Baryon masses and currents in SU(3) BChPT $\times$ 1/Nc

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Outline

1) Motivation for the BChPT x 1/Nc expansion
2) Introduction to the BChPT x 1/Nc expansion (combined) approach
3) Baryon masses
4) Baryon sigma terms
5) Vector currents
6) Axial-vector currents
7) Summary

1 Motivation for BChPT x 1/Nc expansion

PHYS. REV. D **97,** 054010 (2018)

Non relativistic version of the BChPT or HBChPT is based on the expansion in terms of the inverse baryon mass

Derivative expansion for both mesons and baryons become an expansion in powers of (k/Λ_χ)

$$v^\mu v_\mu = 1$$
$$v.k \ll m_{\mathcal{B}}$$

The issue of experiencing a slower rate of convergence compared to the Goldstone Boson Sector

$$p_\mu = m_{\mathcal{B}} v_\mu + k_\mu$$

$$\frac{1}{p^2 - m_{\mathcal{B}}^2} \to \frac{1}{2m_{\mathcal{B}}} \frac{1}{(v.k)} + \mathcal{O}\left(1/m_{\mathcal{B}}^2\right)$$

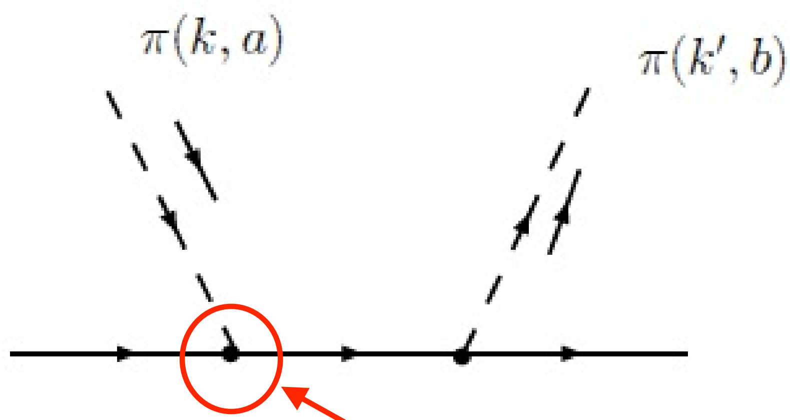
Solution : Inclusion of the decuplet baryons in one-loop corrections to physical observables, has been showing a great improvement!

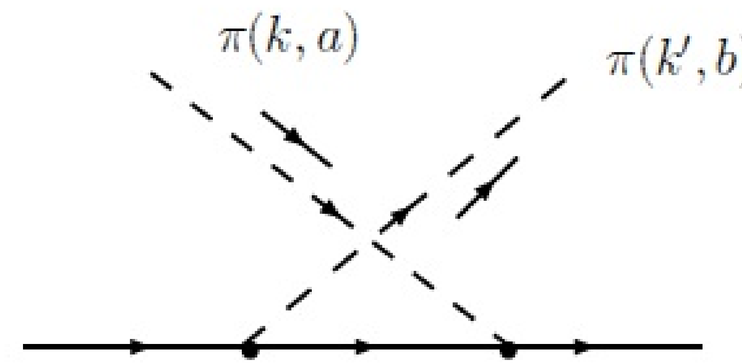
These improvements are due to cancellations between octet and decuplet contributions in loops

On the other hand, studying the baryons in the large Nc limit of QCD emerges a dynamical symmetry called "spin-flavor symmetry" which requires the possibility of having degenerate baryon multiplets of higher spin in the intermediate state/s.

1.1 Spin-flavor symmetry of Baryons in large Nc

Gervais & Sakita; Dashen & Manohar





$\mathcal{O}(\sqrt{N_c})$

$$\frac{\partial_\mu \pi^a}{f_\pi}\langle B'|\bar{q}\gamma^\mu\gamma_5 T^a q|B\rangle \equiv \frac{\partial_i \pi^a}{f_\pi} g_A N_c \langle B'|X^{ia}|B\rangle \Rightarrow \mathcal{O}(\sqrt{N_c})$$

Since, πN amplitude is $\mathcal{O}(N_c^0)$

$$A = -i\frac{k^i k^j}{k_0}\frac{{N_c}^2 g^2}{f_\pi^2}[X^{ia}, X^{ib}] \Rightarrow [X^{ia}, X^{ib}] \leqslant \mathcal{O}(1/N_c)$$

Large N_c consistency condition
$[X_0^{ia}, X_0^{ib}] = 0$

At large Nc, QCD has contracted spin-flavor symmetry $SU_c(2N_f)$ in baryon sector

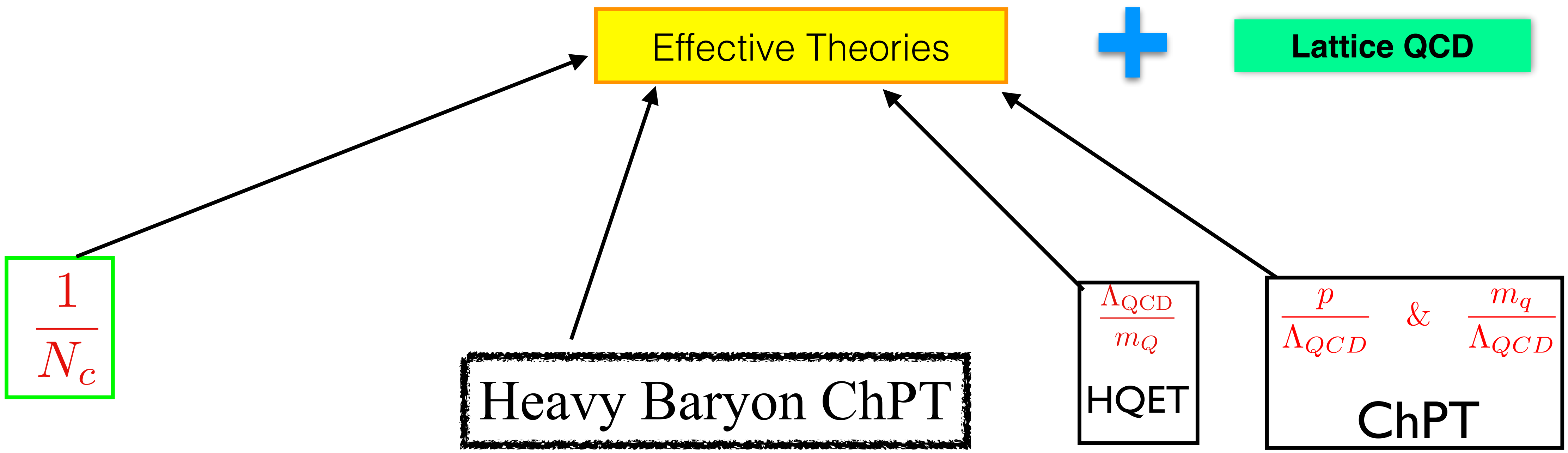
$$X_0^{ia} = \lim_{N_c\to\infty}\frac{G^{ia}}{N_c}$$

This symmetry is broken at sub-leading orders in 1/**Nc**

This spin-flavor symmetry requires the existence of degenerate baryon multiplets with different spins (a dynamical symmetry) : leads to the consideration of both octet and decuplet contributions in the intermediate state

2 Introduction to the combined approach : BChPT x 1/Nc expansion

$$L(\text{Lagrangian}) = \quad L_{LO} + \quad L_{NLO} + \quad L_{NNLO} + \quad L_{NNNLO} + \ldots.$$



Spin-flavor Symmetry + Chiral Symmetry

imposes constraints in the Chiral Lagrangian

Combined approach

Combining the HBChPT with 1/Nc provides a well behaved expansion in the low energy phenomenology (because one cannot expand them independently in low energy)

Link between the Chiral and 1/Nc expansion

$$\xi - \text{expansion}: \; \mathcal{O}\left(1/N_c\right) = \mathcal{O}\left(p\right) = \mathcal{O}\left(\xi\right)$$

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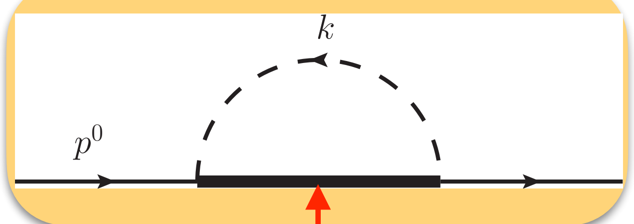
Chiral Symmetry + Spin-flavor Symmetry



$$= \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - M_\pi^2} \frac{i}{p^0 + k^0 - \underbrace{(m_{B'} - m_B)}_{Q \,\sim\, \mathcal{O}(1/N_c)}} \times \text{vertex factors}$$

Intermediate Octet and Decuplet baryon contributions are included

ξ − expansion : $\mathcal{O}\left(1/N_c\right) = \mathcal{O}\left(p\right) = \mathcal{O}\left(\xi\right)$

$$\begin{aligned} I_{1-loop}(Q, M_\pi) &= \int \frac{d^d k}{(2\pi)^d} \frac{\vec{k}^2}{k^2 - M_\pi^2 + i\epsilon} \frac{1}{k^0 - Q + i\epsilon} \\ &= \frac{i}{16\pi^2} \Bigg\{ Q \left((3M_\pi^2 - 2Q^2)(\lambda_\epsilon - \log \frac{M_\pi^2}{\mu^2}) + (5M_\pi^2 - 4Q^2) \right) \\ &+ 2\pi (M_\pi^2 - Q^2)^{3/2} + 4(Q^2 - M_\pi^2)^{3/2} \tanh^{-1} \frac{Q}{\sqrt{Q^2 - M_\pi^2}} \Bigg\} , \end{aligned}$$

Contains both scales: therefore cannot be expanded independently

$Q = \delta m_n - p^0, \; \lambda_\epsilon = \frac{1}{\epsilon} - \gamma + \log 4\pi$

Building blocks :

- Goldstone bosons: pions, kaons, eta

$$u = exp(\frac{i\Pi}{2F_\pi})$$

Meson Fields : $\pi^a T^a$

- Baryons with spin 1/2, 3/2, …, Nc/2

$$B = \begin{pmatrix} N \\ \Delta \\ . \\ . \\ . \end{pmatrix}$$

Degrees of freedom : Hadrons

- Leading order (Spin-flavor symmetry + chiral symmetry)

$$\boxed{L_B = B^\dagger \left(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{HF}}{N_c} \vec{S}^2 - \frac{c_1}{\Lambda} \hat{\chi}_+ \right) B}$$

the axial coupling is at LO $\mathring{g}_A = \frac{6}{5} g_A$, being $g_A = 1.2732(23)$

$$(\partial_0 - i\Gamma_0)$$

$$\boxed{Tr\langle (u^\dagger(i\partial_i + r_i)u - u(i\partial_i + l_i)u^\dagger)\, \lambda^a \rangle}$$

$$\boxed{\left(\frac{1}{2} \left(u^\dagger(i\partial_0 + r_0)u + u(i\partial_0 + l_0)u^\dagger\right) \right)}$$

$$\hat{\chi}_+ \equiv \tilde{\chi}_+ + N_c\, \chi_+^0$$

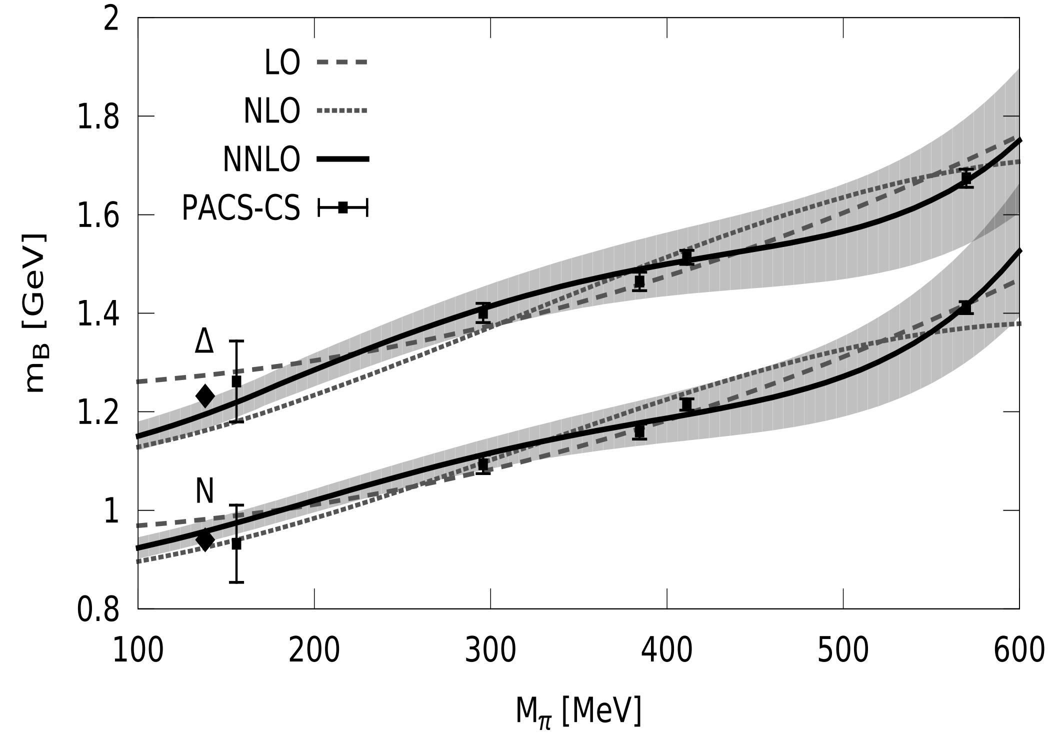
$$\begin{aligned} \chi &= 2B_0(s + ip)\,, \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u \chi^\dagger u\,, \\ \chi_\pm^0 &= \langle \chi_\pm \rangle\,, \\ \tilde{\chi}_\pm &\equiv \chi_\pm^a T^a, \quad \text{where } \chi_\pm^a \equiv \frac{1}{2} \langle \lambda^a \chi_\pm \rangle \end{aligned}$$

3. 1 Baryon masses

Baryon Masses to $\mathcal{O}(\xi^2)$ in SU(2)

$$m_{\mathbf{B}} = N_c M_0 + \frac{C_{HF}}{N_c}\hat{S}^2 + c_1 M_\pi^2 + \delta m_{\mathbf{B}}^{1-\text{loop}+CT}$$

A. Calle-Cordon & J.L. Goity (**PHYSICAL REVIEW D 87, 016019 (2013)**)



Baryon Masses to $\mathcal{O}(\xi^3)$ in SU(3)

I. P. FERNANDO and J. L. GOITY (PHYS. REV. D **97,** 054010 (2018))

$$\mathcal{L}_B = \mathbf{B}^\dagger \Big(iD_0 + \mathring{g}_A u^{ia} G^{ia} - \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{1}{2\Lambda} c_2 \hat{\chi}_+ + \frac{c_3}{N_c \Lambda^3}\hat{\chi}_+^2 + \frac{h_1}{N_c^3}\hat{S}^4 + \frac{h_2}{N_c^2 \Lambda}\hat{\chi}_+ \hat{S}^2 + \frac{h_3}{N_c \Lambda}\chi_+^0 \hat{S}^2 + \frac{h_4}{N_c \Lambda}\chi_+^a \{S^i, G^{ia}\} + \alpha \hat{Q} + \beta \hat{Q}^2 \Big) \mathbf{B}$$

$$\hat{\chi}_+ = N_c\, \chi_+^0 + \tilde{\chi}_+$$
$$\chi_+^0 \to 4B_0 m^0$$
$$\tilde{\chi}_+^a \to 8B_0 \delta^{a8} m^8$$
$$\hat{\chi}_+ \to 4B_0(m^8 T^8 + N_c m^0)$$

$$m^0 = (2\hat{m} + m_s)/3$$
$$m^8 = 2/\sqrt{3}(\hat{m} - m_s)$$

3. 1 Baryon masses

$$
\begin{aligned}
m_B = {} & M_0 + \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{c_1}{\Lambda}2B_0(\sqrt{3}m_8 Y + N_c m_0) - \frac{c_2}{\Lambda}4B_0 m_0 \\
& - \frac{c_3}{N_c\Lambda^3}\left(4B_0(\sqrt{3}m_8 Y + N_c m_0)\right)^2 \\
& - \frac{h_1}{N_c^2\Lambda}\hat{S}^4 - \frac{h_2}{N_c\Lambda}4B_0(\sqrt{3}m_8 Y + N_c m_0)\hat{S}^2 - \frac{h_3}{N_c\Lambda}4B_0 m_0\hat{S}^2 \\
& - \frac{h_4}{N_c\Lambda}\frac{4B_0 m_8}{\sqrt{3}}\left(3\hat{I}^2 - \hat{S}^2 - \frac{1}{12}N_c(N_c+6)\right. \\
& \left. + \frac{1}{2}(N_c+3)Y - \frac{3}{4}Y^2\right) + \delta m_B^{\text{loop}},
\end{aligned}
$$

$$
\delta m_B^{1-loop} = i\,\frac{\mathring{g}_A^2}{F_\pi^2}\frac{1}{d-1}\sum_n G^{ia}\mathcal{P}_n G^{ia}\, I_{1-loop}(\delta m_n - p^0, M_\pi)
$$

$$
\Delta_{GMO} \equiv 3m_\Lambda + m_\Sigma - 2(m_N + m_\Xi)
$$

The breaking to the GMO relation is only coming through the loop corrections and it behaves like 1/Nc in the strict large Nc limit

$$
\begin{aligned}
\Delta_{\mathrm{GMO}} = {} & -\left(\frac{\mathring{g}_A}{4\pi F_\pi}\right)^2\left(\frac{2\pi}{3}\left(M_K^3 - \frac{1}{4}M_\pi^3 - \frac{2}{\sqrt{3}}\left(M_K^2 - \frac{1}{4}M_\pi^2\right)^{\frac{3}{2}}\right)\right. \\
& \left. + \frac{C_{\mathrm{HF}}}{2N_c}\left(4M_K^2\log\left(\frac{4M_K^2 - M_\pi^2}{3M_K^2}\right) - M_\pi^2\log\left(\frac{4M_K^2 - \frac{1}{3}M_\pi^2}{3M_\pi^2}\right)\right)\right) \\
& + \mathcal{O}(1/N_c^3).
\end{aligned}
$$

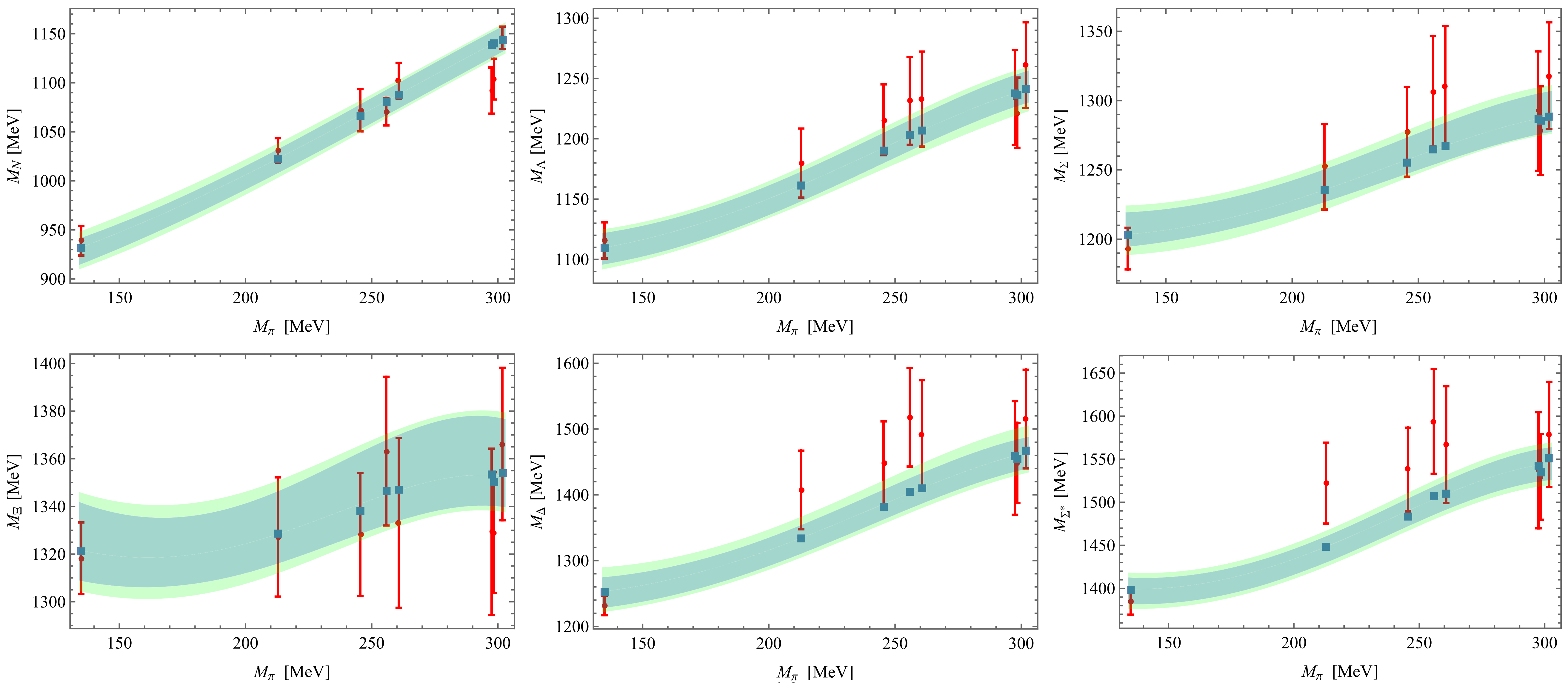
3.2 Fit Results to Baryon Masses

Fit results to Experimental & Lattice QCD masses

I. P. FERNANDO and J. L. GOITY — PHYS. REV. D **97,** 054010 (2018)

TABLE II. Results for LECs: the ratio $\mathring{g}_A/F_\pi = 0.0122\ \mathrm{MeV}^{-1}$ is fixed by using Δ_{GMO}. The first row is the fit to LQCD octet and decuplet baryon masses [48] including results for $M_\pi \leq 303$ MeV (dof = 50), and second row is the fit including also the physical masses (dof = 58). Throughout the $\mu = \Lambda = m_\rho$.

χ^2_{dof}	m_0 [MeV]	C_{HF} [MeV]	c_1	c_2	h_2	h_3	h_4
0.47	221(26)	215(46)	−1.49(1)	−0.83(5)	0.03(3)	0.61(8)	0.59(1)
0.64	191(5)	242(20)	−1.47(1)	−0.99(3)	0.01(1)	0.73(3)	0.56(1)



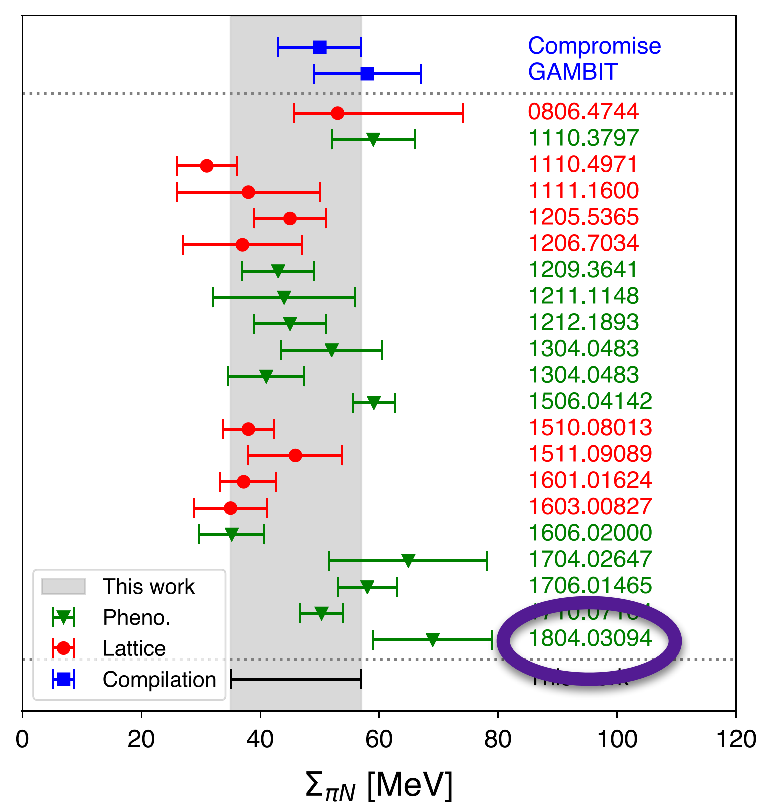
LQCD baryon masses : C. Alexandrou, V. Drach, K. Jansen, C. Kallidonis, and G. Koutsou. *Phys. Rev.*, D90:074501, (2014)

4 Baryon sigma terms

$$\sigma_{f\mathbf{B}}(m_f) = m_f \frac{\partial}{\partial m_f} m_{\mathbf{B}} = \frac{m_f}{2m_{\mathbf{B}}} \langle \mathbf{B} \mid \bar{q}_f q_f \mid \mathbf{B} \rangle$$

1) The value of the pion-Nucleon sigma term ranges from 45 MeV to 64 MeV

Eur. Phys. J. C (2018) 78:569 John Ellis, Natsumi Nagata, Keith A. Olive



$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N} \langle N \mid \bar{u}u + \bar{d}d \mid N \rangle$$

2) There is a long lasting "puzzle" associated with a combination of baryon masses (in SU(3)) in the iso-spin symmetric limit, to obtain the pion-Nucleon sigma term, assuming the contribution by strange quark mass to the nucleon mass is negligible (OZI).

3) The connection between the pion-Nucleon sigma term and size of the correction to the Gell-Mann-Okubo relation

4

Sigma Terms

Baryon masses and σ terms in SU(3) BChPT $\times$ 1/N$_c$

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Baryon matrix elements of scalar quark densities give us the information on the amount of baryon mass originating from the quark masses

Feynman-Hellman theorem

$$\sigma_i(B) = m_i \frac{\partial}{\partial m_i} m_B$$

Baryon mass dependencies on quark masses

$$\sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s$$

m_i indicates a quark mass

$$\sigma_{\pi N} \equiv \frac{\hat{m}}{2m_N}\langle N \mid \bar{u}u + \bar{d}d \mid N\rangle$$

$$\hat{\sigma} = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$

$$\sigma_s = \frac{m_s}{2m_N}\langle N|\bar{s}s|N\rangle$$

$$\sigma_{\pi N} \sim \hat{\sigma}$$

$$|\sigma_s| \lesssim 50 \text{ MeV}$$

4

Sigma Terms

A long lasting puzzle !

$$\sigma_{\pi N} = \hat{\sigma} + 2\frac{\hat{m}}{m_s}\sigma_s$$

$$\hat{\sigma} \equiv \sqrt{3}\frac{\hat{m}}{m_8}\sigma_8$$

$$\sigma_8 = \frac{1}{3}(2m_N - m_\Sigma - m_\Xi)$$

$$\hat{\sigma} = \underbrace{\frac{\hat{m}}{m_s - \hat{m}}}_{\Delta\hat{\sigma}}\overbrace{(m_\Xi + m_\Sigma - 2m_N)}^{\sim\ 26\ \text{MeV}}$$

$$m_3 = m_u - m_d$$

$$m_8 = \frac{1}{\sqrt{3}}(\hat{m} - m_s)$$

There is a (hidden) large correction ~ 44 MeV from non-analytic contributions from baryon self-energies

$$\sigma_8 = \frac{1}{2m_N}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle \qquad \Delta\sigma_8 \equiv \sigma_8 - \frac{1}{3}(2m_N - m_\Sigma - m_\Xi)$$

$$\Delta\sigma_8 = \sigma_8 - \frac{1}{9}\left(\frac{5N_c - 3}{2}m_N - (2N_c - 3)m_\Sigma - \frac{N_c + 3}{2}m_\Xi\right)$$

$$\Delta_{GMO} \equiv 3m_\Lambda + m_\Sigma - 2(m_N + m_\Xi) \sim 25\ \text{MeV}$$

The dominant contributions to Δ_{GMO} and $\Delta\sigma_8$ are calculable non-analytic contributions : $\Delta\sigma_8/\Delta_{\text{GMO}}$ (~ -13.5 for $N_c = 3$)

4

Sigma Terms (Results)

$$\mathcal{L}_B = \mathbf{B}^\dagger\Big(iD_0 + \mathring{g}_A u^{ia}G^{ia} - \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{1}{2\Lambda}c_2\hat{\chi}_+ + \frac{c_3}{N_c\Lambda^3}\hat{\chi}_+^2 + \frac{h_1}{N_c^3}\hat{S}^4 + \frac{h_2}{N_c^2\Lambda}\hat{\chi}_+\hat{S}^2 + \frac{h_3}{N_c\Lambda}\chi_+^0\hat{S}^2 + \frac{h_4}{N_c\Lambda}\chi_+^a\{S^i, G^{ia}\} + \alpha\hat{Q} + \beta\hat{Q}^2\Big)\mathbf{B},$$

$$m_B = M_0 + \frac{C_{HF}}{N_c}\hat{S}^2 - \frac{c_1}{\Lambda}2B_0(\sqrt{3}m_8Y + N_cm_0) - \frac{c_2}{\Lambda}4B_0m_0 - \frac{c_3}{N_c\Lambda^3}\left(4B_0(\sqrt{3}m_8Y + N_cm_0)\right)^2 - \frac{h_1}{N_c^2\Lambda}\hat{S}^4 - \frac{h_2}{N_c\Lambda}4B_0(\sqrt{3}m_8Y + N_cm_0)\hat{S}^2 - \frac{h_3}{N_c\Lambda}4B_0m_0\hat{S}^2 - \frac{h_4}{N_c\Lambda}\frac{4B_0m_8}{\sqrt{3}}\left(3\hat{I}^2 - \hat{S}^2 - \frac{1}{12}N_c(N_c+6) + \frac{1}{2}(N_c+3)Y - \frac{3}{4}Y^2\right) + \delta m_B^{\rm loop},$$

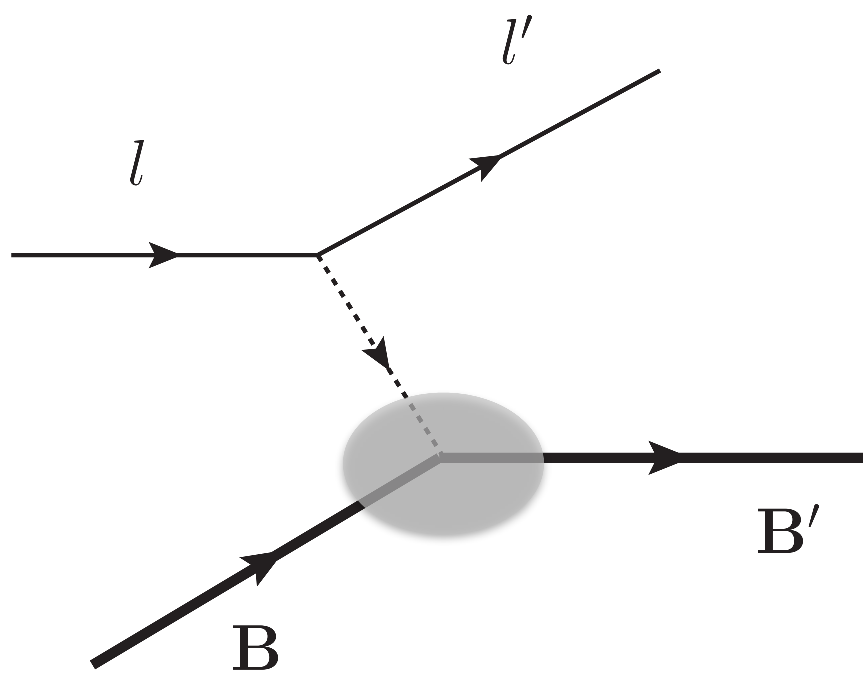
Fit	$\frac{\mathring{g}_A}{F_\pi}$ MeV^{-1}	$\frac{M_0}{N_c}$ MeV	C_{HF} MeV	c_1	c_2	h_2	h_3	h_4	α MeV	β MeV
1	0.0126(2)	364(1)	166(23)	−1.48(4)	0	0	0.67(9)	0.56(2)	−1.63(24)	2.16(22)
2	0.0126(3)	213(1)	179(20)	−1.49(4)	−1.02(5)	−0.018(20)	0.69(7)	0.56(2)	−1.62(24)	2.14(22)
3	0.0126*	262(30)	147(52)	−1.55(3)	−0.67(8)	0	0.64(3)	0.63(3)	−1.63*	2.14*

Fit	$\Delta_{GMO}^{\rm phys}$ MeV	σ_{8N} MeV	$\Delta\sigma_{8N}$ MeV	$\hat{\sigma}_N$ MeV	$\sigma_{\pi N}$ MeV	σ_{sN} MeV	$\sigma_{8\Delta}$ MeV	$\Delta\sigma_{8\Delta}$ MeV	$\hat{\sigma}_\Delta$ MeV
1	25.6(1.1)	−583(24)	−382(13)	70(3)(6)	–	–	−496(46)	−348(16)	59(5)(6)
2	25.5(1.5)	−582(55)	−381(20)	70(7)(6)	69(8)(6)	−3(32)	−511(52)	−352(22)	60(10)(6)
3	25.8*	−615(80)	−384(2)	74(1)(6)	65(15)(6)	−121(15)	−469(26)	350(27)	56(4)(6)

$$\sigma_{\pi N} = 69(10)\ \text{MeV} \qquad \sigma_{\pi\Delta} = 60(10)(6)\ \text{MeV}$$

5 Combined approach : Baryon currents

Hadronic weak currents possess V-A Lorentz structure of the weak interactions



It is important to know these axial and vector couplings in order to extract the standard model parameters for flavor mixings

$$J_\mu = V_\mu - A_\mu$$

$$\begin{aligned} V_\mu &= V_{ud}\bar{u}\gamma_\mu d + V_{us}\bar{u}\gamma_\mu s \\ A_\mu &= V_{ud}\bar{u}\gamma_\mu\gamma_5 d + V_{us}\bar{u}\gamma_\mu\gamma_5 s \end{aligned}$$

$$\begin{aligned} \langle B_2|V_\mu|B_1\rangle &= V_{CKM}\bar{u}_{B_2}(p_2)\left[f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_{B_1}}\sigma_{\mu\nu}q^\nu\right]u_{B_1}(p_1) \\ \langle B_2|A_\mu|B_1\rangle &= V_{CKM}\bar{u}_{B_2}(p_2)\left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_{B_1}}\sigma_{\mu\nu}q^\nu\right]\gamma_5 u_{B_1}(p_1) \end{aligned}$$

5 Combined approach : Baryon vector currents

- $$\mathcal{L}_B^{(2)} = \mathbf{B}^\dagger \left(\frac{c_2}{\Lambda}\chi_+^0 + \frac{C_1^A}{N_c} u^{ia} S^i T^a + \frac{\kappa}{2\Lambda} B_+^{ia} G^{ia} + \cdots \right) \mathbf{B}$$

where the flavor $SU(3)$ electric and magnetic fields are denoted by E_+ and B_+ and given by $E_+^i = F_+^{0i}$ and $B_+^i = \frac{1}{2}\epsilon^{ijk} F_+^{jk}$

- $$\mathcal{L}_B^{(3)} = \mathbf{B}^\dagger \left(\frac{g_1}{\Lambda^2} D_i E_{+i}^a T^a + \frac{\kappa_1}{2\Lambda N_c} B_+^{ia} S^i T^a + \cdots \right) \mathbf{B}$$

- $$\begin{aligned}\mathcal{L}_B^{(4)} = \mathbf{B}^\dagger \Bigg(& \frac{1}{N_c \Lambda^2} (g_2 D_i E_{+i}^a S^j G^{ja} + g_3 D_i E_{+j}^a \{S^i, G^{ja}\}^{\ell=2}) + \frac{\kappa_r}{\Lambda^3} D^2 B_+^{ia} G^{ia} \\ & + \frac{1}{2\Lambda^3} (\kappa_2 \chi_+^0 B_+^{ia} G^{ia} + i\kappa_F f^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_D d^{abc} \chi_+^a B_+^{ib} G^{ic} + \kappa_3 \chi_+^a B_+^{ia} S^i) \\ & + \frac{1}{2\Lambda N_c^2} (\kappa_4 B_+^{ia} \{\hat{S}^2, G^{ia}\} + \kappa_5 B_+^{ia} S^i S^j G^{ja}) + \cdots \Bigg) \mathbf{B}\end{aligned}$$

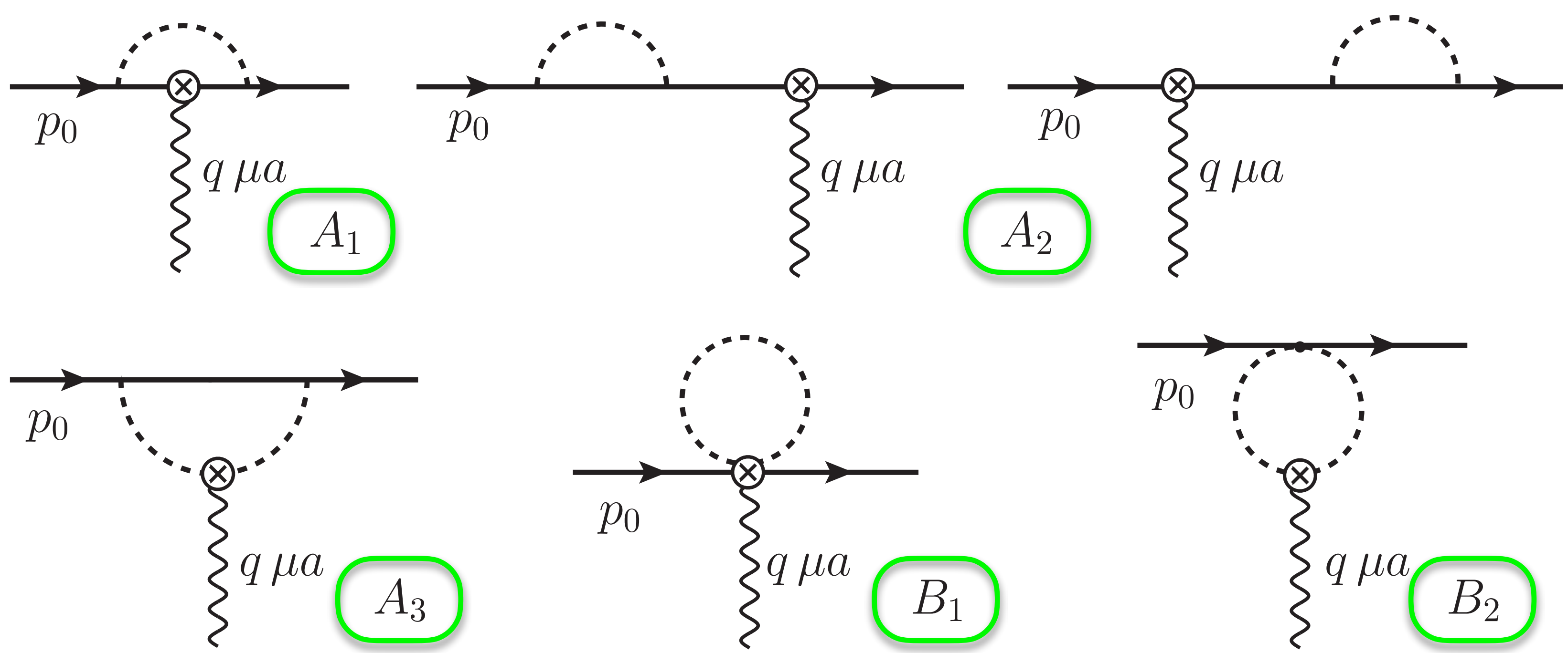
- The LECs g_1 and g_2 will be determined by charge radii
- The term proportional to g_3 gives electric quadrupole moment for $\mathbf{10_B}$ and $\mathbf{10_B} \to \mathbf{8_B}$ transitions.
- The term proportional to κ_r gives contribution to magnetic radii
- The renormalization of the magnetic moments is provided by LECs $\kappa_{D,F,1,\cdots,5}$

5.1 Baryon vector currents

$$\boxed{f_1 = f_1^{SU(3)} + \delta f_1}$$

ONE LOOP CORRECTIONS

RUBÉN FLORES-MENDIETA AND JOSÉ L. GOITY PHYSICAL REVIEW D **90**, 114008 (2014)
I. P. FERNANDO and J. L. GOITY PHYS. REV. D **97**, 054010 (2018)



$$p_0 \to \delta m_{in}$$
$$p_0 + q_0 \to \delta m_{out}.$$

$$\delta \hat{m} \equiv \frac{C_{\mathrm{HF}}}{N_c}\hat{S}^2 - \frac{c_1}{2\Lambda}\hat{\chi}_+$$

$SU(3)$ breaking in $\delta\hat{m}$ are disregarded

$$q_0 = SU(3)\text{breaking mass difference} + \text{kinetic energy}$$

$$\Gamma^{\mu a} = \underbrace{g^{\mu 0} T^a}_{\text{charge}} + i\frac{\kappa}{\Lambda}\underbrace{\epsilon^{0\mu ij} f^{abc} f^{cbd} q^i G^{jd}}_{\text{magnetic moment}}$$

Ademollo-Gatto Theorem (AGT) is "satisfied" : The amplitude of vector currents in the $q^2 \to 0$ limit are uniquely predicted up to first order in symmetry breaking.

5.2 Baryon vector currents charges

At lowest order the charges are represented by the flavor generators T^a

One loop corrections are:

$$f_1 = f_1^{SU(3)} + \delta f_1$$

- UV finite ; $Q^2 \to 0$
- At $Q^2 \to$ finite : UV divergent terms renormalized via g_1 and g_2 in the Lagrangian

Results

- $\delta f_1 / f_1 = \mathcal{O}(1/N_c)$
- Dominant contribution to the Corrections are entirely by the non-analytic pieces

SU(3) breaking corrections to the $\Delta S = 1$ vector charges.

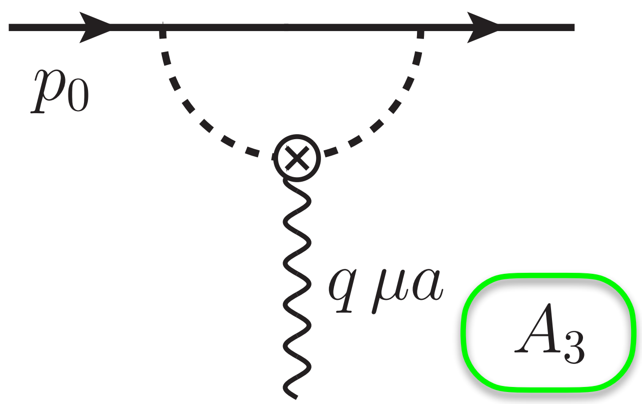
	$\frac{\delta f_1}{f_1}$ One-loop	(P. E. Shanahan et al) LQCD
Λp	$-0.067(15)$	$-0.05(2)$
$\Sigma^- n$	$-0.025(10)$	$-0.02(3)$
$\Xi^- \Lambda$	$-0.053(10)$	$-0.06(4)$
$\Xi^- \Sigma^0$	$-0.068(17)$	$-0.05(2)$

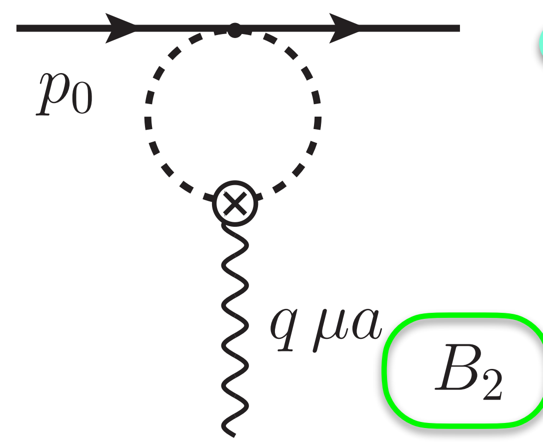
experimentally not enough precision from semi-leptonic hyperon decays to determine

5.3 Baryon charge radii

$$< r^2 >= -6\frac{df_1(Q^2)}{dQ^2}|_{Q^2\to 0}$$

Only the following diagrams contribute





- Charge operator for generic Nc

$$\hat{Q} = T^3 + \frac{1}{\sqrt{3}}T^8 + \frac{3-N_c}{6N_c}B$$

Some important observations (at strict large Nc limit)

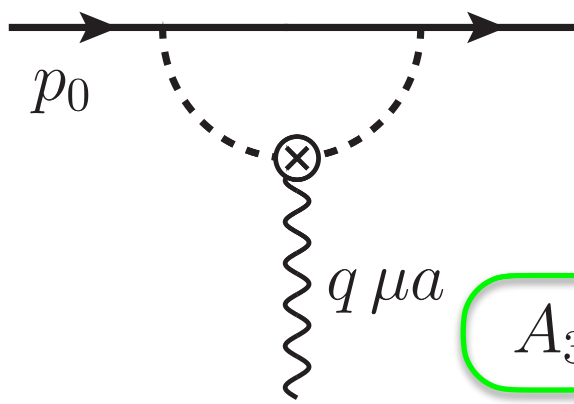
-

loop contributions\ T^a	T^3	T^8
Diagram A3	$\mathcal{O}(N_c^0)$	$\mathcal{O}(N_c^0)$
Diagram B2	$\mathcal{O}(1/N_c)$	$\mathcal{O}(N_c^0)$

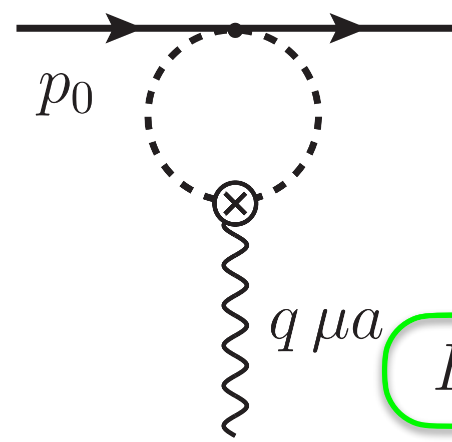
- Dominant non-analytic contributions to the radii are proportional to $\boxed{\log m_q}$

5.3 Baryon charge radii (Cont...)

$$< r^2 >= -6\frac{df_1(Q^2)}{dQ^2}|_{Q^2\to 0}$$



A_3



B_2

Contributions from Counter Terms (CT)
Satisfies the following relation

$$a\Lambda + p + \Sigma^+ + \frac{1}{3}(a-4)(n+\Sigma^0+\Xi^0) + \Sigma^- + \Xi^- = 0$$

resulting from the electric charge being a U-spin singlet

	$\langle r^2\rangle[\mathrm{fm}^2]$		
	Full	CT	Exp
p	0.707	0.596	0.7071(7)
n	−0.116	−0.049	−0.116(2)
Λ	−0.029	−0.024	$\cdots$
Σ^+	0.742	0.596	$\cdots$
Σ^0	0.029	0.024	$\cdots$
Σ^-	0.683	0.548	0.608(156)
Ξ^0	−0.016	−0.049	$\cdots$
Ξ^-	0.633	0.548	$\cdots$

proton radius used is the one resulting from the muonic Hydrogen Lamb shift

5.4 Baryon magnetic moments

LO magnetic moment is given by, $\boxed{\frac{\kappa}{2\Lambda} B_+^{ia} G^{ia}}$ in the $\mathcal{O}(\xi^2)$ Lagrangian

$$e\frac{\kappa}{2\Lambda} = \mu_p = 2.7928\ \mu_N$$

LO magnetic moment operator G^{ia} is proportional to the LO axial currents

LO ratios of magnetic moments

	Exp	LO
p/n	-1.46	-1.5
Σ^+/Σ^-	-2.12	-3
Λ/Σ^+	-0.25	$-\frac{1}{3}$
p/Σ^+	1.14	1
Ξ^0/Ξ^-	1.92	2
p/Ξ^0	-2.23	-1.5
Δ^{++}/Δ^+	$1.4(2.8)$	2
Ω^-/Δ^+	-0.75	-1
p/Δ^+	1.03	1
$p/(\Delta^+ p)$	0.78	$\frac{3}{2\sqrt{2}}$
$p/(\Sigma^{*0}\Lambda)$	1.02	$\sqrt{\frac{3}{2}}$
$p/(\Sigma^{*+}\Sigma^+)$	-0.88	$-\frac{3}{2\sqrt{2}}$

Note that the experimentally available magnetic moment ratios and corresponding LO results shows that the combined approach can describe well these ratios at LO

NLO effects stem from quark masses and spin symmetry breaking

$SU(3)$ breaking corrections $\mathcal{O}((m_s - \hat{m})N_c)$

spin symmetry breaking corrections $\mathcal{O}(1/N_c)$

5.4 Baryon magnetic moments

As an input proton and neutron magnetic moments giving the following relation between LECs

$$\kappa_1 = -19.662 + 6.926\,\kappa - 0.833\left(\kappa_4 + \frac{\kappa_5}{2}\right) + 2.550\,\kappa_D$$
$$\kappa_3 = -5.136 + 1.648\,\kappa - 0.218\left(\kappa_4 + \frac{\kappa_5}{2}\right) + \kappa_D.$$

LEC$\times\frac{m_N}{\Lambda}$	LO	NNLO
κ	2.80	2.887
κ_1	0	3.29
κ_2	0	0.00
κ_D	0	0.397
κ_F	0	$\cdots$
κ_3	0	0.53
κ_4	0	-2.85
κ_5	0	1.05

	μ_{LO}	μ_{NNLO}	μ_{Exp}
p	2.691	Input	2.792847356(23)
n	-1.794	Input	$-1.9130427(5)$
Σ^+	2.691	2.367	2.458(10)
Σ^0	0.897	0.869	$\cdots$
Σ^-	-0.897	-0.629	$-1.160(25)$
Λ	-0.897	-0.611	$-0.613(4)$
Ξ^0	-1.794	-1.275	$-1.250(14)$
Ξ^-	-0.897	-0.652	$-0.6507(25)$
$\Delta^+ p$	2.537	3.65	3.58(10)
$\Sigma^0\Lambda$	1.553	1.57	1.61(8)
$\Sigma^{*0}\Lambda$	2.197	2.68	2.73(25)[a]
$\Sigma^{*+}\Sigma^+$	-2.537	-2.35	$-3.17(36)$[b]

	μ_{LO}	μ_{NNLO}	μ_{Exp}
Δ^{++}	5.381	5.962	$3.7-7.5$
Δ^+	2.691	3.049	2.7(3.6)
Δ^0	0	0.136	$\cdots$
Δ^-	-2.691	-2.777	$\cdots$
Σ^{*+}	2.691	3.151	$\cdots$
Σ^{*0}	0	0.343	$\cdots$
Σ^{*-}	-2.691	-2.465	$\cdots$
Ξ^{*0}	0	0.490	$\cdots$
Ξ^{*-}	-2.691	-2.208	$\cdots$
Ω	-2.691	-2.005	$-2.02(5)$

Coleman Glashow (CG) relation $\mu_p - \mu_n - \mu_{\Sigma^+} + \mu_{\Sigma^-} + \mu_{\Xi^0} - \mu_{\Xi^-} = 0$

valid at tree level NNLO and receives only a finite correction from the one loop contributions.

5.5 Baryon magnetic radii

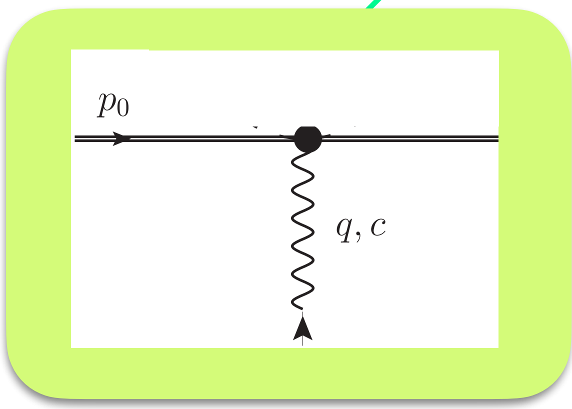
Only the magnetic radii of proton and neutron are experimentally known

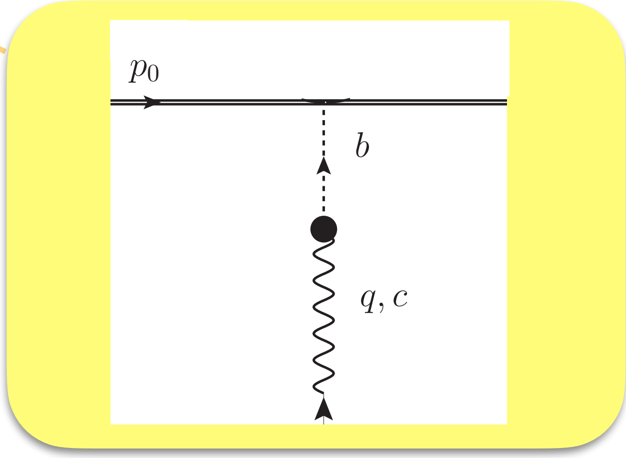
$\kappa_r = -2.63$	$\langle r^2 \rangle [\mathrm{fm}^2]$		
	Exp	Th	Loop
p	0.724	0.718	0.134
n	0.746	0.747	0.179
Σ^+	$\cdots$	0.766	0.100
Σ^0	$\cdots$	0.698	0.061
Σ^-	$\cdots$	0.922	0.189
Λ	$\cdots$	0.895	0.079
Ξ^0	$\cdots$	0.872	0.081
Ξ^-	$\cdots$	0.796	0.035
$\Delta^+ p$	$\cdots$	0.875	0.226

LQCD can test these predictions

6 Baryon axial-vector currents

At tree level $$A^{\mu c} = \mathring{g}_A G^{jc}\left(g^{\mu}_{j} - \frac{q^{\mu}q_j}{q^2 - M_b^2}\delta^{bc}\right) \xrightarrow{\text{In the large Nc limit}} A^{ic}$$





At the leading order, axial couplings are given in terms of $\mathring{g}_A$

Octet : $\boxed{F = \mathring{g}_A/3 \qquad D = \mathring{g}_A/2}$ Decuplet : $\boxed{\mathcal{H} = \mathring{g}_A/6}$

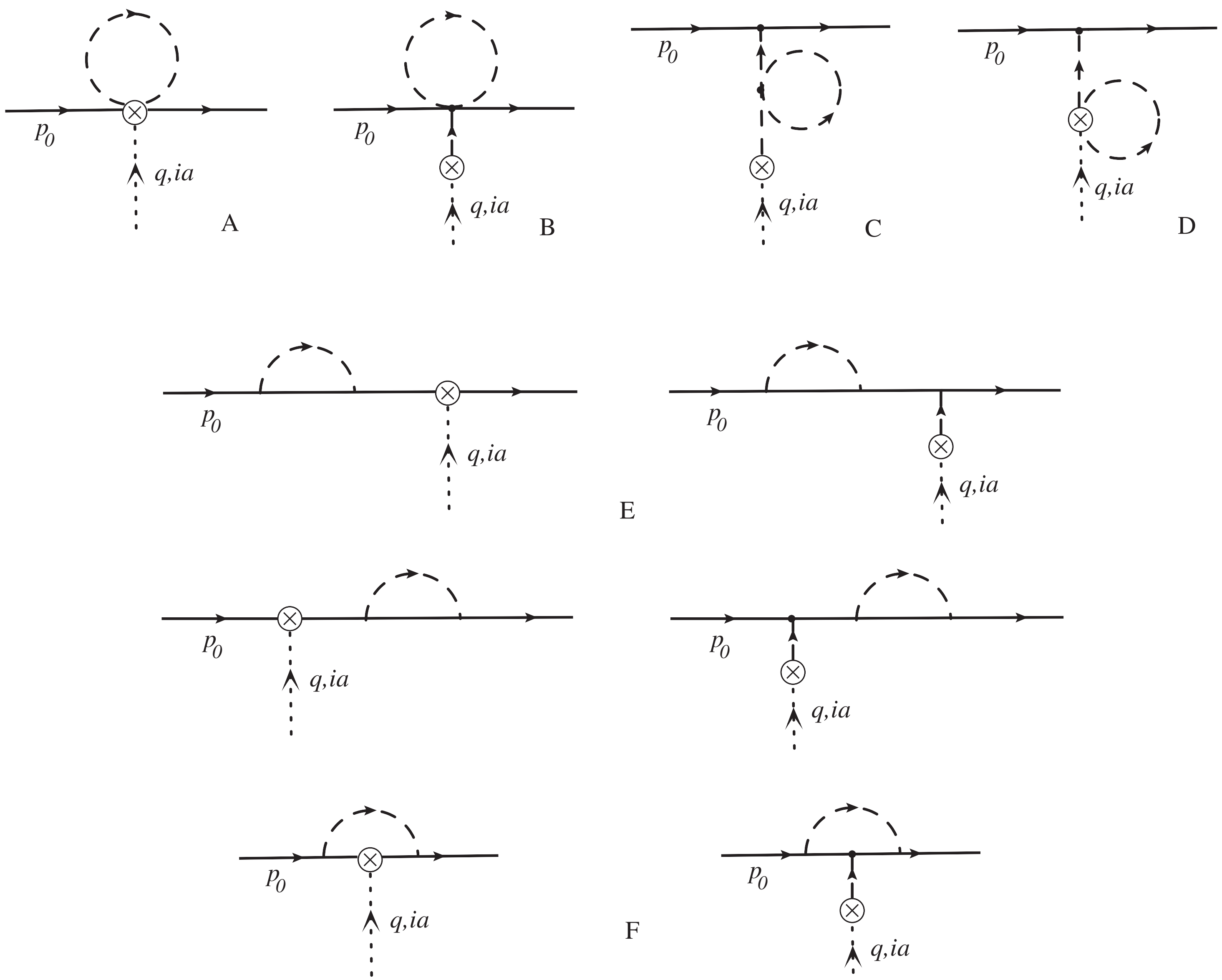
At vanishing 3-momentum $$\langle \mathbf{B}'|A^{ic}|\mathbf{B}\rangle = g_A^{\mathbf{BB}'}\frac{6}{5}\langle \mathbf{B}'|G^{ic}|\mathbf{B}\rangle$$

The calculations up to 1-loop corrections to the axial-currents are performed in SU(3)

$$A^{ic} + \delta A^{ic}_{\text{1-loop}}$$

Baryon axial-vector currents

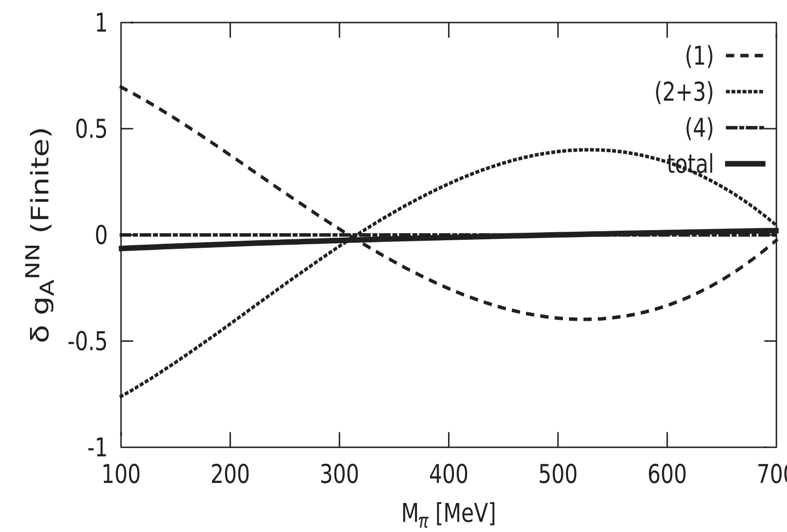
Diagrams contributing to the 1-loop corrections to the axial vector currents in SU(3)

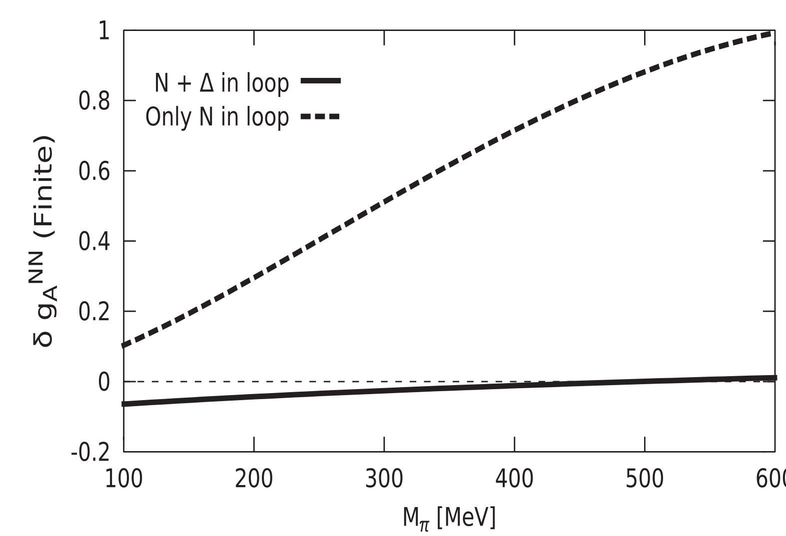


$$+\frac{C_1^A}{N_c} u^{ia} S^i T^a + \frac{C_2^A}{N_c} \epsilon^{ijk} u^{ia} \{S^j, G^{ka}\} + \frac{C_3^A}{N_c^2} u^{ia} \{\hat{S}^2, G^{ia}\} + \frac{C_4^A}{N_c^2} u^{ia} S^i S^j G^{ja}$$

$$+\frac{D_1^A}{\Lambda^2} \chi_+^0 u^{ia} G^{ia} + \frac{D_2^A}{\Lambda^2} \chi_+^a u^{ia} S^i + \frac{D_3^A(d)}{\Lambda^2} d^{abc} \chi_+^a u^{ib} G^{ic} + \frac{D_4^f(f)}{\Lambda^2} f^{abc} \chi_+^a u^{ib} G^{ic}$$

SU(2)





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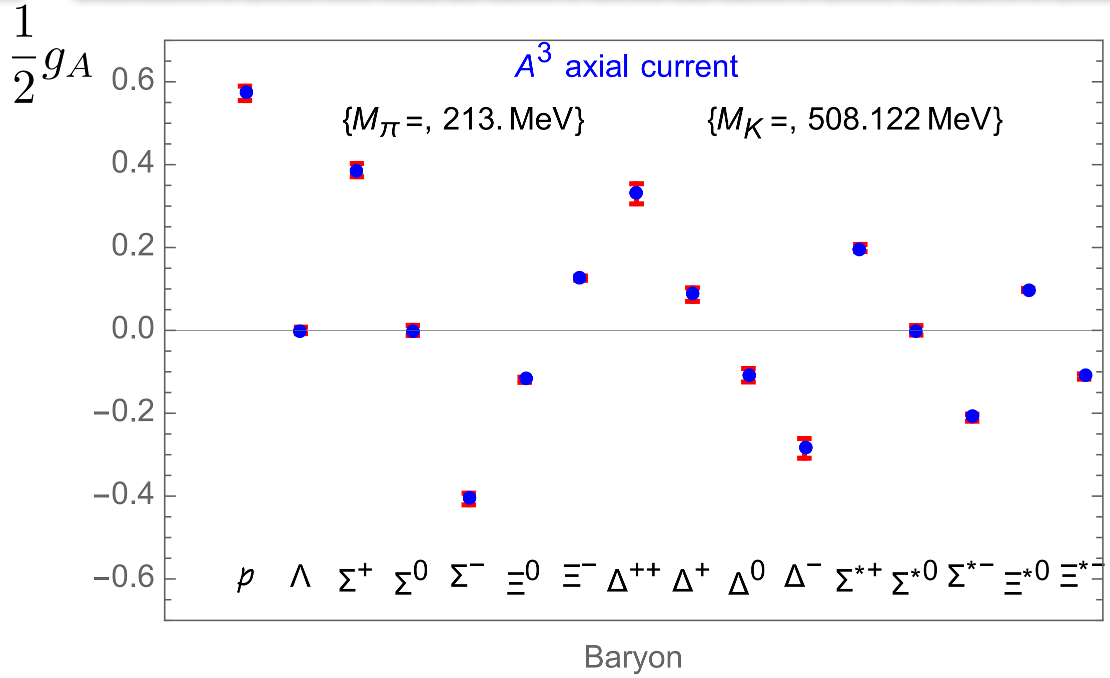
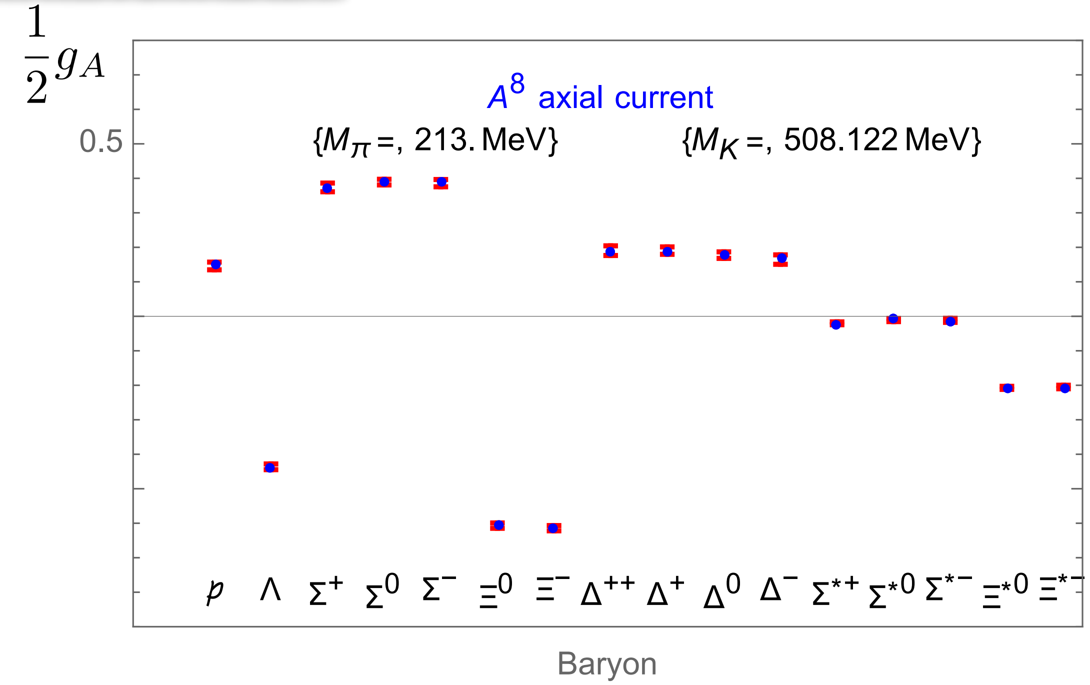
6

Baryon axial-vector currents : Fits to LQCD





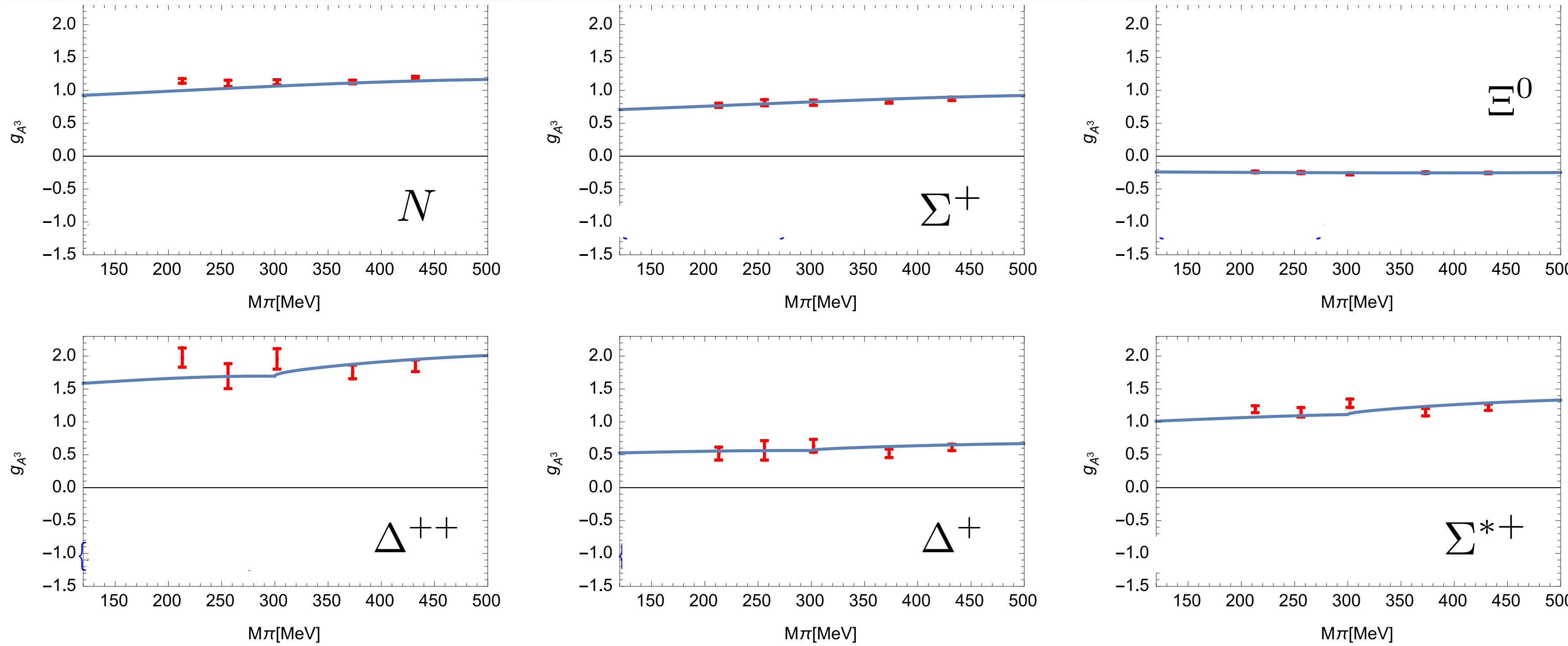
Fit results

C. Alexandrou et al. Phys. Rev. **D94**, 034502 (2016)

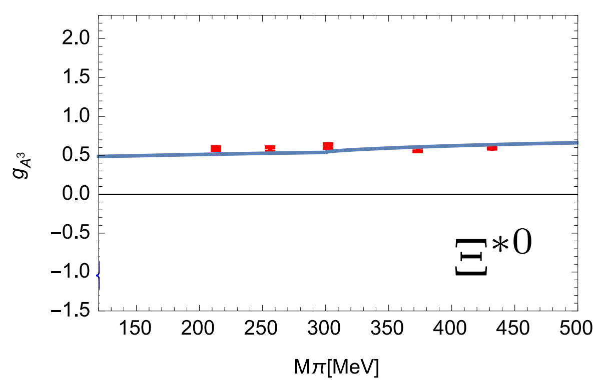
Fit	$\chi^2_{\rm dof}$	$\mathring{g}_A$	$\delta\mathring{g}_A$	C_1^A	C_2^A	C_3^A	C_4^A	D_1^A	D_2^A	D_3^A	D_4^A
LO	3.9	1.35	-	-	-	-	-	-	-	-	-
NLO Tree	0.91	1.42	-	-0.18	-	-	-	-	0.009	-	-
NLO Full	1.08	1.02	0.15	-1.11	0.	1.08	0.	-0.56	-0.02	-0.08	0.
	1.13	1.04	0.08	-1.17	0.	1.15	0.	-0.59	-0.02	-0.09	0.
	1.19	1.06	0.	-1.23	0.	1.21	0.	-0.62	-0.03	-0.09	0.

First LQCD calculations for baryon axial currents including hyperons and the decuplet :

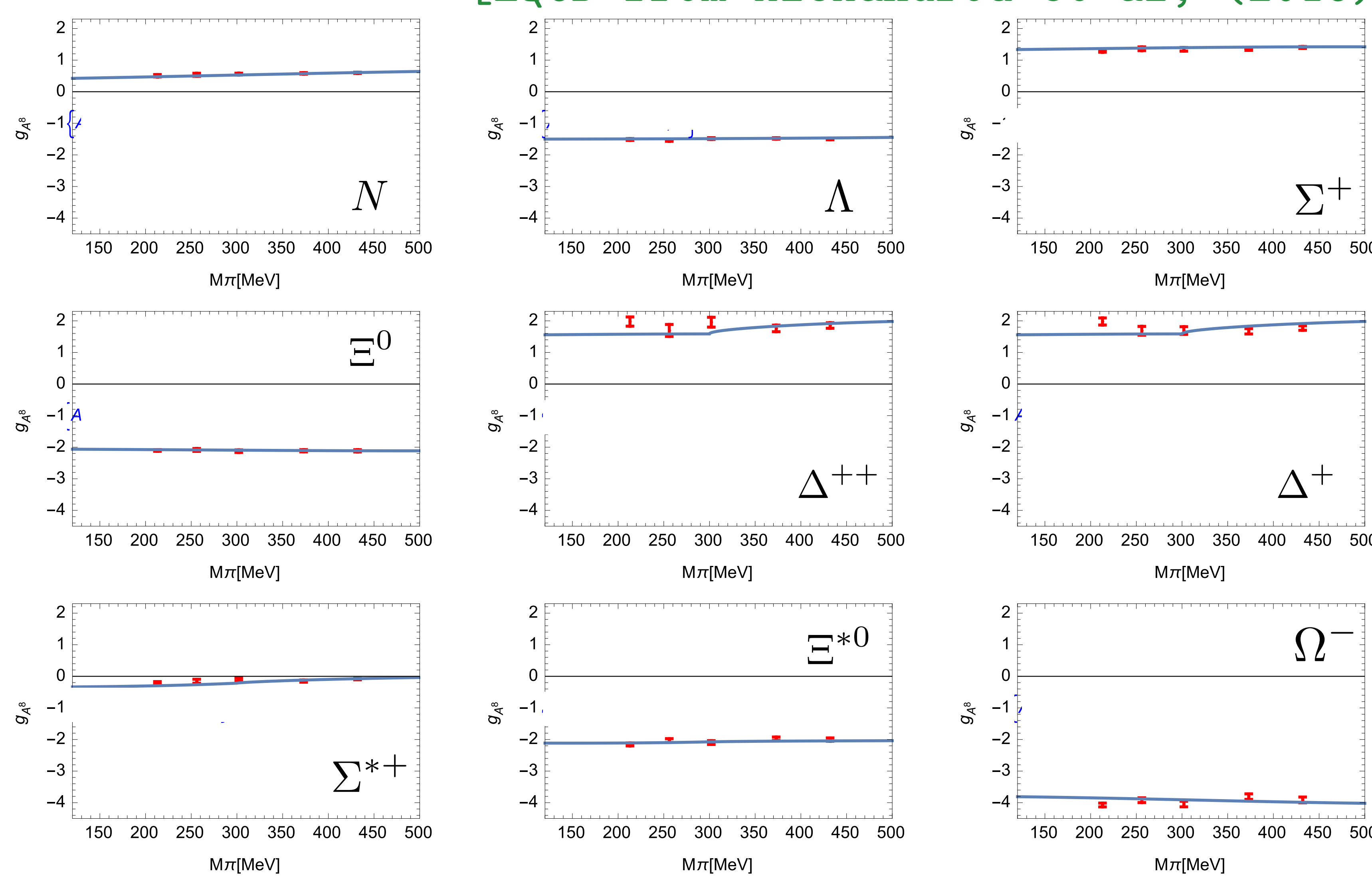
Baryon axial-vector currents : Fits to LQCD



g_A^3



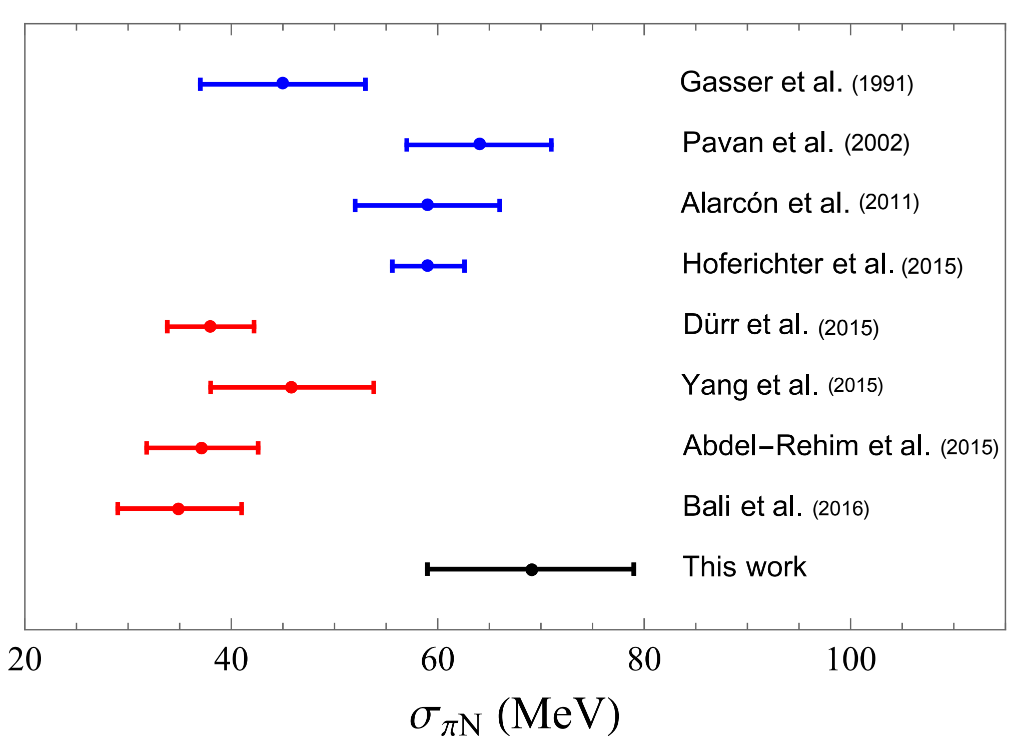
[LQCD from Alexandrou et al, (2016)]

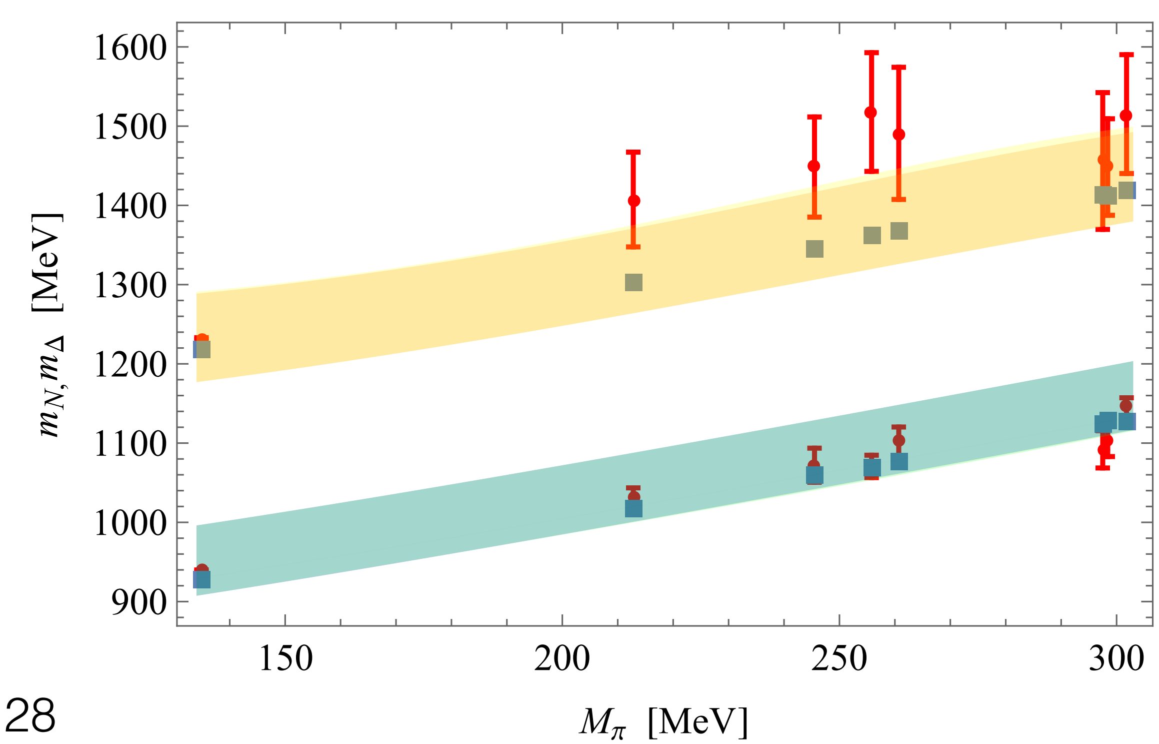


g_A^8

7 Summary

- The σ terms of nucleons were calculated using SU(3) BChPT × 1/Nc
- Our value for sigma Pi-N is in agreement with similar determinations in calculations that included the decuplet baryons as explicit degrees of freedom
- The "σ term puzzle" is understood as the result of large non-analytic contributions to the mass combination, while the higher order corrections to the σ terms have natural magnitude.
- The intermediate spin 3/2 baryons play an important role in enhancing $\hat{\sigma}$ and thus $\sigma_{\pi N}$
- The analysis carried out here shows that there is compatibility in the description of *GMO* and the nucleon σ terms
- The value of $\sigma_{\pi N} = 69 \pm 10$ MeV obtained here from fitting to Physical & LQCD baryon masses agrees with the more recent results from πN analyses





7

Summary (Cont...)

- $BChPT \times 1/N_c$ improves convergence by eliminating large N_c power power violating terms in loop corrections
- $SU(3)$ $BChPT \times 1/N_c$ shows a great improvement in describing charge, charge-radii, magnetic moments, magnetic-radii
- Only two LECs are needed to determine charge-radii of baryons
- Only eight LECs are needed to determine magnetic moments of baryons
- Only one LEC is needed to determine magnetic radii of baryons
- Axial couplings are also an important test of this approach
- More LQCD calculations are welcome, and current predictions can be used to test experimentally as well as in LQCD

Thank you