Fluctuations and phases in baryonic matter

[Brandes, Kaiser and Weise: Eur. Phys. J. A 57 (2021)]

10th international workshop on
chiral dynamics

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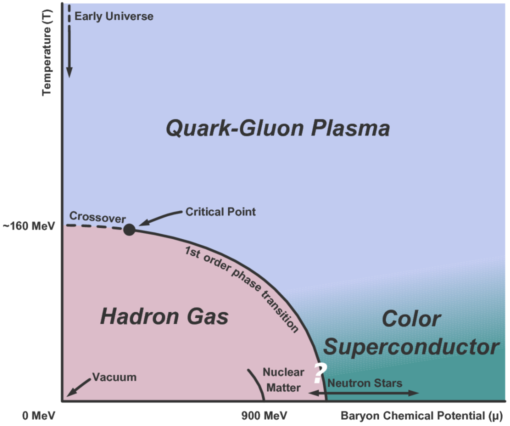
QCD phase diagram



[Kim and Yi: Adv. High Energy Phys. 2011 (2011)]

- **QCD vacuum:** confinement and spontaneous chiral symmetry breaking

  → Hadron degrees of freedom

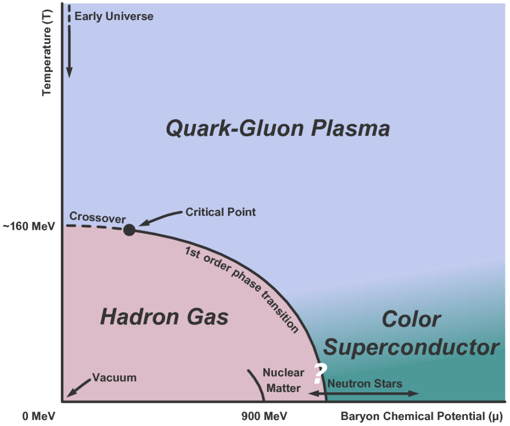
- **Finite temperatures:** lattice QCD and heavy-ion collisions suggest crossover to quark-gluon phase around $T_c \sim 155\,\mathrm{MeV}$
  [Bazavov et al., HotQCD Collaboration, Phys. Rev. D 90 (2014)]

- **Finite densities:** liquid-gas phase transition at $T = 0\,\mathrm{MeV}$ and $\mu_c = 923\,\mathrm{MeV}$

- At very high densities perturbative QCD results imply quark and gluon d.o.f.
  [Fukushima and Hatsuda, Repts. Prog. Phys. 74 (2011)]

  → Transition from nuclear matter to color superconductor still unknown

QCD phase diagram



[Kim and Yi: Adv. High Energy Phys. 2011 (2011)]

- For $\mu \neq 0$ lattice QCD unavailable because of sign problem
- Perturbative approaches such as ChEFT only valid up to $n \lesssim 2\, n_0$
  [Holt, Rho and Weise, Phys. Rept. 621 (2016)]
- Calculations using Nambu & Jona-Lasinio (NJL) and PNJL models suggest chiral first-order phase transition
  [Rößner et al., Nucl. Phys. A 814 (2008)]
- But: thermodynamics strongly influenced by fluctuations not included in mean-field analyses
  [Drews and Weise, Prog.Part.Nucl.Phys. 93 (2017)]
- Measurements of heavy neutron stars $M \sim 2\, M_\odot$ make first-order phase transition unlikely
  [Cromartie et al., Nature Astron. 4 (2019)]

Chiral nucleon-meson model

- Effect of fluctuations on the first-order phase transition in dense baryonic matter?
- $SU(2)_L \times SU(2)_R$ theory of fermion doublet $\Psi = (p, n)$
  [Floerchinger and Wetterich, Nucl. Phys. A 890–891 (2012)]
- Interacting via chiral boson field $\phi = (\sigma, \boldsymbol{\pi})$

$$\mathscr{L} = \bar{\Psi}\left[\gamma_\mu \partial_\mu + g(\sigma + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]\Psi + \frac{1}{2}\left(\partial_\mu\sigma\partial_\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\right) + \mathscr{U}(\sigma, \boldsymbol{\pi}) + \Delta\mathscr{L}$$

- Potential depends on chiral invariant, $\chi \equiv \frac{1}{2}\phi^\dagger\phi = \frac{1}{2}\left(\sigma^2 + \boldsymbol{\pi}^2\right)$ (expanded around v.e.v. χ_0), and explicit symmetry breaking

$$\mathscr{U}(\sigma, \boldsymbol{\pi}) = \sum_{n=1}^{4} \frac{a_n}{n!}\left(\chi - \chi_0\right)^n - m_\pi^2 f_\pi \left(\sigma - f_\pi\right)$$

- Short distance dynamics modeled by massive vector fields v_μ and $\boldsymbol{w}_\mu$
  → Time-independent, homogeneous background fields

$$\Delta\mathscr{L} = -\Psi^\dagger\left[g_v\, v + g_w\, \tau_3\, w\right]\Psi - \frac{1}{2}m_v^2\left(v^2 + w^2\right)$$

Mean-field thermodynamics

- Introduce T and $\mu_{p/n}$ and determine the grand canonical potential in mean-field (MF) approximation

$$\Omega_{MF} = \Omega_F\big(T, \mu_p, \mu_n; \langle\sigma\rangle, v, w\big) + \mathscr{U}\left(\langle\sigma\rangle, \langle\boldsymbol{\pi}\rangle = 0\right) - \frac{1}{2} m_v^2 \left(v^2 + w^2\right)$$

- Assume no pion condensation $\langle\boldsymbol{\pi}\rangle = 0$
- Fermionic part with $E = \sqrt{p^2 + M^2(\sigma)}$ and dynamical nucleon mass $M(\sigma) = g\langle\sigma\rangle$

$$\Omega_F = -2 \sum_{i=p,n} \int \frac{d^3p}{(2\pi)^3} \left[E + \frac{p^2}{3E} \sum_{r=\pm 1} n_F(E - r\bar{\mu}_i) \right]$$

with

$$n_F(E \mp \bar{\mu}_i) = \left[\exp\left(\frac{E \mp \bar{\mu}_i}{T} \right) + 1 \right]^{-1} \qquad \bar{\mu}_{p/n} = \mu_{p/n} - g_v v \mp g_w w$$

- Grand canonical potential at minimum yields thermodynamic observables

$$P = -\Omega_{MF} \qquad s = -\frac{\partial \Omega_{MF}}{\partial T} \qquad n_i = -\frac{\partial \Omega_{MF}}{\partial \mu_i} \qquad \varepsilon = -P + \sum_{i=p,n} \mu_i n_i + Ts$$

Vacuum fluctuations

- Vacuum term in fermionic contribution

$$\delta\Omega_{vac} = -4\int \frac{d^3p}{(2\pi)^3} E$$

→ Neglected in mean-field analyses

- Can be computed via dimensional regularisation
[Skokov et al., Phys. Rev. D 82 (2010)]

$$\delta\Omega_{vac} = \frac{M^4}{8\pi^2}\left(\frac{2}{4-d} + \frac{3}{2} - \gamma_E - \ln\frac{M^2}{4\pi\Lambda^2}\right)$$

- Extended mean-field theory (EMF)

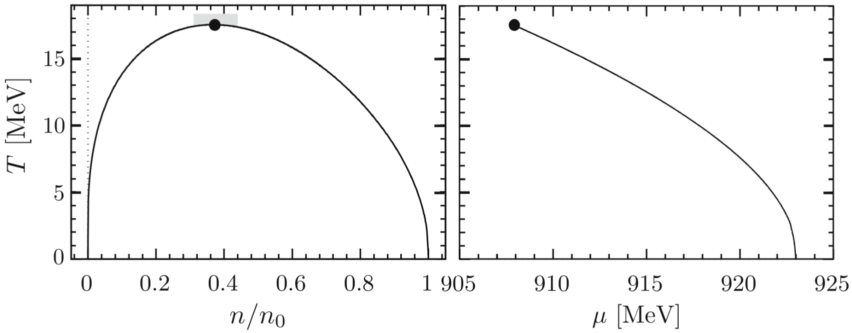
$$\Omega_{EMF} \equiv \Omega_{MF} - \frac{(g\sigma)^4}{4\pi^2}\ln\frac{g\sigma}{\Lambda}$$

- Minimization of Ω_{EMF}/Ω_{MF} leads to coupled differential equations

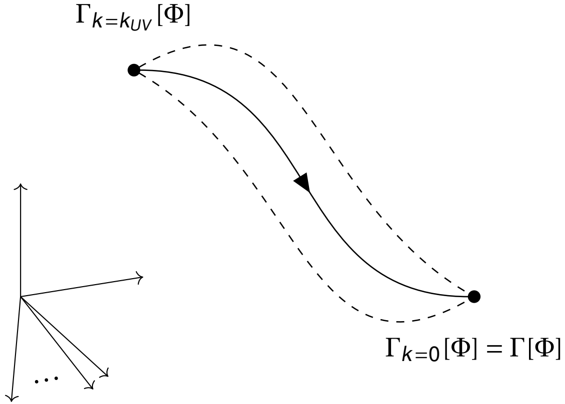
→ Solved at given temperature and chemical potentials

Parameter fixing

- In vacuum:
  - Pressure vanishes at its minimum at $\langle\sigma\rangle_{vac} = f_\pi$
  - $M_N = 939\,\mathrm{MeV}$
- Nuclear phenomenology:
  - $E/A(n_0) = -16\,\mathrm{MeV}$
  - $S(n_0) = 32\,\mathrm{MeV}$
  - Nuclear liquid-gas phase transition with empirical critical parameters [Elliot et al., Phys. Rev. C 87 (2013)]
  - Nuclear surface tension Σ, Landau mass M_L^* and compression modulus K



Functional renormalisation group



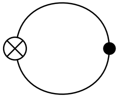
- Other soft degrees of freedom (such as chiral boson and nucleon loops)

  → Use non-perturbative functional renormalization group (FRG) approach

- Initialize effective action $\Gamma_k[\Phi]$ at $k_{UV} \sim 4\pi f_\pi$

- Evolution $k \to 0$ governed by Wetterich's flow equation
  [Wetterich, Phys. Lett. B 301 (1993)]

$$k\frac{\partial \Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(2)}[\Phi] + R_k\right)^{-1}\right] = \frac{1}{2}$$



  → $\Gamma_k[\Phi]$ contains all fluctuations $p^2 \geq k^2$ through regulator $R_k(p)$

- Need truncation to include only relevant operators

Functional renormalisation group

- Chiral nucleon-meson model

  [Drews and Weise: Prog. Part. Nucl. Phys. 93 (2017)]

$$\Gamma_k = \int_0^{1/T} dx_4 \int d^3x \Big\{ \bar{\Psi} \left[\gamma_\mu \partial_\mu + g(\sigma + i\gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \right] \Psi + \Psi^\dagger \left(\boldsymbol{\mu} - g_v v - g_w \tau_3 w \right) \Psi + \frac{1}{2} \left(\partial_\mu \sigma \partial_\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} \right) + \mathcal{U}_k(T, \mu_p, \mu_n; \sigma, \boldsymbol{\pi}, v, w) \Big\}$$

- Effective potential

$$\mathcal{U}_k = \mathcal{U}_k^{(0)}(\chi) - m_\pi^2 f_\pi (\sigma - f_\pi) - \frac{1}{2} m_v^2 (v^2 + w^2)$$

- Chirally symmetric potential

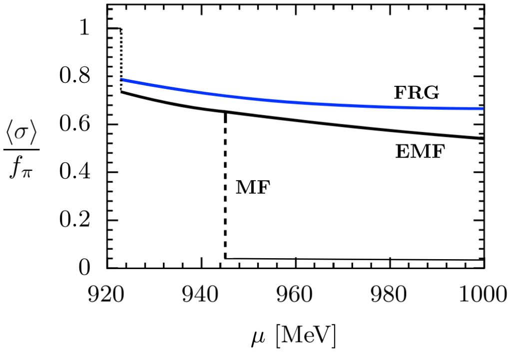
$$\mathcal{U}_k^{(0)}(\chi) = \sum_{n=0}^{4} \frac{a_n(k)}{n!} (\chi - \chi_0)^n$$

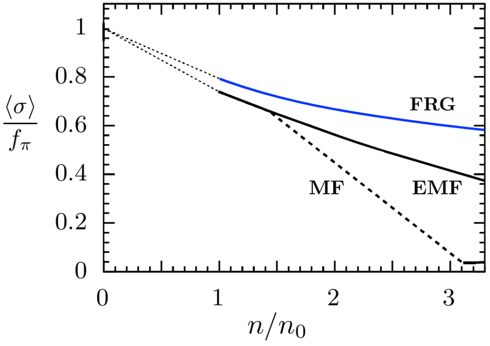
- Flow equation leads to set of coupled differential equations for the coefficients $a_n(k)$
- Determine grand canonical potential

$$\Omega_{FRG}(T, \mu_p, \mu_n) = \mathcal{U}_{k=0}(T, \mu_p, \mu_n; \bar{\sigma}, \bar{v}, \bar{w})$$

Symmetric nuclear matter

- $\langle\sigma\rangle(\mu)$ compared to $\langle\sigma\rangle(\mu=0)=\langle\sigma\rangle_{vac}=f_\pi$ serves as chiral order parameter
- Results for symmetric nuclear matter:
  - **Mean-field:** unphysical first-order phase transition at $n\simeq 1.5\,n_0$
  - **Extended mean-field:** vacuum contribution stabilizes order parameter
  - **FRG:** further stabilization through additional fluctuations





Pure neutron matter

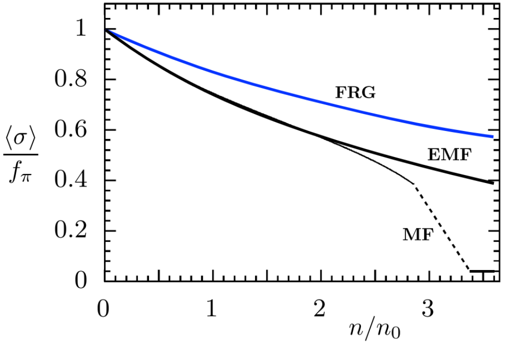
- Similar results for pure neutron matter

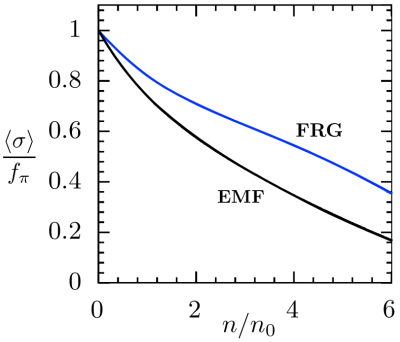
  → Smooth crossover at large densities

- Model adjusted to low-density properties, potential expanded around $\chi_0 = 1/2\, f_\pi^2$

  → For small $\langle\sigma\rangle / f_\pi$ model no longer applicable

- In FRG stays around 40% until $n \sim 6\, n_0$ (central densities in heavy neutron stars!)





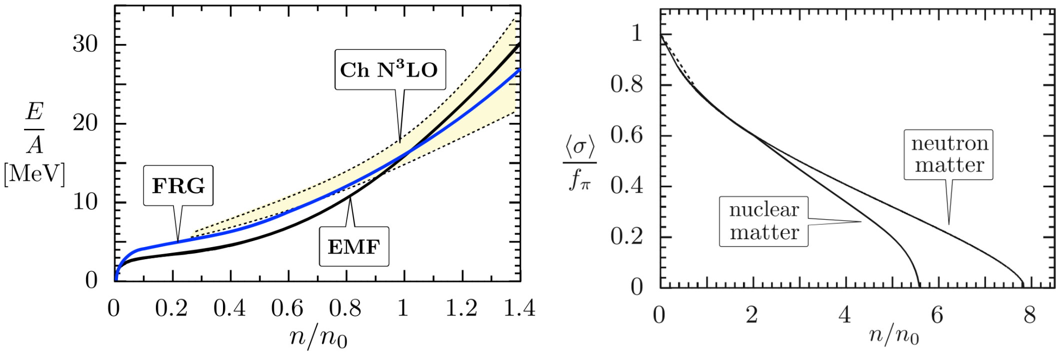
Results

- Good agreement with pure neutron matter E/A from ChEFT calculations
  [Drischler, Hebeler and Schwenk, Phys. Rev. Lett. 122 (2019)]

- In chiral limit $m_\pi \to 0$ crossover turns into second-order phase transition

- Very similar behaviour in chiral quark-meson models (with Polyakov loop)

  → Mean-field chiral restoration avoided by vacuum fluctuations and in FRG
  [Zacchi and Schaffner-Bielich, Phys. Rev. D 97 (2018)] [Gupta and Tiwari, Phys. Rev. D 85 (2012)]



Summary

- Chiral $SU(2)_L \times SU(2)_R$ nucleon-meson model reproduces empirical nuclear properties and liquid-gas phase transition
- **Mean-field**: chiral first-order phase transition at unphysical low densities
- **Extended mean-field**: included vacuum contribution

  → Chiral symmetry remains spontaneously broken up to higher densities
- **Functional renormalisation group**: even stronger stabilization against chiral restoration
- Similar results for chiral quark-meson models

  → Fluctuations convert first-order phase transitions to a crossover

Parameters

- Extended mean-field:

$$m_\sigma = 617.6\,\text{MeV} \qquad G_v = 5.88\,\text{fm}^2 \qquad G_w = 0.97\,\text{fm}^2 \qquad a_3 = 2.16\cdot 10^{-1}\,\text{MeV}^{-2} \qquad a_4 = -5.29\cdot 10^{-5}\,\text{MeV}^{-4}$$

- Functional renormalisation group:

[Drews and Weise, Prog.Part.Nucl.Phys. 93 (2017)]

$$m_\sigma = 770\,\text{MeV} \qquad G_v = 4.04\,\text{fm}^2 \qquad G_w = 1.12\,\text{fm}^2 \qquad a_3 = 5.55\cdot 10^{-3}\text{MeV}^{-2} \qquad a_4 = 8.38\cdot 10^{-5}\,\text{MeV}^{-4}$$

- Evolution $k \to 0$ results in downward shift of sigma mass

$$m_\sigma^{IR} = \sqrt{\mathcal{U}'_{k=0}(\chi_0) + 2\chi_0\,\mathcal{U}''_{k=0}(\chi_0)} \simeq 0.6\,\text{GeV}$$

→ Pole in isoscalar s-wave $\pi\pi$ scattering amplitude $m_\sigma \simeq 0.44\,\text{GeV}$

[Caprini, Colangelo and Leutwyler, Phys. Rev. Lett. 96 (2006)]

- Repulsive short-range nucleon-nucleon interaction G_v larger in EMF

→ Approach does not account for high-momentum fluctuation effects

Phenomenology

- Empirical critical parameters of liquid-gas phase transition: [Elliot et al., Phys. Rev. C 87 (2013)]

$$T_{\text{crit}} = 17.9 \pm 0.4\,\text{MeV} \qquad P_{\text{crit}} = 0.31 \pm 0.07\,\text{MeV/fm}^3 \qquad n_{\text{crit}} = 0.06 \pm 0.01\,\text{fm}^{-3}$$

- EMF:

$$T_{\text{crit}} = 17.5\,\text{MeV} \qquad P_{\text{crit}} = 0.33\,\text{MeV/fm}^3 \qquad n_{\text{crit}} = 0.06\,\text{fm}^{-3} \qquad \mu_{\text{crit}} = 908\,\text{MeV}$$

- Nuclear surface tension:

$$\Sigma = \int_{\langle\sigma\rangle_0}^{f_\pi} d\sigma \, \sqrt{2\Omega_{EMF}(\sigma)} = 1.1\,\text{MeV/fm}^2$$

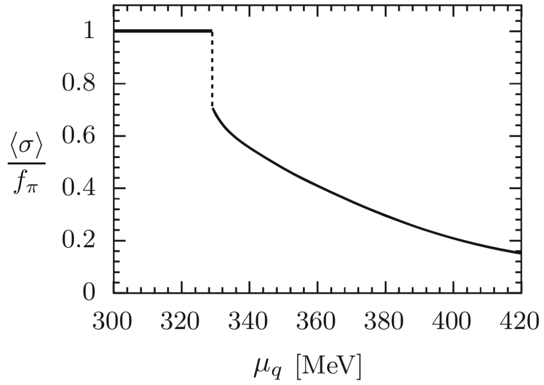
- Landau effective mass:

$$M_L^* = \sqrt{p_F^2 + (g\,\langle\sigma\rangle_0)^2} = \mu_0 - G_v\,n_0 = 0.79\,M_N$$

- Compression modulus:

$$K = 9n\left(\frac{dn}{d\mu}\right)^{-1}\Bigg|_{n=n_0} = 282\,\text{MeV}$$

Chiral quark-meson model



- Isospin $SU(2)$ doublet quark field, $\psi = (u, d)$ replaces the nucleon field Ψ

- Effective potential:

$$\mathcal{U}_k = \mathcal{U}_k^{(0)}(\chi) - c\,\sigma - \frac{1}{2} m_v^2 (v^2 + w^2)$$
$$\mathcal{U}_{k=\Lambda}^{(0)} = m_\Lambda^2\, \chi + \lambda\, \chi^2$$

→ Parameters fixed to reproduce m_π and f_π

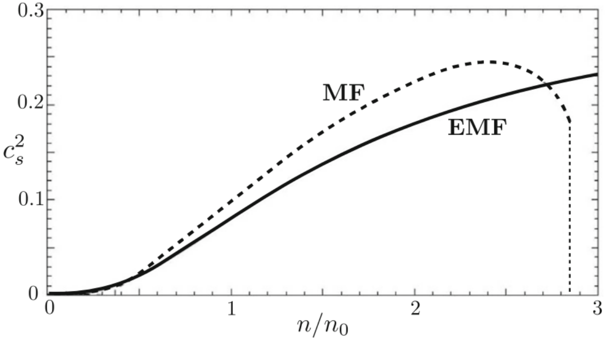
- Liquid-gas transition in matter formed by constituent quarks

- Chiral first-order phase transition converted to smooth crossover by fluctuations

  [Tripolt et al. Phys. Rev. D 97 (2018)]

Neutron stars

- Speed of sound $c_s^2 = \frac{\partial P}{\partial \varepsilon}$



- Extended mean-field supports only $M \lesssim 1.84\, M_\odot$

  → Adding two orders to polynomial expansion of the potential gives $M \lesssim 2.14\, M_\odot$

- FRG can support $M \lesssim 1.97\, M_\odot$ (without additional parameters!)
  [Drews and Weise, Prog. Part. Nucl. Phys. 93 (2017)]