Charmonium and Charm physics on lattice (II)

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Outline

I. An overview of lattice QCD
II. Present Status of LQCD
III. Charm quark on lattice
IV. Charmonium and D meson spectrum
V. Charmonium radiative transitions and decays
VI. Summary and perspectives

III. Charm quark on lattice

- **Discretization error of lattice fermion actions** $\propto O((m_q a)^n)$

Generally, tested by spectral quantities

Naïve fermion (r=0): $E = m + O(m^2 a^2)$

Wilson fermion (r=1): $Ea = \log(1 + ma)$

For the typical lattice spacing of current dynamical simulations,

$$a \sim 0.1\, fm$$

$$m_c a \sim (1.4 GeV)(0.1\, fm) \sim 0.7$$

$$(1\, fm = (1.973 GeV)^{-1})$$

- **So, it is difficult to put heavy quarks on lattice straightforwardly**

Recent large-scale Nf=2+1 calculations

			(fm)	(fm)	(MeV)
■	MILC	Kogut-Susskind	0.06	4	220
■	PACS-CS	wilson-clover	0.09	3	155
■	BMW	wilson-clover	0.09	4	190
■	RBC-UKQCD	domain-wall	0.085	4	220
■	JLQCD	overlap	0.11	1.8	290

- **Non-relativistic QCD prescription**

$$S_{NRQCD} = \sum_x \phi^\dagger(x) \left(-\nabla_0^{(+)} + \frac{1}{2m}\sum_i \Delta_i + \frac{1}{2m}\sigma \cdot B(x) + \frac{1}{8m^3}(\sum_i \Delta_i)^2 + \dots \right) \phi(x),$$

where

$$\nabla_\mu^{(+)}\psi(x) \equiv \frac{1}{a}\left(U_\mu(x)\psi(x+a\hat{\mu}) - \psi(x)\right),$$

- **Improve relativistic fermion action**

In order to do high precision calculations, it is necessary to correct the fermion action to a higher order of ma by including more terms in the action.

Taking Wilson fermion for example, this can be realized by including a clover term (Sheikholeslami and Wohlert, 1985)

$$S_{SW} = \frac{iag}{4} c_{SW} \sum_x \bar{\psi}(x)\sigma_{\mu\nu}\mathcal{F}_{\mu\nu}(x)\psi(x)$$

Can be tuned or calculated in PT

$$O(ma) \rightarrow O((ma)^2)$$

• FermiLab fermion action for heavy quark on the lattice

$$S = \sum_n \bar{\psi}_n \psi_n - \kappa \sum_n \left[\bar{\psi}_n (1-\gamma_4) U_{n,4} \psi_{n+\hat{4}} + \bar{\psi}_{n+\hat{4}} (1+\gamma_4) U^\dagger_{n,4} \psi_n \right]$$
$$- \kappa\zeta \sum_{n,i} \left[\bar{\psi}_n (r_s - \gamma_i) U_{n,i} \psi_{n+\hat{\imath}} + \bar{\psi}_{n+\hat{\imath}} (r_s + \gamma_i) U^\dagger_{n,i} \psi_n \right]$$
$$- c_B \kappa\zeta \sum_n \bar{\psi}_n i \Sigma \cdot B_n \psi_n - c_E \kappa\zeta \sum_{n;i} \bar{\psi}_n \alpha \cdot E_n \psi_n,$$

In the Fermilab method, a version of the clover Wilson action, but the results are interpreted with (dimensionally regulated and MS-renormalized) NRQCD with modified short-distance coefficients. This is possible because Wilson fermions possess heavy-quark symmetry, and the proposed improvements preserve this feature. Then the parallel structure of the NRQCD descriptions of QCD and lattice gauge theory are used to match the latter to the former. In both frameworks, the lattice action can be systematically improved via the nonrelativistic expansion (FermiLab & MILC Collab. arXiv:09122701)

Breaking the space and time interchange symmetry

IV. Charmonium spectrum

- Charmonium and bottomium properties are relatively well understood by potential models and effective theories.

- Charmonium and bottomium spectrum can be a good testbed to check techniques of lattice QCD.

- **Charmonium spectrum**

$$C(t) = \langle 0 | \sum_{x} \mathcal{O}_f(\boldsymbol{x}, t) \mathcal{O}_i(0) | 0 \rangle = \sum_{n} \frac{\langle 0 | \mathcal{O}_f | n \rangle \langle n | \mathcal{O}_i | 0 \rangle}{2 E_n} \, e^{-E_n t} \, .$$

Charmonium spectrum for quenched LQCD

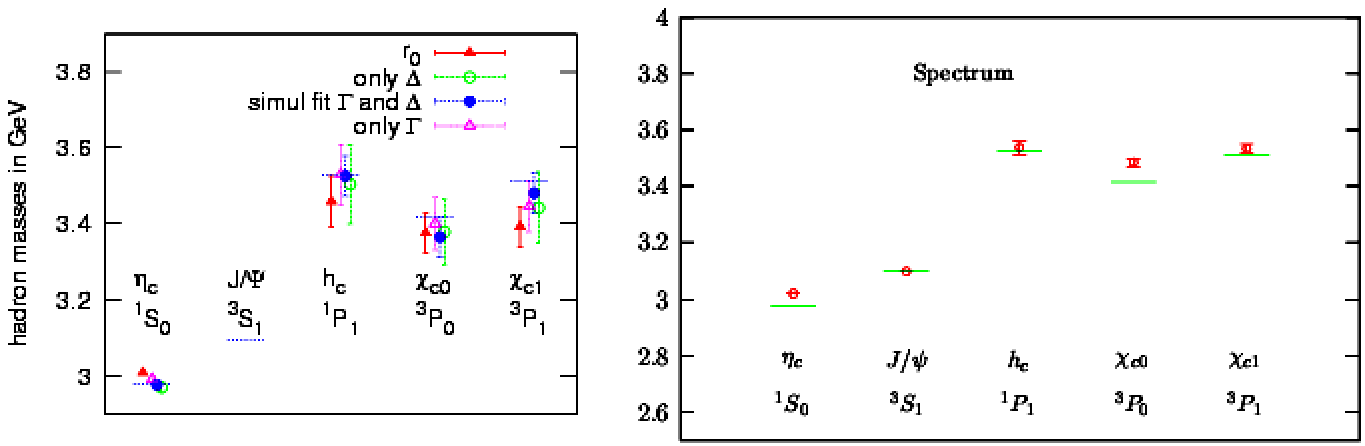


Figure 10: The left panel is chiQCD Collab.'s result from overlap fermion(hep-lat/0407027). The left panel is the result of QCD-TARO(hep-lat/0307004) using Wilson type fermions.

Latest results of onium mass splittings (Fermilab&MILC, arXiv:09122701)

TABLE IX: Continuum extrapolations of splittings in charmonium and bottomonium in MeV. The first error comes from statistics and accumulated extrapolation systematics; the second comes from the uncertainty in scale setting with $r_1 = 0.318^{+0.000}_{-0.007}$ fm.

Splitting	Charmonium		Bottomonium	
	This work	Experiment	This work	Experiment
$\overline{1P}$-$\overline{1S}$	$473 \pm 12^{+10}_{-0}$	457.5 ± 0.3	$446 \pm 18^{+10}_{-0}$	456.9 ± 0.8
$^1P_1$-$\overline{1S}$	$469 \pm 11^{+10}_{-0}$	457.9 ± 0.4	$440 \pm 17^{+10}_{-0}$	—
$\overline{2S}$-$\overline{1S}$	$792 \pm 42^{+17}_{-0}$	606 ± 1	$599 \pm 36^{+13}_{-0}$	$(580.3 \pm 0.8)^a$
1^3S_1-1^1S_0	$116.0 \pm 7.4^{+2.6}_{-0}$	116.4 ± 1.2	$54.0 \pm 12.4^{+1.2}_{-0}$	69.4 ± 2.8
$1P$ tensor	$15.0 \pm 2.3^{+0.3}_{-0}$	16.25 ± 0.07	$4.5 \pm 2.2^{+0.1}_{-0}$	5.25 ± 0.13
$1P$ spin-orbit	$43.3 \pm 6.6^{+1.0}_{-0}$	46.61 ± 0.09	$16.9 \pm 7.0^{+0.4}_{-0}$	18.2 ± 0.2
$1S$ $\bar{s}Q$-$\bar{Q}Q$	$1058 \pm 13^{+24}_{-0}$	1084.8 ± 0.8	$1359 \pm 304^{+31}_{-0}$	1363.3 ± 2.2

[a] $\Upsilon(2S)$-$\overline{1S}$ instead of $\overline{2S}$-$\overline{1S}$.

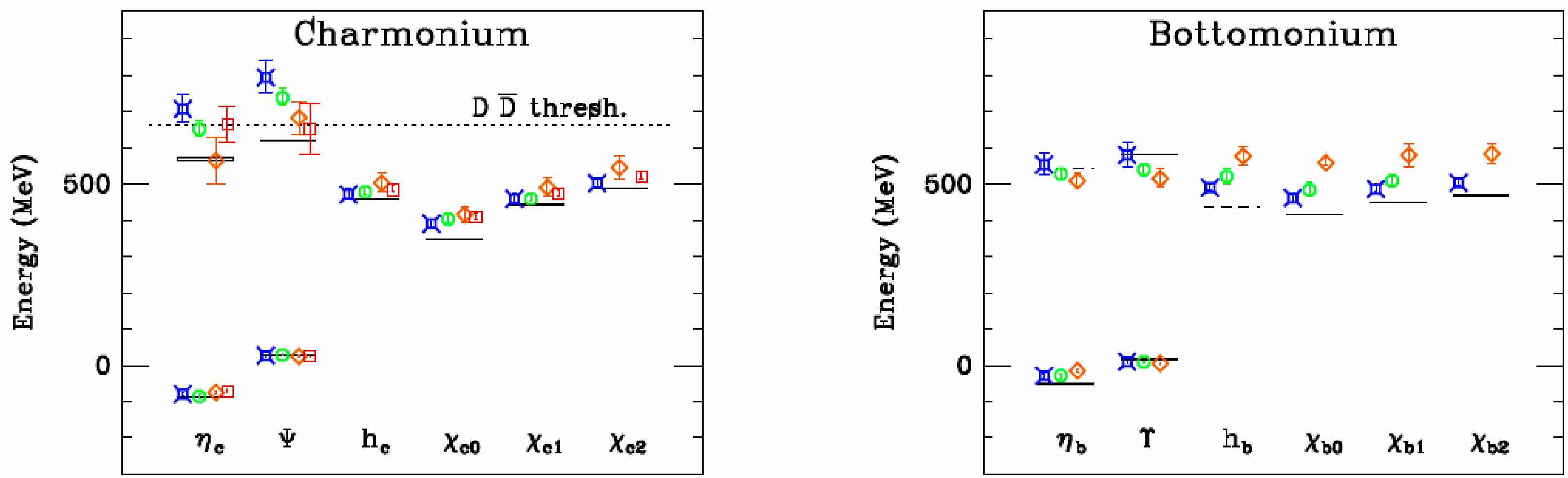
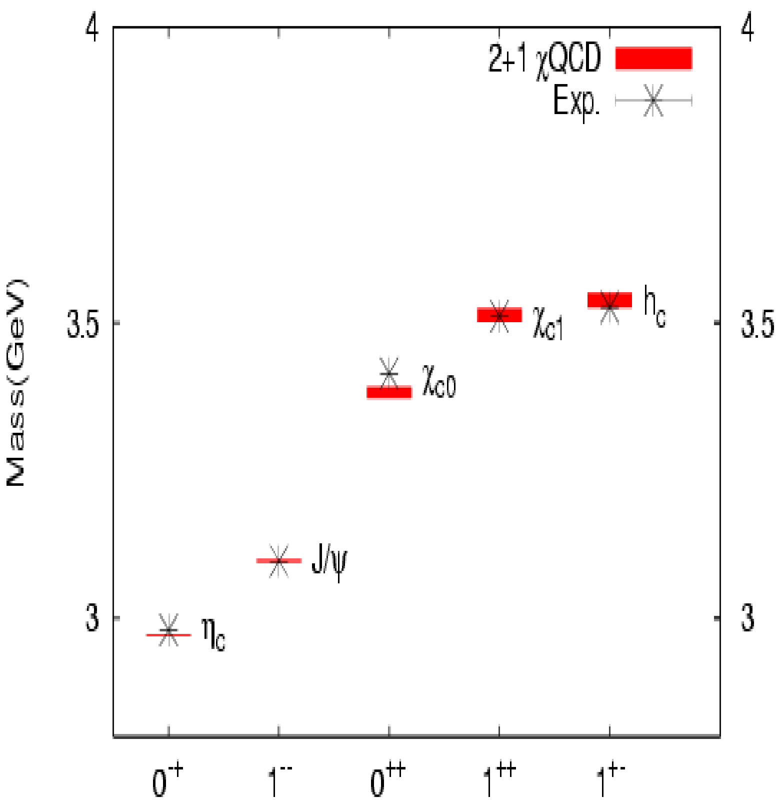
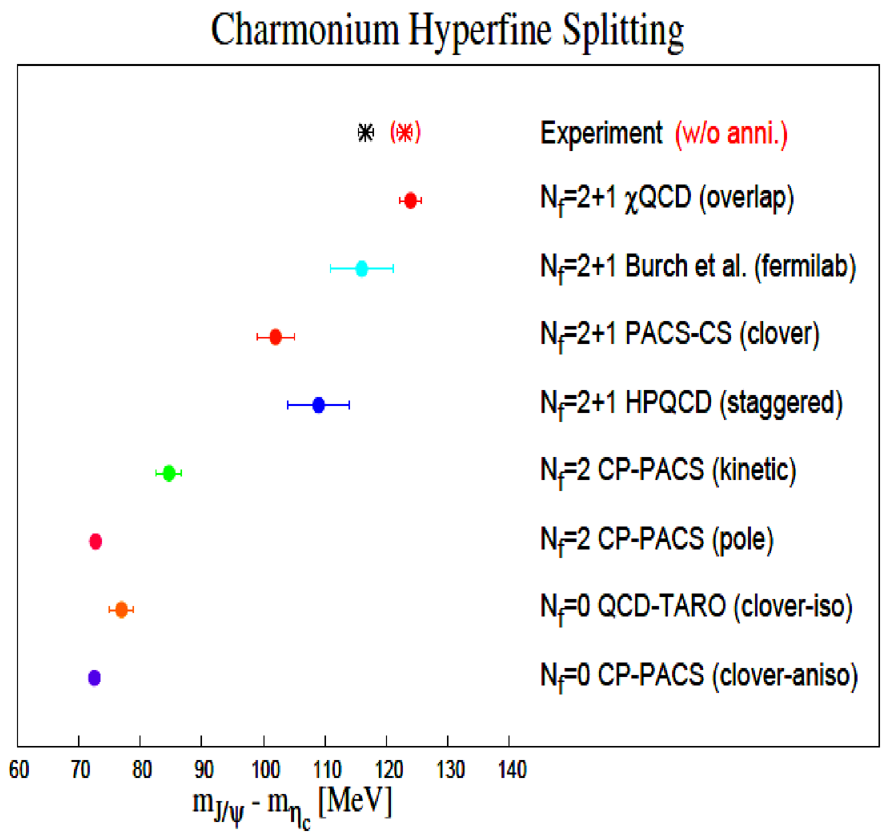


FIG. 17: Quarkonium spectrum as splittings from the $\overline{1S}$ level for $\bar{c}c$ (left) and $\bar{b}b$ (right). The fine-ensemble results are in blue fancy squares, the coarse in green circles, the medium-coarse in orange diamonds and the extra-coarse in red squares. Solid lines show the experimental values, and dashed lines estimates from potential models. The dotted line in the left panel indicates the physical open-charm threshold. The error on the data points combines statistical, κ-tuning, and r_1 uncertainties.

- Hyperfine splitting

2+1 DMF sea and overlap valence (chiQCD preliminary)





The exp't result of hpf is corrected by considering the omission of c-barc annihalation effect in LQCD calculation.

- **Hybrid charmonia**

$$\bar{c}cg$$

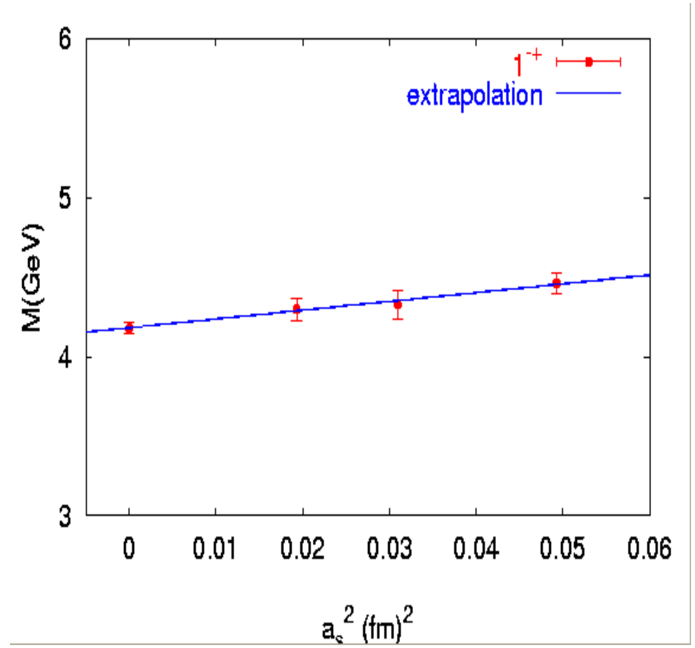
Operators always used to calculate 'hybrid' mesons on lattice

0_H^{-+}	0^{-+}	$\epsilon_{ijk}\bar{c}^a\gamma_i c^b F^{ab}_{jk}$
1_H^{--}	1^{--}	$\epsilon_{ijk}\bar{c}^a\gamma_5 c^b F^{ab}_{jk}$
0_H^{+-}	0^{+-}	$\bar{c}^a\gamma_5\gamma_i c^b \epsilon_{ijk} F^{ab}_{jk}$
0_H^{--}	0^{--}	$\bar{c}^a\gamma_5\gamma_i c^b F^{ab}_{i0}$
1^{-+}	1^{-+}	$\bar{c}^a\gamma_j c^b F^{ab}_{ji}$

'exotic' quantum number

For q-barq sys.

$$P = (-)^{L+1}$$
$$C = (-)^{L+S}$$



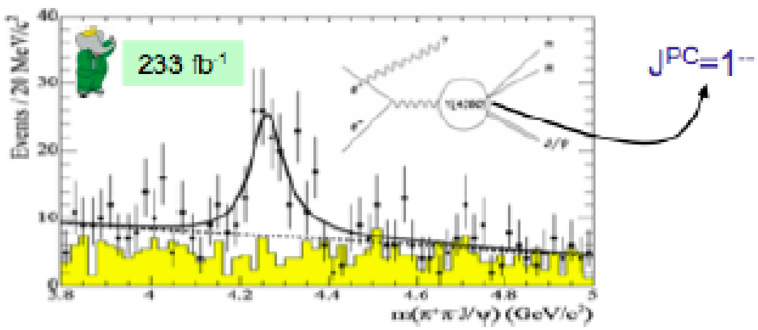
$$M(1^{-+}{}_{c\bar{c}}) = 4.18(3)GeV$$

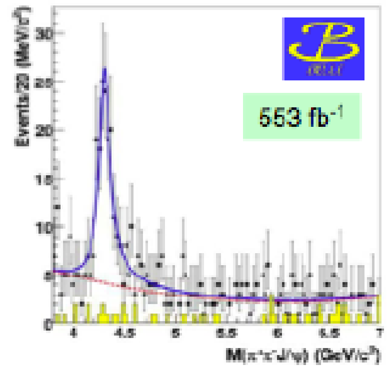
(CLQCD,in quenched approximation)

Y(4260)——A hybrid charmonium?

Experimental facts:

- New resonance discovered in $e^+e^- \to \gamma_{ISR}(J/\psi\pi^+\pi^-)$ by **BaBar**





- BaBar measures: $M = (4259 \pm 8)$ MeV/c^2, $\Gamma = (88 \pm 23)$ MeV
- Belle measures: $M = (4295 \pm 10\ ^{+10}_{-3})$ MeV/c^2, $\Gamma = (133\ ^{+26}_{-22}\ ^{+13}_{-6})$ MeV
- Confirmed by CLEO: $M = (4283\ ^{+17}_{-16} \pm 4)$ MeV/c^2

- No evidence for:
  - $e^+e^- \to \gamma_{ISR}(D\bar{D})$, $e^+e^- \to \gamma_{ISR}(\phi\pi^+\pi^-)$, $e^+e^- \to \gamma_{ISR}(p\bar{p})$, $e^+e^- \to \gamma_{ISR}(J/\psi\gamma\gamma)$

- 3σ enhancement in B decays
  - $B^- \to YK^-$, $Y \to J/\psi\pi^+\pi^-$
  - **Needs confirmation**

BaBar: Phys. Rev. Lett. 95 (2005) 142001
Belle: hep-ex/0612006
BaBar: hep-ex/0607083
BaBar: PRD 73, 011101 (2006)

- $\bar{c}cg$ hybrids in 0^{-+} and 1^{--} channels

0-+ channel,

hybrid-hybrid correlation function (H-H);
meson-meson correlation function (M-M)

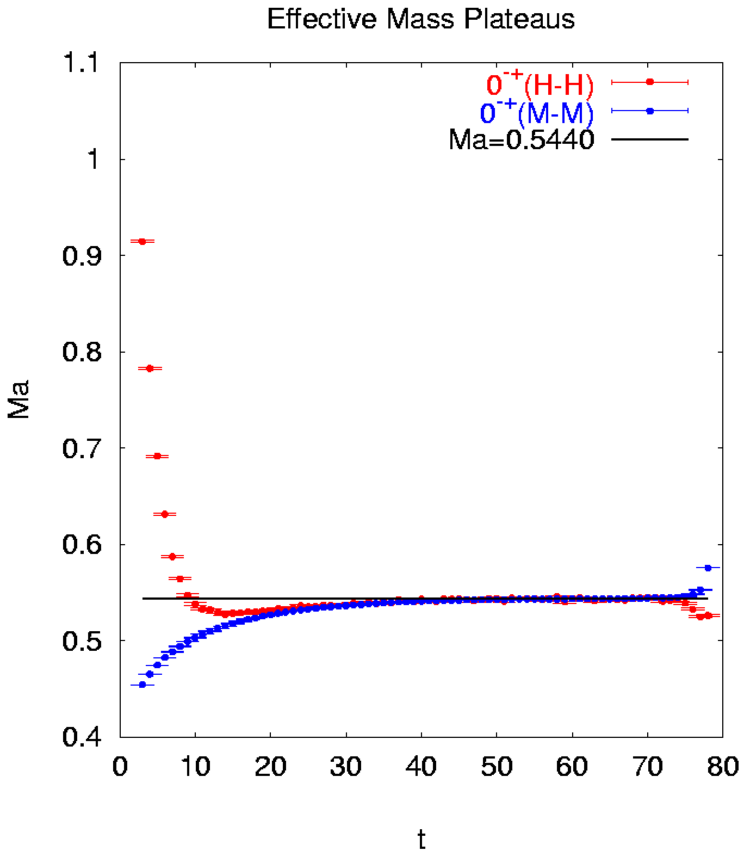
1-- channel,

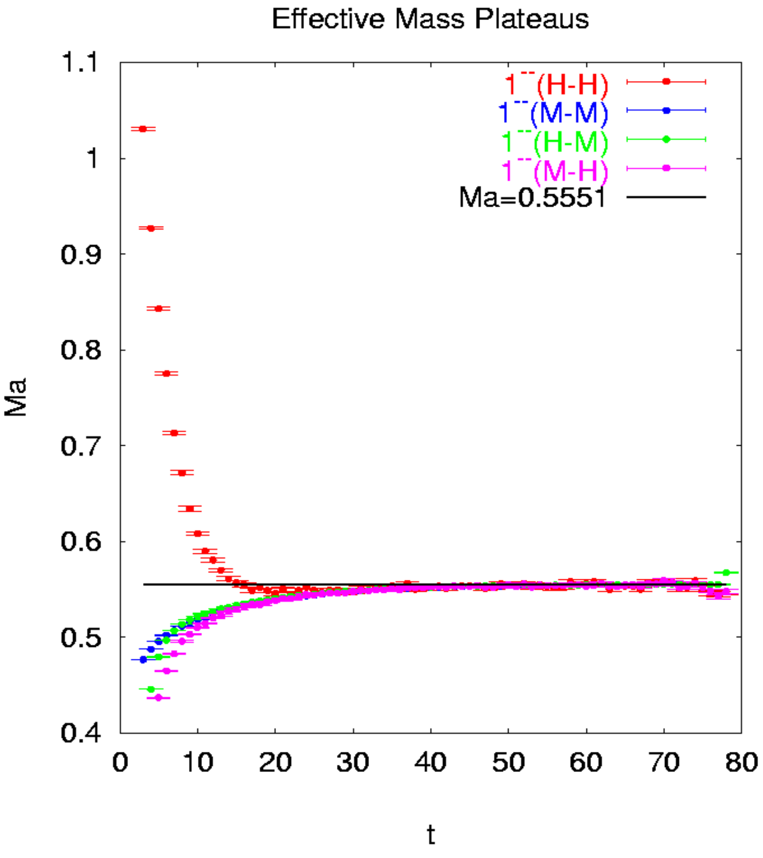
hybrid-hybrid correlation function (H-H);
meson-meson correlation function (M-M);
meson-hybrid correlation function (M-H);
hybrid-meson correlation function (H-M)

Intuitively, an operator can overlap to all the hadron states with the same quantum number of the operator. Therefore, These hybrid operators can couple to the conventional Charmonium states and also the hybrid states if they exist.

$$C(t) = \langle 0| \sum_{x} \mathcal{O}_f(\boldsymbol{x}, t)\mathcal{O}_i(0)|0\rangle = \sum_{n} \frac{\langle 0|\mathcal{O}_f|n\rangle\langle n|\mathcal{O}_i|0\rangle}{2E_n} e^{-E_n t}.$$

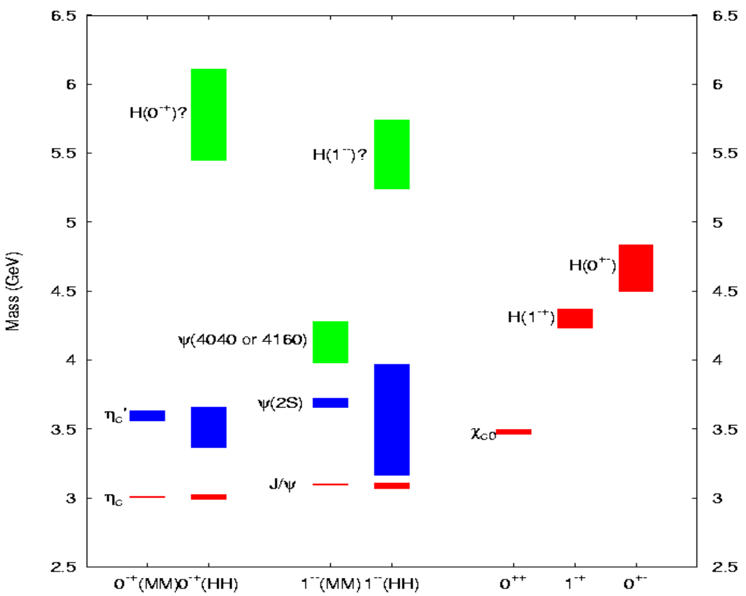
$$m_{eff} = \frac{1}{a} \ln \frac{C(t)}{C(t+1)}$$





Quenched results (CLQCD preliminary, 2006)

	MM (GeV)	HH (GeV)	Exp't (GeV)
η_c	3.009(2)	3.008(14)	2.980
J/ψ	3.100(2)	3.089(21)	3.097
χ_{c0}	3.480(17)		3.411
$\eta_c(2S)$	3.597(38)	3.51(15)	3.642
$\psi(2S)$	3.690(35)	3.57(40)	3.686
$\psi(3S???)$	4.13(15)		4.040
			4.160
$H(0^{-+})$		5.78(33)	
$H(1^{--})$		5.49(25)	
1^{-+}		4.30(7)	
0^{+-}		4.67(17)	



- Masses from the four-mass-term SEB fitting of hybrid-hybrid (HH) and meson-meson (MM) correlation functions in 1-- and 0-+ channels.
- It is understandable that the masses of the ground states are almost the same and the masses of the first excited states are consistent with each other, because the operators with the same quantum numbers can overlap to the same hadron states.
- The masses of the second excited state of HH are very different from those of MM
- No convincing results of masses of non-exotic hybrids can be derived in our work.

V. D(Ds) leptonic decay constants and semi-leptonic decay form factors

Decay constants: $\langle 0|A_\mu|H_q(p)\rangle = i f_{H_q} p_\mu$

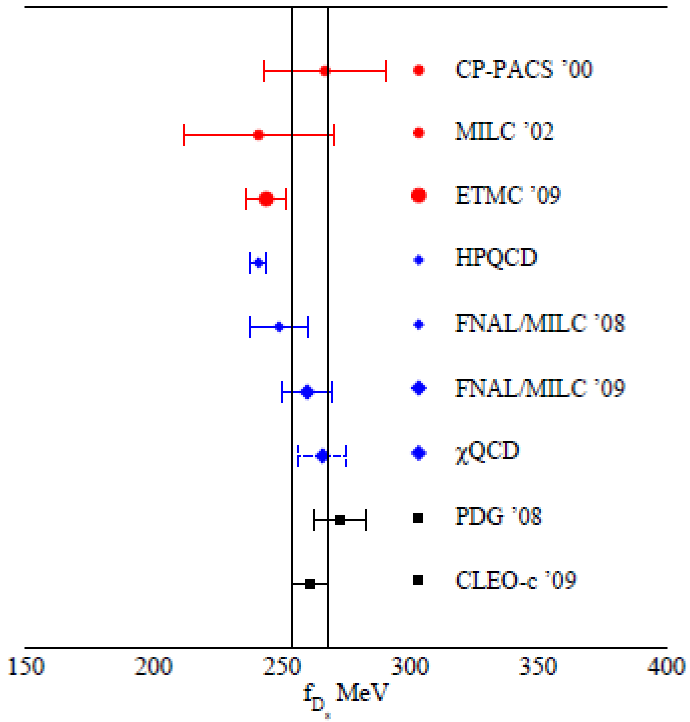
Which can be derived from the two-point function,

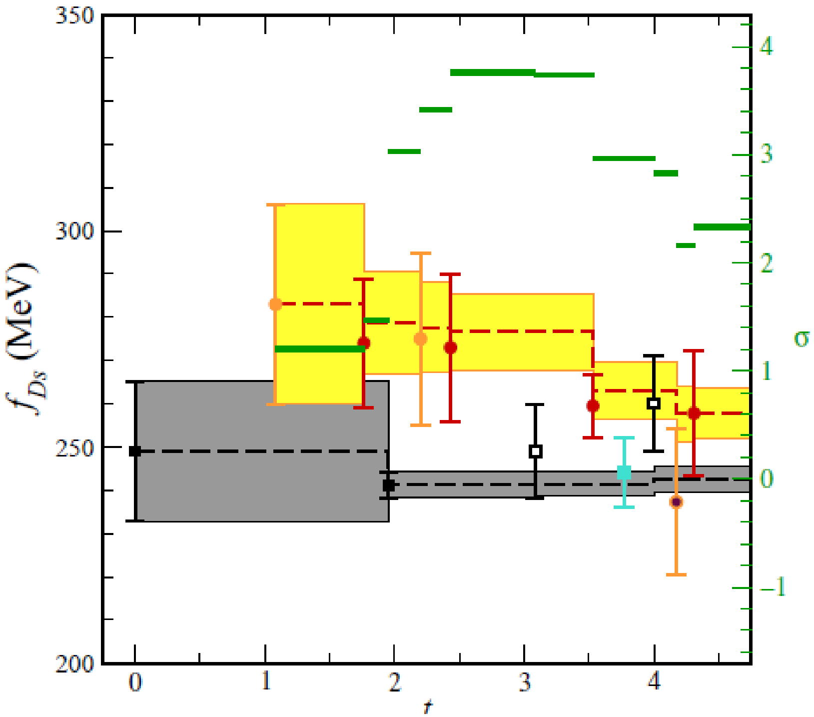
$$C_{A_4}(t) = \langle A_4(t) O^\dagger_{H_q}(0)\rangle$$

The axial current should be renormalized

$$\langle 0|\bar{s}\gamma_\mu\gamma_5 c|D_s\rangle = i f_{D_s} p_\mu,$$
$$(m_c + m_s)\langle 0|\bar{s}\gamma_5 c|D_s\rangle = -m^2_{D_s} f_{D_s}.$$

f_Ds puzzle or not?





Puzzle?

Grey bands indicate the HPQCD results (Kronfeld 2009)

TABLE IV: Recent absolute measurements of f_{D_s} from CLEO-c

Experiment	Mode	$\mathcal{B}$ (%)	f_{D_s} (MeV)
This result	$\tau^+\nu$ $(\rho^+\overline{\nu})$	$(5.52 \pm 0.57 \pm 0.21)$	$257.8 \pm 13.3 \pm 5.2$
CLEO-c [9]	$\tau^+\nu$, $(\pi^+\overline{\nu})$	$(6.42 \pm 0.81 \pm 0.18)$	$278.0 \pm 17.5 \pm 4.4$
CLEO-c [16]	$\tau^+\nu$ $(e^+\nu\overline{\nu})$	$(5.30 \pm 0.47 \pm 0.22)$	$252.6 \pm 11.2 \pm 5.6$
Average	$\tau^+\nu$	$(5.58 \pm 0.33 \pm 0.13)$	$259.7 \pm 7.8 \pm 3.4$
CLEO-c [9]	$\mu^+\nu$	$(0.565 \pm 0.045 \pm 0.017)$	$257.6 \pm 10.3 \pm 4.3$
Average	$\tau^+\nu$+$\mu^+\nu$		$259.0 \pm 6.2 \pm 3.0$

$$B(D_s \to \ell\nu) = \frac{m_{D_s}\tau_{D_s}}{8\pi} f_{D_s}^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 |G_F V_{cs}^* m_\ell|^2 ,$$

Latest results given by HPQCD

$$m_{D_s} = 1.9691(32) GeV$$
$$f_{D_s} = 0.2480(25) GeV$$
$$f_{\eta_c} = 0.3947(24) GeV$$

HPQCD 2010
arXiv:1008.4018

Error budget of HPQCD2010

TABLE V: Full error budget for m_{D_s}, f_{D_s} and f_{η_c} given as a percentage of the final fitted value. Note that in the case of f_{η_c} the top six errors are those to be considered for a lattice QCD calculation that matches this one. As discussed in the text, the bottom three errors are included for completeness.

Error	m_{D_s}	f_{D_s}	f_{η_c}
statistical/valence tuning	0.094%	0.57%	0.45%
r_1/a	0.025%	0.15%	0.16%
r_1	0.051%	0.57%	0.27%
a^2 extrapoln	0.044%	0.40%	0.24%
$m_{q,sea}$ extrapoln	0.048%	0.34%	0.09%
finite volume	0%	0.10%	0%
m_{η_s}	0.056%	0.13%	–
em effects in D_s	0.036%	0.10%	–
em and annihln in m_{η_c}	0.076%	0.00%	0.05%
em effects in η_c	–	–	0.40%
missing c in sea	0.01%	0%	0.01%
Total	0.16%	1.0%	0.6% (top 6)

- Form factors of D to K or pion

CKM matrix and PDG number (Amsler2008)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.9742 & 0.2257 & 3.59 \times 10^{-3} \\ 0.2256 & 0.9733 & 41.5 \times 10^{-3} \\ 8.74 \times 10^{-3} & 40.7 \times 10^{-3} & 0.9991 \end{pmatrix}$$

Lattice gold-plated processes which can be used to determine the CKM matrix elements

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell\nu & K \to \ell\nu & B \to \pi\ell\nu \\ & K \to \pi\ell\nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\ D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

Master equation:

$$Expt.\# = (\text{SM parameters})\,(\text{matrix elements})\,(\text{kinematic factors})$$

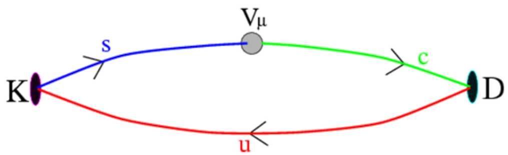
Decay constants,

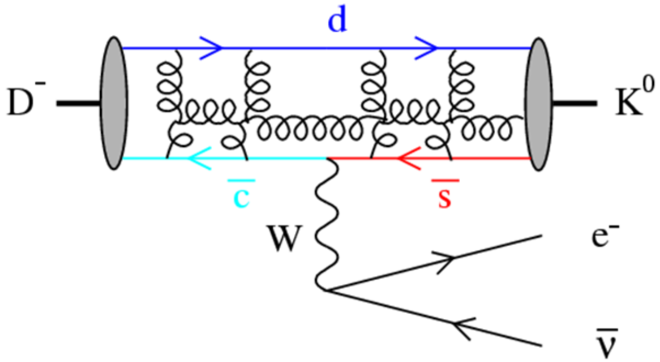
Form factors

$$B(D_s \to \ell\nu) = \frac{m_{D_s}\tau_{D_s}}{8\pi} f_{D_s}^2 \left(1 - \frac{m_\ell^2}{m_{D_s}^2}\right)^2 \left|G_F V_{cs}^* m_\ell\right|^2,$$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \left|V_{q_2 q_1}\right|^2}{192\pi^3 m_H^3} \left[\left(m_H^2 + m_P^2 - q^2\right)^2 - 4m_H^2 m_P^2\right]^{3/2} \left|f_+(q^2)\right|^2$$

Form factors





$$
\begin{aligned}
C_{3,\mu}(t_x, \mathbf{p}, t_y; \mathbf{q}) &= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+q\cdot y)} \langle 0 | T \left[K^+ V_\mu D^0 \right] | 0 \rangle \\
&= \sum_{\mathbf{x},\mathbf{y}} e^{-i(px+q\cdot y)} \langle 0 | \bar{s}(0)\gamma_5 u(0) \bar{c}(y)\gamma_\mu s(y) \bar{u}(x)\gamma_5 c(x) | 0 \rangle \\
&\sim \frac{\exp\left(E_K(\mathbf{p}+\mathbf{q})t_y - E_D(\mathbf{p})(t_x - t_y)\right)}{4E_K(\mathbf{p}+\mathbf{q})E_D(\mathbf{q})} \\
&\times \langle 0 | \bar{s}\gamma_5 u | K(\mathbf{p}+\mathbf{q}) \rangle \langle K(\mathbf{p}+\mathbf{q}) | \bar{c}\gamma_\mu s | D(\mathbf{p}) \rangle \langle D(\mathbf{p} | \bar{u}\gamma_5 c | 0 \rangle
\end{aligned}
$$

- The connection between the matrix elements and the form factors

$$\langle K|V^\mu|D\rangle = f_+^{D\to K}(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right] + f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu \qquad (\text{:}$$

Multiplying $q_\mu = p_\mu^D - p_\mu^K$, we have the identity $f_+(0) = f_0(0)$

- Alternatively, if we forcus on f+(0) for deriving Vcs

$$\langle K|S|D\rangle = \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}} f_0^{D\to K}(q^2) \quad \rightarrow \quad f_0^{D\to K}(q^2) = \frac{(m_{0c} - m_{0s})\langle K|S|D\rangle}{M_D^2 - M_K^2}.$$

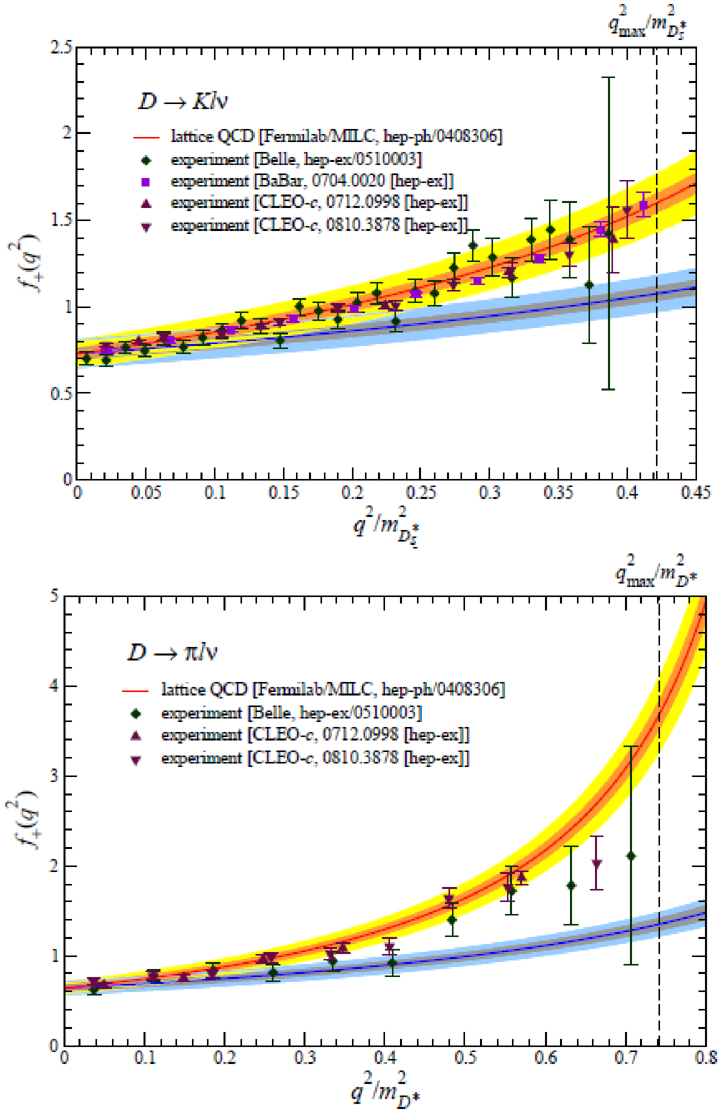
PCAC qurantees the renormaliztion invariant

- For parameterization

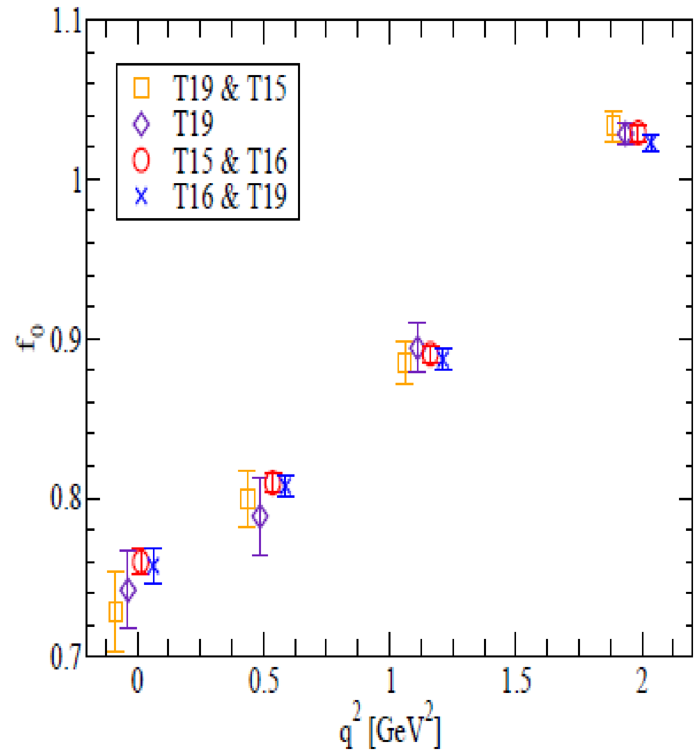
One choice is the Bećirević-Kaidalov (BK) ansatz

$$f_+(q^2) = \frac{F}{(1-\tilde{q}^2)(1-\alpha\tilde{q}^2)},$$

$$f_0(q^2) = \frac{F}{1-\tilde{q}^2/\beta},$$



Fermilab&MILC, 2009

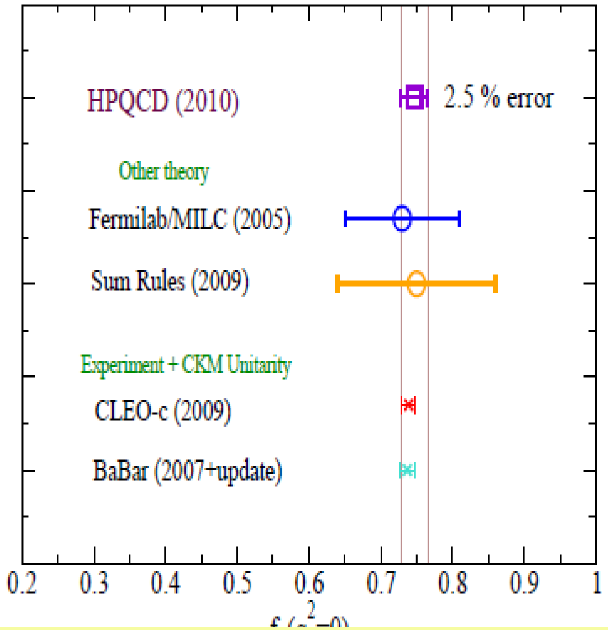


HPQCD 2010 (arXiv:1008.4562)

The final lattice result

$$f_+^{D\to K}(0) = 0.747 \pm 0.011 \pm 0.015.$$

Type	Error
Statistical	1.5 %
Lattice scale (r_1 and r_1/a)	0.2 %
Input meson mass	0.1 %
Light quark dependence	0.6 %
Strange quark dependence	0.7 %
Sea quark dependence	0.4 %
am_c extrapolation	1.4 %
aE_K extrapolation	1.0 %
Finite volume	0.01 %
Charm quark tuning	0.05 %
Total	2.5 %

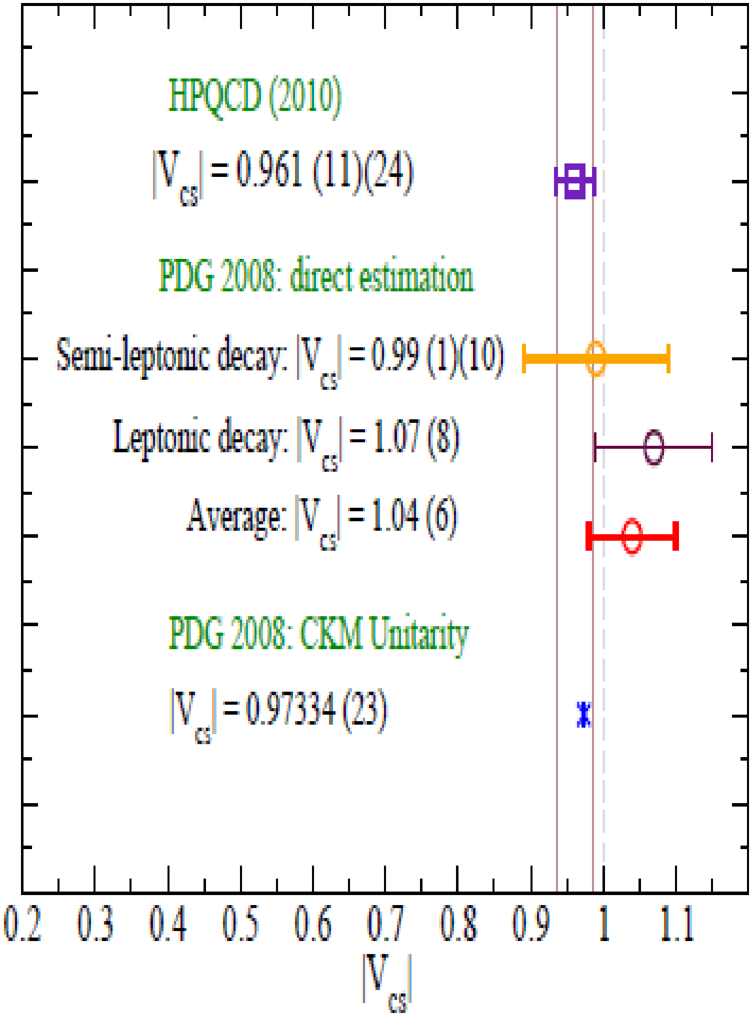


Using the lattice form factor as input, and combining CLEO_c and BaBar experiments,

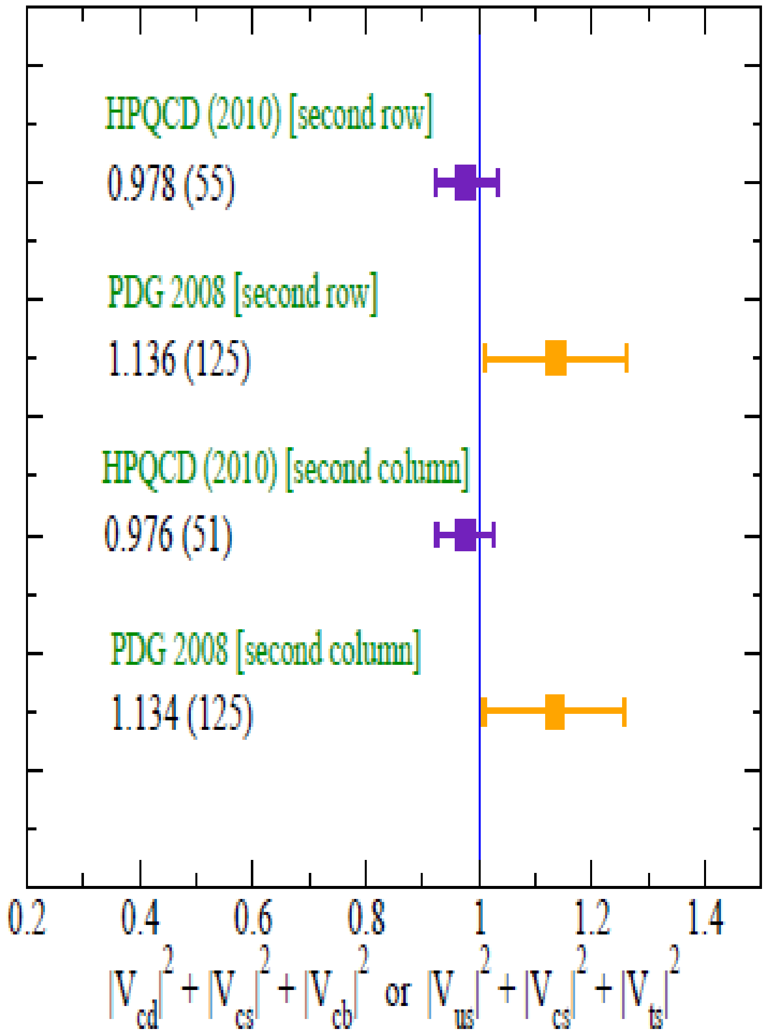
$$|V_{cs}| = 0.961 \pm 0.011 \pm 0.024,$$

CKM unitarity value

$$|V_{cs}| = 0.97334(23)$$



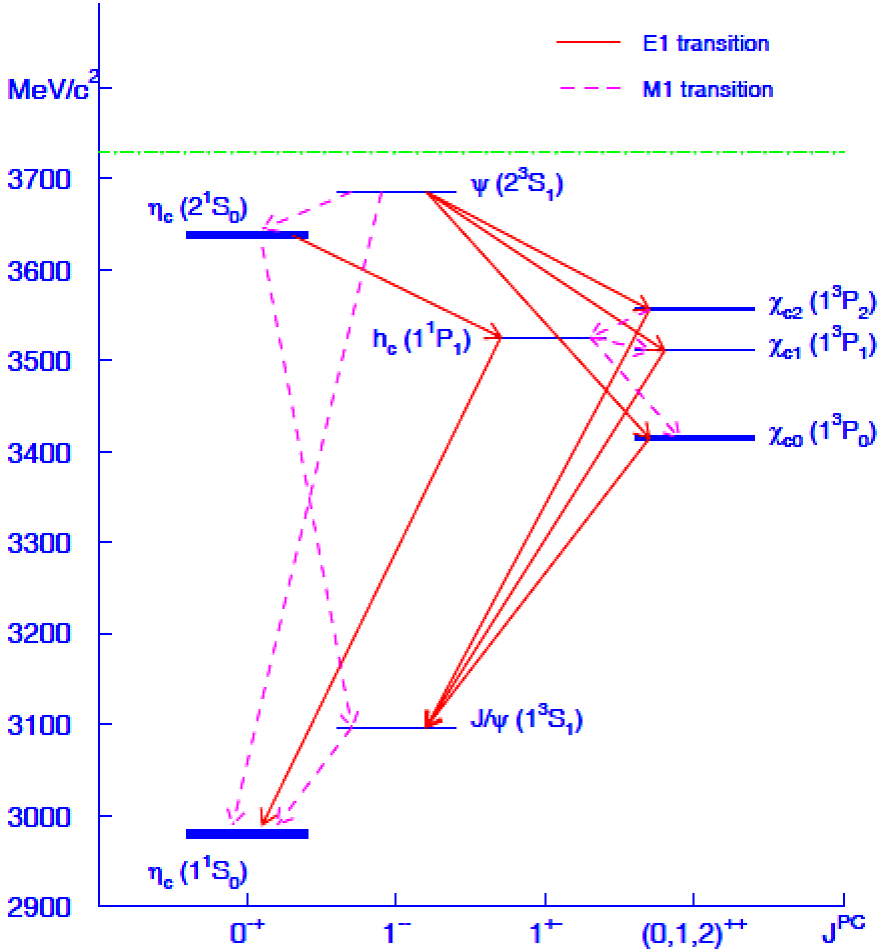
HPQCD (2010)
$|V_{cs}| = 0.961\ (11)(24)$
PDG 2008: direct estimation
Semi-leptonic decay: $|V_{cs}| = 0.99\ (1)(10)$
Leptonic decay: $|V_{cs}| = 1.07\ (8)$
Average: $|V_{cs}| = 1.04\ (6)$
PDG 2008: CKM Unitarity
$|V_{cs}| = 0.97334\ (23)$
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1
$|V_{cs}|$



HPQCD (2010) [second row]
0.978 (55)
PDG 2008 [second row]
1.136 (125)
HPQCD (2010) [second column]
0.976 (51)
PDG 2008 [second column]
1.134 (125)
0.2 0.4 0.6 0.8 1 1.2 1.4
$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2$ or $|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2$

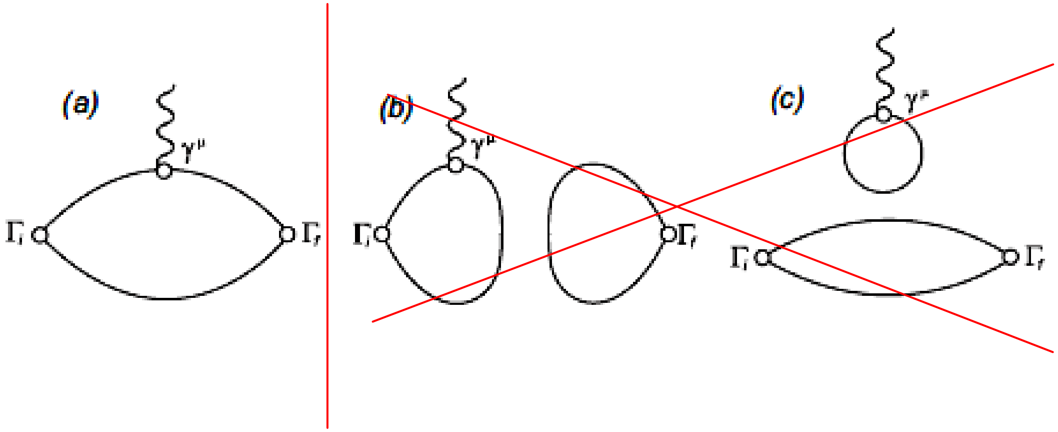
V. Charmonium radiative transitions and decays

- Charmonium radiative trnasitions



The Charmonium System

- Can be studied on lattice



$$\Gamma^{(3)}_{f\Gamma i}(\vec{p}_f, \vec{q}; t_f, t) = \sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f.\vec{x}} e^{+i\vec{q}.\vec{y}} \langle O_f(\vec{x}, t_f)\ \bar{\psi}\Gamma\psi(\vec{y}, t)\ O_i^\dagger(\vec{0}, 0)\rangle$$

State	$I^G(J^{PC})$	Operator
Scalar(σ)	$1^-(0^{++})$	$\overline{u}(x)d(x)$
	$1^-(0^{++})$	$\overline{u}(x)\gamma_4 d(x)$
Pseudoscalar	$1^-(0^{-+})$	$\overline{u}(x)\gamma_5 d(x)$
	$1^-(0^{-+})$	$\overline{u}(x)\gamma_4\gamma_5 d(x)$
Vector	$1^+(1^{--})$	$\overline{u}(x)\gamma_i d(x)$
	$1^+(1^{--})$	$\overline{u}(x)\gamma_i\gamma_4 d(x)$
Axial (a_1)	$1^-(1^{++})$	$\overline{u}(x)\gamma_i\gamma_5 d(x)$
Tensor(b_1)	$1^+(1^{+-})$	$\overline{u}(x)\gamma_i\gamma_j d(x)$

After the intermediate insertion, the three-point function takes the form,

$$\Gamma^{(3)}_{f\Gamma i}(\vec{p}_f, \vec{q}; t_f, t) = \sum_{f,i} \frac{e^{-E_f t_f} e^{-(E_i - E_f)t}}{2E_f(\vec{p}_f)\, 2E_i(\vec{p}_i)} \langle 0|\bar{\psi}\Gamma_f \psi(\vec{0},0)|f(\vec{p}_f, r_f)\rangle$$

$$\langle f(\vec{p}_f, r_f)|\bar{\psi}\Gamma\psi(\vec{0},0)|i(\vec{p}_i, r_i)\rangle \left(\langle 0|\bar{\psi}\Gamma_i\psi(\vec{0},0)|i(\vec{p}_i, r_i)\rangle\right)^*$$

This matrix element can be derived by combining the relevant two-point function,

$$\Gamma^{(2)}_{ij}(\vec{p}; t) \equiv \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle O_i(\vec{x}, t) O_j^\dagger(0)\rangle$$

$$= \sum_N \frac{Z_i^{(N)}(\vec{p}) Z_j^{(N)*}(\vec{p})}{2E^{(N)}} e^{-E^{(N)}t}$$

where

$$Z_i^{(N)}(\vec{p}) = \langle 0|O_i|N(\vec{p})\rangle.$$

- Some of the charmonium radiative transitions have been numerically studied by quenched lattice QCD

[1] Jozef Dudek, Robert Edwards, Christopher Thomas
'Radiative Transitions in Charmonium from Lattice QCD',
Phys. Rev. D 73:074507, 2006
arXiv: hep-ph/0601137

[2] Jozef Dudek, Robert Edwards, Christopher Thomas
'Exotic and excited-state radiative transitions
in charmonium from lattice QCD',
Phys. Rev. D 79:074504, 2009
arXiv:0902.2241(hep-lat)

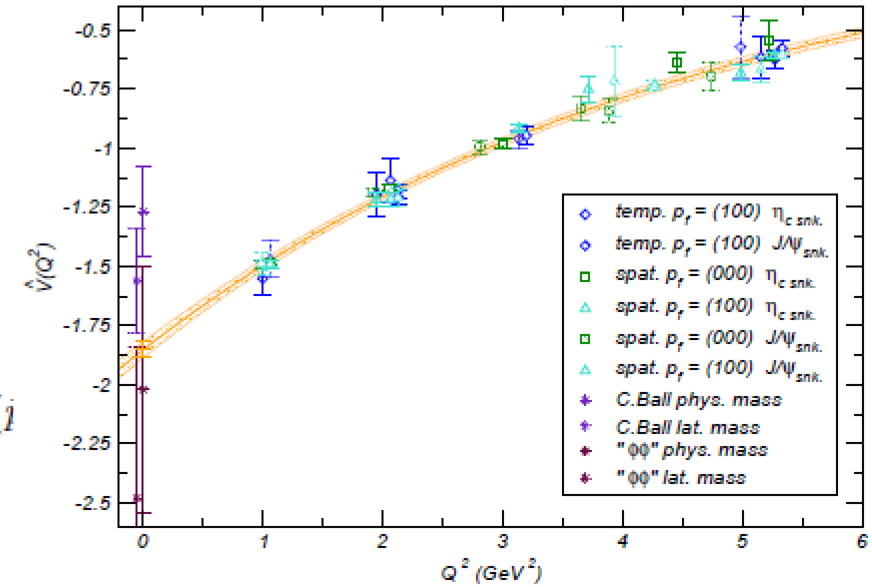
Charm quark prescription: aniotropic lattice with a_s/a_t>>1, such that

$$m_c a_t \sim 0.2 << 1$$

$J/\psi \to \eta_c \gamma$

$$\Gamma(J/\psi \to \eta_c\gamma) = \frac{1}{4\pi}\frac{|\vec{q}|^3}{(m_{\eta_c}+m_\psi)^2}\frac{4}{3}|V(0)|^2.$$

$$\langle \eta_c(\vec{p}\,')|j^\mu(0)|J/\psi(\vec{p},r)\rangle = \frac{2V(Q^2)}{m_{\eta_c}+m_\psi}\epsilon^{\mu\alpha\beta\gamma}p'_\alpha p_\beta \epsilon_\gamma(i$$



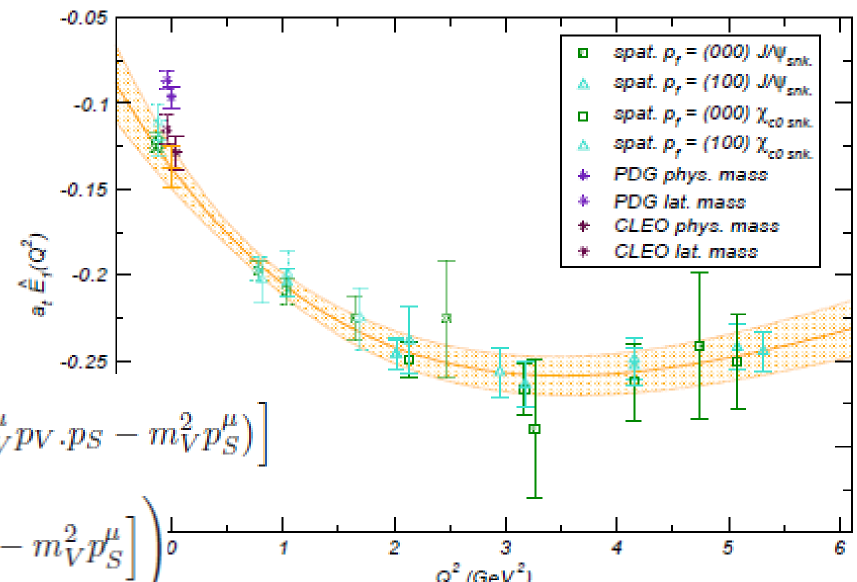
$\chi_{c0} \to \gamma J/\psi$

$$\Gamma(\chi_{c0} \to J/\psi\gamma) = \alpha\frac{|\vec{q}|}{m^2_{\chi_{c0}}}\frac{16}{9}|\hat{E}_1(0)|^2;$$

$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V,r)\rangle =$$

$$\Omega^{-1}(Q^2)\Bigg(E_1(Q^2)\Big[\Omega(Q^2)\epsilon^\mu(\vec{p}_V,r) - \epsilon(\vec{p}_V,r).p_S\left(p_V^\mu p_V.p_S - m_V^2 p_S^\mu\right)\Big]$$

$$+\frac{C_1(Q^2)}{\sqrt{q^2}}m_V\epsilon(\vec{p}_V,r).p_S\Big[p_V.p_S(p_V+p_S)^\mu - m_S^2 p_V^\mu - m_V^2 p_S^\mu\Big]\Bigg)$$



E1	$\chi_{c0} \to J/\psi\gamma$	$\chi_{c1} \to J/\psi\gamma$	$h_c \to \eta_c\gamma$
β/MeV	542(35)	555(113)	689(133)
ρ/MeV	1080(130)	1650(590)	∞
$\Gamma^{\mathrm{lat.mass}}_{\mathrm{phys.mass}}$/keV	288(60) 232(41)	600(178) 487(122)	663(132) 601(55)
$\Gamma^{\mathrm{PDG}}_{\mathrm{CLEO}}$/keV	115(14) 204(31)	303(44) 364(31)	-

M1	$J/\psi \to \eta_c\gamma$	**M2**	$\chi_{c1} \to J/\psi\gamma$
β/MeV	540(10)	β/MeV	617(142)
$\Gamma^{\mathrm{lat.mass}}_{\mathrm{phys.mass}}$/keV	1.61(7) 2.57(11)	$\frac{M2}{E1}$	$-0.199(121)$
$\Gamma^{\mathrm{PDG}}_{\phi\phi}$/keV	1.14(33) 2.9(1.5)	expt.	$-0.002(^{+8}_{-17})$

C1	$\chi_{c0} \to J/\psi\gamma$	$\chi_{c1} \to J/\psi\gamma$	$h_c \to \eta_c\gamma$
β/MeV	501(33)	502(38)	545(49)
$\lvert\tilde{c}\rvert$/GeV	11(1)	17.6(1.6)	17.5(1.1)

TABLE II: Radiative transitions

sink level	suggested transition	$a_t\hat{E}_1(0)$	β/MeV λ/GeV^{-2}	$\Gamma_{\text{lat}}/\text{keV}$	$\Gamma_{\text{expt}}/\text{keV}$
0	$\chi_{c0} \to J/\psi\gamma$	0.127(2)	409(12) 1.14(5)	199(6)	131(14)
1	$\psi' \to \chi_{c0}\gamma$	0.092(19)	164(55) 0[fixed]	26(11)	30(2)
3	$\psi'' \to \chi_{c0}\gamma$	0.265(33)	324(77) 0.58(56)	265(66)	199(26)
5	$Y_{\text{hyb.}} \to \chi_{c0}\gamma$	0.00(3)	linear fit	$\lesssim 20$	-

sink level	suggested transition	$\hat{V}(0)$	β/MeV λ/GeV^{-2}	$\Gamma_{\text{lat}}/\text{keV}$	$\Gamma_{\text{expt}}/\text{keV}$
0	$J/\psi \to \eta_c\gamma$	1.89(3)	513(7) 0[fixed]	2.51(8)	1.85(29)
1	$\psi' \to \eta_c\gamma$	0.062(64)	530(110) 4(6)	0.4(8)	0.95(16) 1.37(20)
3	$\psi'' \to \eta_c\gamma$	0.27(15)	367(55) $-1.25(30)$	10(11)	-
5	$Y_{\text{hyb.}} \to \eta_c\gamma$	0.28(6)	250(200) 0[fixed]	42(18)	-

- J/psi radiative decays to glueballs

CLQCD Collab.: Y. Chen et al, in progress.

- QCD predicts the existence of glueballs
- Quenched LQCD predicts glueball spectrum
  - Lowest-lying glueballs have masses in the range 1~3GeV
- Experimentally, f0(1370), f0(1500), f0(1710), etc., are glueball candidates, but decisive conclusion cannot be drawn.
- Due to its abundance of gluons, J/psi radiative decay can be the best hunting ground.
- BESIII in Beijing is producing 10^{10} J/psi events

J^{PC}	$r_0 M_G$	M_G (MeV)
0^{++}	4.16(11)(4)	1710(50)(80)
2^{++}	5.83(5)(6)	2390(30)(120)
0^{-+}	6.25(6)(6)	2560(35)(120)
1^{+-}	7.27(4)(7)	2980(30)(140)
2^{-+}	7.42(7)(7)	3040(40)(150)
3^{+-}	8.79(3)(9)	3600(40)(170)
3^{++}	8.94(6)(9)	3670(50)(180)
1^{--}	9.34(4)(9)	3830(40)(190)
2^{--}	9.77(4)(10)	4010(45)(200)
3^{--}	10.25(4))(10)	4200(45)(200)
2^{+-}	10.32(7)(10)	4230(50)(200)
0^{+-}	11.66(7)(12)	4780(60)(230)

Y. Chen et al,
Phys. Rev. D 73, 014516 (2006)

Numerical details

• Formalism

The decay width of J/psi radiatively decaying to the scalar glueball can be derived from the formular

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2$$

where E1(0) is the on-shell form factor, which appears in the matrix elements (J.J. Dudek, hep-lat/0601137)

$$\langle S(\vec{p}_S)|j^\mu(0)|V(\vec{p}_V,r)\rangle = \left(E_1(q^2)\left[\varepsilon^\mu(\vec{p}_V,r) - \varepsilon(\vec{p}_V,r)\cdot p_S \frac{p_V^\mu p_V\cdot p_S - m_V^2 p_S^\mu}{\Omega(q^2)}\right] + \frac{C_1(q^2)}{\sqrt{q^2\Omega(q^2)}} m_V \varepsilon(\vec{p}_V,r)\cdot p_S\left[p_V\cdot p_S(p_V+p_S)^\mu - m_S^2 p_V^\mu - m_V^2 p_S^\mu\right]\right)$$

With the vector current insertion $j^\mu = \bar{c}\gamma^\mu c$, these Matrix elements can be calculated through the three point function,

$$\Gamma^{(3)}(\vec{p}_f,\vec{q};t_f,t) = -\sum_{\vec{x},\vec{y}} e^{-i\vec{p}_f\cdot\vec{x}} e^{+i\vec{q}\cdot\vec{y}} \langle O_S(\vec{x},t_f) j^\mu(\vec{y},t) O_V^\dagger(0,0)\rangle$$

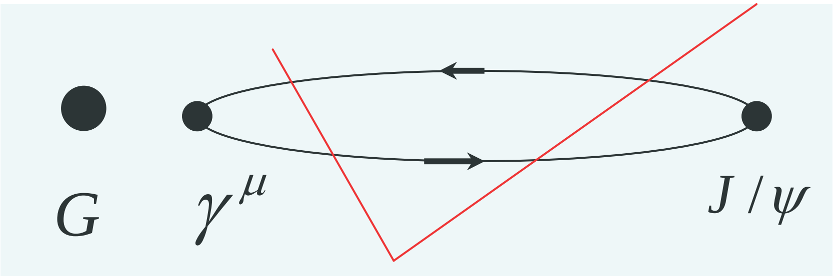
$$(t_f \geq t \geq 0)$$

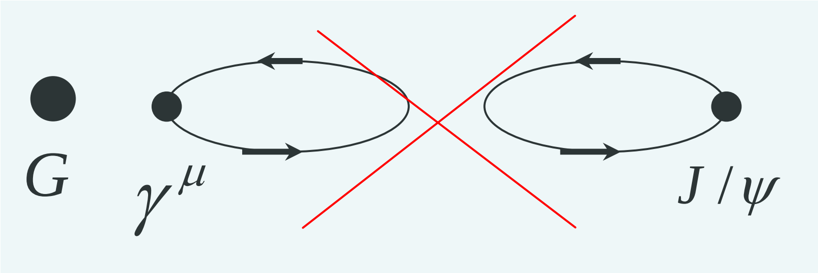
- **Lattice and parameters**

Anisotropic lattice: $L^3 \times T = 8^3 \times 96 \qquad \xi = a_s / a_t = 5$

Strong coupling: $\beta = 2.4 \qquad a_s = 0.222(2)\, fm$

- 5000 gauge configurations, separated by 100 HB sweeps
- Charm quark mass is set by the physical mass of J/psi
- On each configuration, 96 charm quark propagators are calculated with point sources on all the 96 time slices. The periodic boundary conditions are used both for the spatial and temporal directions.



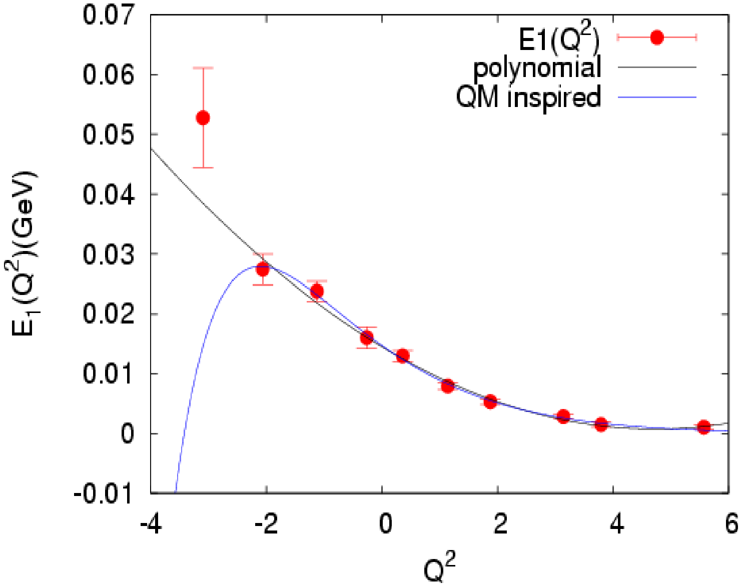


- **The form factor and the decay width**

Polynomial fit:

$$E_1(Q^2) = E_1(0) + aQ^2 + bQ^4$$

$$E_1(0) = 0.0145(13) GeV$$



The branch ratio is

$$\Gamma(J/\psi \to \gamma G_{0^+}) = \frac{4}{27}\alpha \frac{|p|}{M_{J/\psi}^2}|E_1(0)|^2 = 0.030(5) keV$$

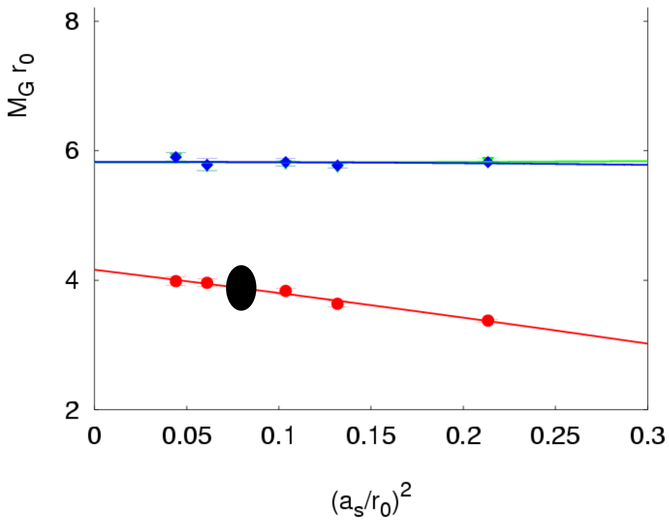
$$\frac{\Gamma}{\Gamma_{tot}} = 0.030(5)/93.2 = 3.2(5)\times 10^{-4}$$

Experimental results for J/psi radiatively decaying to scalars

C. Amsler et al., Phy. Lett. B667, 1 (2008)

Decay modes	Branch ratio (Γ_i/Γ)
$J/\psi \to \gamma f_0(1710) \to \gamma K\bar{K}$	$(8.5^{+1.2}_{-0.9}) \times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma \pi\pi$	$(4.0 \pm 1.0) \times 10^{-4}$
$J/\psi \to \gamma f_0(1710) \to \gamma \omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$J/\psi \to \gamma f_0(1500)$	$> (5.7 \pm 0.8) \times 10^{-4}$
$J/\psi \to \gamma f_0(1370)$	**N/A**

- The systematic uncertainties



- The continuum extrapolation has not been carried out. (The same calculation on a finer lattice is undergoing.)

- The lattice vector current has not been renormalized. (We are working on it.)

- The uncertainty owing to the quenched approximation. (Cannot be resolved in the near future.)

- The analyses of tensor channel and pseudoscalar are under the way.

VI. Summary

- The results of dynamical lattice QCD are impressive.
- Lattice QCD is promising.

Thank You!