Chiral effective theory in the Higgs sector

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Outline

1.) EW Chiral Symmetry and the SM?

2.) Building the EW eff.th. (= EWχL = HEFT)

a) Particle content, symmetry & counting

b) LO Lagrangian, $O(p^2)$

c) NLO Lagrangian $O(p^4)$

d) 1-loop corrections – $O(p^4)$ renormalization

3.) A glimpse on pheno

4.) Conclusions

1) EW chiral symmetry and the SM

• Let us examine the **SM scalar sector**: $(g = g' = \lambda_{Yuk} = 0)$

$$\mathcal{L}_{SM}^{scalar\ sector} = \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

• Global $SU(2)_L$ **invariance**:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{\varphi_1 - i\varphi_2}{\sqrt{2}} \\ \frac{\varphi_0 + i\varphi_3}{\sqrt{2}} \end{pmatrix} \xrightarrow{SU(2)_L} g_L \Phi$$

$$\mathcal{L}_{SM}^{scalar} \longrightarrow \mathcal{L}_{SM}^{scalar}$$

• An $SU(2)$ peculiarity:

$$\Phi^c = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix} = \begin{pmatrix} \frac{\varphi_0 - i\varphi_3}{\sqrt{2}} \\ \frac{-\varphi_1 + i\varphi_2}{\sqrt{2}} \end{pmatrix}$$

also transforms like a doublet under this same $SU(2)_L$ group

1ST WAY

- However, there is **OTHER $SU(2)$ invariance hidden** in this Lagrangian,

$$\mathcal{L}_{SM}^{\text{scalar sector}} = \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 =$$

$$= \frac{1}{2}(\partial_\mu \varphi_0^2 + \partial_\mu \varphi_1^2 + \partial_\mu \varphi_2^2 + \partial_\mu \varphi_3^2)$$

$$+ \frac{\mu^2}{2}(\varphi_0^2 + \varphi_1^2 + \varphi_2^2 + \varphi_3^2) - \frac{\lambda}{4}(\varphi_0^2 + \varphi_1^2 + \varphi_2^2 + \varphi_3)^2 =$$

$$= \partial_\mu \chi_R^\dagger \partial^\mu \chi_R + \mu^2 \chi_R^\dagger \chi_R - \lambda (\chi_R^\dagger \chi_R)^2$$

, with $\chi_R = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} = \begin{pmatrix} \frac{\varphi^0 + i\varphi^3}{\sqrt{2}} \\ \frac{\varphi^1 - i\varphi^2}{\sqrt{2}} \end{pmatrix}$

NOTICE THIS IS NOT A DOUBLET UNDER $SU(2)_L$!!

such that we have the new symmetry

$$\chi_R \xrightarrow{SU(2)' \equiv SU(2)_R} g_R \chi_R$$

$$\mathcal{L}_{SM}^{\text{scalar}} \longrightarrow \mathcal{L}_{SM}^{\text{scalar}}$$

- The SM has then a wider **global chiral symmetry,**

$$\mathcal{G} = SU(2)_L \times SU(2)_R$$

- **This chiral symmetry is spontenously broken** by the vacuum,

  which only invariant under the diagonal subgroup with $g_L = g_R$

$$\mathcal{H} = SU(2)_{L+R}$$ custodial symmetry / (weak) isospin

↳ $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$ COSET PATTERN

↓

GOLDSTONE'S

CHIRAL PATTERN

2ND WAY

- This can be better understood if we understand the **complex doublet Φ as a real 4-plet $\vec{\varphi}$**:

$$\Phi = \begin{pmatrix} \frac{\varphi_1 - i\varphi_2}{\sqrt{2}} \\ \frac{\varphi_0 + i\varphi_3}{\sqrt{2}} \end{pmatrix} \Longrightarrow \vec{\varphi} = \begin{pmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \xrightarrow{\ g\in \mathcal{G}=SO(4)\ } g\vec{\varphi}$$

$$\mathcal{L}^{\rm Scalar}_{SM} = \frac{1}{2}\partial_\mu\vec{\varphi}^{\,2} + \frac{\mu^2}{2}\vec{\varphi}^{\,2} - \frac{\lambda}{4}\vec{\varphi}^{\,4} \xrightarrow{\ g\in \mathcal{G}=SO(4)\ } \mathcal{L}^{\rm Scalar}_{SM}$$

but with the vacuum, e.g., $\langle\vec{\varphi}\rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ 0 \end{pmatrix}$, being only invariant under $\mathcal{H}=SO(3)$

SO(3) Rot. in these axes

$\hookrightarrow$ $SO(4)/SO(3)$ COSET PATTERN

(with the local isomorphisms $SO(4)\sim SU(2)\times SU(2)$, $SO(3)\sim SU(2)$)

3RD WAY

• It is also useful to express ***Φ* in the bi-fundamental representation**,

$$\mathcal{L}_{SM}^{scalar} = \frac{1}{2}\langle \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \rangle + \frac{\mu^2}{2}\langle \Sigma^\dagger \Sigma \rangle - \frac{\lambda}{4}\langle \Sigma^\dagger \Sigma \rangle^2$$

with $\Sigma = (\Phi^c, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$

$\Sigma^\dagger = \begin{pmatrix} \Phi^{c\dagger} \\ \Phi^\dagger \end{pmatrix} = \begin{pmatrix} \phi^0 & -\phi^+ \\ \phi^- & \phi^{0*} \end{pmatrix}$

where $\langle \Sigma^\dagger \Sigma \rangle = \Phi^{c\dagger}\Phi^c + \Phi^\dagger \Phi = 2\Phi^\dagger \Phi$, etc.

$\Phi^{c\dagger}\Phi^c = \Phi^\dagger \Phi$

with the $SU(2)_L \times SU(2)_R$ symmetry,

$$\Sigma \longrightarrow g_L \Sigma g_R^\dagger$$

$$\mathcal{L}_{SM}^{scalar} \longrightarrow \mathcal{L}_{SM}^{scalar}$$

- It is convenient to better understand the dynamics:

  ➔ separate module (Higgs boson + vev, $h + v$)

  ➔ and "phase" (Goldstone matrix, $U(\omega)$)

$$\Sigma = \frac{(v+h)}{\sqrt{2}}\, U(\omega)$$

MODULE — UNITARY MATRIX

$$\mathcal{L}_{SM}^{scalar} = \frac{(v+h)^2}{4}\langle \partial_\mu U^\dagger \partial^\mu U\rangle + \frac{1}{2}(\partial_\mu h)^2 - V_{SM}(h)$$

with transformation properties:

$$h \longrightarrow h$$
$$U \longrightarrow g_L U g_R^\dagger$$
$$\mathcal{L}_{SM}^{scalar} \longrightarrow \mathcal{L}_{SM}^{scalar}$$

with the SM decomposition of the Higgs multiplet Φ:

$$4 = 1 + 3$$

(SM renormalizability ⇓ $h + U(\omega)$ COMBINED in a 4-plet)

BUT, if you give up renormalizability $\{h, U(\omega)\}$ CAN BE SEPARATE MULTIPLETS

• This is the essential idea behind the EW effective theory (= Higgs EFT = EW Chiral Lagr.)

• Additional details to complete the SM:

➔ Gauge the scalar sector:

but only the $SU(2)_L \times U(1)_Y$ gauge fields are physical;

the remaining ones in $\mathcal{G}$ are auxiliary spurionic fields to keep chiral invariance

➡ **explicit chiral breaking**

➔ Add SM fermion fields:

use auxiliary spurionic Yukawa fields to keep chiral invariance

➡ **explicit chiral breaking**

2.) Building the EW effective theory

aka EWχL

aka HEFT

2.a) content, symmetry, counting

In order to construct your EFT, please specify:

(i) <u>What is your particle content?</u>

- What are the soft modes / light dof in $\mathcal{L}_{EFT}$?

(i) <u>What is your symmetry?</u>

- What symmetry invariance you use to construct the EFT operators in $\mathcal{L}_{EFT}$?

(i) <u>Chiral power counting?</u>

- Classify the infinite series of EFT operators in $\mathcal{L}_{EFT}$:

1^{st} most important ops. (LO); 2^{nd} most important ops. (NLO); etc.

(i) SM content:

- Bosons χ: Higgs h + gauge bosons W^a_μ, B_μ *(and QCD)*
  + EW Golsdtones $\omega^\pm$,z [non-linearly realized via $U(\omega^a)$]
- Fermions ψ: (t,b)-type doublets

(ii) Symmetries:

- **SM symmetry:** **Gauge sym. group** $G_{SM} = SU(2)_L \times U(1)_Y$ *(and QCD)*
  Spont. Breaking (EWSB) $G_{SM} \rightarrow H_{SM} = U(1)_{EM}$
- **Symmetry of the SM scalar sector:**
  Global CHIRAL sym. $G = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \supset G_{SM}$
  Sp.S.Breaking to Cust.sym. $G \rightarrow H = SU(2)_{L+R} \times U(1)_{B-L} \supset H_{SM}$
  Explicit Breaking: L⇔R asymmetry of the gauge sector $(g,g'\neq 0)$
  t⇔b splitting $(\lambda_t \neq \lambda_b)$

(iii) Chiral power counting:

[boson]	⇔	order 0	($\sim p^0$)
$[\, g W^\mu \,] = [\, g' B^\mu \,] = [\, d_\mu \,] = [\, g \,] = [\, \lambda_\psi \,] = [m_{\chi,\psi}] =$ ~~$[\,\psi\psi\,]$~~	⇔	order 1	($\sim p^1$)
weak SM fermion coupling $[\,\psi\psi\,]$	⇔	order 2	($\sim p^2$)

* See, e.g., rev: HXSWG Yellow Report (non-linear EFT Sec.), arXiv:1610.07922 [hep-ph]
* Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012; Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

- **Chiral expansión in powers of p^d for the EFT Lagrangian:**

$$\mathcal{L}_{\rm EWET} = \sum_{\hat{d}\geq 2} \mathcal{L}_{\rm EWET}^{(\hat{d})}$$

with the same usual chiral counting rule for observables:

$$D = 2L + 2 + \sum_d N_d\,(d-2)$$

O(p^4) Lagrangian:
(x) Buchalla, Cata, JHEP 1207 (2012) 101; Buchalla,Catà,Krause, NPB 880 (2014) 552-573
(x) Alonso,Gavela,Merlo,Rigolin,Yepes, PLB 722 (2013) 330-335; Brivio et al, JHEP 1403 (2014) 024
(x) Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012;
(x) Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

Basic Works:
(*) Apelquist,Bernard ’80; Longhitano ‘80, ’81
(*) Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937
(*) Grinstein,Trott, PRD 76 (2007) 073002

Counting:
* Weinberg ‘79
* Manohar,Georgi, NPB234 (1984) 189
* Georgi,Manohar NPB234 (1984) 189
* Hirn,Stern ’05
* Pich,Rosell,Santos,SC JHEP 1704 (2017) 012
* Buchalla,Catá,Krause PLB 731 (2014) 80-86

- EFT Lagrangian:

$$\mathcal{L}_{ECLh} = \mathcal{L}_{p^2} + \mathcal{L}_{p^4} + \ldots$$

$$\mathcal{L}^{SM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \not{D} \psi + h.c. + \bar{\Psi}_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

SM < LO Lagrangian

Examples of BSM terms:

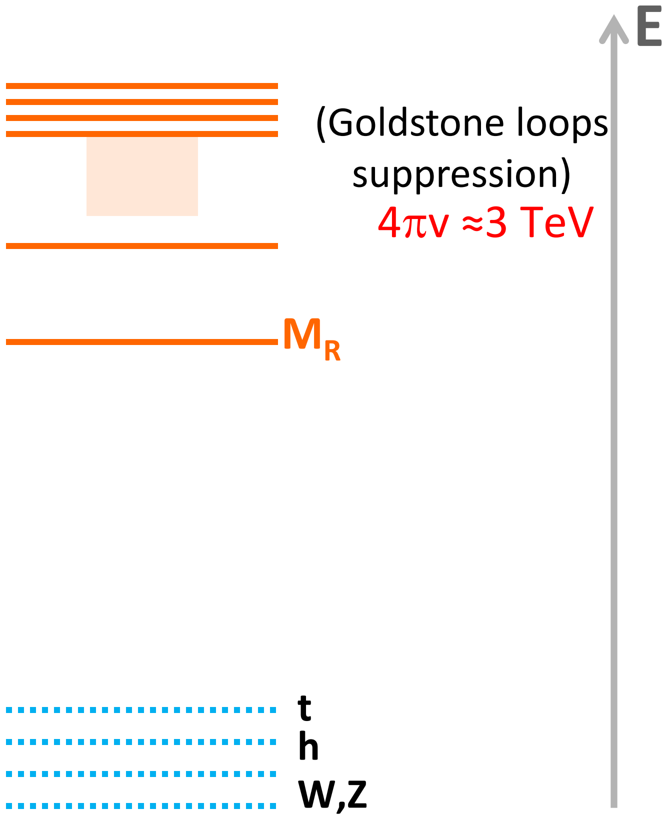
$$\mathcal{L}_{p^2}^{\rm BSM} = \frac{(a-1)\,h}{2v} \mathrm{Tr}\{D_\mu U^\dagger D^\mu U\} + \ldots$$

$$\mathcal{L}_{p^4}^{\rm BSM} = \frac{i}{2}(a_2 - a_3)\,\mathrm{Tr}\{f_+^{\mu\nu}[u_\mu, u_\nu]\} + \mathcal{F}_{X\psi\psi}\,\mathrm{Tr}\{f_{+\,\mu\nu} d^\mu J_V^\nu\} + \ldots$$

which leads to a chiral exp. in the sctatering (and other amplitudes)

$$T(2\to 2) = \frac{p^2}{v^2} + \underbrace{\frac{a_{(4)} p^4}{v^4}}_{tree-NLO} + \underbrace{\frac{p^4}{16\pi^2 v^4}}_{1\ell oop-NLO} + \ldots$$

- **Naive expectation of the EW effective theory scales?**



Naïve rescaling
from QCD to EW scale

QCD		**EW**
$F_\pi=0.090$ GeV	→	$v=0.246$ TeV
$\Lambda_{\chi PT}=4\pi F_\pi \approx 1.2$ GeV	→	$\Lambda_{EW}=4\pi v \approx 3.1$ TeV
$M_\rho=0.770$ GeV	→	$M_{V1}=2.1$ TeV
$M_{a1}=1.260$ GeV	→	$M_{A1}=3.4$ TeV

(WARNING:
This is just a naive expectation
EXPERIMENT has the last word!!!)

2.b) LO Lagrangian

$u(\varphi) = \exp\{i\vec{\sigma}\,\vec{\varphi}/(2v)\}$
$U(\varphi) \equiv u(\varphi)^2$

• O(p²), LO (⊃ SM):

$$\mathcal{L}_{\rm EWET}^{(2)} = \sum_{\xi} \left[i\,\bar{\xi}\gamma^\mu d_\mu \xi - v\,\left(\,\bar{\xi}_L \mathcal{Y} \xi_R + \text{h.c.}\right)\right]$$
$$- \frac{1}{2g^2}\langle \hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\rangle_2 - \frac{1}{2g'^2}\langle \hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\rangle_2 - \frac{1}{2g_s^2}\langle \hat{G}_{\mu\nu}\hat{G}^{\mu\nu}\rangle_3$$
$$+ \frac{1}{2}\partial_\mu h\,\partial^\mu h - \frac{1}{2}m_h^2 h^2 - V(h/v) + \frac{v^2}{4}\mathcal{F}_u(h/v)\,\langle u_\mu u^\mu\rangle_2$$

$\mathcal{Y}(h)$ YUKAWAS

$V(h)$ HIGGS POTENTIAL

$\mathcal{F}_u(h)$ HIGGS-GAUGE INTERACTION/DECAYS

with $\mathcal{F}_{\rm u} = 1 + \frac{2{\rm ah}}{\rm v} + \frac{{\rm bh}^2}{{\rm v}^2} + \mathcal{O}({\rm h}^3)$ *, being* $\rm a_{SM} = b_{SM} = 1$

Goldstone dynamics:

(*) Weinberg ’79; Apelquist,Bernard ’80;
Longhitano ‘80, ’81

Inclusion of the Higgs as a singlet INDEPENDENT field:

(*) Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937

(*) Grinstein,Trott, PRD 76 (2007) 073002

For notation:

* See, e.g., rev: HXSWG Yellow Report (non-linear EFT Sec.), arXiv:1610.07922 [hep-ph]
* Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012
* Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

2.c) NLO Lagrangian

• **O(p⁴), NLO** (pure BSM):

(Here only CP-even)

$u(\varphi) = \exp\{i\vec{\sigma}\,\vec{\varphi}/(2v)\}$
$U(\varphi) \equiv u(\varphi)^2$

$$\mathcal{L}_{\rm EWET}^{(4)} = \sum_{i=1}^{12} \mathcal{F}_i(h/v)\, \mathcal{O}_i + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i(h/v)\, \widetilde{\mathcal{O}}_i$$

$$+ \sum_{i=1}^{8} \mathcal{F}_i^{\psi^2}(h/v)\, \mathcal{O}_i^{\psi^2} + \sum_{i=1}^{3} \widetilde{\mathcal{F}}_i^{\psi^2}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^2}$$

$$+ \sum_{i=1}^{10} \mathcal{F}_i^{\psi^4}(h/v)\, \mathcal{O}_i^{\psi^4} + \sum_{i=1}^{2} \widetilde{\mathcal{F}}_i^{\psi^4}(h/v)\, \widetilde{\mathcal{O}}_i^{\psi^4}$$

(x) Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012; Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

• List of CP even operators : *[Caveat: no flavour]*

P-even

i	$\mathcal{O}_i$	$\mathcal{O}_i^{\psi^2}$	$\mathcal{O}_i^{\psi^4}$
1	$\frac{1}{4}\langle f_+^{\mu\nu}f_{+\,\mu\nu} - f_-^{\mu\nu}f_{-\,\mu\nu}\rangle_2$	$\langle J_S\rangle_2\langle u_\mu u^\mu\rangle_2$	$\langle J_S J_S\rangle_2$
2	$\frac{1}{2}\langle f_+^{\mu\nu}f_{+\,\mu\nu} + f_-^{\mu\nu}f_{-\,\mu\nu}\rangle_2$	$i\langle J_T^{\mu\nu}[u_\mu,u_\nu]\rangle_2$	$\langle J_P J_P\rangle_2$
3	$\frac{i}{2}\langle f_+^{\mu\nu}[u_\mu,u_\nu]\rangle_2$	$\langle J_T^{\mu\nu}f_{+\,\mu\nu}\rangle_2$	$\langle J_S\rangle_2\langle J_S\rangle_2$
4	$\langle u_\mu u_\nu\rangle_2\langle u^\mu u^\nu\rangle_2$	$\hat{X}_{\mu\nu}\langle J_T^{\mu\nu}\rangle_2$	$\langle J_P\rangle_2\langle J_P\rangle_2$
5	$\langle u_\mu u^\mu\rangle_2\langle u_\nu u^\nu\rangle_2$	$\frac{\partial_\mu h}{v}\langle u^\mu J_P\rangle_2$	$\langle J_V^\mu J_{V,\mu}\rangle_2$
6	$\frac{(\partial_\mu h)(\partial^\mu h)}{v^2}\langle u_\nu u^\nu\rangle_2$	$\langle J_A^\mu\rangle_2\langle u_\mu \mathcal{T}\rangle_2$	$\langle J_A^\mu J_{A,\mu}\rangle_2$
7	$\frac{(\partial_\mu h)(\partial_\nu h)}{v^2}\langle u^\mu u^\nu\rangle_2$	$\frac{(\partial_\mu h)(\partial^\mu h)}{v^2}\langle J_S\rangle_2$	$\langle J_V^\mu\rangle_2\langle J_{V,\mu}\rangle_2$
8	$\frac{(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)}{v^4}$	$\langle \hat{G}_{\mu\nu}J_T^{8\,\mu\nu}\rangle_{2,3}$	$\langle J_A^\mu\rangle_2\langle J_{A,\mu}\rangle_2$
9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu}u_\nu\rangle_2$	—	$\langle J_T^{\mu\nu}J_{T\,\mu\nu}\rangle_2$
10	$\langle \mathcal{T}u_\mu\rangle_2\langle \mathcal{T}u^\mu\rangle_2$	—	$\langle J_T^{\mu\nu}\rangle_2\langle J_{T\,\mu\nu}\rangle_2$
11	$\hat{X}_{\mu\nu}\hat{X}^{\mu\nu}$	—	—
12	$\langle \hat{G}_{\mu\nu}\hat{G}^{\mu\nu}\rangle_3$	—	—

P-odd

i	$\widetilde{\mathcal{O}}_i$	$\widetilde{\mathcal{O}}_i^{\psi^2}$	$\widetilde{\mathcal{O}}_i^{\psi^4}$
1	$\frac{i}{2}\langle f_-^{\mu\nu}[u_\mu,u_\nu]\rangle_2$	$\langle J_T^{\mu\nu}f_{-\,\mu\nu}\rangle_2$	$\langle J_V^\mu J_{A,\mu}\rangle_2$
2	$\langle f_+^{\mu\nu}f_{-\,\mu\nu}\rangle_2$	$\frac{\partial_\mu h}{v}\langle u_\nu J_T^{\mu\nu}\rangle_2$	$\langle J_V^\mu\rangle_2\langle J_{A,\mu}\rangle_2$
3	$\frac{(\partial_\mu h)}{v}\langle f_+^{\mu\nu}u_\nu\rangle_2$	$\langle J_V^\mu\rangle_2\langle u_\mu \mathcal{T}\rangle_2$	—

(x) Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012; Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

• A history recollection on the $\mathcal{L}_{p^4}$ construction (1):

Gauged EW Goldstones

(*) Gauged EW Goldstones → Longhitano PRD 22 (1980) 1166;

NPB 188 (1981) 118

(*) Not to be forgotten → development of ChPT in QCD:

Weinberg Physica A 96 (1979) 1-2, 327-340

Gasser,Leutwyler Annals Phys. 158 (1984) 142;

NPB 250 (1985) 465

Higgs as a singlet: $\mathcal{F}(h)$ FUNCTIONS

(*) Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937

(*) Grinstein,Trott, PRD 76 (2007) 073002

• A history recollection on the $\mathcal{L}_{p^4}$ construction (2):

Full HIGSS-LESS $\mathcal{O}(p^4)$
(Goldstones + gauge + fermions)

(x) Alonso,Gavela,Merlo,Rigolin,Yepes, PLB 722 (2013) 330-335
(x) Buchalla,Cata JHEP 07 (2012) 101

Full HIGGS-FULL $\mathcal{O}(p^4)$
(h + Goldstones + gauge + fermions)

(x) Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937
(x) Grinstein,Trott, PRD 76 (2007) 073002
(x) Buchalla,Catà,Krause NPB 880 (2014) 552
(x) Brivio,Corbett,Éboli,Gavela,Gonzalez-Fraile, Gonzalez-Garcia,Merlo,Rigolin, JHEP 03 (2014) 024

Full HIGGS-FULL $\mathcal{O}(p^4)$
+
SUPRESSION of CUSTODIAL Sym. Breaking

(x) Pich,Rosell,Santos,SC, JHEP 1704 (2017) 012
(x) Krause,Pich,Rosell,Santos,SC, JHEP 1905 (2019) 092

2.d) 1 loop corrections

&

1-loop renormalization of $\mathcal{L}^{p^4}$

- **Expansion** in non-linear EFT's: *

Finite pieces from loops (amplitude dependent) (+)

$$\mathcal{M}(2\to 2) \approx \frac{p^2}{v^2} + \left(\frac{\mathcal{F}_k(\mu)\, p^4}{v^2} - \frac{\Gamma_k\, p^4}{16\pi^2 v^2} \ln\frac{p^2}{\mu^2} + \ldots \right) + \mathcal{O}(p^6)$$

LO (tree)

NLO (tree)

suppression
$\sim 1/M^2$ + ...
(heavier states)

** Catà, EPJC74 (2014) 8, 2991
** Pich,Rosell,Santos,SC, [1501.07249]; 'forthcoming FTUAM-15-20
** Pich,Rosell and SC, JHEP 1208 (2012) 106;
PRL 110 (2013) 181801

NLO (1-loop)

Typical loop suppression
$\sim \Gamma_k/(16\pi^2 v^2)$
(non-linearity)

100% determined
by $\mathcal{L}_2$
[Guo,Ruiz-Femenia,SC,
PRD92 (2015) 074005]

*** Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342
*** Alonso,Kanshin,Saa, PRD 97 (2018) no.3, 035010
*** Buchalla,Cata,Celis,Knecht,Krause, NPB 928 (2018) 93-106

- Indeed, the SM has this arrangement but with $\frac{p^2}{16\pi^2 v^2} \sim \frac{g^{(\prime)\,2}}{(4\pi)^2}, \frac{\lambda}{(4\pi)^2}, \frac{\lambda_f^2}{(4\pi)^2} \ll 1$; **hence**

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (1):

Higgs-less 1-loop RENORMALIZATION

(*) Herrero,Ruiz Morales, NPB 418 (1994) 431-455

Higgs-full: 1-LOOP CALCULATIONS of PARTICULAR OBSERVABLES

A small sample of 1-loop HEFT observable computations:

(x) Delgado,Dobado,Llanes-Estrada, PRL114 (2015) 22, 221803
(x) Espriu,Mescia,Yencho, PRD88 (2013) 055002
(x) Delgado,Garcia-Garcia,Herrero, JHEP 11 (2019) 065
(x) Fabbrichesi,Pinamonti(,Tonero,Urbano, PRD93 (2016) 1, 015004
(x) Corbett,Éboli,Gonzalez-Garcia, PRD 93 (2016) 1, 015005
(x) de Blas,Eberhardt,Krause, JHEP 07 (2018) 048
(x) Quezada,Dobado,SC, PoS ICHEP2020 (2021) 076; in preparation

• A history recollection on the $\mathcal{L}^{p^4}$ renormalization (2):

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar loops
& true $\mathcal{O}(D^4)$ divergences

(*) Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

$\mathcal{L} = \dots + \frac{v^2}{4}\mathcal{F}_C(h)\langle D_\mu U^\dagger D^\mu U\rangle$

c_k	Operator $\mathcal{O}_k$	Γ_k	$\Gamma_{k,0}$
c_1	$\frac{1}{4}\langle f_+^{\mu\nu}f_{+\mu\nu} - f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
(c_2-c_3)	$\frac{i}{2}\langle f_+^{\mu\nu}[u_\mu,u_\nu]\rangle$	$\frac{1}{24}(\mathcal{K}^2-4)$	$-\frac{1}{6}(1-a^2)$
c_4	$\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$	$\frac{1}{96}(\mathcal{K}^2-4)^2$	$\frac{1}{6}(1-a^2)^2$
c_5	$\langle u_\mu u^\mu\rangle^2$	$\frac{1}{192}(\mathcal{K}^2-4)^2+\frac{1}{128}\mathcal{F}_C^2\Omega^2$	$\frac{1}{8}(a^2-b)^2+\frac{1}{12}(1-a^2)^2$
c_6	$\frac{1}{v^2}(\partial_\mu h)(\partial^\mu h)\langle u_\nu u^\nu\rangle$	$\frac{1}{16}\Omega(\mathcal{K}^2-4)-\frac{1}{96}\mathcal{F}_C\Omega^2$	$-\frac{1}{6}(a^2-b)(7a^2-b-6)$
c_7	$\frac{1}{v^2}(\partial_\mu h)(\partial_\nu h)\langle u^\mu u^\nu\rangle$	$\frac{1}{24}\mathcal{F}_C\Omega^2$	$\frac{2}{3}(a^2-b)^2$
c_8	$\frac{1}{v^4}(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	$\frac{3}{32}\Omega^2$	$\frac{3}{2}(a^2-b)^2$
c_9	$\frac{(\partial_\mu h)}{v}\langle f_-^{\mu\nu}u_\nu\rangle$	$\frac{1}{24}\mathcal{F}'_C\Omega$	$-\frac{1}{3}a(a^2-b)$
c_{10}	$\frac{1}{2}\langle f_+^{\mu\nu}f_{+\mu\nu} + f_-^{\mu\nu}f_{-\mu\nu}\rangle$	$-\frac{1}{48}(\mathcal{K}^2+4)$	$-\frac{1}{12}(1+a^2)$

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar loops
& GEOMETRIC APPROACH

A deeper understanding through geometry:
(x) Alonso,Jenkins,Manohar, PLB 754 (2016) 335-342;
PLB 756 (2016) 358-364; JHEP 08 (2016) 101

- Beautiful geometric connection to this result *
  provided by the curvature (x) of the scalar manifold metric $g_{ij}(\phi) = \begin{bmatrix} F(h)^2 g_{ab}(\varphi) & 0 \\ 0 & 1 \end{bmatrix}$, with $\mathcal{L} = \frac{1}{2} g_{ij} D_\mu \phi^i D^\mu \phi^j$

$$\mathcal{R}_4 = \left(1 - v^2 (F')^2\right) F^2 \quad = \quad \left(1 - \mathcal{K}^2/4\right) \mathcal{F}_C \,,$$

$$\mathcal{R}_2 = \left(1 - v^2 (F')^2\right) - \frac{v^2 F'' F}{(N_\varphi - 1)} \quad = \quad \left(1 - \mathcal{K}^2/4\right) - \frac{\mathcal{F}_C \Omega}{8} \,,$$

$$\mathcal{R}_0 = 2\mathcal{F}_C^{-1} \mathcal{R}_2 - \mathcal{F}_C^{-2} \mathcal{R}_4 \,,$$

$$F = \mathcal{F}_C^{1/2} \qquad N_\varphi = 3$$

with Λ^{-2} = the Riemann $\mathbb{R}_{ijmn} \propto \mathscr{R}_{0,2,4} / v^2$ *(loosely speaking, the curvature $\mathbb{R}$)*

- NDA gives you the suppression of individual diagrams ~ $1 / (4\pi v)^2$
  but the <u>full</u> loop suppression is **~ $g^2 \mathbb{R} / (4\pi)^2$ & ~ $\mathbb{R}^2 / (4\pi)^2$**

  EFT as an expansion $\mathcal{M} \sim R\, \mathbf{p}^2 + \frac{R^2\, \mathbf{p}^4}{(4\pi)^2} + \frac{R^3\, \mathbf{p}^6}{(4\pi)^4} + \ldots$ in the curvature?

- **SM:** $\mathbb{R}_{ijmn} = 0$ ➔ No $O(p^4)$ renormalization

* Guo,Ruiz-Femenia,SC, PRD92 (2015) 074005

(x) Alonso,Jenkins,Manohar, PLB754 (2016) 335; PLB756 (2016) 358; JHEP 1608 (2016) 101

• A history recollection on the $\mathcal{L}_{p^4}$ renormalization (3):

$\mathcal{O}(p^4)$ HEFT renormalization:
scalar + gauge + fermion loops
(FULL)

(*) Buchalla,Cata,Celis,Knecht,Krause, NPB 928 (2018) 93-106

(*) Alonso,Kanshin,Saa, PRD 97 (2018) 3, 035010

(*) Buchalla,Catà,Celis,Knecht,Krause, PRD 104 (2021) 7, 076005

Low-energy EFT (SM + …): representations

- Higgs field representation: SMEFT vs HEFT, a matter of taste? (+)

1) Linear* (SMEFT): in terms of a doublet $\phi = (1+h/v)\, U(\omega^a) \langle\phi\rangle$

$$\mathcal{L}^{L}_{EFT} = (D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \ldots$$
$$= \frac{(v+h)^2}{4}\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(1+P(h))\,(\partial_\mu h)^2 + \ldots$$

$$\frac{dh^{NL}}{dh^{L}} = \sqrt{1+P(h^L)}$$

$$h^{NL} = \int_0^{h^L}\sqrt{1+P(h)}\,dh$$

???

$$\frac{v^2}{2}\mathcal{F}_C(h^{NL}) = \frac{(v+h^L)^2}{2} = \phi^\dagger\phi$$

if there exists an $SU(2)_L \times SU(2)_R$ fixed point $\mathcal{F}_C(h^*)=0$ (x)

$$\mathcal{L}^{NL}_{EFT} = \frac{v^2}{4}\mathcal{F}_c(h)\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(\partial_\mu h)^2 + \ldots$$

$$\mathcal{F}_C(h) = 1 + \frac{2ah}{v} + \frac{bh^2}{v^2} + \mathcal{O}(h^3)$$

2) Non-linear* (HEFT or EWχL): in terms of 1 singlet h + 3 NGB in $U(\omega^a)$

(+) SC, arXiv:1710.07611 [hep-ph]; PoS EPS-HEP2017 (2017) 460
* Jenkins,Manohar,Trott, JHEP 1310 (2013) 087
* LHCHXSWG Yellow Report [1610.07922]

(x) Transformations:
Giudice,Grojean,Pomarol,Rattazzi, JHEP 0706 (2007) 045
Alonso,Jenkins,Manohar, JHEP 1608 (2016) 101

• It is not a question about how you write it: (+)

- **SMEFT → HEFT :***

$$\mathcal{L}^{L}_{EFT} = (D_\mu\phi)^\dagger D_\mu\phi - \frac{1}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \ldots$$
$$= \frac{(v+h)^2}{4}\langle (D_\mu U)^\dagger D_\mu U\rangle + \frac{1}{2}(1+P(h))(\partial_\mu h)^2 + \ldots$$

$$\mathcal{F}_C(h) = 1 + \frac{2ah}{v} + \frac{bh^2}{v^2} + \mathcal{O}(h^3)$$

(if no ~~Custodial~~) $a^2 = 1+\Delta(a^2) = 1-\frac{2v^2}{\Lambda^2}+\ldots,\quad b = 1+\Delta b = 1-\frac{4v^2}{\Lambda^2}+\ldots \quad \Rightarrow \quad 2\Delta(a^2) = \Delta b$

(D≥8 operators: corrections v^4/Λ^4, v^6/Λ^6...)

- **Non-linear scenarios:** e.g., dilaton models (x) ⟶ $\Delta(a^2) = \Delta b$

if you want to write it in the SMEFT form, large "…" needed (D ≥ 8 operators!!) → SMEFT exp. breakdown

(+) SC, PoS EPS-HEP2017 (2017) 460
(+) Agrawal,Saha,Xu,Yu,Yuan,PRD 101 (2020) 7, 075023

* Jenkins,Manohar,Trott, [1308.2627]
* LHCHXSWG Yellow Report [1610.07922]
* Buchalla,Catà,Celis,Krause,NPB917 (2017) 209-233

(x) Goldberger,Grinstein,Skiba, PRL100 (2008) 111802

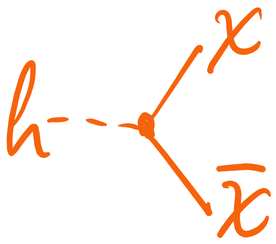
SMEFT vs HEFT?
(x) Cohen,Craig,Lu,Sutherland, JHEP 03 (2021) 237
(x) Falkowski,Rattazzi, JHEP 10 (2019) 255
(x) Agrawal,Saha,Xu,Yu,Yuan,PRD 101 (2020) 7, 075023
(x) Alonso,West, arXiv:2109.13290 [hep-ph]
(x) Salas,Gómez-Ambrosio,Llanes-Estrada,SC, in preparation

3) A glimpse on phenomenology

Summary of current $h \to \chi\bar{\chi}$ couplings

$h \to \chi\bar{\chi}$ coupling, mofifications wrt the SM

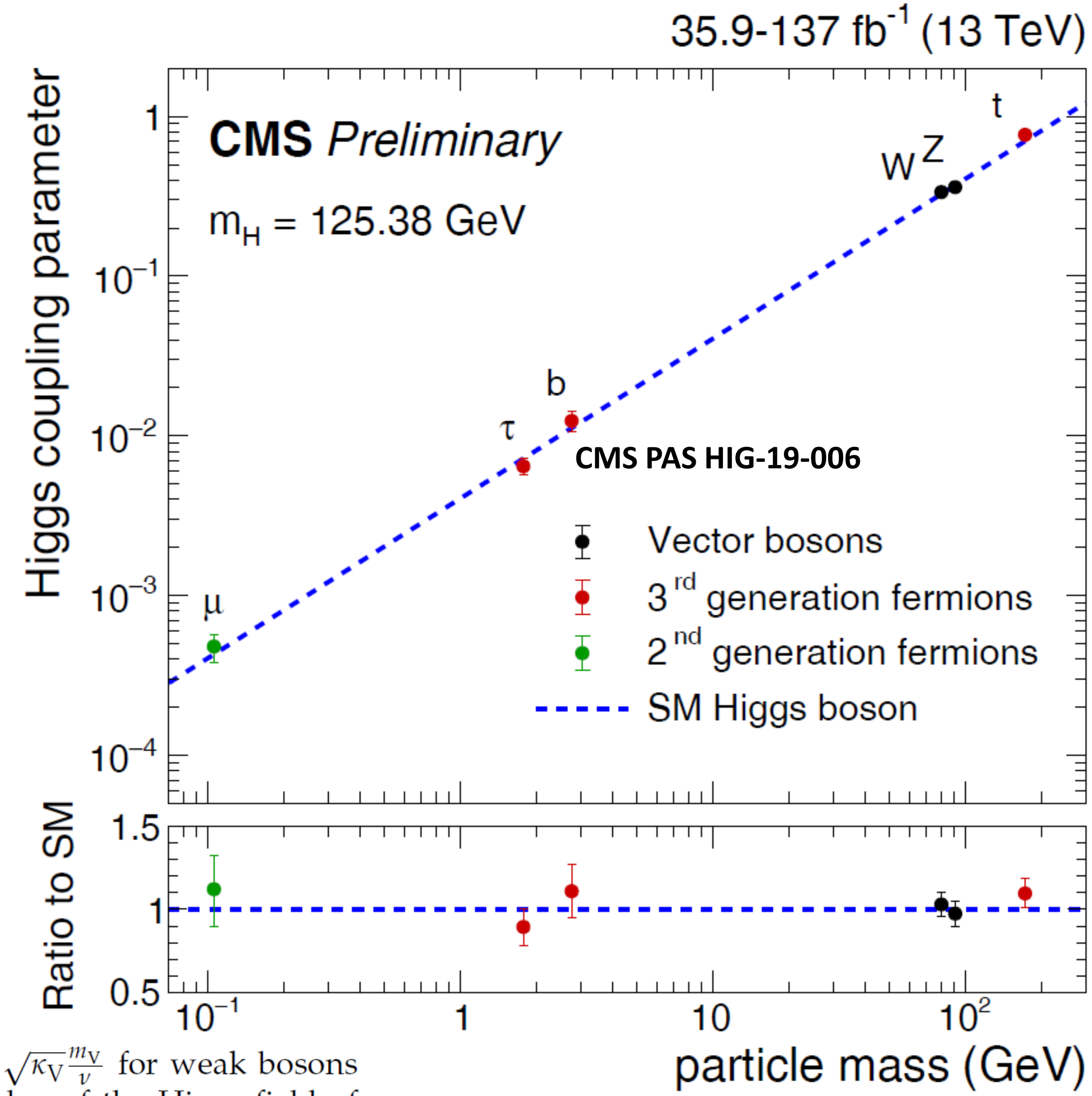
$$\kappa_\chi \equiv \frac{g_{h\to\chi\bar{\chi}}}{(g_{h\to\chi\bar{\chi}})_{SM}}$$



The corresponding CL intervals for κ_μ are:

$0.91 < \kappa_\mu < 1.34$ @68%CL

$0.65 < \kappa_\mu < 1.53$ @95%CL



in terms of reduced coupling strength modifiers, defined as $y_V = \sqrt{\kappa_V}\frac{m_V}{v}$ for weak bosons and $y_F = \kappa_F\frac{m_F}{v}$ for fermions, where v is the vacuum expectation value of the Higgs field of 246.22 GeV. Figure 14 (right) shows the best-fit estimates for the six reduced coupling strength

• HEFT calculation:

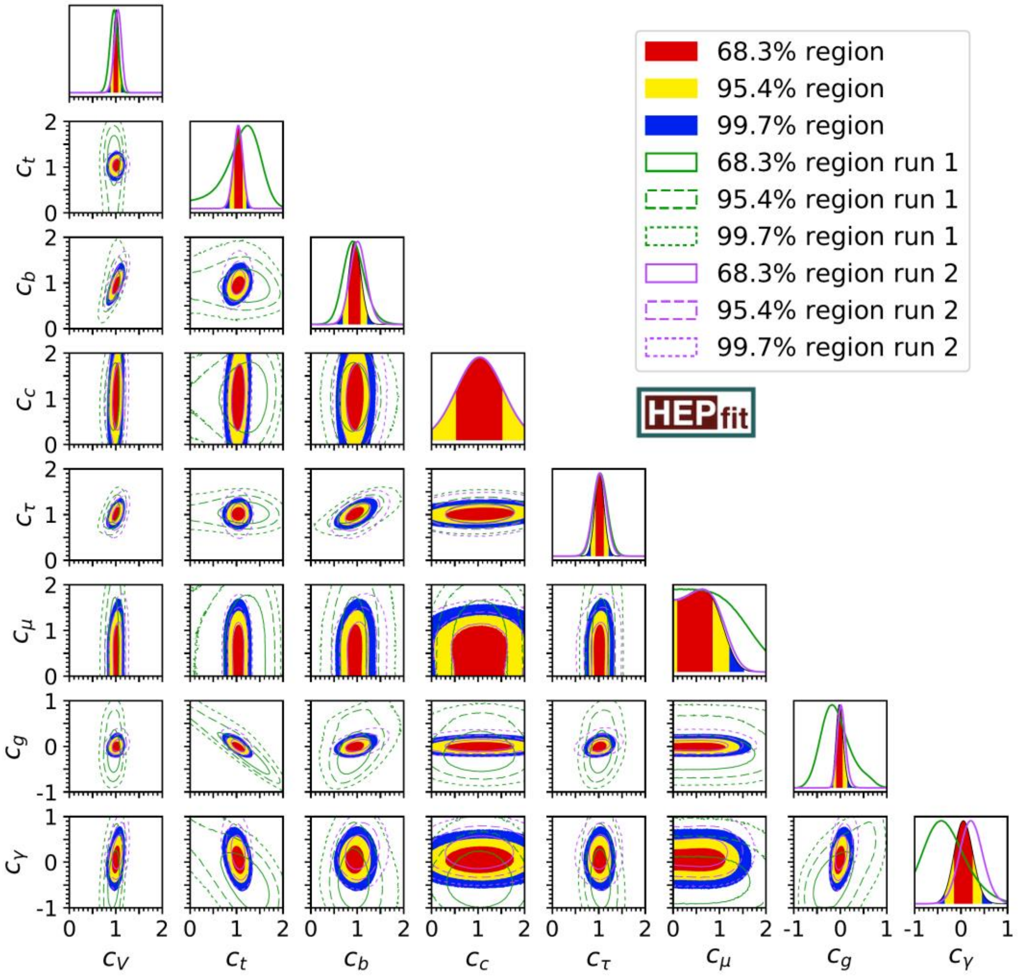
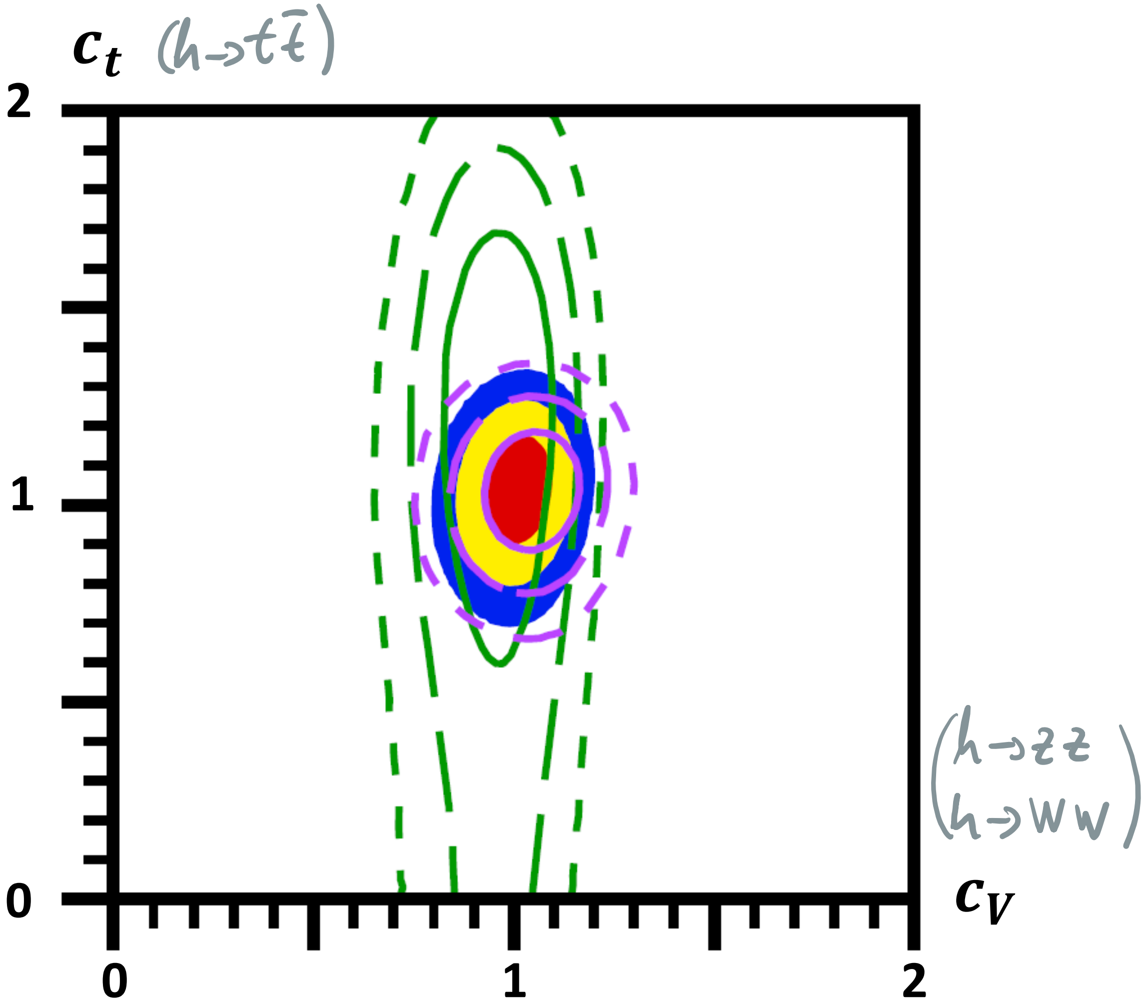


Figure 5. For the parameters c_i with $i = V, t, b, c, \tau, \mu, g, \gamma$ we display the one-dimensional posterior distribution as well as their two-dimensional correlations. The regions allowed at 68.3%, 95.4% and 99.7% probability by current Higgs data are represented by the red, yellow and blue filled contours, respectively. Additionally, we show the single contributions from pre-13 TeV run data (green) and LHC run 2 data (purple).

c_V	1.01 ± 0.06
c_t	$1.04^{+0.09}_{-0.10}$



* Current and Future Constraints on Higgs Couplings in the Nonlinear Effective Theory, Jorge de Blas, Otto Eberhardt, Claudius Krause, *JHEP* 07 (2018) 048 ;e-Print: 1803.00939 [hep-ph]

$hh \to VV$ couplings

$$C_{2V} \equiv b \equiv \frac{g_{hh\to WW}}{g^{SM}_{hh\to WW}}$$

$$-1.02 < C_{2V} < 2.71$$

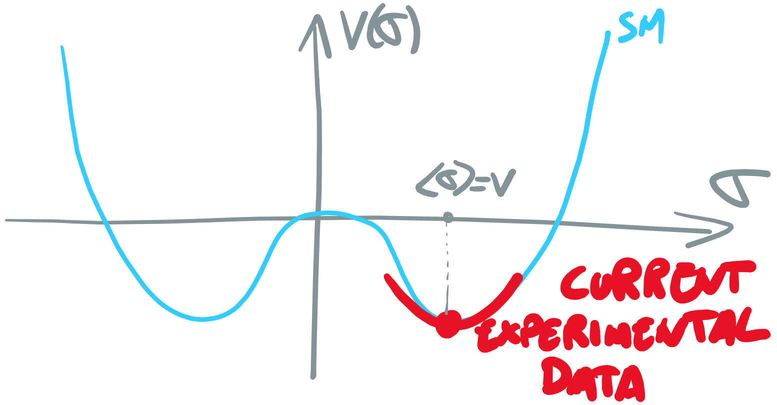
(@95% CL)

← The ATLAS collaboration [ATLAS Collaboration],“Search for theHH→b ̄bb ̄bprocess via vector bosonfusion production using proton-proton collisions at√s=13 TeV with the ATLAS detector,” ATLAS-CONF-2019-030.

Higgs self-couplings

$$\left.\frac{d^2V(\sigma)}{d\sigma^2}\right|_{\langle\sigma\rangle} = m_h^2 = 125.10 \pm 0.14\ \text{GeV}$$

$$\langle\sigma\rangle = v = (\sqrt{2}\, G_F)^{-1/2} \approx 246.22\ \text{GeV}$$, with $\sigma \equiv \sqrt{2}\,\|\Phi\|$



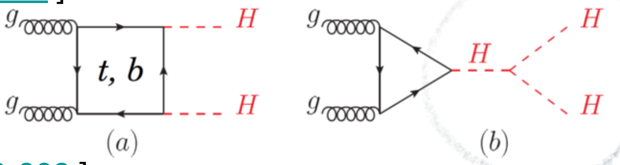
This is what we really know, so far, about the Higgs potential

- Some first results on Higgs self-interaction - CUBIC TERM - (but still poor bounds):

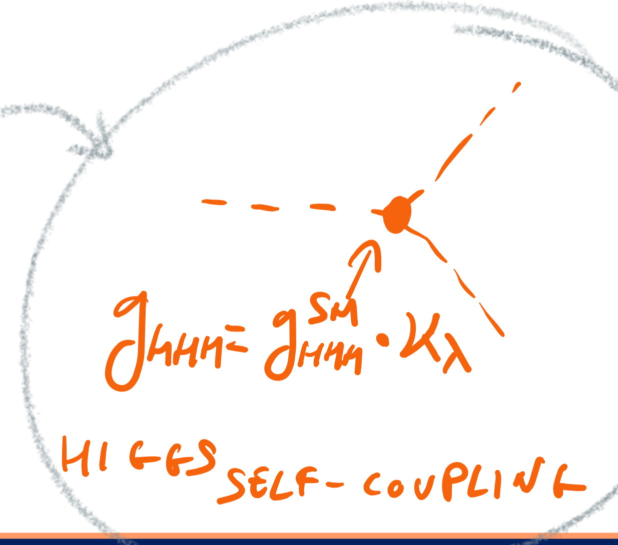
$$-4.1 < \kappa_\lambda < 14.1, \text{ at the } 95\%$$ [CMS PAS FTR-18-020]

$$-2.3 < \lambda_{HHH}/\lambda^{SM}_{HHH} < 10.3$$ [ATLAS-CONF-2019-049]

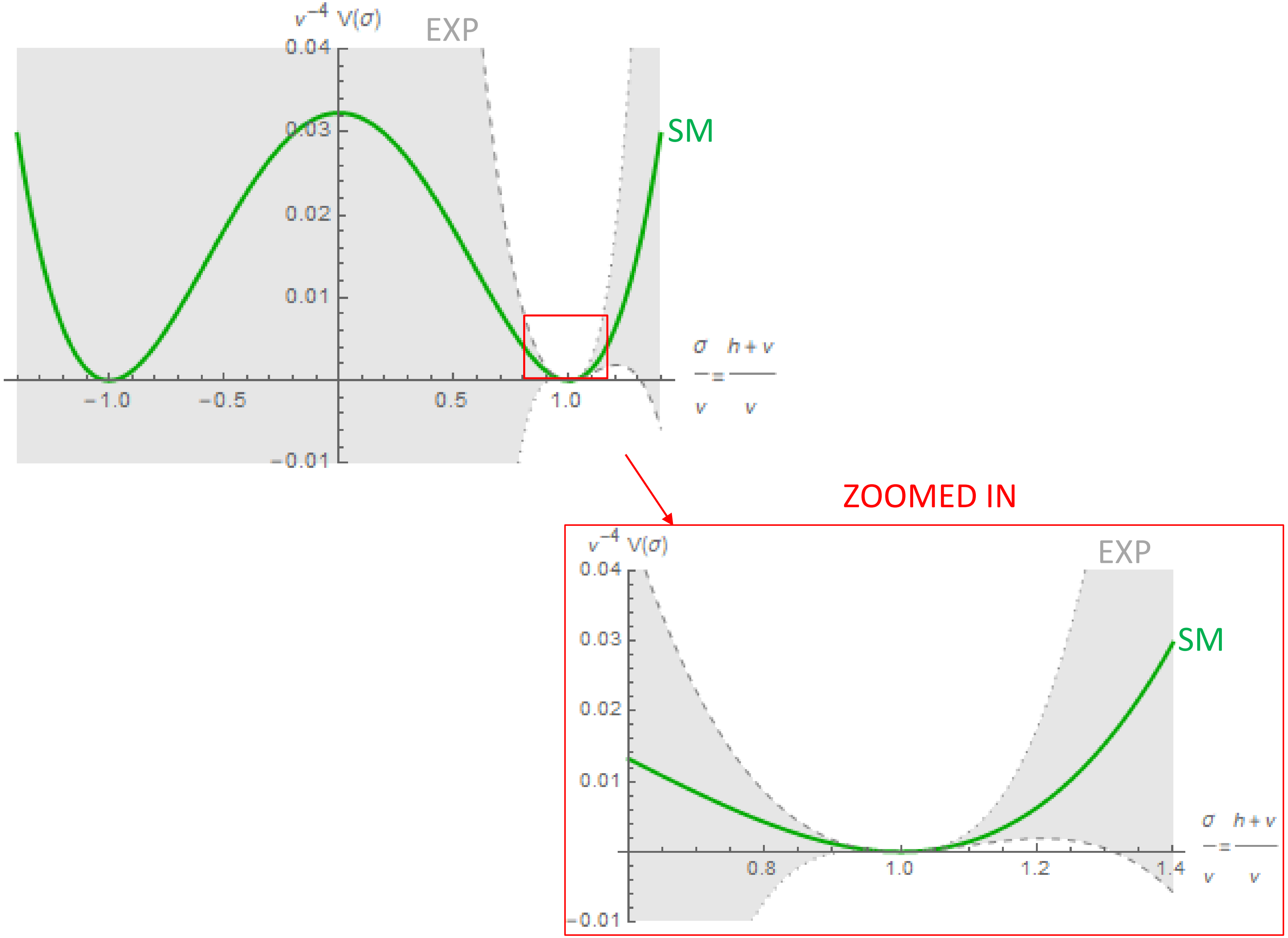
at 95% confidence level



at $\sqrt{s}$ = 13 TeV, corresponding to an integrated luminosity of up to 80 fb^{-1}. With the assumption that new physics affects only the Higgs boson self-coupling (λ_{HHH}), the ratio $\lambda_{HHH}/\lambda^{SM}_{HHH}$ is determined to be $\lambda_{HHH}/\lambda^{SM}_{HHH} = 4.0^{+4.3}_{-4.1}$, excluding values outside the interval $-3.2 < \lambda_{HHH}/\lambda^{SM}_{HHH} < 11.9$ at the 95% C.L. Results with less stringent assumptions are also provided. [ATL-PHYS-PUB-2019-009]



* See Khosa & Sanz, e-Print: 2102.13429 [hep-ph], and references therein]



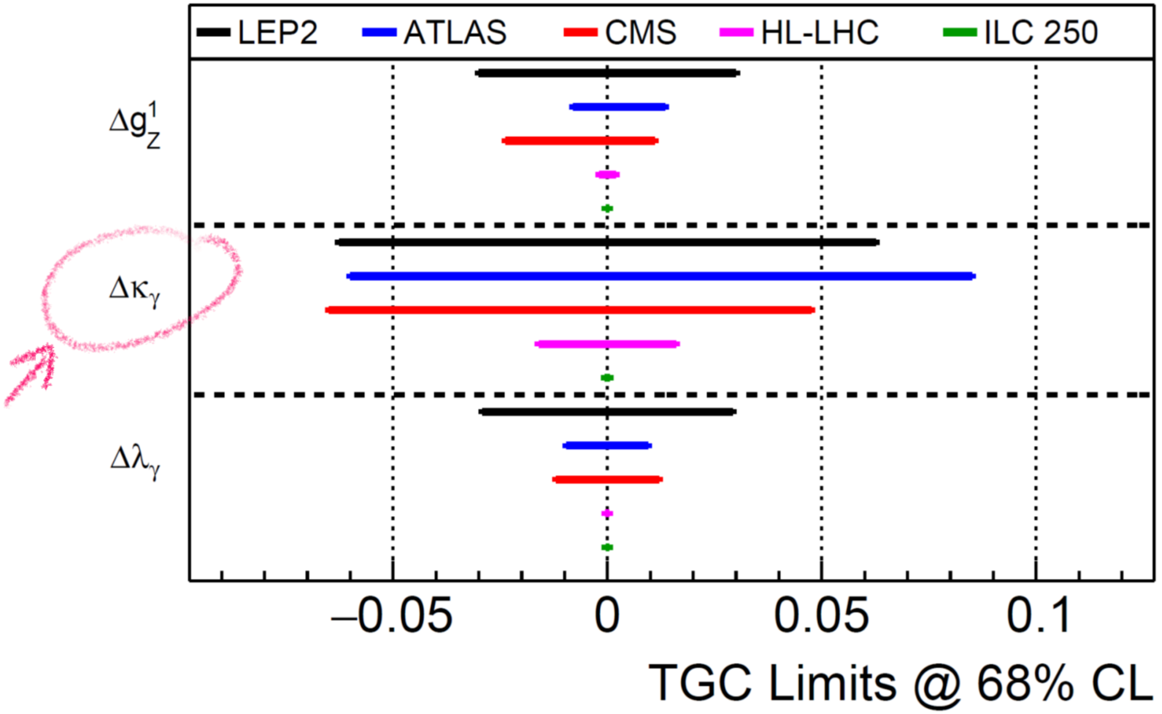
For prospects in HEFT at future colliders:

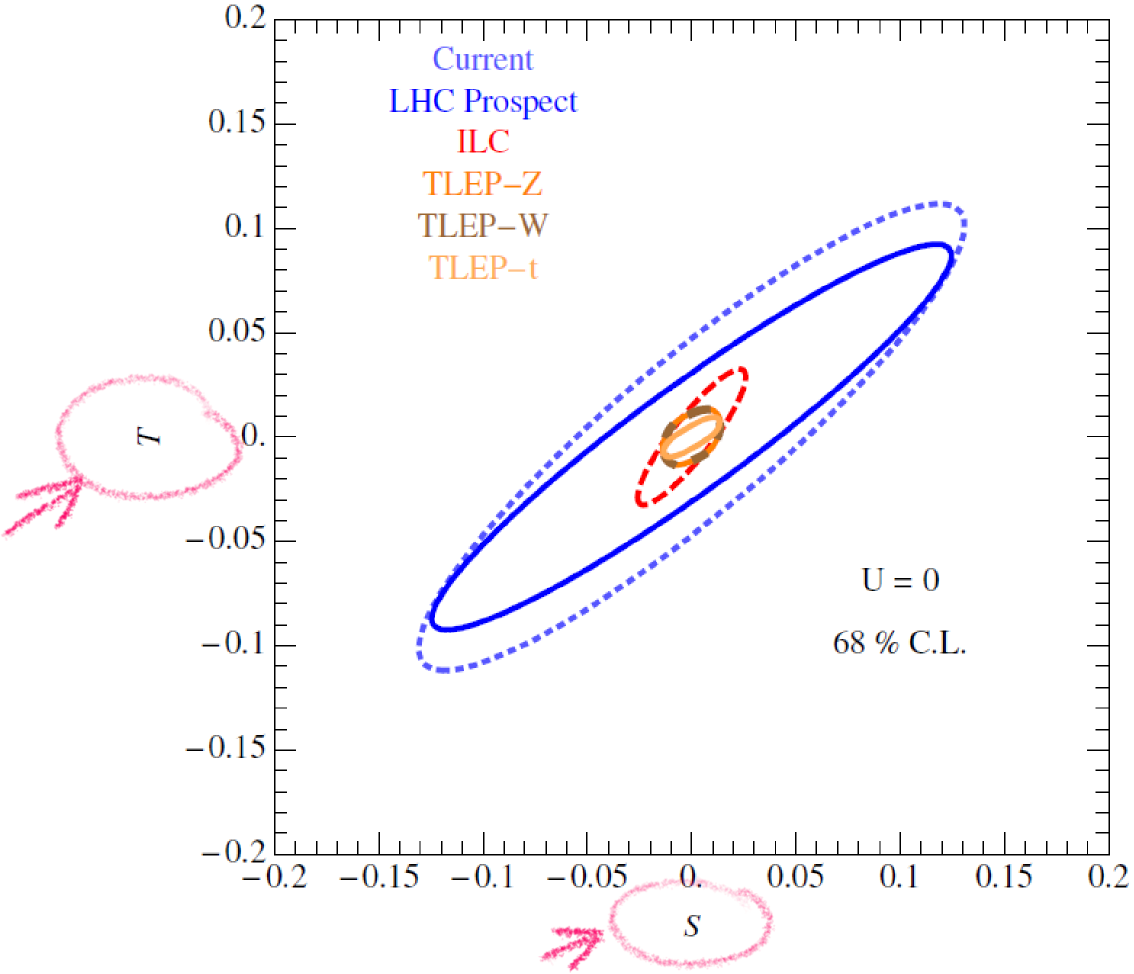
(*) Arganda,Garcia-Garcia,Herrero, NPB 945 (2019) 114687

Prospects at future colliders

- Significant presition improvement: TGC, oblique parameters, etc

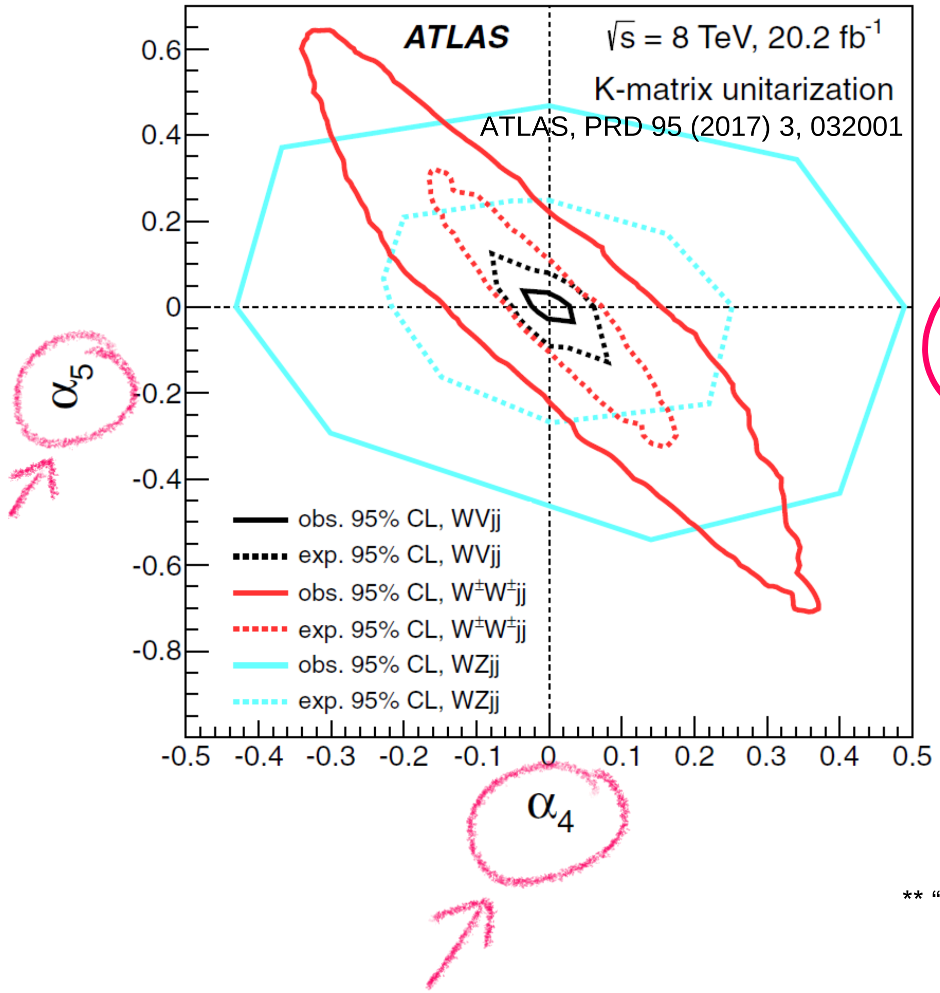
* "The International Linear Collider: A Global Project", Bambade et al., [1903.01629 [hep-ex]]

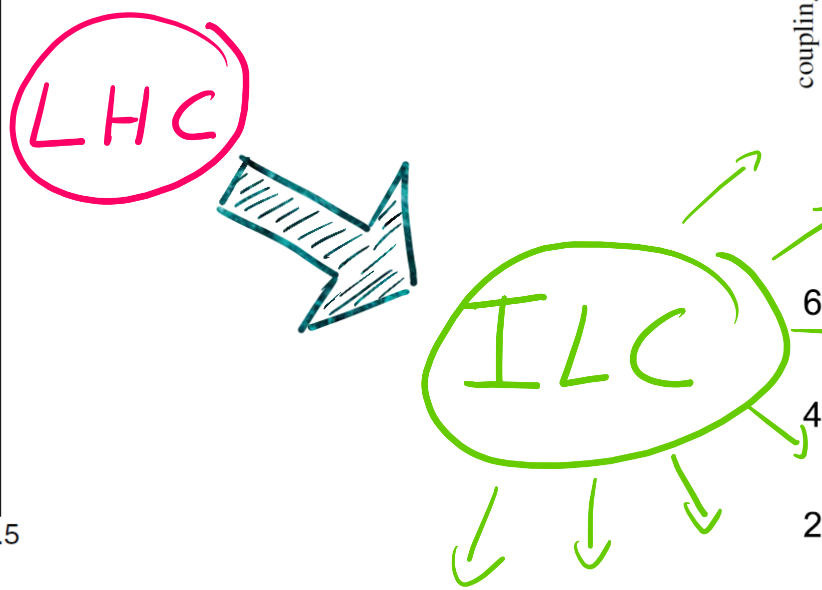


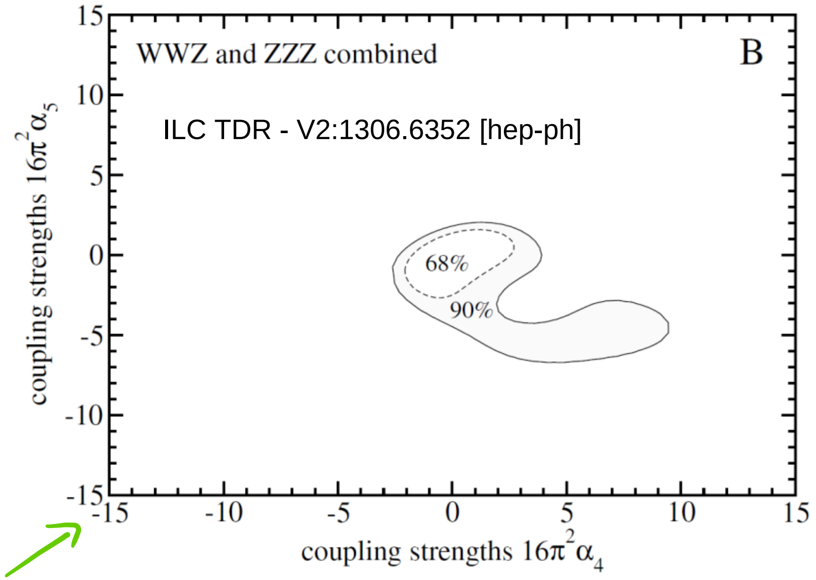


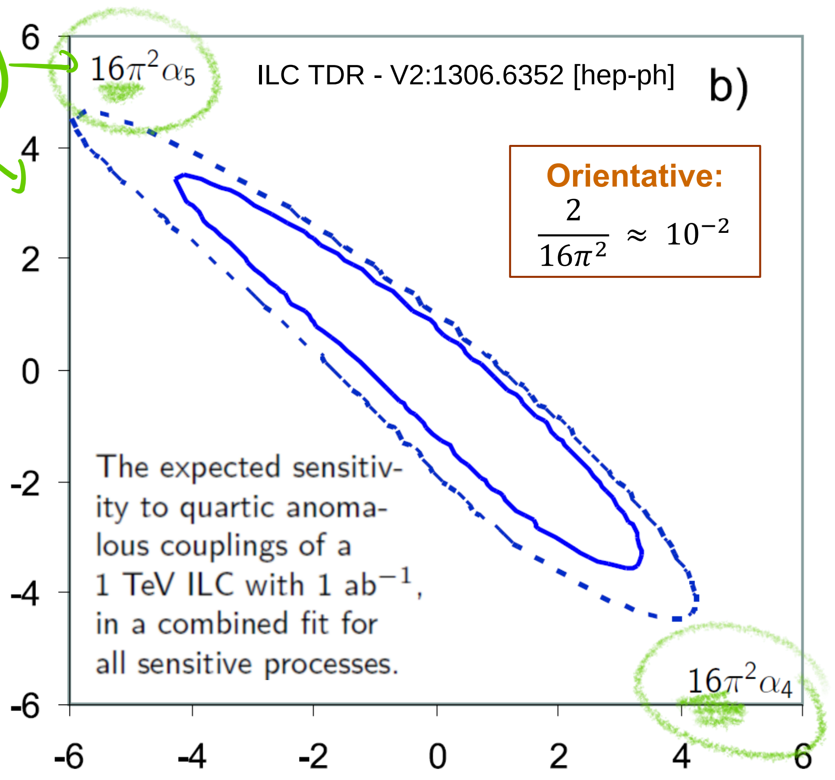
** "Possible Futures of Electroweak Precision: ILC, FCC-ee, and CEPC", Fan,Reece,Wang, JHEP 09 (2015) 196

• VV→VV gauge boson scat.









** "ILC TDR - V2: Physics", Baer et al., [1306.6352 [hep-ph]]

- Vector boson pair production:

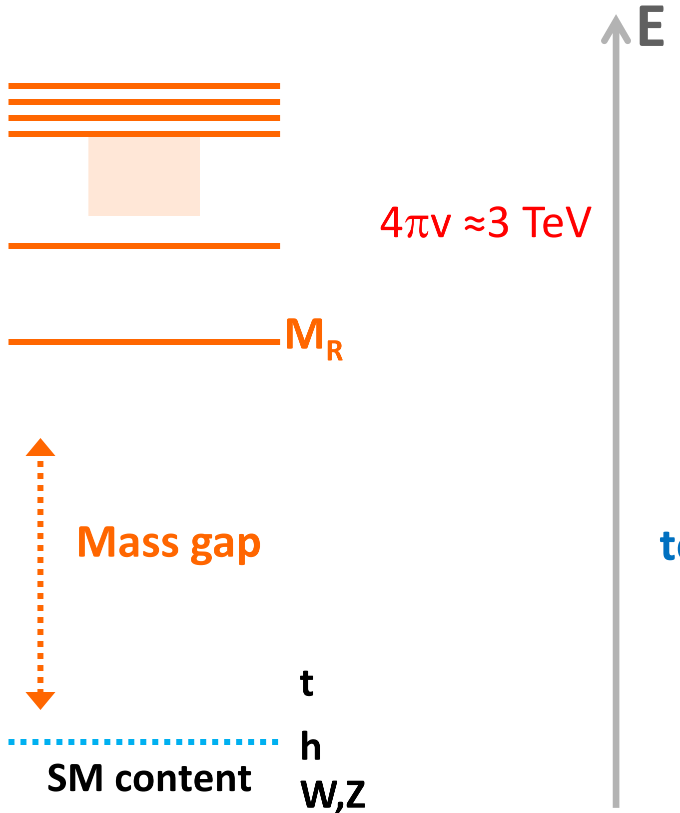
  $e^+e^- \rightarrow W^+W^-$

  $e^+e^- \rightarrow ZZ$

  $\gamma\gamma \rightarrow W^+W^-$

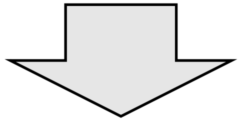
- WW, ZZ scattering at high energy

Resonance contributions to $\mathcal{L}_4$ at tree level *



SM content
- **Bosons:**
  singlet h,
  EW Goldstones $U(\omega^a)$,
  gauge bosons
- **Fermion $\psi_{L,R}$**

Custodial symmetry
+
Resonance Lagrangian
+
UV completion hypothesis

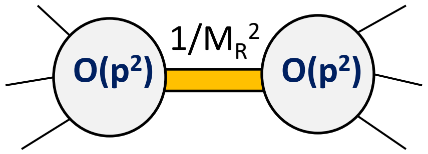


Resonance contributions *,**
to the NLO low-energy couplings

$SU(2)_L \otimes SU(2)_R / SU(2)_{L+R}$

V,A,S,P, Fermionic R
EW singlet, doublet & triplet
Colour singlet & octet

Sum-rules

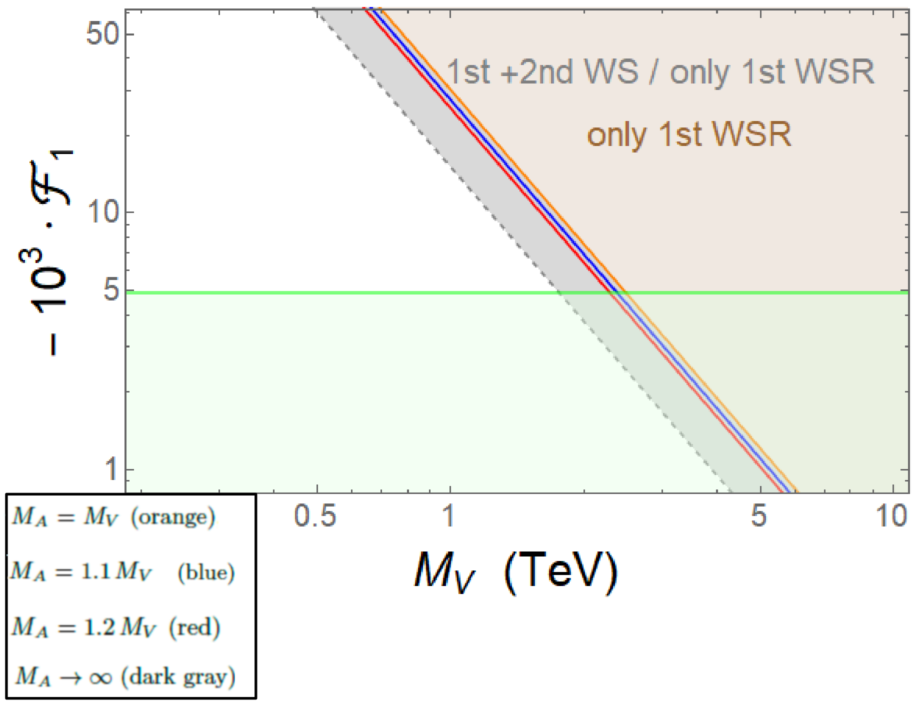


$$e^{i\,S[\chi,\psi]_{\rm EFT}} = \int [dR]\; e^{i\,S[\chi,\psi,R]} \overset{\rm tree-level}{=} e^{i\,S[\chi,\psi,R_{\rm cl.}]}$$

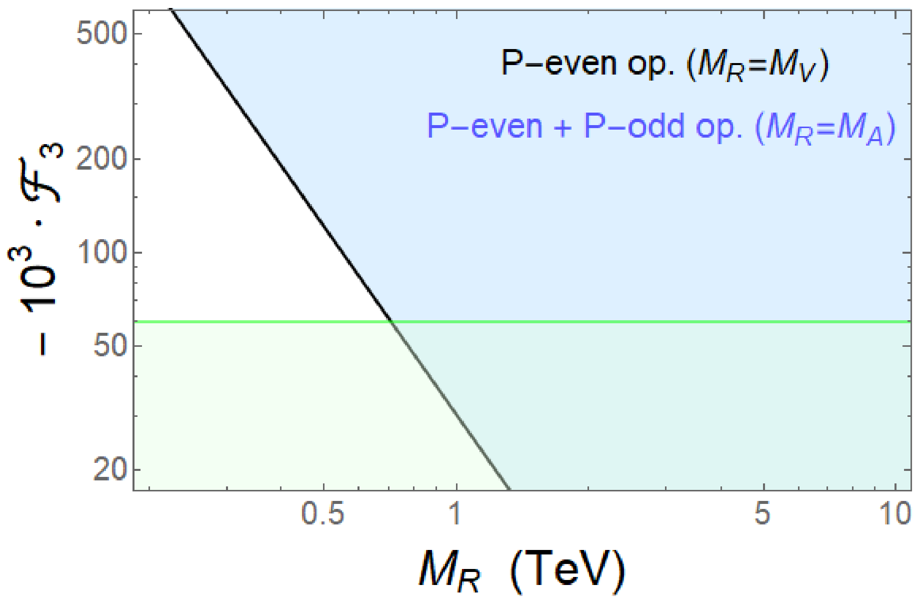
* Pich,Rosell,Santos,SC, PRD93 (2016) no.5, 055041; JHEP 1704 (2017) 012;
Krause,Pich,Rosell,Santos,SC [arXiv:1810.10544 [hep-ph]]
** See also: Alboteanu, Kilian, Reuter, JHEP 0811 (2008) 010; Pappadopulo, Thamm, Torre, Wulzer, JHEP 1409 (2014) 060; Corbett, Joglekar, Li, Yu, [arXiv:1705.02551 [hep-ph]]; Corbett,Éboli,Gonzalez-Garcia,PRD93 (2016) no.1, 015005; Buchalla, Cata, Celis, Krause, NPB917 (2017) 209;
de Blas,Criado,Perez-Victoria,Santiago, JHEP 1803 (2018) 109; Dobado,Pelaez, PRD 56 (1997) 3057

• **PREDICTIONS vs DATA:**

S-parameter

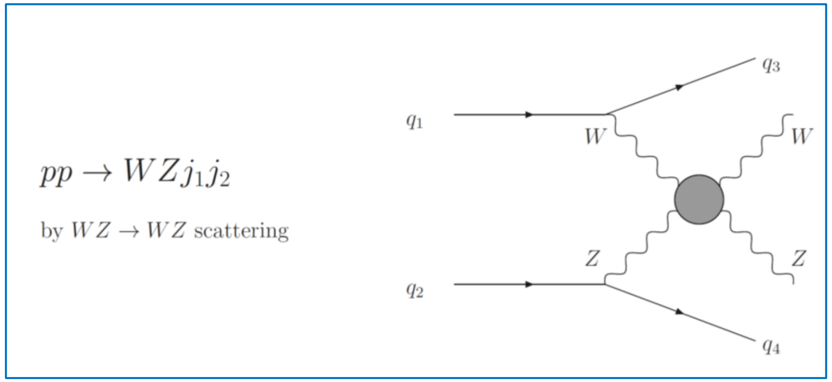


Anomalous TGC



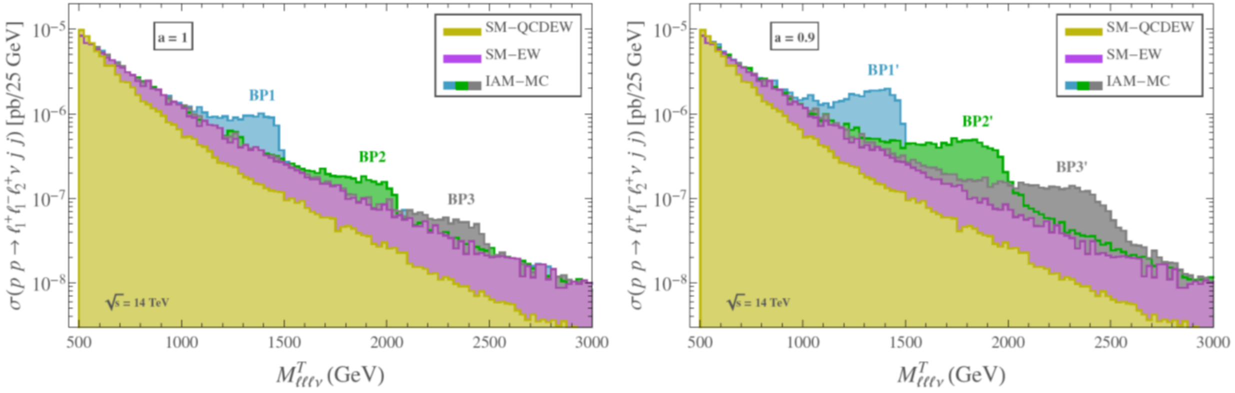
* Pich, Rosell,SC, Phys.Rev.D 102 (2020) 3, 035012

HEFT → RESONANCES: WW scattering



14 TeV		BP1	BP2	BP3	BP1’	BP2’	BP3’
$\mathcal{L} = 300\ \text{fb}^{-1}$	N_ℓ^{IAM-MC}	2	0.5	0.1	5	2	0.7
	N_ℓ^{SM}	1	0.4	0.1	2	0.6	0.3
	σ_ℓ^{stat}	0.9	–	–	2.8	1.4	–
$\mathcal{L} = 1000\ \text{fb}^{-1}$	N_ℓ^{IAM-MC}	7	2	0.4	18	5	2
	N_ℓ^{SM}	4	1	0.3	6	2	1
	σ_ℓ^{stat}	1.6	0.3	–	5.1	2.5	1.4
$\mathcal{L} = 3000\ \text{fb}^{-1}$	N_ℓ^{IAM-MC}	22	5	1	53	16	7
	N_ℓ^{SM}	12	4	1	17	6	3
	σ_ℓ^{stat}	2.7	0.6	0.3	8.9	4.4	2.4

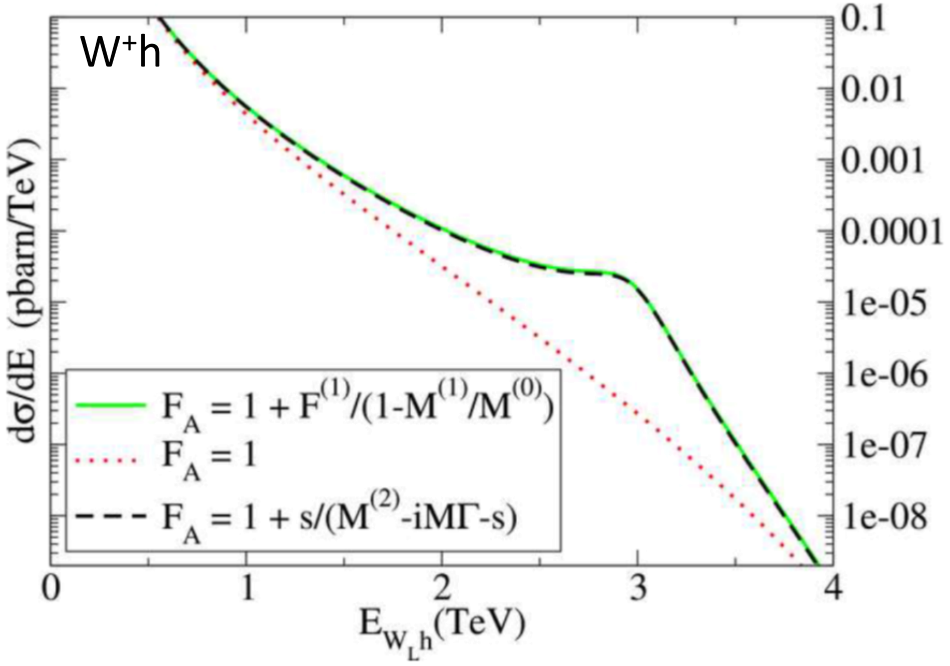
- Fully leptonic decays: 14 TeV

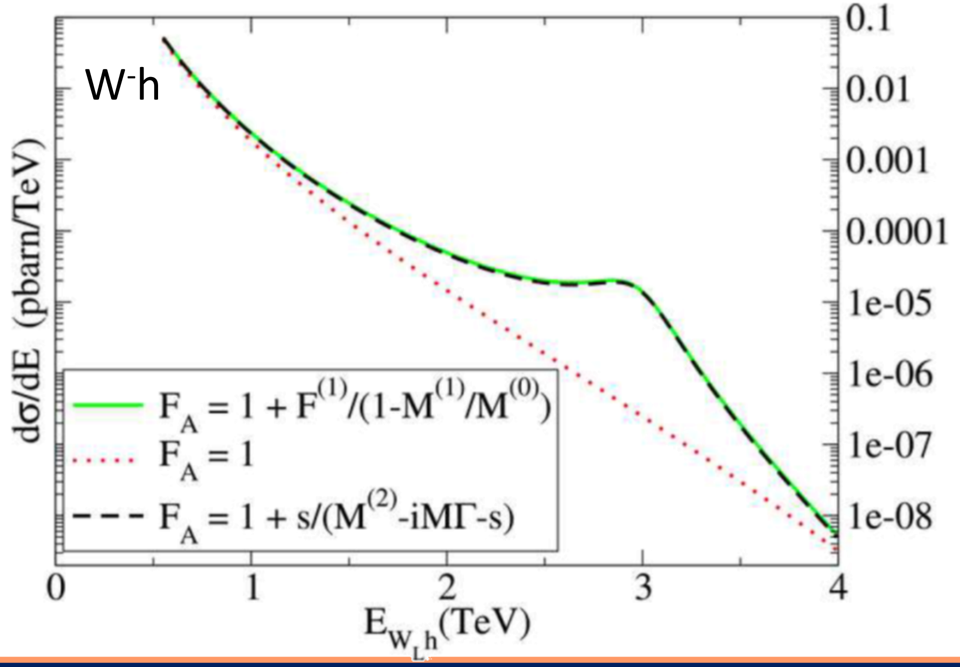


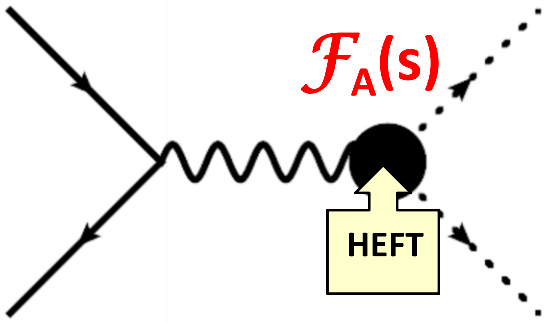
BP	M_V (GeV)	Γ_V (GeV)	$g_V(M_V^2)$	a	$a_4 \cdot 10^4$	$a_5 \cdot 10^4$
BP1	1476	14	0.033	1	3.5	−3
BP2	2039	21	0.018	1	1	−1
BP3	2472	27	0.013	1	0.5	−0.5
BP1’	1479	42	0.058	0.9	9.5	−6.5
BP2’	1980	97	0.042	0.9	5.5	−2.5
BP3’	2480	183	0.033	0.9	4	−1

* Delgado, Dobado, Espriu, Garcia-Garcia, Herrero, Marcano, SC, JHEP 11 (2017) 098

HEFT
↓
RESONANCES:
DRELL-YAN







BENCHMARK point

HEFT: $a=0.95$, $b=0.7\,a^2$, $\mu = 3$ TeV

$\mathcal{M}_{11}(s)$ PWA→ $e(\mu) - 2d(\mu) = 1.64 \cdot 10^{-3}$

$\mathcal{F}_A(s)$ AFF→ $f_9(\mu) = -6 \cdot 10^{-3}$

IAM unitarization

HEFT+R: $M_A = 3$ TeV, $\Gamma_A = 0.4$ TeV

HEFT predict: BSM excess ~ 10^{-2} fb

* Dobado,Llanes-Estrada,SC, JHEP 1803 (2018) 159

Conclusions

- **Chiral symmetry** ➔ A crucial ingrediet of the SM & BSM extensions

- **EFT extension of the SM based on chiral symmetry:**

  SM particles with a singlet h + triplet Goldstones

  SM chiral and gauge symmetries

  Chiral expansion in p^d

- **HEFT:**

  LO Lagrangian + NLO Lagrangian + 1-loop corrections (NLO)

- **Phenomenology:** so far, nothing...

  but many results point out the possibility of new heavy states in the few TeV

  (related to NLO couplings $\mathcal{F}_j \sim 10^{-3}$)