^{-2}$$ imaginary part negligibly small

- Dalitz-plot asymmetry in $10^{-3}$ ($g_1$ and $\delta g_1$ in $\mathrm{GeV}^{-2}$):

$$A_{LR} = -31.0\,\mathrm{Re}\,g_1\big(1+0.09\,\delta g_1\big) - 6.6\,\mathrm{Im}\,g_1\big(1+0.10\,\delta g_1\big) = -2.3(1.7)$$

The dispersive framework for TOPE forces in $\eta \to \pi^0\pi^+\pi^-$ and $\eta' \to \eta\pi^+\pi^-$

- Based on fundamental principles of analyticity, unitarity and crossing
- Derived $C$- and $CP$-odd contributions driven by $\Delta I = 0, 2$ and $\Delta I = 1$ transitions
- Extracted effective BSM couplings $g_0$, $g_2$ and $g_1$
- Current experimental precision:
  - Upper limit on $C$-odd signals in the relative per mille and cent level
  - In both cases BSM signals vanish within 1-2$\sigma$

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From experimental point of view:

- *JLab Eta Factory* (JEF)
- *Rare Eta Decays with a TPC for Optical Photons* (REDTOP)

Thank you very much for your attention!