Nuclear structure corrections in muonic atoms from chiral effective field theory

Simone Salvatore Li Muli

Sonia Bacca

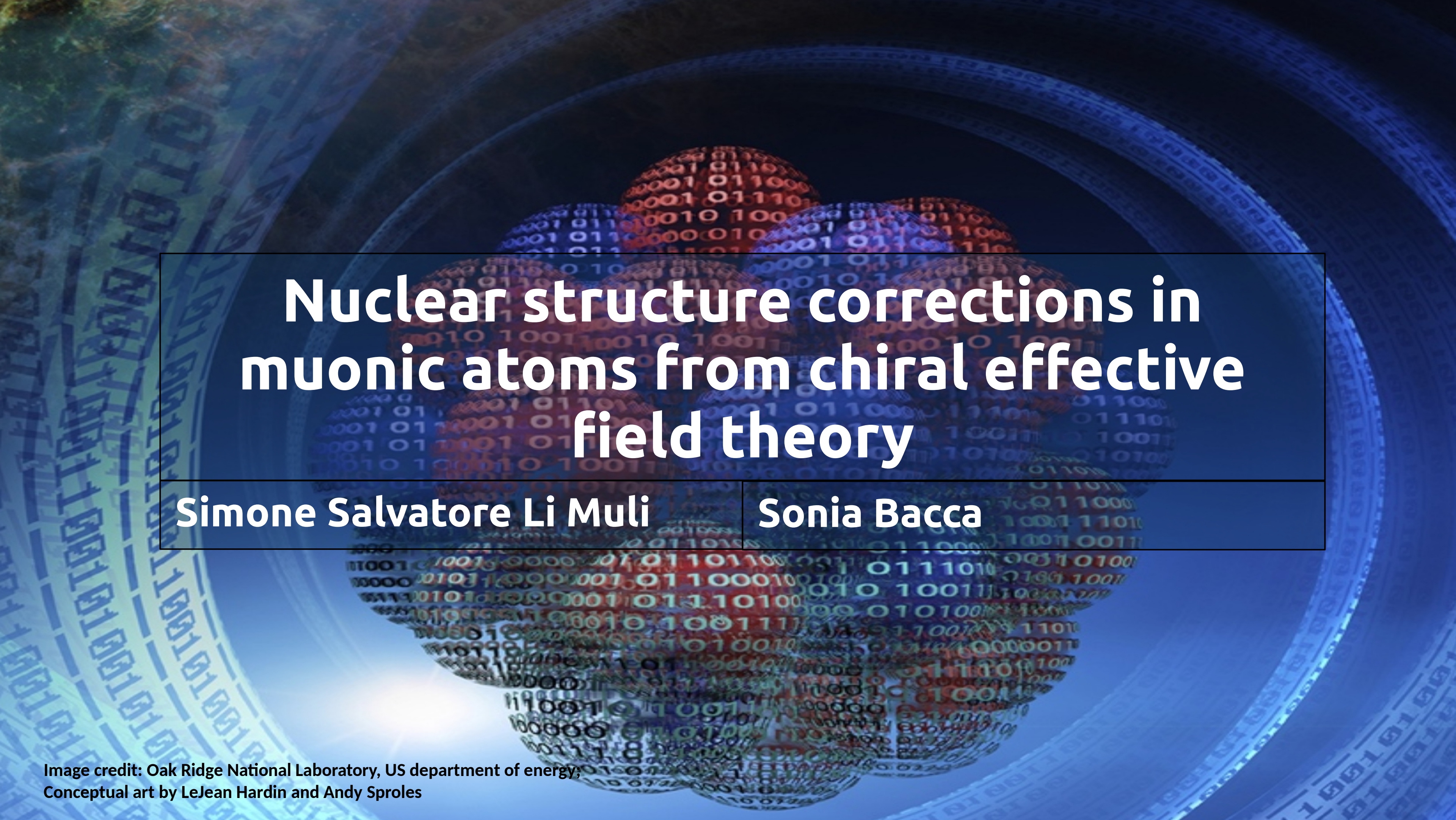

Image credit: Oak Ridge National Laboratory, US department of energy;
Conceptual art by LeJean Hardin and Andy Sproles

Outline

- Introduction to muonic atoms and LS-spectroscopy
- Theory and numerical methods
- Results
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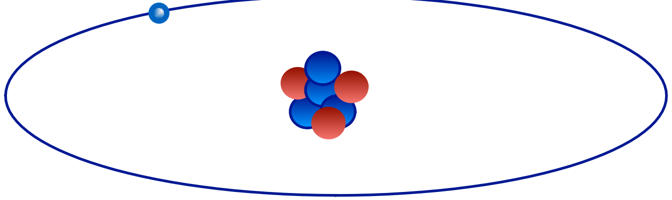
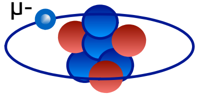
Muonic atoms

Hydrogen-like systems

Ordinary atoms



Muonic atoms



The muon is more sensitive to the nucleus

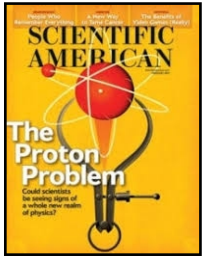
Excellent precision probe for the nucleus

Experimental program at **PSI** of the **CREMA** collaboration

Muonic Hydrogen
- Pohl et al., Nature (2010)
- Antognini et al., Science (2013)

Muonic Deuterium
- Pohl et al., Science (2016)

Muonic Helium-4
- Krauth et al., Nature (2021)



Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}$$

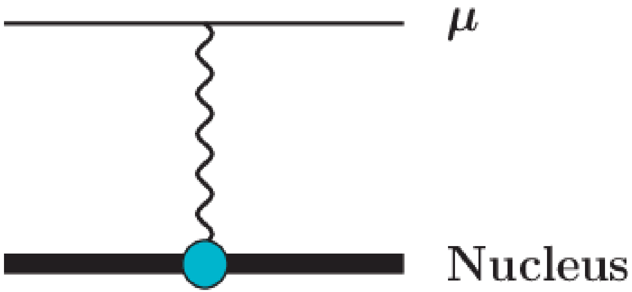
Measured

Extracted

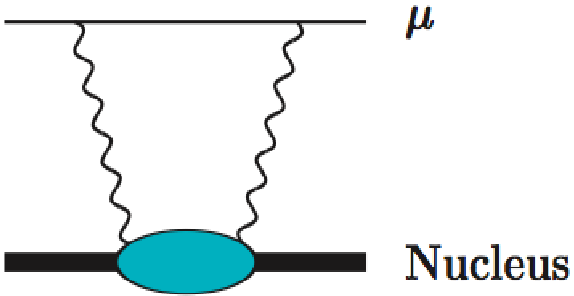
Lamb-shift and charge radius

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \boxed{\mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}}$$

Nuclear structure corrections



$$\mathcal{A}_{\mathrm{OPE}} \approx m_r^3 (Z\alpha)^4 / 12$$

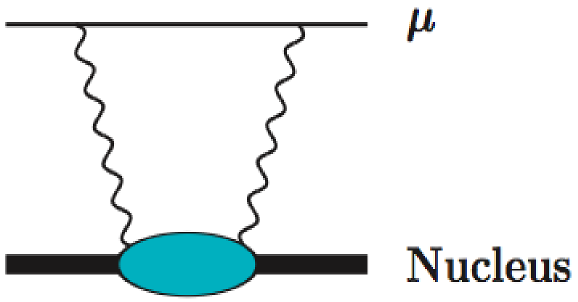


Starts at $(Z\alpha)^5$

A matter of precision

$$\delta_{\mathrm{LS}} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\, r_c^2 + \delta_{\mathrm{TPE}}$$

	δ_{QED}	$\mathcal{A}_{\mathrm{OPE}}\, r_c^2$	δ_{TPE}
Relative contribution	95 %	4 %	1 %
Uncertainty budget (theo)	5 %	3 %	92 %



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Evaluation of the TPE amplitude

$$\delta_{TPE} = \sum_i \left[C_i(Z\alpha, m_r) \int_0^\infty \mathcal{F}_i(\omega/m_r) \, S_{O_i}(\omega) \, d\omega \right]$$

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Coefficient of expansion

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Weight function

Evaluation of the TPE amplitude

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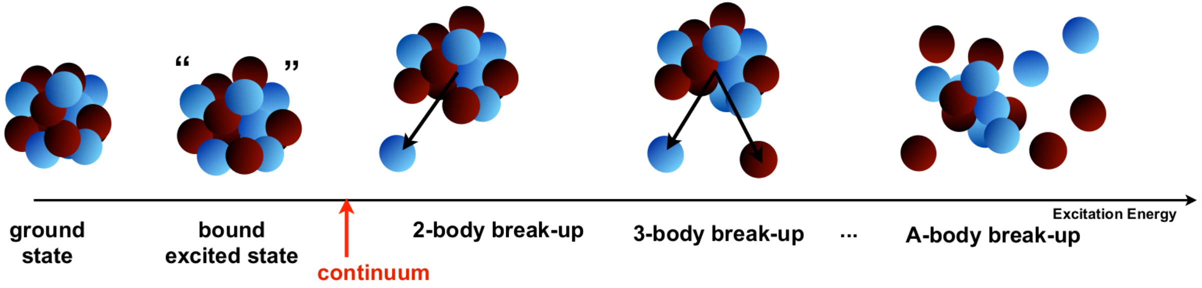
Response function

$$S_O(\omega) = \frac{1}{2J_0+1} \sum_{N \neq N_0, J} |\langle NJ || \hat{O} || N_0 J_0 \rangle|^2 \, \delta(\omega - \omega_N)$$



The response function is computed numerically in an ab-initio framework.

Ab-initio Nuclear Theory

Ab-initio methods: Solutions of the time-independent Schrödinger equation for the nuclear states

$$\widehat{H}\Psi = E\Psi$$

With controlled approximations.

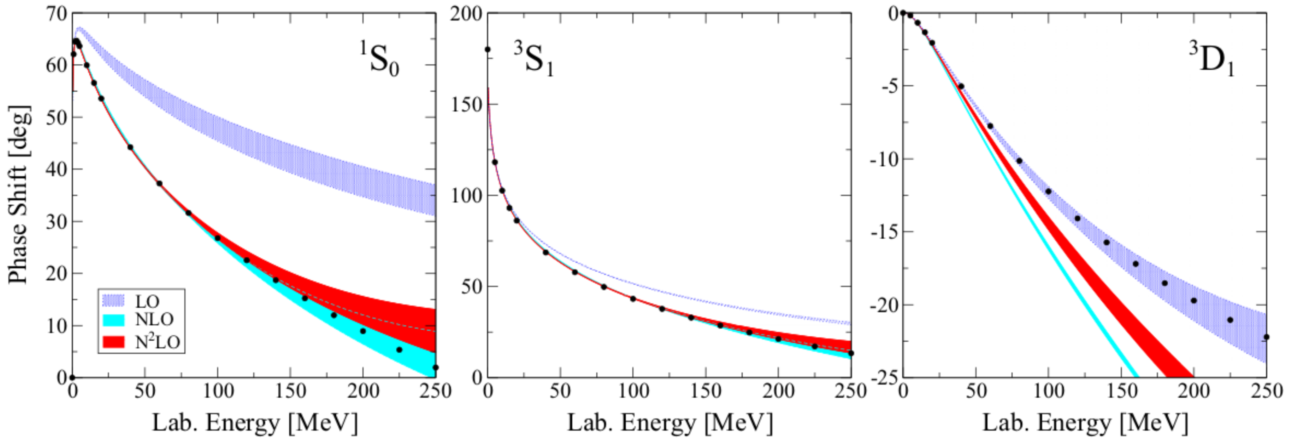
Controlled approximations come from two sides:

1) **Hamiltonian** $\widehat{H}$

2) **Nuclear wavefunction** Ψ

The Hamiltonians

Interactions derived from the Chiral effective field theory, written in coordinate space up to N2LO with only one non-local operator.



A. Gezerlis et. al. PRC 90, 054323 (2014)
J. Lynn et. al. PRL 116, 062501 (2016).

Our few-body methods take advantage on working with local interactions in coordinate space.

The Wavefunctions

First we do a Jacobi transformation of the coordinate-system to remove the center-of-mass motion and then we move to hyperspherical coordinates.

$$\{\vec{r}_1, \ \dots \ , \vec{r}_A\} \xrightarrow{N=A-1} \{\rho_N, \ \Omega_N\}$$

3A coordinates → **3A-3 coordinates**

3N-1 Hyperangular coordinates

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Then the nuclear wavefunction can be separated in a hyper-radial part and an hyper-angular part

$$\Psi = \mathcal{R}_{(K_N)}(\rho_N)\ \mathcal{H}_{(K_N)}(\Omega_N)$$

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$$\Psi = \mathcal{R}_{(K_N)}(\rho_N)\ \mathcal{H}_{(K_N)}(\Omega_N)$$

Which are expanded using complete sets of functions

$$\mathcal{H}_{(K_N)}(\Omega_N) = \sum_{(K_N)}^{K_{max}} \sum_{Y_N} C_{Y_N}\ [\mathcal{Y}_{[K_N]}\ \chi_{[S_A]}]^{JJ_z}$$

Hyperspherical Harmonics functions

$$\mathcal{R}_{(K_N)}(\rho_N) = \sum_{n=0}^{n_{max}} C^n_{(K_N)}\ L^v_n(\rho_N)$$

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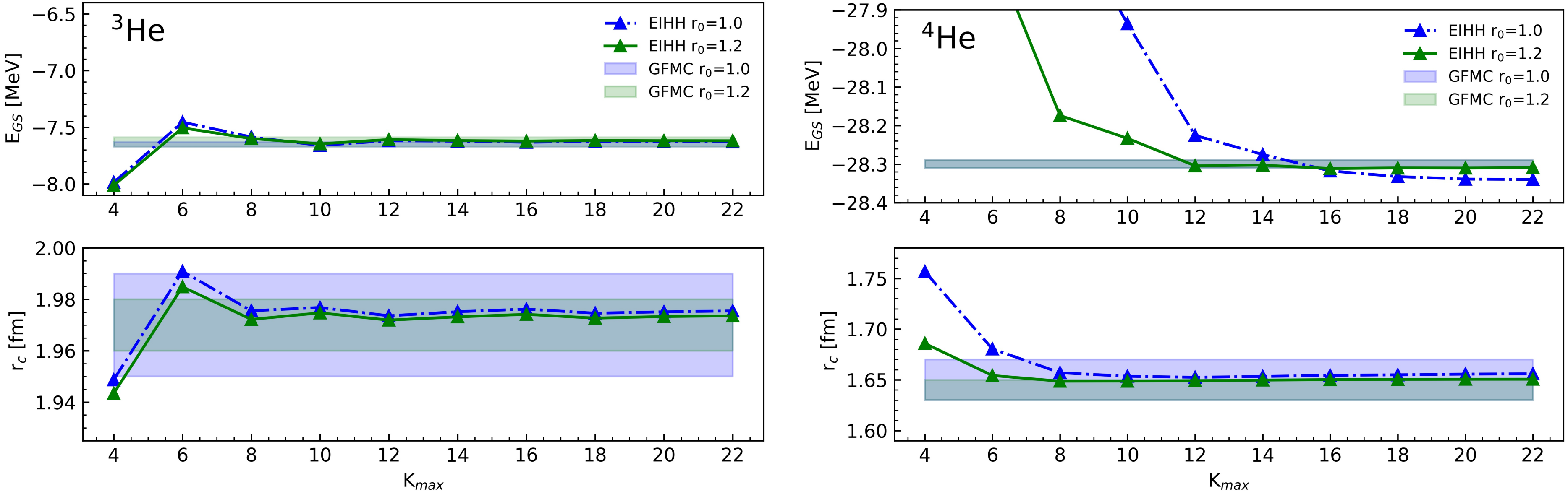
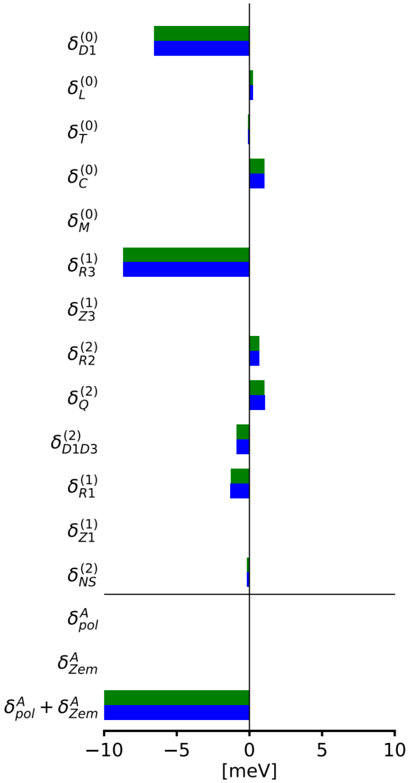
Benchmark tests

S.S. LM, S. Bacca , N. Barnea, Front. Phys. 9, 671869 (2021)

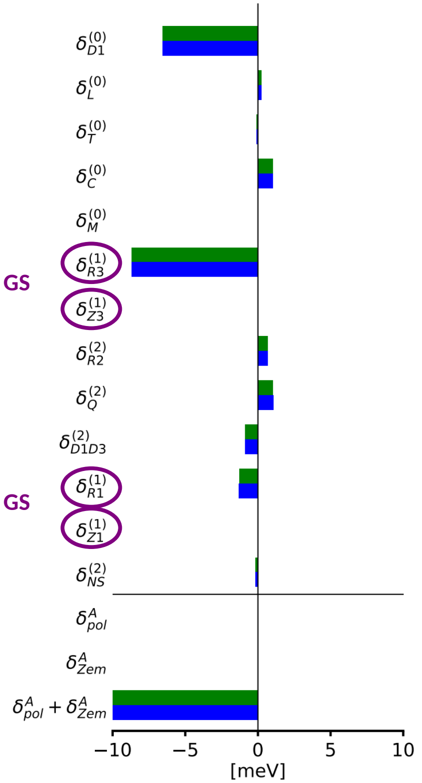


TPE in He-3



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta_{D1}^{(0)}$	-6.562	-6.552
$\delta_{L}^{(0)}$	0.232	0.232
$\delta_{T}^{(0)}$	-0.104	-0.104
$\delta_{C}^{(0)}$	1.018	1.015
$\delta_{M}^{(0)}$	0.008	0.008
$\delta_{R3}^{(1)}$	-8.674	-8.691
$\delta_{Z3}^{(1)}$	-	-
$\delta_{R2}^{(2)}$	0.67	0.696
$\delta_{Q}^{(2)}$	1.043	1.062
$\delta_{D1D3}^{(2)}$	-0.877	-0.899
$\delta_{R1}^{(1)}$	-1.297	-1.302
$\delta_{Z1}^{(1)}$	-	-
$\delta_{NS}^{(2)}$	-0.191	-0.193
δ_{pol}^{A}	-	-
δ_{Zem}^{A}	-	-
$\delta_{pol}^{A} + \delta_{Zem}^{A}$	-14.735	-14.723

TPE in He-3



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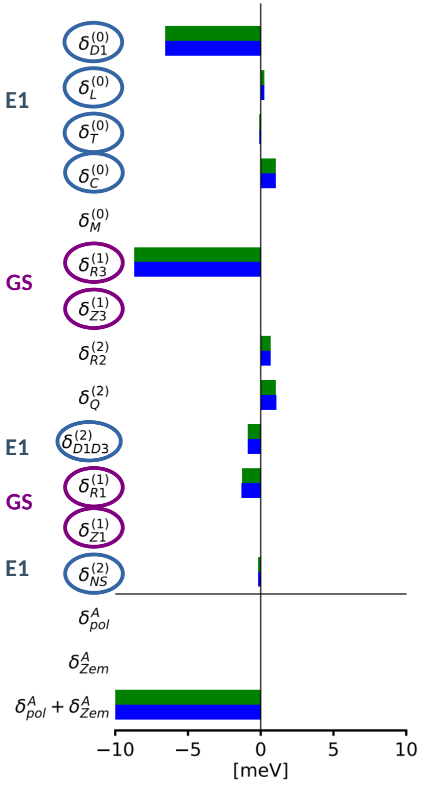

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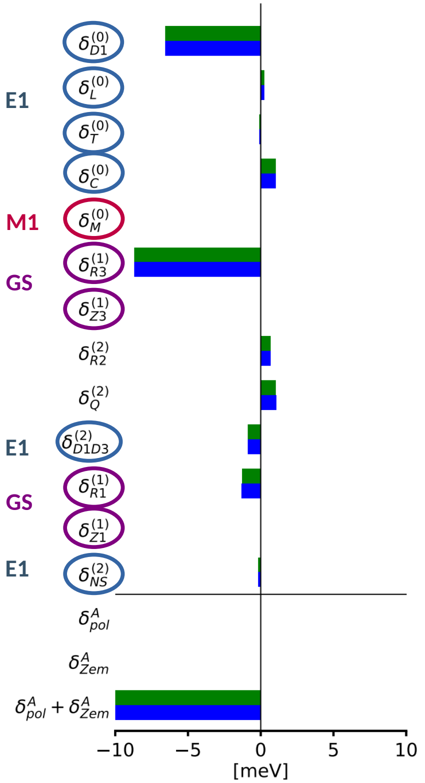

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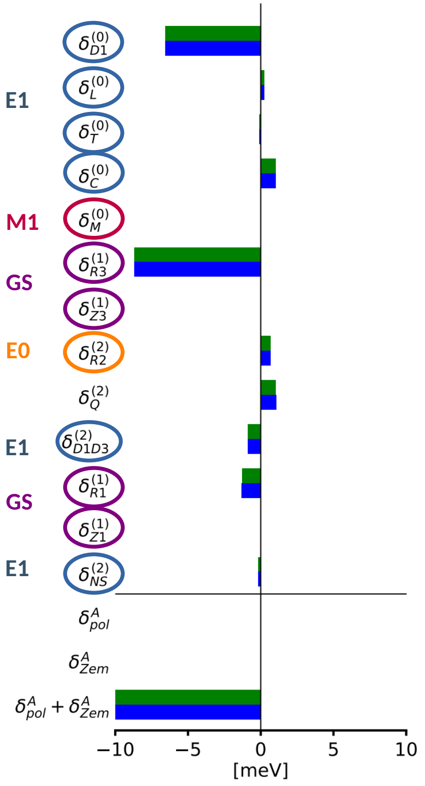

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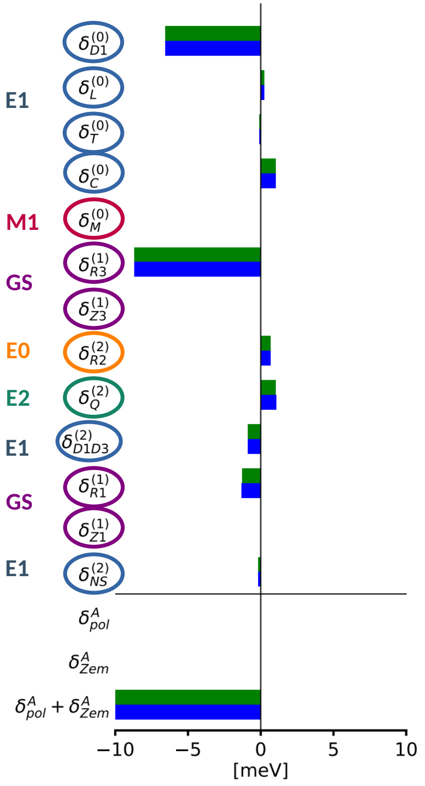

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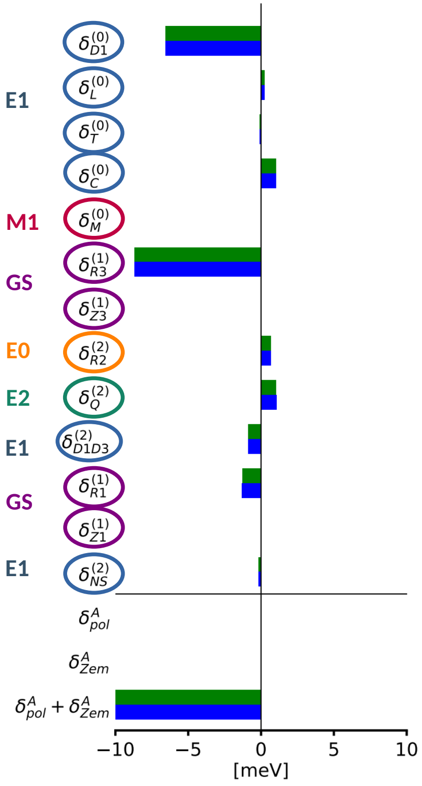

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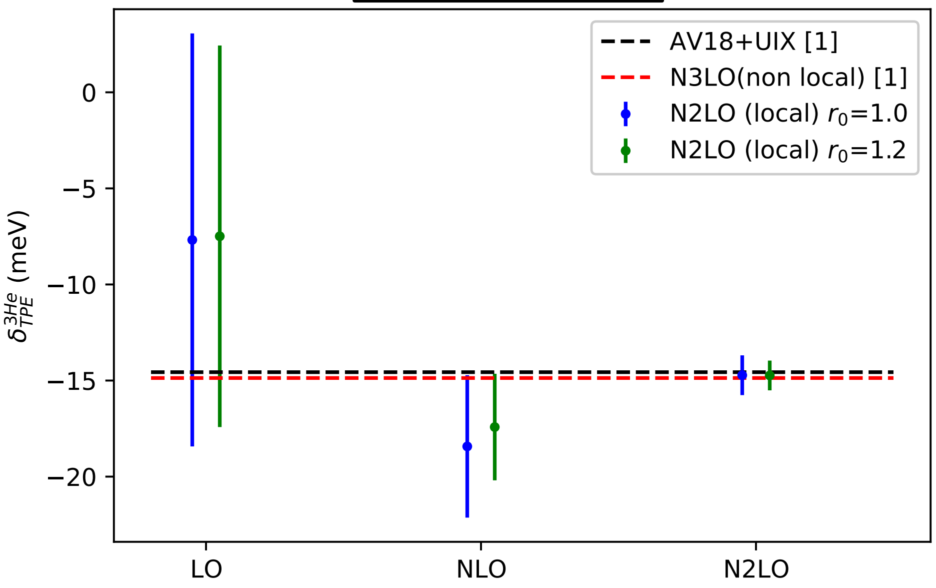
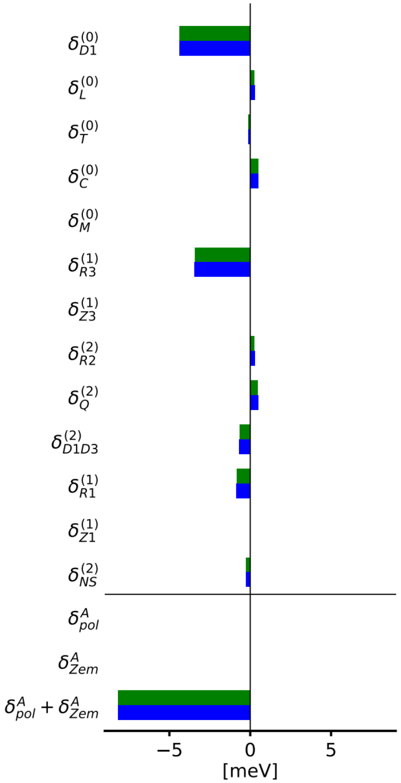
In preparation



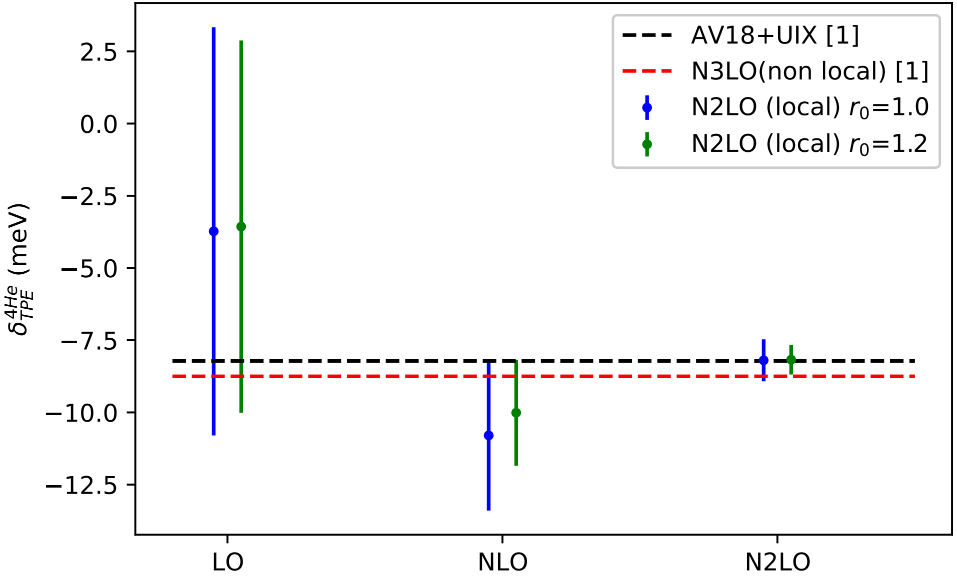
[1] C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

TPE in He-4



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta^{(0)}_{D1}$	-4.386	-4.373
$\delta^{(0)}_{L}$	0.272	0.287
$\delta^{(0)}_{T}$	-0.124	-0.125
$\delta^{(0)}_{C}$	0.517	0.514
$\delta^{(0)}_{M}$	0.011	0.011
$\delta^{(1)}_{R3}$	-3.422	-3.477
$\delta^{(1)}_{Z3}$	-	-
$\delta^{(2)}_{R2}$	0.267	0.285
$\delta^{(2)}_{Q}$	0.484	0.505
$\delta^{(2)}_{D1D3}$	-0.668	-0.69
$\delta^{(1)}_{R1}$	-0.846	-0.856
$\delta^{(1)}_{Z1}$	-	-
$\delta^{(2)}_{NS}$	-0.272	-0.277
δ^{A}_{pol}	-	-
δ^{A}_{Zem}	-	-
$\delta^{A}_{pol} + \delta^{A}_{Zem}$	-8.174	-8.196

In preparation



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Conclusions

- **Ab-initio theories represent an excellent framework to calculate nuclear structure effects in muonic atoms.**

- **We quantified the nuclear structure uncertainties to the Lamb-shift in muonic He-3 and He-4 in an order-by-order chiral truncation, validating the previous estimates.**

- **At N2LO we get uncertainties that are larger than the previous estimates; a calculation at N3LO could reduce the current uncertainties.**